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Essays on Collective Decision-Making

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"We have an agreement in principle.
The question is, do we all have the same principles?"

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Essays on collective decision-making

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Contents

List of Figures

List of Tables

1

Introduction

1.1 General introduction

Every day we make *choices* over some alternatives, from the simplest to the more important ones. We make choices either individually or collectively. Be it individually or collectively, however, making a choice involves evaluating alternatives. Rational evaluation of the alternatives is usually based on one or more criteria, measured either subjectively or objectively. Hence, from the simplest examples of individual choices to more complex collective choices, rational decision-making depends on measurement. The measurement can take different forms: we can measure the alternatives by placing one alternative over the other, that is, rank them; or we can measure the payoffs we get from the outcomes that appear as a consequence of choosing the alternatives, i.e., get utility. The former method of measurement is known as ordinal, while the latter is cardinal.

Regardless of the form that the evaluation of the alternatives takes, i.e., ordinal or cardinal, it is related to *social choice theory*, which is primarily concerned with mechanisms of *aggregation*. In the standard social choice settings we deal with the aggregation of preferences in order to produce a *collective preference*. However, dealing with collective choices is not only about preference aggregation, but also about aggregation of collective actions. The social choice or collective choice occurs when there is a group of individuals facing a collective decision problem of any kind with any form of evaluation of alternatives.

The main components then of the social choice setting is: (1) a group of *individuals* (or agents, voters), (2) a set of *alternatives* (or candidates, options) evaluated by each individual and (3) an *aggregation procedure*, which aggregates either preferences or actions into a *collective outcome*. The problem is then how to aggregate what each individual wants into what a group as a collective wants; or how to aggregate what each individual wants into what a group gets.

When all individuals in a group have identical preferences, the collective outcome is found trivially: the alternative that is most preferred by all individuals is chosen unanimously. However, the problem arises when the group is no longer homogeneous and the preferences are conflicting. How to aggregate conflicting preferences? Or how could the conflict be resolved? What if the group of individuals is a political party that faces the task of choosing a candidate (or a policy) to represent the party in the forthcoming elections? Each party member belongs to a faction inside the party and wants his own faction's candidate to be the party's nominee for the general election. How could this intra-party conflict be resolved? What if the party unity is at stake and the party's electoral performance depends on the candidate chosen (the outcome of candidate's selection may influence the party's chances of winning)? This problem motivates Chapter 2.

In a collective decision-making problem with conflicting preferences, some individuals may be tempted to manipulate the collective outcome to serve their selfish preferences. Manipulation can take various forms; for example, an individual when asked about his true preferences may lie if in such a way he can influence the final outcome in his favour. In some cases the individuals may possess valuable information concerning the optimal decision. Consider a committee of experts who must award a prize; or a jury of a contest who must choose a winner. Being experts or juries, they possess information as to who truly deserves to get the prize or to be the winner. However, being selfish some (or all) of them may try to manipulate the outcome by misrepresenting the correct information.

This manipulation may be detrimental for the group and produce a socially suboptimal decision (or socially detrimental outcome). The goal then is to design the aggregation procedure in order to elicit true preferences or information from the individuals with the aim of obtaining a socially desirable outcome. Chapter 3 deals with this problem.

It is sensible requirement that in a collective decision-making, every individual's preferences should be taken into account, when making the collective choice. The expression "collective choice" itself is meant to produce a "collective will". Having only one individual decide on the outcome seems to contradict the principle of the "collective will". Of course, being selfish some individuals would prefer to dictate their favourite outcome; however, what everyone wants is that there is no *dictator*. Thus, the aggregation procedure where the outcome is dictated by a dictator, i.e. a *dictatorial voting rule*, is usually deemed as bad and undesirable. However, are dictatorial rules that bad? Could we obtain a *good* voting rule if we get away from a *bad* dictatorial rule? What voting rule could be *least-dictatorial* in some sense and what properties could it possess? From the opposite point of view, what voting rule could be *most-dictatorial*, or stated differently, that creates *collective dictators*? Chapter 4 investigates these questions.

Is there an aggregation procedure (a voting rule) that is immune to manipulation? The classic result of the Gibbard-Satterthwaite theorem (Gibbard [33]; Satterthwaite [73]) gives us a negative answer: every voting rule that is immune to manipulation (or strategy-proof) must be dictatorial. Thus, whenever a group of individuals decides to employ any reasonable voting rule to find a collective outcome, there always must be a dilemma: either any of the individuals could manipulate the outcome or one of them should dictate the outcome. Since all voting rules are either manipulable and non-dictatorial, or non-manipulable and dictatorial, are there voting rules that are *least* manipulable? And are there voting rules that are *least* dictatorial? Is there any relationship between both incompatible properties, for example, less manipulable and more dictatorial? To see to what degree a voting rule is manipulable and dictatorial might be helpful, when we have to make our choice of which voting rule to employ. There is a positive answer to the first question in the literature: we can ”quantify the evil” and measure the degree of manipulability of the voting rules by employing certain indices of manipulability. Chapter 5 tries to answer the second and third questions by introducing the index of non-dictatorship based on the results found in Chapter 4, investigating the relationship between manipulability and dictatorship.

To summarize, in this dissertation we deal with four different problems in collective decision making. Thus the thesis presents a collection of independent research articles. Depending on the problem under study, we apply a different approach in resolving it. In Chapter 2 we use a game-theoretical approach, in Chapter 3 we adopt a mechanism design approach, Chapter 4 we rely on an optimization-based methodology and Chapter 5 we follow a computational approach. We now briefly review each chapter separately.

1.2 Summary of chapter 2

Chapter 2 analyses a collective decision problem in a model of intra-party politics. In particular, we go beyond the unitary actor assumption usually considered in the literature and view a party as a coalition of factions, namely, a party elite and a dissenting faction, with conflicting policy preferences. We characterize the outcomes of the intra-party conflict in a formal model of the intra-party game between the elite and the dissenting faction. We study the conditions that are most conducive to the adoption of primaries through which the intra-party conflict becomes resolved.

This chapter is motivated by the recent importance of primaries. More and more parties throughout the world are changing their internal organization by adopting primary elections. Having its birthplace in the US (Ware [84]), party primaries are getting increasingly common also in Latin America, Asia, Israel and in many countries of Europe, including Belgium, Italy, France, Portugal and Spain among others (Rahat and Sher-Hadar [71]; Carey and Polga-Hecimovich [21]; Kemahlioglu et al. [43]; Wauters [85]; Lisi [49]; Sandri and Seddone [72];

De Luca [27]). A recent example is the 2017 French presidential election, where the two major parties - the conservative *Les Republicains* and the Socialist party - chose their candidates in primaries for the first time. Even more recently, in 2018, the Spanish conservative Popular Party (PP) for the first time in its history conducted party primaries in order to choose its leader, a situation that was unthinkable just some years ago.

In this regard, the question *Why do parties adopt primaries?* becomes relevant. However, the answer to this question is not a straightforward one. Parties are usually thought to be conservative organizations that resist changes (Harmel and Janda [35]) and are mostly controlled by a small group of members called *elite*, who is in charge of personnel recruitment (candidate or party leader nomination) and control of policies. Democratizing candidate and leadership selection methods by widening up selectorate, granting the party base the power to make a decision, supposes for the party elite to give away (some) of its own *power*. Why would the party elite be willing to change the internal rules of the party relinquishing its power?

Hortalá-Vallve and Mueller [38] argue that primaries may have a unifying role and prevent the party from splitting. The authors view the party as a coalition of factions: the party elite and the dissenting faction, with heterogeneous policy preferences. When party heterogeneity is too large, parties are in danger of splitting into smaller yet more homogeneous groups. In this context primaries may have a unifying role if the party elite cannot commit to policy concessions. Their model shows that three factors create incentives for the party elite to adopt primaries: (1) the alignment in policy preferences between factions; (2) the relative weight of each of these factions inside the party; (3) the electoral system. The important point is that primaries are only adopted when the exit threat of the dissidents is credible.

Hortalá-Vallve and Mueller [38] build a simple game-theoretical model to analyse the strategic interaction of both factions in the adoption of primaries. In their model the elite faction is the first-mover in the game and decides on the institutional set-up of the party (namely, the candidate selection method), leaving the dissenting faction to decide afterwards whether to stay in the party or to split. That is, the democratization of the candidate selection method happens as a result of the strategic top-down calculations of the party elite. However, several cases are documented where the adoption of primaries happened as a result of the internal pressure of the dissatisfied faction from within the party. For example, in 1988 dissatisfied with the presidential nominee of the Institutional Revolutionary Party (PRI) of Mexico, some prominent politicians split from the party. Nevertheless, before the actual split, the discontented politicians attempted to "democratize" PRI by launching a challenge to the party leader and reclaiming the adoption of primaries. Once they realized that their efforts had failed, they opted to exit the party. Another example is the Belgian Flemish nationalist party VU, a faction of which forced the party elite to introduce primaries with the aim to reduce the ideological gap between the grassroot members and the party elite (Wauters [85]).

In Chapter 2 we show that the democratization of candidate selection can also occur as a consequence of bottom-up, demand-side pressure from within the party. Members who feel not represented by their leadership (candidates) any more may be willing to organize in a collective action and to launch the challenge to the party leader. To capture this scenario, we add an additional stage to the model of Hortalá-Vallve and Mueller [38], where it is the dissenting faction who moves first. By changing the order of moves we grant more freedom to the dissidents to influence the party elite's decision towards the democratization of candidate selection. When moving first the dissenters decide whether to accept the current internal organization of the party, and, consequently, the elite's candidate, or voicing their discontent by demanding primaries, influencing the party elite's decision. In addition, we introduce a new variable capturing the public perception of party unity. This allows us to ask additional questions. How would the intra-party candidate selection procedure influence the voters' decision on whom to vote for (Hazan and Voerman [37])? How much would the party's unity influence voters' decisions (Cox and Rosenbluth [22]; Greene and Haber [34])?

The magnitude of how united the public (or voters) perceive the party is important for the latter's electoral success, as the voters may punish the parties who are (or appear to be) internally divided (Kam [40]; Greene and Haber [34]). We explore how the dimension of this variable influences the adoption of primaries.

Changing the order of moves of the players allows us to have conditions for the adoption of primaries richer than in the benchmark model of Hortalá-Vallve and Mueller [38]. In particular, we find two equilibria when primaries are adopted: (1) *Primaries with threat*, which are adopted under the credible exit threat of the dissidents and (2) *Primaries with no threat*, where primaries are adopted when there is a strong ideological cohesion between both factions (Serra [76] also finds that the primaries are more likely the closer both factions are ideologically). The latter type of primaries is absent in the benchmark model of Hortalá-Vallve and Mueller [38], as in their work when both factions are too much aligned in their policy preferences, the primaries never happen.

We characterize the conditions when the primaries can resolve the intra-party conflict with respect to the level of the intra-party conflict, the relative strength of both factions, the characteristics of electoral system (proportionality, disproportionality) and the public perception of party unity.

1.3 Summary of chapter 3

One of the typical settings in social choice theory consists of a set of *individuals* (also known as voters or agents) and a set of *alternatives* (also known as candidates or opinions). The individuals express their preferences over the alternatives, which are then aggregated by some aggregation procedure or a *social choice function* into a single outcome. In Chapter 3 we consider a special case when the set of individuals and the set of alternatives coincide, that is, the voters are the candidates themselves. The collective decision problem the individuals face is that they have to choose a winner among themselves.

Think of a group of peers or a committee of experts who have to award a prize among themselves; or a jury who have to choose a winner of the contest and where each jury member is biased towards one of the contestants. The goal is to select the winner (being him a peer or an expert), who truly deserves to receive the prize (or to win) based on the information provided by each agent (peer, jury, expert). In this context, the individuals may behave strategically and be tempted to provide other than the true opinion or furnish misinformation to influence the final outcome and serve their selfish preferences.

In this chapter we deal with this situation. In particular, a group of agents must choose one of them to be the winner. Each agent knows who deserves to win, called the *deserving winner*, but each agent is *selfish* in the sense that he always wants to be the winner. At the same time, he is *impartial* towards the rest: if an agent cannot be chosen as the winner, he prefers the deserving winner to be chosen. The socially desirable outcome is that the deserving winner wins. Our goal is to design a mechanism which leads to the socially desirable outcome.

To solve this problem we follow a mechanism design perspective. Mechanism design uses the framework of non-cooperative game theory with incomplete information and seeks to study how the privately held preference information can be elicited. It is concerned with the settings where a policy maker (or a social planner) faces the problem of aggregating the announced preferences of multiple agents into a collective (or social) decision when the actual preferences are not publicly known. In fact, mechanism design can be viewed as the reverse engineering of games or equivalently the art of designing the rules of the game to achieve a specific desired outcome.

The analysis of this chapter was inspired by a similar problem considered in Amorós [8]. Amorós [8] proposes a sequential-form mechanism in which the agents take turns announcing whom should be the deserving winner. The socially desirable outcome is implemented; yet, the mechanism needs at least four agents to work. In this chapter we propose another sequential-form mechanism that implements the socially desirable outcome, improving upon Amorós's [8] result as now the mechanism works for at least three individuals. In addition, instead of asking the agents to name who deserves to win, we ask the agents to express their negative preferences by *vetoing* an individual, thus employing a veto rule.

1.4 Summary of chapter 4

Two cornerstone theorems of social choice theory, Arrow's Impossibility Theorem (Arrow [11]) and the Gibbard-Satterthwaite theorem (Gibbard [33]; Satterthwaite [73]) involve dictatorship and manipulability. While Arrow's theorem roughly states that whenever we deal with at least three alternatives only a dictatorial preference aggregation rule satisfies apparently reasonable properties, the Gibbard-Satterthwaite theorem shows that with three or more eligible alternatives, a non-manipulable (or strategy-proof) voting rule must be dictatorial. In this regard, there is a dilemma between dictatorship and manipulability: for at least three alternatives every universal and resolute social choice function is either manipulable or dictatorial.

Chapter 4 is motivated by the negative interpretations of those theorems, in particular with dictatorship. Since there is an implicit assumption that dictatorial voting rules are 'bad', we may expect to obtain a 'good' voting rule by being as far as possible from the 'bad' voting rule. By constructing a simple and natural distance function between social choice functions, we find that this might not be the case. The rule we obtain lies indeed the further away from the closest dictatorial rule but violates some important desirable properties. We call this rule the *reverse-plurality rule* as it never chooses the top alternative of any voter.

Alternatively, we might conjecture that getting as close as possible to all dictators at the same time could be considered as a kind of neutral or balanced solution with respect to all dictators. We search for the set of *balanced rules* by minimizing the sum of the distances to all dictators. We find that the plurality rule and the balanced rule are the same. This result provides an additional characterization of the plurality rule (the most widely used one), as the rule that can be considered as a kind of compromise between all dictatorial rules. Our results question the necessity of complete elimination of the dictators, which appears to be a too strong condition.

Our results contribute to the literature on distance rationalizability of voting rules (Farkas and Nitzan [30]; Nitzan [63]; Lerer and Nitzan [48]; Meskanen and Nurmi [56]; Andjiga et al. [9]; Elkind et al. [29]). Instead of minimizing the distance to some plausible criterion such as unanimity or Condorcet criterion, usually considered in the literature, we are optimizing the distance to the "bad" dictatorial voting rule.

1.5 Summary of chapter 5

Based on the classic Gibbard-Satterthwaite's impossibility theorem the properties of strategy-proofness and non-dictatorship are incompatible if there are at least three alternatives, any preference profile is possible and the social choice function has to be onto. Therefore, whenever a decision has to be made of which social choice function should be employed, there must always be a dilemma between dictatorship and manipulability: for at least three alternatives every universal and resolute social choice function is either manipulable or dictatorial. Chapter 5 is motivated by these negative implications and aims to explore the relationship between manipulability and dictatorship.

Both incompatible properties are undesirable. Yet, if we are to decide on which voting rule to employ, can we measure to what degree a given voting rule is manipulable? And to what degree a voting rule is dictatorial? The literature gives us a positive answer to the first question (Aleskerov [3]; Favardin et al. [31]; Fristrup and Keiding [32]; Maus et al. [52], [53], [54], [55]). It is possible to "quantify the evil" and to measure the strategy-proofness of a social choice function by evaluating its "degree of manipulability". Although, there is no universally accepted way to measure this degree, one of the most common approaches is to consider the ratio of preference profiles where manipulation is possible to the total number of profiles. This measure is called the Nitzan-Kelly index (NKI hereinafter) of manipulability, since it was first introduced in Nitzan [64] and Kelly [42]. A voting rule is considered to be least manipulable if it has the smallest NKI index.

However, there is no apparent answer to the second question, except for the works of Tangian [81], [82], [83] and Quesada [70]. In particular, Tangian [81] evaluates numerically the representative capacity of Arrow's dictators. He reports that the quantitative evaluation enables us to find "good" dictators who can be rather considered as representatives. His result was reinforced further in Tangian [82] where he finds that the average power of a dictator over any two alternatives is larger than 50%. This shows that Arrow's condition of the prohibition of dictators "is stronger than commonly supposed, excluding "good" dictators together with "bad" ones" (Tangian [83]). Quesada [70] further investigates computationally the prohibition of dictators. He comes to a similar conclusion that the harmfulness of Arrow's dictators is overemphasized. Having studied the influence sharing among dictators and other individuals, he finds that the dictator was only twice powerful than any other voter. Tangian [83] introduces indices of popularity and universality of representatives. All those works deal with the average power of a dictator.

In this work, we follow a different route and introduce the non-dictatorship index, with which we can measure the "degree of dictatorship" of a social choice function. We aim to explore the relationship between the indices of manipulability, represented by NKI, and of non-dictatorship for some common social choice functions. We investigate whether classifying voting rules according to both indices may help us to base our decision of which voting rule to choose. By employing computer simulations, we first calculate the non-dictatorship indices for some common social choice functions. Secondly, we put both manipulability and non-dictatorship indices into a common framework for those social choice functions and investigate their relationship. We find that the plurality rule performs the worst in terms of both Nitzan-Kelly and non-dictatorship indices. However, there is no voting rule that performs the best based on the two indices.

2

Primaries on demand

Abstract

We develop a formal model of the internal party dynamics to explain the adoption of primaries. We view a party composed of two factions: the elite faction and the dissenting faction. Both factions must choose the party's nominee to compete in a general election; however, they have conflicting preferences. We show that primaries may resolve the intra-party conflict. We also analyse how the public perception of party (dis)unity influences the adoption of primaries. When there is a high demand for party unity, or equivalently, the voters punish internally divided parties, the party elite is willing to introduce primaries in order to conceal factional divisions from the public.

2.1 Introduction

Political parties play a central role in modern democracies: they form governments, dominate legislatures, develop policies, run election campaigns, mobilize and persuade voters to elect their candidates in office and serve as a crucial link between citizens and their governments. Despite the long list of functions that parties perform, there is a widespread agreement in the literature that the most important task is the selection of personnel, i.e., party leaders and legislative candidates, to serve as representatives of the voters (Bille [17]; Besley [16]; Poguntke and Webb [69]). "Who is empowered to participate in leadership selection", as noted in Cross and Blais [23, 9], "speaks directly to the issue where power lies in [a] party". Perlin [66, 2] points to the importance of party leader as "the choice of the leader sets the course of the major Canadian parties in virtually everything they do". Thus, leaders play an essential role within political parties (Michels [57]; Pilet and Cross [68]).

The leaders and candidates once elected have a major influence on the policy direction of political parties as well as their composition. The individuals who are responsible for the party's personnel nomination have an indirect ability to shape the party, by choosing "candidates who are most in line with their own views of the party" (Cross et al. [25]).

Given the importance of the leaders and candidates, the methods that parties use to select them are important as well. As noted in Cross and Blais [23, 145-146]:

> "The influence leaders have within their parties, and more broadly on public decision-making, makes the question of who selects them crucial to any enquiry about who wields democratic influence. Given the changing norms of intra-party democracy and the growing influence of party leaders, it is not surprising that we find significant change in selection methods in recent years. While not universal, the trend is away from selection by a small group of party elites towards empowerment of a party's rank and file members".

There are several ways the parties can select their leaders and candidates, ranging from the less participative methods, where a small group of elites decides who will become a party leader and/or candidate, to more open and democratic procedures, such as primaries, allowing only party members (closed primaries) and/or all voters (open primaries) to participate in the selection of a party's personnel (Hazan and Rahat [36]). Figure 2.1 shows the leadership and/or selection methods according to their degree of inclusiveness (Kenig [44]). At the left end point of a continuum a small group of elites decides who will become the party's leader or candidate (exclusive degree), while moving to the right the leadership and candidate selection methods become more inclusive, ending at the left end point where all party members or even all voters are allowed to decide (inclusive degree).

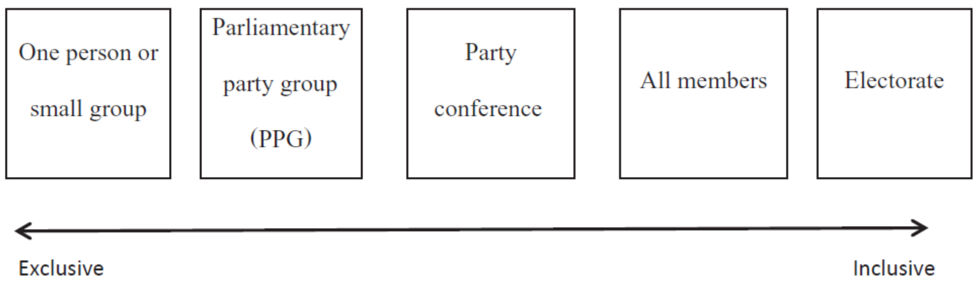

Figure 2.1: Degree of inclusiveness of leadership and candidate selection methods (Kenig [44])

Over the last two decades a significant number of parties in advanced and new democracies clearly has moved towards the right end of the continuum adopting primaries (Bille [17]; Kittilson and Scarrow [47]; Cross et al. [25]). Originally borrowed from the US, the primary elections are becoming popular outside of the US continent as well: in Europe, Asia and Latin America (Cross et al. [25]). Recent examples from the 2017 French presidential election, where the two major parties, the conservative *Les Republicains* and the Socialist party, chose their candidates in primaries; and that of the Spanish Popular Party (PP) choosing its leader for the first time, make a question of "*Why do parties adopt primaries*?"

quite relevant.

Nevertheless, the answer is not a straightforward one. Parties are thought to be rather conservative organization reluctant to changes (Harmel and Janda [35]; Cross and Blais [23]). The expressions of Schattschneider [74, p.100], "he who has the power to make nominations owns the party" and William M. Tweed: "I don't care who does the electing, so long as I can do the nominating" (Stewart and Archer [80, p.3] just reinforce the reluctance of those in power to lose that power. As the authority to nominate is usually concentrated in the hands of the selected few within political parties, known as the party elite, and identified by Michels as the "law of oligarchy" (Michels [57]), the democratization of the leadership and candidate selection methods makes the party elites to voluntarily give away this power. What drives political elites to give away the power? What are the determinants of the adoption of primaries?

These questions gave rise to a literature exploring the reasons for the adoption of primaries. Three main explanations are highlighted. First, party primaries may help to elicit the voters' preferences and to choose the most attractive candidate (Adams and Merrill [2]; Aragon [10]; Serra [75], [76]). Second, primaries may increase the internal competition among candidates, creating incentives among them to exert more effort during the electoral campaign and to better target the median voter's interests (Caillaud and Tirole [20]; Crutzen et al. [26]). Finally, a complementary view was suggested that primary elections may avoid costly intra-party conflict and serve as a unifying device for the party (Kemahlioglu et al. [43]; Hortalá-Vallve and Mueller [38]).

In particular, abandoning the usually held assumption in the literature of a party acting as a unitary actor, and considering the party to be composed of heterogeneous members with (possibly) conflicting preferences, primaries may unify the party and save it from splitting. Therefore, intra-party conflict may matter for the adoption of primaries.

What is the intra-party conflict? The intra-party conflict entails a disagreement between members of a political party and often arises "when members of the same political party pursue incompatible political goals or try to influence the decision making process of the party to their advantage" (Momodu and Matudi [58, 3]. Although a political party is a group of people bound in policy and opinion, with similar views on how to run a state, still the lack of homogeneity on some issues may arise leading to intra-party conflicts and provoking party factionalization.

On the other hand, a political party must aggregate the divergent preferences of its members in order to present a united front and achieve its goals. Otherwise, the inability to resolve the intra-party conflict could lead to destructive consequences. Hence, the way how the party manages to resolve the conflict may matter for the further functioning of the party. Being a social group, the intra-party conflict may not be an uncommon phenomenon inside the party; yet, depending on how it is resolved by the party, it can be constructive or destructive. In the former case, the party can benefit from the conflict, as it can bring new ideas, reconcile or unite its members; in the latter case, it can be harmful for the party and in the worst scenario may lead to the party split.

The purpose of this chapter is to study the outcomes of the intra-party conflict and show how the incorporation of internal democracy within the party may resolve it. To this end, we follow the view of Hortalá-Vallve and Mueller [38] (HM, hereinafter) and take their model as a benchmark to enrich it with several extensions, which allows us to investigate additional questions. We first give a short overview of the HM model.

In the HM model, a party is viewed as a coalition of heterogeneous individuals grouped into two factions: a party *elite* faction and a *dissenting* faction. The collective decision problem that both factions face is that they need to come to an agreement of who the party's nominee will be in a forthcoming general election. Both factions want to view their own faction's candidate representing the party in the general election, given that each faction gets the highest policy payoff from its own faction candidate's winning. We show the intra-party game of HM in the Figure 2.2 below.

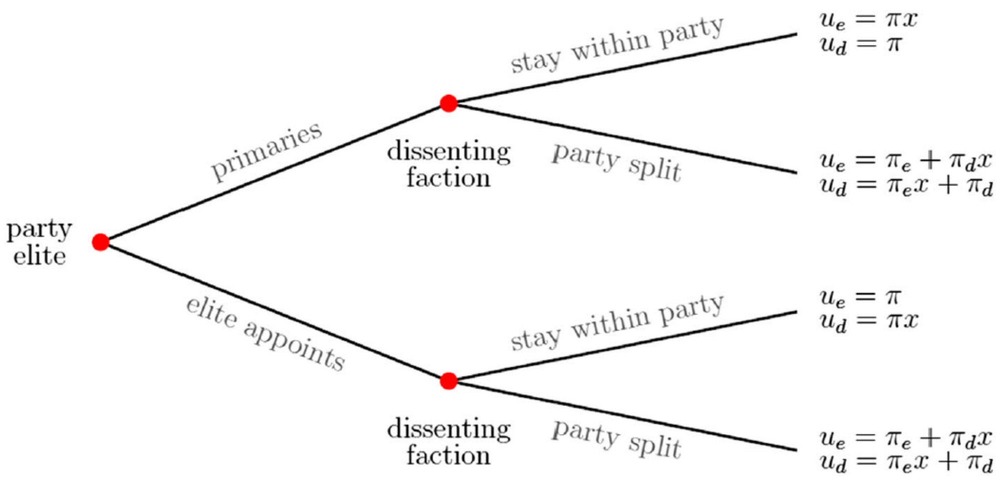

Figure 2.2: The intra-party game (Hortalá-Vallve and Mueller [38])

In the status quo, the party's candidate belongs to the elite faction, as it controls the party. The elite decides whether to keep the status quo or introduce primaries. The dissenting faction then either agrees to stay in the party, accepting the candidate nomination procedure, or chooses to exit the party. The model of HM points to the two conditions that must hold for the party elite to adopt primaries. First, the exit threat of the dissidents must be credible (i.e., the dissenting faction must prefer running separately than jointly) and the party elite should be inclined to give up on the selection of the party candidate in order to avoid the party split. The key factors that influence the adoption of primaries are the dimension of the intra-party conflict, the relative strength of both factions within the party and the electoral bonus of running jointly, which depends on the characteristics of the electoral system. In the HM model, the party elite moves first deciding on the institutional set-up for candidate selection (whether to adopt primaries or not) leaving the dissenting faction with a dilemma to decide afterwards whether to stay in the party or to leave. Thus, the adoption primaries is a result of the strategic top-down calculations by the party elite.

We now describe how we modify the HM model and the reasons motivating the new model. First, as pointed out earlier, parties are quite conservative organizations. Hence, we can presume that party elites may be unwilling to give up their power easily. Changes in the internal organization could be the result of factional pressure from within the party. We want to explicitly model this scenario and to this end we add an additional stage to the HM game, where the dissenting faction moves first. The dissenting faction can either accept the status quo, where the party's nominee is the elite faction's candidate, or voice their discontent and demand that the party's candidate be chosen through primaries. In this way we model the internal pressure which comes from within the party.

This situation is not uncommon in real world situations. Recall explained earlier cases of Mexican PRI and Belgian VU. Thus, by adding an additional stage to the HM game, we add a further option to the dissenting faction: the possibility of influencing the party elite's decision by voicing their discontent. Choosing to use voice means that the dissenting faction does not accept the elite faction's candidate (and consequently his policy) and instead seeks to persuade the elite to resolve the policy conflict through primaries. Furthermore, we introduce a new variable capturing the public perception of party unity, which we call the *cost of disunity*. The intra-party conflict is a key variable in the adoption of primaries. On the other hand, does it also matter in the voters' decision whether to support that party in the general election? How would the intra-party candidate selection procedure influence the voters' decision whom to vote for (Hazan and Voerman [37])? Does the strong and unified party (or party's coherent brand name) influence voters' decision (Cox and Rosenbluth [22]; Greene and Haber [34])?

After the dissenting faction's decision whether to stay loyal or demand primaries is made, the elite faction then decides whether to accept the dissidents' demand by adopting primaries or reject it. If the demand is accepted, the game ends and primaries are introduced. Similarly to the HM model, we assume that, in this case, the dissenting faction's candidates wins the primaries. The fact that the party elite accepts the dissidents' demand sends a signal to voters that the party is internally democratic and that all the party's members views are taken into account, which might increase the party's electoral performance. For example, Shomer et al. [78] show that the introduction of primaries increases the trust in parties among voters which in its turn increases their electoral performance. Although there is a policy conflict between both factions, it may be resolved through primaries. The party still appears to be united, both factions run jointly and the party gets an electoral bonus. As long as the party appears to be united, their policy platform will be more credible. Therefore, the parties who postulate the united front to the voters might improve their electoral performance as the party unity may be essential for electoral success (Boucek [19]; Greene and Haber [34]).

If, on the contrary, the elite faction rejects the dissenting faction's demand, then the dissidents decide whether to stay in the party after the failed attempt of voice or exit the party. In the event of exit, the party splits. In the event of stay, the party still remains united, but the whole party incurs a loss in their winning probability due to an unresolved internal conflict that becomes known to the public, which can happen through media, and/or with party members themselves or the opposition conducting a negative campaign to highlight its rival's internal divisions. The appearance of public internal divisions likely influences voters' evaluation of parties (Green and Haber [34]). The voters may view negatively the internally divided parties and so switch their vote to the opposition or simply abstain. The party cohesion influences electoral success while the lack of cohesion brings failure among the electorate (Kam [40]). Indeed, party commitments may seem less credible if internal disagreements exist, and as a consequence voters may punish parties if they show evidence of being internally divided (Greene and Haber [34]).

Several new insights are brought with these new changes in comparison with the benchmark model of HM. First, we find two types of primaries: (1) *Primaries with threat*, when there is a credible exit threat of the dissenting faction; and (2) *Primaries no threat*, when there is no exit threat from the dissidents.

The first type of primaries is present in the model of HM, while the second type of primaries is a new result, obtained by extending the game of HM in which the dissenting faction moves first and voters observe the intra-party conflict. The *Primaries no threat* type only exists when both factions are close ideologically, the cost of disunity is sufficiently low, or equivalently, there is no high demand for party unity among the electorate, and the electoral bonus is sufficiently high. The fact that the party elite is willing to adopt primaries when both factions are very close ideologically is absent in the model of HM, as in their model primaries are never introduced in this case. Serra [76] obtains a result similar to ours: the party elite is willing to adopt primaries when the rank-and-file members are very close to it ideologically.

Second, we show how voters' perception of party unity influences adoption of primaries. In particular, when there is a high demand for strong and united parties, there always exists a credible exit threat of the dissenters, and the party elite is willing to democratize internally the candidate selection procedure in order to save the party from splitting and conceal factional divisions from public. The important result is that the cost of disunity is inversely related to the disproportionality of the electoral system: the higher the system's disproportionality, the higher the cost of disunity is, or equivalently, the higher the demand for party unity is.

Which type of primaries prevails depends on the level of intra-party conflict, the relative strength of both factions, the characteristics of the electoral system (whether there is a bonus of running jointly) and the public perception of party unity. We may infer that in majoritarian electoral systems, where there is a high demand for strong and united parties, or equivalently, the cost of disunity is high, the type *Primaries with threat* is the most likely outcome. In that event, public perceptions of party (dis)unity may explain why parties in majoritarian democracies try to eliminate or conceal factional divisions within the party (Boucek [19]), and in our case, by responding positively to the demand of the dissidents to adopt primaries. This case may also represent the countries with relatively new democracies. In the countries with relatively new democracies, there is a pressure from the public towards the parties to be internally democratic.

As long as the electoral system becomes more proportional, the demand for party unity decreases, and, consequently, the cost of disunity falls, the likelihood of the primaries under the credible exit threat (*Primaries with threat*) decreases as well, but another type of primaries appears: the one which only exists when there is a high ideological cohesion within the party (*Primaries no threat*).

The remainder of the chapter is as follows. Section 2.2 describes the model. Section 2.3 provides the characterization of subgame perfect equilibria. Section 2.4 analyses the comparative statics. Section 2.5 summarizes the results.

2.2 Model

We follow closely the framework of HM. There are two groups of identical individuals, which are factions of one political party P: the *elite* faction E and the *dissenting* faction D. A general election is to be held. At issue is a policy (to be implemented by a candidate in case of victory), and over which there is a conflict of interest between E and D, measured by parameter $x \in (0, 1)$. The value of x reads as follows: x close to 0 represents a high discrepancy between E and D on policy issues, while x close to 1 means that E and D are much aligned in their policy preferences. Each faction would like to implement its own favourite policy (or equivalently to choose its own faction's candidate to run in the general election). Therefore, if D's (E's) candidate wins the election, D (E) gets the highest payoff normalized to 1. If the winning candidate belongs to D (E), then E (D) gets the in-between payoff of $x \in (0, 1)$ Finally, if the winning candidate belongs to some opposing party (whose internal strategic dynamic is not modelled and taken as given), both factions get the minimum payoff of 0. Given that the candidates are identified by their ideology, choosing a candidate is equivalent to choosing a policy.

By assumption, E currently controls the candidate nomination process, and so will impose its own faction's candidate to represent P in the general election, who implements his or her preferred policy in case of winning the election. D can respond to this situation by choosing between two options: either choose loyalty and accept E's candidate (and as a result, the party runs united with E's candidate representing P in the general election); or to voice discontent and demand primaries. We assume that the dissenting faction has overcome the collective-action problem and do not impose any costs on the dissidents for having voiced their discontent. By demanding primaries, D *believes* that, by holding them, the internal conflict concerning the policy issues will be resolved. By assumption, if primaries are held, the winner is D's candidate.

In case D demands primaries, E, in its turn, may respond positively by accepting D's demand and adopt primary elections, or reject it, at which point D must decide whether to exit or stay in the party. In the former case, the party splits and both factions run separately. In the latter case, the party still runs united but loses a share of its winning probability due to an unresolved internal conflict that is revealed to public. This situation is modelled as an extensive form game in Figure 2.3.

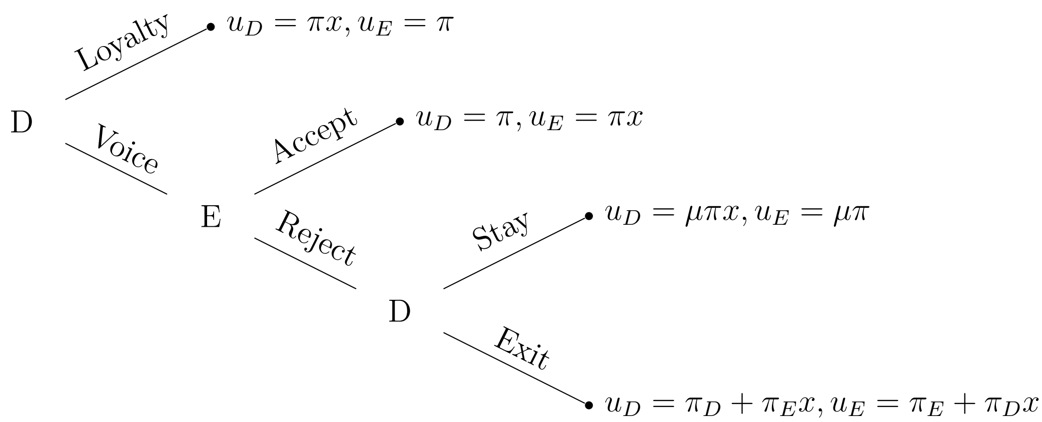

Figure 2.3: The intra-party game

The players' payoffs presented in Figure 2.3 are justified as follows. We adopt the genericity assumption according to which, when making a choice, no faction may get the same payoff from the two choices. If both factions run jointly under the party P, its probability of winning the general election is $\pi \in (0, 1)$; the probability that some opposing party wins is $(1 - \pi)$. In case P splits, each faction runs separately and each wins the election with probability $\pi_i \in (0, 1)$, where $i \in \{E, D\}$; some opposing party wins with the remaining probability $(1 - \pi_E - \pi_D)$. Similarly to the model of HM, there is an *electoral bonus* of running jointly denoted by α, where $\alpha > 1$, which is defined as the relative improvement both factions enjoy when running together relative to the sum of probabilities when running separately. Formally,

$$\alpha = \frac{\pi}{\pi_E + \pi_D}.$$

The interpretation is that α captures the characteristics of the electoral system, namely, its degree of proportionality: the higher α, the more disproportional the electoral system is. A high α describes a highly disproportional electoral system, which tends to reward large parties (majoritarian). Contrariwise, in a highly proportional systems (proportional representation) the electoral bonus of running jointly is minimal (Balinski and Young [15]). Note that in the absence of the assumption that both factions are better off in terms of the winning probability when running jointly than separately, keeping the party unity with primaries would make little sense as both E and D would be better off through exit.

In case E rejects D's demand and D decides to remain in the party, both factions still run jointly, P's nominee belongs to the elite faction, and the whole party incurs a loss in its winning probability measured by μ, where $\mu \in (0, 1)$. This loss occurs due to an unresolved internal conflict that becomes known to voters. Thus, a publicised intra-party conflict discounts the party's winning probability. We interpret μ as the *cost of disunity* or, conversely, the benefit of party unity: when μ is low, the cost of disunity is high, that is, voters value united parties. Alternatively, μ could define the share of voters who are dissatisfied with the candidate from the elite faction, and so decide to abandon the party in order to vote for the opposition or abstain.

We also denote by y, where $y \in (0, 1)$, the *relative strength* of the party elite by considering its winning probability when running alone relative to the sum of the probabilities when both factions run separately. Formally,

$$y = \frac{\pi_E}{\pi_E + \pi_D}.$$

Therefore, y denotes the relative strength of the elite faction after the party split, which can be represented as the share of party supporters (or voters) the elite faction can mobilize when running separately. The relative strength of the dissenting faction, D, is $1 - y$, respectively. When $y > \frac{1}{2}$, we say that the elite faction is *stronger* than D, that is, $\pi_E > \pi_D$. In case $y < \frac{1}{2}$, the dissenting faction is stronger than the elite, that is, $\pi_D > \pi_E$.

Next, we describe in detail each strategy of the players and the relevant payoffs obtained by playing those strategies, as depicted in Figure 2.3. The game begins with D deciding how to respond to E's choice of the candidate. Recall that E is in charge of P's policy; hence, by default the candidate from P belongs to E's faction. If D decides to remain loyal, the game ends and E's candidate wins the general election with probability π, in which case D gets an expected payoff of $u_D = \pi x + (1 - \pi) \times 0 = \pi x$ and E gets an expected payoff of $u_E = \pi \times 1 + (1 - \pi) \times 0 = \pi$, where u_D and u_E are expected utilities of D and E respectively. Specifically, u_D is defined as the winning probability with which P wins the election multiplied by the utility D gets from E's candidate policy, measured by x; and u_E is E's expected utility which equals the probability with which E's candidate wins the election multiplied by a maximum payoff of 1, given that E implements its preferred policy.

Should D voice discontent and demand primary elections, the game moves to the next stage, where E decides whether to accept D's demand and adopt primaries, or reject it. If accepted, the party runs united and we assume that D's candidate wins the primary and also the subsequent general election with probability π. In this case, D gets the highest expected payoff corresponding to

$u_D = \pi$ and E gets $u_E = \pi x$. If E rejects D's demand, the game moves to the last stage, where D decides between exiting or staying in the party P. If D exits, both factions run separately in the election, in which case D gets $u_D = \pi_D + \pi_E x$ and E gets $u_E = \pi_E + \pi_D x$. If D stays, the candidate belongs to E's faction, and the whole party incurs a loss in terms of the winning probability, as the internal conflict, not resolved through primaries, is revealed to voters, measured by μ. As a result, D gets $u_D = \mu\pi x$ and E gets $u_E = \mu\pi$, where $\mu < 1$.

2.3 Results I: equilibria

We use the subgame perfect Nash equilibrium (SPNE) solution concept to solve the family of the extensive form games depicted in Figure 2.3. Accordingly, we proceed by backward induction.

There are five types of SPNE grouped in Propositions 2.1 to 2.5 next. Equilibria are written in the following form: (D's first action, E's action, D's second action). We present the results in terms of our key parameters of the game: the level of intra-party conflict x, the relative strength of the party elite y, the electoral bonus α and the cost of disunity μ. We recall the following restrictions on our parameters: $0 < x < 1$, $0 < y < 1$, $\alpha > 1$ and $0 < \mu < 1$.

Before presenting the main results as in propositions, for the ease of exposition we first introduce some lemmas characterizing the best replies of the players in each node of the game.

Lemma 2.1. *Exit is D's best reply if and only if*
(a) $\alpha < \frac{1}{\mu}$*; or*
(b) $\alpha > \frac{1}{\mu}$ *and* $x < \frac{1-y}{\mu\alpha - y}$.

Proof. At the last decision node, D chooses *Exit* rather than *Stay* if $\pi_D + \pi_E x > \mu\pi x$. After dividing both sides of the inequality by $\pi_E + \pi_D$, we can rewrite it in terms of x, y, α and μ as $1 - y + yx > \mu\alpha x$, which is rearranged into

$$(\mu\alpha - y)x < 1 - y \tag{2.1}$$

From (2.1) it follows that:
(A) if $\alpha < \frac{y}{\mu}$ (such that, $\mu\alpha - y < 0$), then (2.1) holds for any values of $0 < x < 1$. Since $\alpha > 1$, it must be that $y > \mu$.
(B) if $\frac{y}{\mu} < \alpha < \frac{1}{\mu}$ (such that, $\mu\alpha - y > 0$ and $\mu\alpha - y < 1 - y$), then (2.1) holds for any $0 < x < 1$.
(C) if $\alpha > \frac{1}{\mu}$ (such that, $\mu\alpha - y > 0$ and $\mu\alpha - y > 1 - y$), then (2.1) holds if

$$x < \frac{1 - y}{\mu\alpha - y} \tag{2.2}$$

□

Lemma 2.2. *Stay is D's best reply if and only if* $\alpha > \frac{1}{\mu}$ *and* $x > \frac{1-y}{\mu\alpha - y}$.

Proof. At the last decision node, D chooses *Stay* rather than *Exit* if $\mu\pi x > \pi_D + \pi_E x$. After dividing both sides of the last inequality by $\pi_E + \pi_D$, we rewrite and rearrange it as $(\mu\alpha - y)x > 1 - y$, which holds if

$$x > \frac{1 - y}{\mu\alpha - y} \tag{2.3}$$

Since $x < 1$, (2.3) requires that $\alpha > \frac{1}{\mu}$. □

Lemma 2.3. *Accept is E's best reply if and only if*
(a) D has chosen Exit and $x > \frac{y}{\alpha - 1 + y}$*; or*
(b) D has chosen Stay and $x > \mu$.

Proof. (a) *Accept* is E's best reply, when D has chosen *Exit*, if $\pi x > \pi_E + \pi_D x$. After dividing both sides of the inequality by $\pi_E + \pi_D$, we rewrite and rearrange it into $(\alpha - 1 + y)x > y$, which holds if

$$x > \frac{y}{\alpha - 1 + y} \tag{2.4}$$

(b) *Accept* is E's best reply, when D has chosen *Stay*, if $\pi x > \mu\pi$, that is, if $x > \mu$. □

Lemma 2.4. *Reject is E's best reply if and only if*
(a) D has chosen Exit and $x < \frac{y}{\alpha-1+y}$*; or*
(b) D has chosen Stay and $x < \mu$*.*

Proof. (a) *Reject* is E's best reply, when D has chosen *Exit*, if $\pi x < \pi_E + \pi_D x$. After dividing both sides of the inequality by $\pi_E + \pi_D$, we rewrite and rearrange it into $(\alpha - 1 + y)x < y$, which holds if

$$x < \frac{y}{\alpha - 1 + y} \tag{2.5}$$

(b) *Reject* is E's best reply, when D has chosen *Stay*, if $\pi x < \mu\pi$, that is, if $x < \mu$. □

Lemma 2.5. *Voice is D's best reply if and only if*
(a) The sequence of the play is (Accept, Exit) or (Accept, Stay); or
(b) The sequence of the play is (Reject, Exit) and $x < \frac{1-y}{\alpha-y}$*.*

Proof. (a) The proof is straightforward: since $x < 1$ whenever E accepts D's demand, D gets the highest expected payoff, $\pi > \pi x$.
(b) Given the sequence of the play (*Reject*, *Exit*), *Voice* is D's best reply, if

$\pi x < \pi_D + \pi_E x$. After dividing both sides of the inequality by $\pi_E + \pi_D$, we rewrite and rearrange it as $(\alpha - y)x < 1 - y$, which holds if

$$x < \frac{1-y}{\alpha - y} \tag{2.6}$$

□

Lemma 2.6. *Loyalty is D's best reply if and only if*
(a) The sequence of the play is (Reject, Exit) and $x > \frac{1-y}{\alpha-y}$*; or*
(b) The sequence of the play is (Reject, Stay).

Proof. (a) Given the sequence of the play (*Reject*, *Exit*), *Loyalty* is D's best reply, if $\pi x > \pi_D + \pi_E x$. After dividing both sides of the inequality by $\pi_E + \pi_D$, we rewrite and rearrange it as $(\alpha - y)x > 1 - y$, which holds if

$$x > \frac{1-y}{\alpha - y} \tag{2.7}$$

(b) Given the sequence of the play (*Reject*, *Stay*), *Loyalty* is always D's best reply as $\pi x > \mu\pi x$ since $\mu < 1$. □

Having characterized the best replies of our players in each node of the game, we present next the SPNE in which the party elite adopts primaries. There are two such SPNE grouped in Propositions 2.1 and 2.2.

Proposition 2.1 (*Primaries with threat*)**.** *(Voice, Accept, Exit) is a SPNE if and only if:*

(i) $\alpha < \frac{1}{\mu}$ *and* $x > \frac{y}{\alpha-1+y}$*; or*

(ii) $\alpha > \frac{1}{\mu}$ *and* $\frac{y}{\alpha-1+y} < x < \frac{1-y}{\mu\alpha-y}$.

Proof. By Lemma 2.1, D's best reply is *Exit* if and only if (a) $\alpha < \frac{1}{\mu}$; or (b) $\alpha > \frac{1}{\mu}$ and $x < \frac{1-y}{\mu\alpha - y}$. By Lemma 2.3(a), E's best reply is *Accept* if and only if $x > \frac{y}{\alpha-1+y}$. By Lemma 2.5(a), D's best reply is *Voice.* □

Remark 2.1. Condition on $0 < y < \frac{\alpha-1}{\alpha(1+\mu)-2}$ guarantees that $\frac{y}{\alpha-1+y} < x < \frac{1-y}{\mu\alpha-y}$ when $\alpha > \frac{1}{\mu}$.

Proof. The proof follows from resolving the inequality $\frac{y}{\alpha-1+y} < \frac{1-y}{\mu\alpha-y}$, which is simplified and rearranged into

$$y(\alpha(1+\mu)-2) < \alpha - 1 \tag{2.8}$$

Since $\mu < 1$ and $\alpha > \frac{1}{\mu}$, it follows that $\alpha(1+\mu) - 2 > 0$, and, as a result, (2.8) holds if $y < \frac{\alpha-1}{\alpha(1+\mu)-2}$. Observe that since $\mu < 1$, $\frac{\alpha-1}{\alpha(1+\mu)-2} > \frac{1}{2}$. □

Proposition 2.1 divides the conditions for *Primaries with threat* in two cases depending on the constraint on α. Condition (i) corresponds to the case when the cost of party disunity is high, or $\mu\pi < \pi_E + \pi_D$, which is equivalent to $\alpha < \frac{1}{\mu}$ or $\mu < \frac{1}{\alpha}$, i.e., in the presence of the outside pressure from the voters, who punish internally divided parties. In this case, D prefers to exit the party, irrespective of the level of the intra-party conflict, as remaining within the party after failed attempt to challenge brings the high loss to D's expected utility. Given D's credible exit threat, E accepts primaries for a certain threshold of the intra-party conflict x in order to preserve the party unity and to hide the factional divisions within the party.

Condition (ii) describes the case when α is less constrained and the cost of disunity ranges from intermediate to low values, i.e. $\mu\pi > \pi_E + \pi_D$, or equivalently, $\alpha > \frac{1}{\mu}$ or $\mu > \frac{1}{\alpha}$. In this case, the threat of the dissenting faction is credible but there is no such outside pressure from the voters. In this case, there is inside pressure from the dissidents and the party elite is willing to adopt primaries if the level of the intra-party conflict is above a certain threshold.

We now turn to the cases when the party elite is willing to accept primaries even when there is no credible exit threat from the dissidents.

Proposition 2.2 (*Primaries no threat*)**.** *(Voice, Accept, Stay) is a SPNE if and only if* $\alpha > \frac{1}{\mu}$ *and* $x > max\{\mu, \frac{1-y}{\mu\alpha - y}\}$.

Proof. From Lemma 2.2 we know that D's best reply is *Stay* if $\alpha > \frac{1}{\mu}$ and $x > \frac{1-y}{\mu\alpha - y}$. From Lemma 2.3(b) we know that E's best reply is *Accept* if $x > \mu$. From Lemma 2.5(a) we know that D's best reply is *Voice*. □

Remark 2.2. (a) If $\alpha > \frac{1}{\mu^2}$, then SPNE (*Voice*, *Accept*, *Stay*) exists if $x > \mu$ for $0 < y < 1$;

(b) If $\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$, then SPNE (*Voice*, *Accept*, *Stay*) exists if $x > \mu$ for $\frac{1-\mu^2\alpha}{1-\mu} < y < 1$ and $x > \frac{1-y}{\mu\alpha - y}$ for $0 < y < \frac{1-\mu^2\alpha}{1-\mu}$.

Proof. To see this, observe that $x > max\{\mu, \frac{1-y}{\mu\alpha - y}\}$ leads to either (i) $\mu > \frac{1-y}{\mu\alpha - y}$ or (ii) $\mu < \frac{1-y}{\mu\alpha - y}$. After rearranging the inequality (i) we obtain

$$y(1-\mu) > 1 - \mu^2\alpha \tag{2.9}$$

From (2.9) it follows that:
(1) if $\alpha > \frac{1}{\mu^2}$ (such that, $1-\mu^2\alpha < 0$), then (2.9) holds for any values of $0 < y < 1$. In this case, $x > \mu$ implies $x > \frac{1-y}{\mu\alpha - y}$, making $x > \frac{1-y}{\mu\alpha - y}$ insignificant. As a result, (*Voice*, *Accept Stay*) is a SPNE if $x > \mu$, proving (a).

(2) if $\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$ (such that, $1 - \mu^2\alpha > 0$), then for (2.9) to hold it must be that $y > \frac{1-\mu^2\alpha}{1-\mu}$. In this case, $x > \mu$ implies $x > \frac{1-y}{\mu\alpha - y}$, thus making $x > \frac{1-y}{\mu\alpha - y}$

insignificant. As a result, (*Voice*, *Accept*, *Stay*) is a SPNE if $x > \mu$.

The inequality (ii) is the opposite of (i) and, consequently, is true if $y < \frac{1-\mu^2\alpha}{1-\mu}$. As a result, $x > \frac{1-y}{\mu\alpha-y}$ implies $x > \mu$, thus making $x > \mu$ insignificant. Hence, (*Voice*, *Accept*, *Stay*) is a SPNE if $x > \frac{1-y}{\mu\alpha-y}$. This proves (b). □

The results of the Proposition 2.2 describe the case when the party elite is willing to cooperate with the dissenting faction and change the internal organization of the party. Although the dissenting faction stays within the party, the primaries are still adopted. This case only happens when both factions are much aligned in their policy preferences (x is high) and for the party elite to accept D's candidate is the same as to accept E's candidate. The necessary condition for this type of primaries is the cost of disunity be sufficiently low or the electoral bonus is sufficiently high ($\alpha > \frac{1}{\mu}$ or $\mu > \frac{1}{\alpha}$).

We next characterize SPNE when the dissenting faction stays loyal to the party. There are two such SPNE described in the Propositions 2.3 and 2.4.

Proposition 2.3 (*Loyalty with threat*)**.** *(Loyalty, Reject, Exit) is a SPNE if and only if:*

(a) $\alpha < \frac{1}{\mu}$ *and* $\frac{1-y}{\alpha-y} < x < \frac{y}{\alpha-1+y}$*; or*

(b) $\alpha > \frac{1}{\mu}$ *and* $\frac{1-y}{\alpha-y} < x < min\{\frac{1-y}{\mu\alpha-y}, \frac{y}{\alpha-1+y}\}$.

Proof. By Lemma 2.1 D's best reply is *Exit* if and only if (a) $\alpha < \frac{1}{\mu}$; or (b) $\alpha > \frac{1}{\mu}$ and $x < \frac{1-y}{\mu\alpha-y}$. By Lemma 2.4(a), E's best reply is *Reject* if and only if $x < \frac{y}{\alpha-1+y}$. By Lemma 2.6(a), D's best reply is *Loyalty* if and only if $x > \frac{1-y}{\alpha-y}$. □

Remark 2.3. Condition on $y < \frac{1}{2}$ guarantees that $\frac{1-y}{\alpha-y} < x < \frac{y}{\alpha-1+y}$.

Proof. For $\frac{1-y}{\alpha-y} < x < \frac{y}{\alpha-1+y}$ to hold, $\frac{y}{\alpha-1+y} > \frac{1-y}{\alpha-y}$ must be satisfied, which happens if $y > \frac{1}{2}$. □

Remark 2.4. If $\alpha > \frac{1}{\mu}$, then SPNE (*Loyalty*, *Reject*, *Exit*) exists if $\frac{1-y}{\alpha-y} < x < \frac{1-y}{\mu\alpha-y}$ for $\frac{\alpha-1}{\alpha(1+\mu)-2} < y < 1$ and $\frac{1-y}{\alpha-y} < x < \frac{y}{\alpha-1+y}$ for $\frac{1}{2} < y < \frac{\alpha-1}{\alpha(1+\mu)-2}$.

Proof. Observe that condition $x < min\{\frac{1-y}{\mu\alpha-y}, \frac{y}{\alpha-1+y}\}$ implies either (i) $\frac{y}{\alpha-1+y} < \frac{1-y}{\mu\alpha-y}$ or (ii) $\frac{y}{\alpha-1+y} > \frac{1-y}{\mu\alpha-y}$. After rearranging the inequality (i) we obtain

$$y(\alpha(1+\mu)-2) < \alpha - 1 \tag{2.10}$$

Since $\alpha > 1$ and $\alpha > \frac{1}{\mu}$, then $\alpha(1+\mu)-2 > 0$, hence (2.10) holds for $y < \frac{\alpha-1}{\alpha(1+\mu)-2}$. In this case, $x < \frac{y}{\alpha-1+y}$ implies $x < \frac{1-y}{\mu\alpha-y}$. Inequality (ii) is the opposite of (i) and since $\alpha > \frac{1}{\mu}$ is true for $y > \frac{\alpha-1}{\alpha(1+\mu)-2}$. In this case, $x < \frac{1-y}{\mu\alpha-y}$ implies $x < \frac{y}{\alpha-1+y}$. □

From the Proposition 2.3 it follows that the outcome of *Loyalty with threat* only exists when the dissenting faction is in the minority ($y > \frac{1}{2}$), that is, the dissenting faction chances to mobilize enough voters to win when running separately are low and so it prefers to stay inside the party.

Proposition 2.4 (*Loyalty no threat*)**.** *(Loyalty, Reject, Stay) is a SPNE if and only if* $\alpha > \frac{1}{\mu}$ *and* $\frac{1-y}{\mu\alpha-y} < x < \mu$.

Proof. By Lemma 2.2 D's best reply is *Stay* if and only if $\alpha > \frac{1}{\mu}$ and $x > \frac{1-y}{\mu\alpha-y}$. By Lemma 2.4(b), E's best reply is *Reject* if and only if $x < \mu$. By Lemma 2.6(b), D's best reply is *Loyalty.* □

Remark 2.5. (a) If $\alpha > \frac{1}{\mu^2}$, then SPNE (*Loyalty*, *Reject*, *Stay*) exists if $\frac{1-y}{\mu\alpha-y} < x < \mu$ for $0 < y < 1$.

(b) If $\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$, then SPNE (*Loyalty*, *Reject*, *Stay*) exists if $\frac{1-y}{\mu\alpha - y} < x < \mu$ for $\frac{1-\mu^2\alpha}{1-\mu} < y < 1$.

Proof. For condition $\frac{1-y}{\mu\alpha-y} < x < \mu$ to hold, the inequality $\mu > \frac{1-y}{\mu\alpha-y}$ must be satisfied, which is rearranged into

$$y(1-\mu) > 1 - \mu^2\alpha \tag{2.11}$$

From which it follows that if $\alpha > \frac{1}{\mu^2}$ (such that, $1-\mu^2\alpha < 0$), then (2.11) holds for any $0 < y < 1$, proving part (a).
If $\alpha < \frac{1}{\mu^2}$ (such that, $1-\mu^2\alpha > 0$) and since $\alpha > \frac{1}{\mu}$, then (2.11) holds if $y > \frac{1-\mu^2\alpha}{1-\mu}$. □

From Proposition 2.4 we can see that the dissidents prefer to stay loyal to the party without threatening to exit it. Observe that this case requires the electoral bonus, α, to be sufficiently high ($\alpha > \frac{1}{\mu}$) and the cost of disunity to be sufficiently low ($\mu > \frac{1}{\alpha}$). Moreover, by Remark 2.5(a) when α is very high ($\alpha > \frac{1}{\mu^2}$), both factions remain in the party with D staying loyal for all values of y, that is, the relative strength of both factions plays no role.

Proposition 2.5 (*Party split*)**.** *(Voice, Reject, Exit) is a SPNE if and only if* $x < min\{\frac{1-y}{\alpha-y}, \frac{y}{\alpha-1+y}\}$.

Proof. By Lemma 2.1, D's best reply is *Exit* if and only if (a) $\alpha < \frac{1}{\mu}$; or (b) $\alpha > \frac{1}{\mu}$ and $x < \frac{1-y}{\mu\alpha-y}$. By Lemma 2.4(a), E's best reply is *Reject* if and only if $x < \frac{y}{\alpha-1+y}$. By Lemma 2.5(b), D's best reply is *Voice* if and only if $x < \frac{1-y}{\alpha-y}$.

Observe that $x < \frac{1-y}{\alpha-y}$ implies $x < \frac{1-y}{\mu\alpha-y}$, since $\mu < 1$. Therefore, condition $x < \frac{1-y}{\mu\alpha-y}$ becomes insignificant. □

From Proposition 2.5 it follows that for high cost of party disunity (low values of μ) and high intra-party conflict (low values of x), the dissidents prefer to exit the party. In its turn, the elite faction prefers to accept the party's split, as both factions are in a strong ideological disagreement (x is low).

Table 2.1 next summarizes the results of Propositions 2.1 - 2.5. The table reads as follows: given the value of $0 < \mu < 1$, we choose the value of α, which can be either low, belonging to Case (1) $1 < \alpha < \frac{1}{\mu}$, or high, belonging to Case (2) $\alpha > \frac{1}{\mu}$. Conditions on μ and α translate into the necessary values of y and x to produce a certain SPNE.

<table>
<tr><th rowspan="3">Proposition</th><th colspan="6">Conditions on α given $0 < \mu < 1$</th></tr>
<tr><th colspan="2">Case (1) $1 < \alpha < \frac{1}{\mu}$</th><th colspan="4">Case (2) $\alpha > \frac{1}{\mu}$</th></tr>
<tr><th>Conditions on y</th><th>Conditions on x</th><th colspan="2">Conditions on y</th><th colspan="2">Conditions on x</th></tr>
<tr><td>P1: Primaries with threat</td><td>$0 < y < 1$</td><td>$\frac{y}{\alpha-1+y} < x < 1$</td><td colspan="2">$0 < y < \frac{\alpha-1}{\alpha(1+\mu)-2}$</td><td colspan="2">$\frac{y}{\alpha-1+y} < x < \frac{1-y}{\mu\alpha-y}$</td></tr>
<tr><td rowspan="2">P2: Primaries no threat</td><td colspan="2" rowspan="2">–</td><td colspan="2" rowspan="2">$0 < y < 1$</td><td>$\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$</td><td>$\alpha > \frac{1}{\mu^2}$</td></tr>
<tr><td>$max\{\mu, \frac{1-y}{\mu\alpha-y}\} < x < 1$</td><td>$\mu < x < 1$</td></tr>
<tr><td>P3: Loyalty with threat</td><td>$\frac{1}{2} < y < 1$</td><td>$\frac{1-y}{\alpha-y} < x < \frac{y}{\alpha-1+y}$</td><td colspan="2">$\frac{1}{2} < y < 1$</td><td colspan="2">$\frac{1-y}{\alpha-y} < x < min\{\frac{1-y}{\mu\alpha-y}, \frac{y}{\alpha-1+y}\}$</td></tr>
<tr><td rowspan="2">P4: Loyalty no threat</td><td colspan="2" rowspan="2">–</td><td>$\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$</td><td>$\alpha > \frac{1}{\mu^2}$</td><td colspan="2" rowspan="2">$\frac{1-y}{\mu\alpha-y} < x < \mu$</td></tr>
<tr><td>$\frac{1-\mu^2\alpha}{1-\mu} < y < 1$</td><td>$0 < y < 1$</td></tr>
<tr><td rowspan="2">P5: Party split</td><td>$0 < y < \frac{1}{2}$</td><td>$0 < x < \frac{y}{\alpha-1+y}$</td><td colspan="2">$0 < y < \frac{1}{2}$</td><td colspan="2">$0 < x < \frac{y}{\alpha-1+y}$</td></tr>
<tr><td>$\frac{1}{2} < y < 1$</td><td>$0 < x < \frac{1-y}{\alpha-y}$</td><td colspan="2">$\frac{1}{2} < y < 1$</td><td colspan="2">$0 < x < \frac{1-y}{\alpha-y}$</td></tr>
</table>

Table 2.1: SPNE conditions described in Propositions 2.1 to 2.5

We next show graphically the results of Propositions 2.1 to 2.5 resumed in Table 2.1. To this end, we define the indifference curves of the players as follows. Let $f(y) = \frac{y}{\alpha-1+y}$ define E's indifference curve between accepting or rejecting primaries under the credible exit threat of D. Observe that the derivative $\frac{df}{dy}$ is positive and the second-order derivative $\frac{d^2f}{dy^2}$ is negative; therefore, $f(y)$ is increasing and concave. Let $g(y) = \frac{1-y}{\alpha-y}$ define D's indifference curve between being loyal to party P or voicing demand for primaries. The derivative $\frac{dg}{dy}$ is negative as well as the second-order derivative $\frac{d^2g}{dy^2}$; hence, $g(y)$ is decreasing and concave.

Let $\phi(y) = \frac{1-y}{\mu\alpha-y}$ define D's indifference curve between exiting or staying in the party P after the failed attempt of voice. The derivative $\frac{d\phi}{dy} = \frac{1-\mu\alpha}{(\mu\alpha-y)^2}$ is positive if $\mu < \frac{1}{\alpha}$ and in this case the second-order derivative $\frac{d^2\phi}{dy^2}$ is also positive; consequently, $\phi(y)$ is increasing and convex. If $\mu > \frac{1}{\alpha}$, then the derivative $\frac{d\phi}{dy}$ is negative, as well as the second-order derivative $\frac{d^2\phi}{dy^2}$; as a result, $\phi(y)$ is decreasing and concave.

We divide the graphs according to the two constraints on the values of α: case (1) and case (2) of Table 2.1. However, case (2) consists of two subcases, when (2.1) $\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$ and (2.2) $\alpha > \frac{1}{\mu^2}$. There are three graphs showing the regions in the space (x, y). We start with the case when the cost of disunity is high $\mu < \frac{1}{\alpha}$, which puts a constraint on α, $1 < \alpha < \frac{1}{\mu}$ as shown on Figure 2.4. We increase the cost of disunity until $0 < \mu < \frac{1}{\sqrt{\alpha}}$ or, equivalently, $\frac{1}{\mu} < \alpha < \frac{1}{\mu^2}$ as shown on Figure 2.5. Finally, the case when the cost of disunity is sufficiently small, $\mu > \frac{1}{\sqrt{\alpha}}$ and there is no constraint on α, i.e. $\alpha > \frac{1}{\mu^2}$, is shown on Figure 2.6.

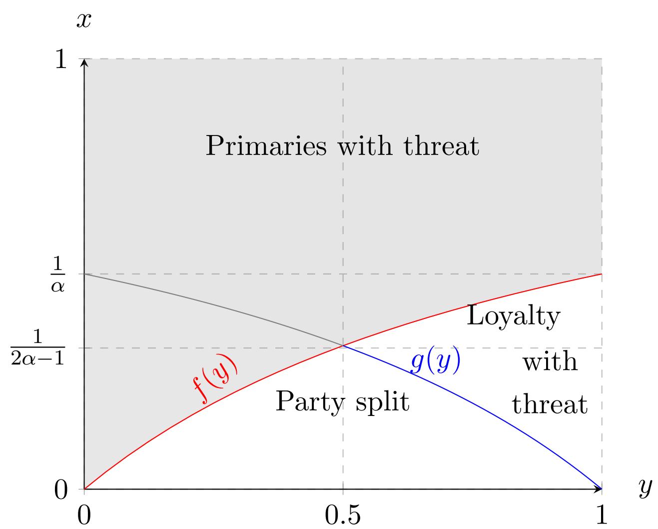

Figure 2.4: The SPNE when there is a high cost of disunity ($\mu < \frac{1}{\alpha}$), where $f(y) = \frac{y}{\alpha-1+y}$, $g(y) = \frac{1-y}{\alpha-y}$

Figure 2.4 shows the case when the cost of disunity is high ($\mu < \frac{1}{\alpha}$), i.e., there is a high demand for strong and united parties, and the electoral bonus of running jointly is constrained ($\alpha < \frac{1}{\mu}$). The $f(y)$ line represents the curve of indifference of the elite faction between accepting and rejecting the primaries which leads to party split. Above it, the party elite is willing to accept primaries, while below this line it prefers to split from the party. The $g(y)$ line represents the indifference curve of the dissenting faction between splitting from the party and staying loyal. Below this line, the dissenters prefer to split from the party, while above this line they prefer to remain loyal. Observe that in this case D's indifference curve between exiting the party and staying after the failed attempt to challenge is absent.

Under this case only three outcomes are possible: *Party split*, *Primaries with threat* and *Loyalty with threat*. Let us focus on the left-hand side of the figure, which corresponds to the case when the dissenting faction is in the majority ($y < \frac{1}{2}$). In this case only two outcomes are possible: either the elite decides to reject the demand of primaries leading to the party split (which happens when x is low) or the elite accepts the demand and introduces primaries. As long as both factions are getting closer ideologically (x is increasing), the likelihood of primaries increases.

Now let us focus on the right-hand side of the Figure 2.4, which corresponds to the case when the party elite is strong ($y > \frac{1}{2}$). Again, for sufficiently low values of x, the dissidents decide to split from the party after the elite rejects their demand. As long as x increases, the dissenting faction still threatens to leave the party, but after the elite's rejection, they decide to remain loyal to the party. This requires intermediate values of x. When x increases, the elite is willing to accept primaries, as now both factions are much aligned ideologically.

This case is absent in the model of HM, where the party elite never accepts the adoption of primaries when it commands the majority support in the party. In comparison, when the electoral bonus is constrained and there is a cost of party disunity, the party elite is willing to accept primaries even when it commands the majority support inside the party. Although the party elite represents the median party member, it gets higher utility by accepting primaries, in which case a candidate from the dissenting faction wins. D's candidate may represent better the median general voter and have higher chances of beating the opposition party's candidate.

We now turn to subcase (2.1), when α is less constrained and the cost of disunity is moderate, i.e. $\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$, which is shown in Figure 2.5 next.

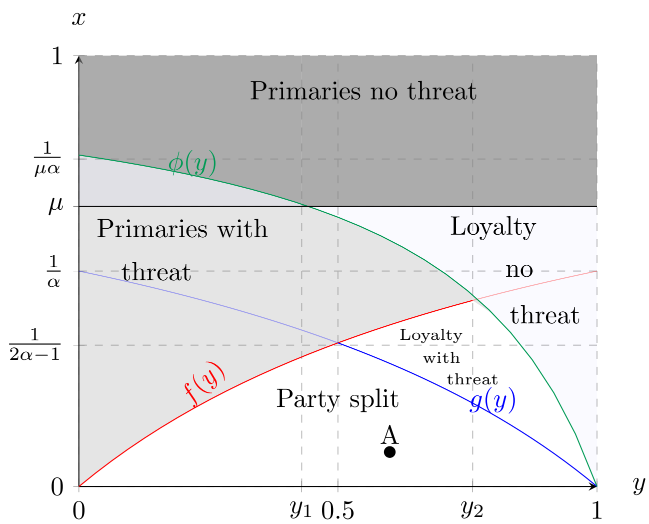

Figure 2.5: The SPNE when the cost of disunity is moderate ($\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$), where $f(y) = \frac{y}{\alpha-1+y}$, $g(y) = \frac{1-y}{\alpha-y}$, $\phi(y) = \frac{1-y}{\mu\alpha-y}$, $y_1 = \frac{1-\mu^2\alpha}{1-\mu}$ and $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$

Let us focus on the left-hand case, when $0 < y < y_1$, i.e., the dissenting faction is relatively stronger than the elite. In this case, keeping other variables constant and varying x, the party can end up in two cases: the party split or the primaries. For low values of x the party elite rejects primaries and the party splits. With x increasing, the party elite accepts primaries under the credible exit threat of the dissidents. With further increase in x, the dissidents do not threaten to leave the party any more, and the party elite accepts primaries. This case requires high ideological cohesion between both factions.

Let us focus on the right-hand side of the Figure 2.5, in particular when $y > y_2$, i.e. the elite is relatively stronger than the dissidents. In this case, three outcomes are possible. For low values of x the party splits. As long as x increases, the dissenters choose stay loyal to the party. And lastly for sufficiently large x the elite accepts primaries with no credible exit threat of the dissidents.

Finally, the most interesting case, where all five outcomes are possible is when $y_1 < y < y_2$. Note that y_2 is always greater than $\frac{1}{2}$, and that y_1 can be smaller or greater than $\frac{1}{2}$. Observe that, when μ is approaching $\frac{1}{\mu\alpha}$, y_1 decreases. As a consequence, the region where all five equilibria are possible increases. Here both factions are relatively equal in their strength inside the party. This case represents all the five outcomes the party can go through. Imagine the level of the intra-party conflict x is located at point A in Figure 2.5. At this point, the dissenters split from the party as there is a high ideological discrepancy between both factions. If the ideological alignment between both factions sufficiently rises (x increases), the dissenters prefer to stay loyal to the party after the elite's rejection of their demand. With further increase in x, the party elite accepts the primaries without any exit threat from the dissidents, as now both factions find themselves in a strong ideological agreement ($x > \mu$).

Now let us observe how the decrease in the cost of disunity μ affects the likelihood of each type of primaries, which is shown in Figure 2.6 below.

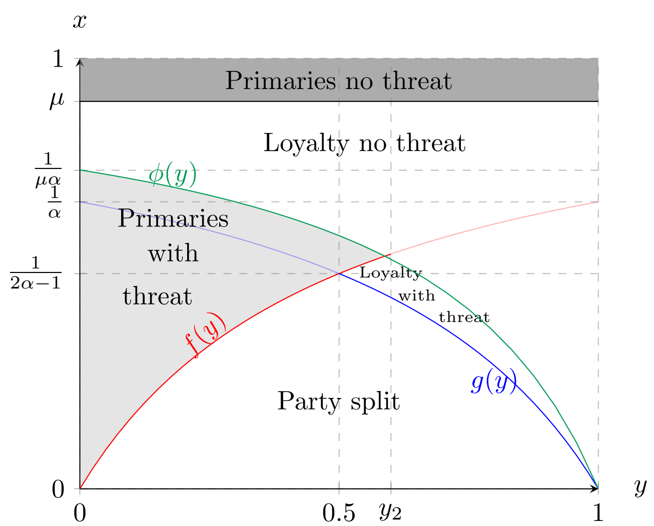

Figure 2.6: The SPNE when the cost of disunity is low ($\mu > \frac{1}{\sqrt{\alpha}}$), where $f(y) = \frac{y}{\alpha-1+y}$, $g(y) = \frac{1-y}{\alpha-y}$, $\phi(y) = \frac{1-y}{\mu\alpha-y}$, $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$

Under this case, the likelihood of both equilibia when primaries are adopted (*Primaries with threat* and *Primaries no threat*) decreases. This happens because now the elite is more inclined to reject the dissidents' demand, as it brings no cost. As μ approaches 1 (that is, there is no demand for party unity among the electorate, or there is no demand for internally democratic parties), the likelihood of *Primaries with threat* decreases, while the likelihood of *Primaries no threat* almost disappears. This case is almost identical to HM's one, however, the difference between our model and theirs is that under their model if the dissidents move first, the primaries are never introduced.

The difference of Figure 2.6 with Figure 2.5, is that now, when D is relatively stronger than E ($y < \frac{1}{2}$), the dissidents stay loyal to party without exit threat when $\frac{1}{\mu\alpha} < x < \mu$.

The interesting question is how changes in our key variables of interest, namely, x, y, α and μ lead to a transition from one equilibria to another. To this end, let us focus on the intermediate case shown on Figure 2.5, where $1 < \alpha < \frac{1}{\mu^2}$ or $\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$, which is described in the next section.

2.4 Results II: comparative statics

In this section we analyse how changes in our key variables of interest influence the likelihood of the adoption of primaries and possible equilibrium transitions when the ideological affinity x and the party elite strength y vary.

2.4.1 Changes in the cost of disunity μ

Let us focus on Figures 2.7 and 2.8 next. Imagine both factions are close ideologically (high x, e.g. $x > \mu$) and the dissenting faction commands the majority support inside the party ($y < \frac{1}{2}$); that is, we are at point B captured on the graph. In this case, the elite accepts primaries and there is no exit threat from the dissidents. We start with $\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$. As long as μ is decreasing and is approaching $\frac{1}{\alpha}$, the party elite still accepts primaries but now under the credible exit threat of the dissidents. This case is shown on Figure 2.8.

Now we analyse the case when the elite commands the majority support inside the party ($y > \frac{1}{2}$). Imagine we are located at point C on Figure 2.7. In this case, the level of intra-party conflict is moderate and the dissenting faction stays loyal to the party without the credible exit threat. As long as μ decreases and approaches $\frac{1}{\alpha}$, the dissenting faction threatens to split from the party forcing the party elite to accept primaries. The likelihood of both equilibria *Primaries no threat* and *Loyalty no threat* decreases.

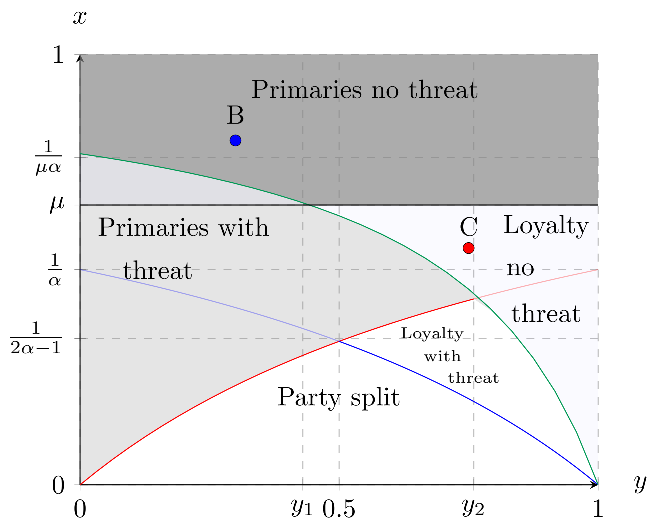

Figure 2.7: The SPNE when the cost of disunity is moderate ($\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$)

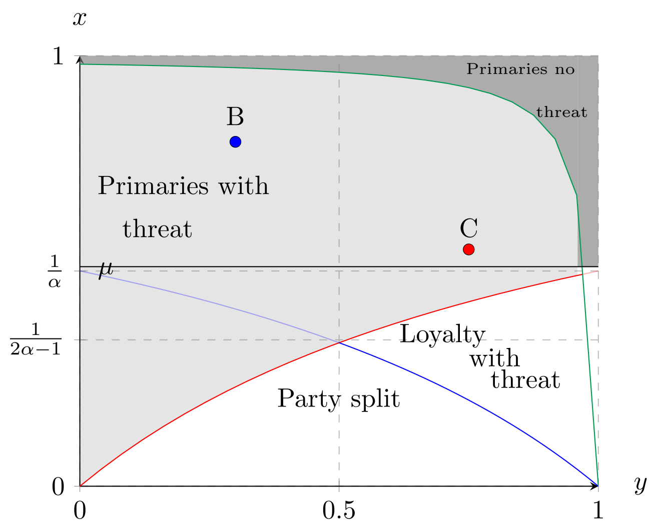

Figure 2.8: The SPNE when the cost of disunity μ is approaching $\frac{1}{\alpha}$

With further decrease in the value of μ we find ourselves in the case shown in

Figure 2.4. Observe that the changes of μ do not influence directly the party's split.

2.4.2 Changes in the electoral bonus α

Next we analyse how changes in α affect the likelihood of primaries. Figure 2.9 shows the case when α is low, which may characterize the proportional electoral system, and the cost of party disunity is moderate $\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$. We can see, that since now the bonus of running jointly is sufficiently low, the likelihood of the party split is high. Both types of primaries require high internal affinity inside the party.

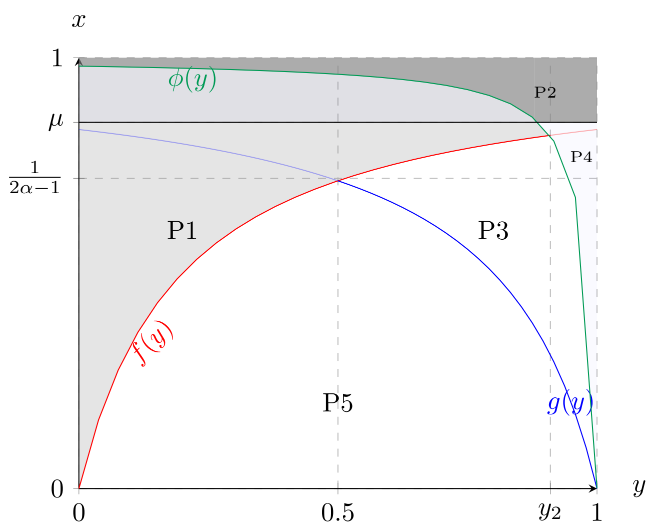

Figure 2.9: The SPNE when the cost of disunity is moderate ($\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$) and α is low
$f(y) = \frac{y}{\alpha-1+y}$, $g(y) = \frac{1-y}{\alpha-y}$, $\phi(y) = \frac{1-y}{\mu\alpha-y}$, $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$
P1 = Primaries with threat, P2 = Primaries no threat, P3 = Loyalty with threat, P4 = Loyalty no threat, P5 = Party split

Figure 2.10 next depicts the case, when α increases, while μ stays the same. We can easily observe that the likelihood of *Party split* decreases substantially, as well as the likelihood of *Primaries with threat* and *Loyalty with threat*. On the contrary, the likelihood of *Loyalty no threat* increases. Since the electoral bonus of running jointly is high, the dissidents are more inclined to stay inside the party without threatening. Alternatively, there is a high cost of splitting, which may be the case of majoritarian electoral system, that prevents the dissenting faction from threatening the party elite. Observe that *Primaries no threat* is unaffected with the change in α for this case. The party elite is willing to adopt primaries when there is a high ideological cohesion inside the party.

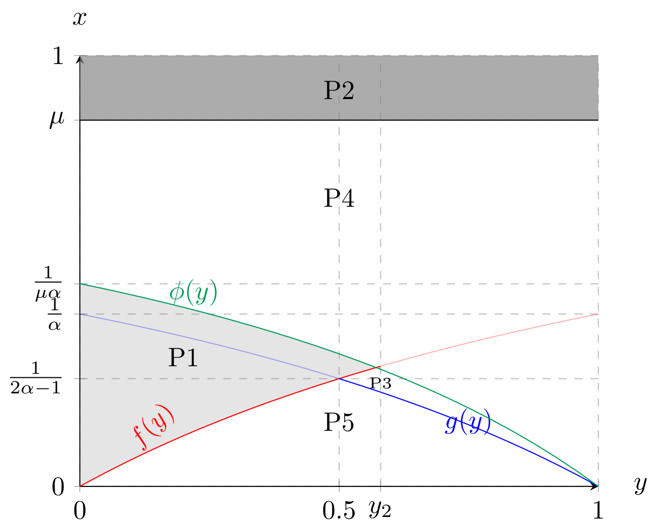

Figure 2.10: The SPNE when the cost of disunity is moderate ($\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$) and α increases
$f(y) = \frac{y}{\alpha-1+y}$, $g(y) = \frac{1-y}{\alpha-y}$, $\phi(y) = \frac{1-y}{\mu\alpha-y}$, $y_1 = \frac{1-\mu^2\alpha}{1-\mu}$ and $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$
P1 = Primaries with threat, P2 = Primaries no threat, P3 = Loyalty with threat, P4 = Loyalty no threat, P5 = Party split

2.4.3 Changes in ideological affinity x and elite strength y

We now analyse how the changes in x and y, given the constraints on μ and α, create possible transitions between the SPNE. Table 2.2 presents the results when the ideological cohesion between the elite and dissenting faction increases (x increases). Table 2.2 reads as follows. We start from low values of x and analyze how the increase in x leads to changes in SPNE. The results depend on the values of μ and y. Expression "P5 $\rightarrow$ P1" in the right-hand column means that we started from the SPNE *Party split* and the increase in x leads to the SPNE *Primaries with threat.*

Case	Range of values of μ	Range of values of y	Transition between SPNE
1	$\mu < \frac{1}{\alpha}$	$y < \frac{1}{2}$ $y > \frac{1}{2}$	P5 → P1 P5 → P3 → P2
2	$\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$	$y < \frac{1-\mu^2\alpha}{1-\mu}$ $\frac{1-\mu^2\alpha}{1-\mu} < y < \frac{\alpha-1}{\alpha(1+\mu)-2}$ $y > \frac{\alpha-1}{\alpha(1+\mu)-2}$	P5 → P3 → P2 P5 → P3 → P1 → P4 → P2 P5 → P3 → P4 → P2
3	$\mu > \frac{1}{\sqrt{\alpha}}$	$y < \frac{1}{2}$ $\frac{1}{2} < y < \frac{\alpha-1}{\alpha(1+\mu)-2}$ $y > \frac{\alpha-1}{\alpha(1+\mu)-2}$	P5 → P1 → P4 → P2 P5 → P3 → P1 → P4 → P2 P5 → P3 → P4 → P2

Table 2.2: Transitions between equilibria when x increases, starting from low values
P1 = Primaries with threat, P2 = Primaries no threat, P3 = Loyalty with threat, P4 = Loyalty no threat, P5 = Party split

Table 2.2 shows that as long as both factions become closer ideologically, the party's internal dynamics can pass through a limited range of outcomes. The most diverse case is when the cost of disunity is moderate to low (Cases 2 and 3). In all cases, the high level of the intra-party conflict (x is low) guarantees the party split. On the contrary, high ideological cohesion between the factions guarantees the adoption of primaries.

Table 2.3 captures how the increase in the relative strength of the elite (y increases) creates transitions between different equilibria. Table 2.3 reads as follows. We start from the low values of y and analyse how an increase in the relative strength of the party elite leads to different SPNE. For example, looking at the first column, when μ is sufficiently low ($\mu < \frac{1}{\alpha}$) and x is sufficiently low as well ($x < \frac{1}{2\alpha-1}$), the party initially finds itself in the SPNE *Primaries with threat.* As y increases, the party's internal dynamics goes through the SPNE *Party split* and the SPNE *Loyalty with threat.*

Case	Range of values of μ	Range of values of x	Transition between SPNE
1	$\mu < \frac{1}{\alpha}$	$x < \frac{1}{2\alpha-1}$ $\frac{1}{2\alpha-1} < x < \frac{1}{\alpha}$ $x > \frac{1}{\alpha}$	P1 → P5 → P3 P1 → P3 P1
2	$\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$	$x < \frac{1}{2\alpha-1}$ $\frac{1}{2\alpha-1} < x < \frac{1}{\alpha}$ $\frac{1}{\mu} < x < \mu$ $\mu < x < \frac{1}{\mu\alpha}$ $x > \frac{1}{\mu\alpha}$	P1 → P5 → P3 → P4 P1 → P3 → P4 P1 → P4 P1 → P2 P2
3	$\mu > \frac{1}{\sqrt{\alpha}}$	$x < \frac{1}{2\alpha-1}$ $\frac{1}{2\alpha-1} < x < \frac{1}{\alpha}$ $\frac{1}{\alpha} < x < \frac{1}{\mu\alpha}$ $\frac{1}{\mu\alpha} < x < \mu$ $x > \mu$	P1 → P5 → P3 → P4 P1 → P3 → P4 P1 → P4 P3 P2

Table 2.3: Transitions between equilibria when y increases, starting from low values
P1 = Primaries with threat, P2 = Primaries no threat, P3 = Loyalty with threat, P4 = Loyalty no threat, P5 = Party split

From Table 2.3 we can see that the increase in the relative strength of the elite faction (increase in y) has no effect on the changes in the SPNE, when there is a high ideological cohesion between both factions (x is sufficiently high) and the cost of disunity is high ($\mu < \frac{1}{\alpha}$) (Case 1), or when there is a high ideological cohesion between both factions (x is sufficiently high) and the cost of disunity is moderate to low (Case 2). In the former case, the party ends up in *Primaries with threat*, while in the latter case the outcome is *Primaries no threat*. For intermediate levels of intra-party conflict x, as the elite's relative strength increases, the intra-party dynamics passes from the case when primaries are introduced to the case when the dissenting faction stays loyal.

Figures 2.11, 2.12 and 2.13 show graphically the results of Tables 2.2 and 2.3.

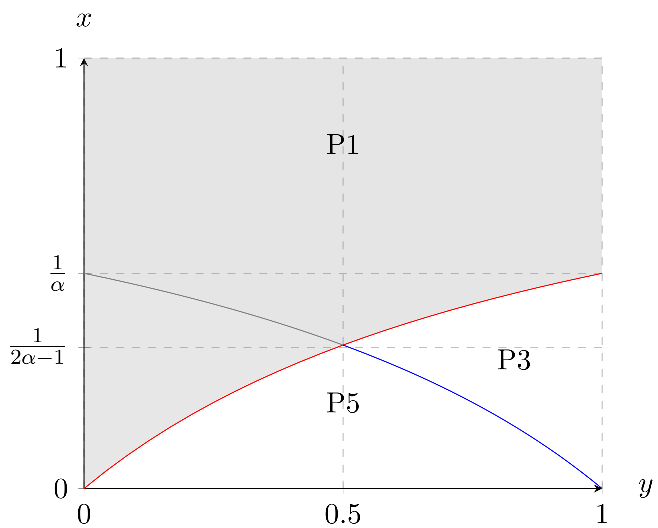

Figure 2.11: Transition between the SPNE when there is a high cost of disunity $\mu < \frac{1}{\alpha}$

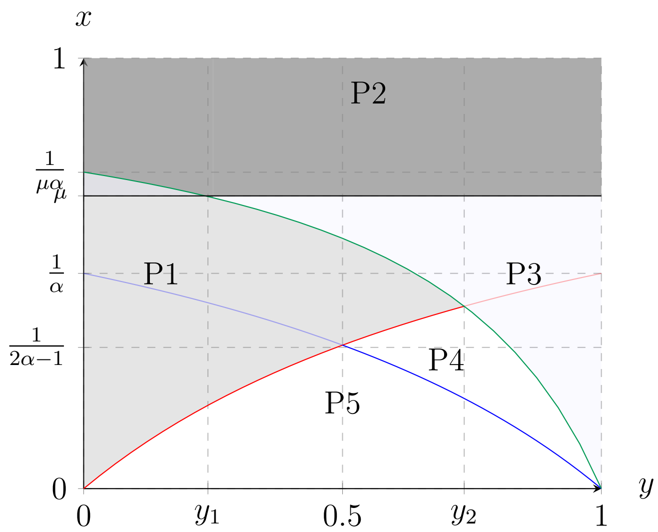

Figure 2.12: Transition between the SPNE when the cost of disunity is moderate $\frac{1}{\alpha} < \mu < \frac{1}{\sqrt{\alpha}}$, where $y_1 = \frac{1-\mu^2\alpha}{1-\mu}$, $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$

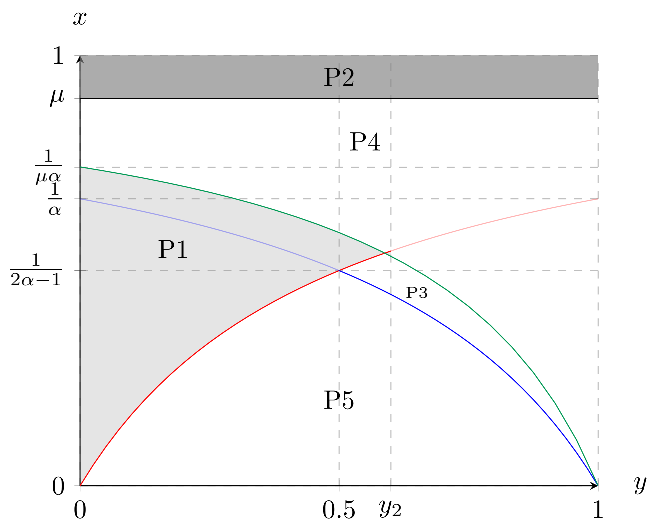

Figure 2.13: The SPNE when the cost of disunity is low $\mu > \frac{1}{\sqrt{\alpha}}$, where $y_2 = \frac{\alpha-1}{\alpha(1+\mu)-2}$

From the analysis we can observe that the increase in the ideological cohesion between the party elite and the dissenting faction leads to the adoption of primaries, while the increase in the relative strength of the elite generally leads to the loyalty of the dissenting faction. As the cost of disunity increases, the likelihood of *Primaries with threat* increases, while the likelihood of *Primaries no threat* decreases. This happens because when there is a high demand for strong and united parties among the electorate, the rejection of primaries brings a cost to the party; moreover, the dissenting faction always threatens to leave the party, and so the party elite prefers to conceal factional divisions inside the party and appear united in the eyes of the voters.

When the cost of disunity decreases, the likelihood of *Primaries with threat* decreases, while the likelihood of *Primaries no threat* increases. In this case, there is no threat from the dissidents to split, but the party elite is willing to cooperate with the dissidents in adoption of primaries. This latter type of primaries only happens when both factions stand in a strong ideological agreement.

Finally, when there is almost no cost of disunity, both types of primaries decrease, and in addition, *Primaries no threat* almost disappears. As now the party elite is not constrained by the cost of rejection, it accepts primaries only when it is weaker than the dissenting faction in terms of its relative strength inside the party and the level of the intra-party conflict takes the intermediate levels.

2.5 Conclusion

Democratizing candidate selection is getting common among many political parties all over the world. The reasons of why political elites are willing to concede their power in nominating candidates are not yet well understood. In this chapter we try to shed light on the reasons of why the party elites adopt primary elections by examining the intra-party factional dynamics.

Following the work of HM, we view a party as a coalition of factions, composed of the party elite and the dissenting faction. Extending the work of HM, we analyse the strategic interplay between both factions. We show that the primaries are adopted in two cases. In the first case, there is a credible threat of the dissenting faction to split from the party, and as a consequence, the party elite finds itself in a weak position and is forced to adopt primaries in order to preserve the party unity and to hide from the public the party's internal divisions. In the second case, the party elite adopts primaries even when there is no threat from the dissenting faction to split. This case happens only when cohesion towards the policy issues between both factions is strong.

The major changes in the results compared to the benchmark model of HM are brought by the changed order of the moves of the players and the introduction of the variable capturing public perception towards party's internal (dis)unity. In the cases where there is a high demand for strong and united parties among voters (majoritarian electoral systems), the party elites are more willing to respond positively to the demands of the dissenters in order to prevent the factional disagreements from becoming publicly known. To be perceived less united as the opponent may be damaging for political parties. We have seen that in our case, when the cost of disunity is high (which captures high demand for party unity), the likelihood that the primaries are adopted becomes the highest.

In contrast, in the proportional electoral systems (consensus democracies), intra-party disagreements may be viewed more positively; for example, they may be seen as solutions to coalition bargaining games or as moderating influences in building balanced governments. As long as the cost of disunity decreases and, consequently, the demand for party unity decreases, the need to conceal factional divisions becomes less necessary.

In future research it would be interesting to incorporate the opposing party as a strategic actor into the game. So far we have analysed only the strategic interaction of both factions inside the party without taking into account the strategic decision of the opposition party. It would be interesting to see how the likelihood of primaries will depend on whether the opposition party adopts primaries or not.

Another avenue for research would be to endogenize the probability of winning when the party runs jointly depending on the candidate chosen. So far we have assumed that the winning probability of the party when it runs united, π, is the same regardless of whether the party's candidate belongs to the elite faction or the dissenting faction.

Finally, we could also incorporate the continuous policy space and asymmetric payoffs, and assume that one faction can be more extreme than the other. In such a case, it would be interesting to analyse how the degree of extremism of the party elite or the dissenting faction affects the likelihood of the adoption of primaries. Primaries might then have an effect on the probability of winning the election, given the position of D and the one of the median voter. The trade-off the party elite would have to resolve is that of the party unity and the proximity to the median voter.

3

A mechanism to pick the deserving winner

*

Abstract

A group of individuals is choosing an individual (the winner) among themselves, when the identity of the deserving winner is common knowledge among individuals. A simple mechanism of voting by veto is proposed as an alternative to the mechanism studied by Amorós [8]. Like that of Amorós [8], the suggested mechanism implements the socially desirable outcome (the deserving winner is chosen) in subgame perfect equilibria.

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3.1 Introduction

A typical social choice setting involves a set of *alternatives* (also known as *candidates*) and a set of *agents* (also known as *voters*). The agents express their preferences over the alternatives and then they are mapped by some social choice function (also known as a *voting rule*) to output a winner. In this chapter we consider a special case, where the number of alternatives equal the number of agents, i.e. the voters are the candidates themselves. The collective decision problem the agents face is that they have to choose a winner among themselves in the presence of a *deserving winner* who is common knowledge among all agents.

The fact that the set of agents and the set of alternatives coincides allows us to make several assumptions about the agents' preferences. In particular, we assume that each agent is selfish: he always wants to be the winner. But at the same time, he is impartial towards the rest: if an agent cannot be chosen as the winner, he prefers the deserving winner to be chosen.

Think of a contest where the jury has to choose a winner among themselves. Each jury member knows who deserves to win, and yet, he wants to be the winner. Or, for instance, a group of agents that have to choose a leader. Each member of the group knows who deserves to be the leader (for example, according to his experience or knowledge) but at the same time each one wants to be the leader.

In a collective decision-making problem, assuming rationality, we can encounter that some individuals may behave in a selfish way. In most cases the individuals only care about their own private interests towards different outcomes, and each individual makes decisions to pursue his own individual objectives. Here the question arises: Is it possible to design a mechanism or institution (or when speaking about voting issues, a voting mechanism) so that no matter how selfish the individuals are, their actions will always lead to the outcome that is socially desirable? In other words, given the socially desirable outcome, is it possible to create the conditions according to which every (in some sense, optimal) action of individuals result in it?

It is to handle this problem that the implementation theory or mechanism design intervenes. The issue of the implementation theory or mechanism design consists in designing a mechanism (or a game form) in which agents (individuals) interact. One can think of a mechanism design as a reverse game theory. A mechanism specifies the rules of a game. The players are the members of society (agents, individuals), who interact according to the rules of the game. The interactions of agents (individuals) result in an outcome that the mechanism generates in equilibrium. The question is then whether the equilibrium outcomes will be socially optimal. The problem is how to design the mechanism such that the equilibrium behaviour of the players will lead to socially desirable outcomes, no matter how selfish the individuals are. The socially desirable outcome is prescribed by a social choice rule. If, in each possible state of the world, the equilibrium outcome of the mechanism equals the set of optimal outcomes prescribed by the social choice function, then this mechanism is said to *implement* the social choice function.

The literature on implementation theory, like that on game theory, uses the game-theoretic solution concepts which describe the agents' behaviour within the game: the notion of dominant strategies, Nash equilibrium (in case of complete information), Bayesian Nash equilibrium (in case of incomplete information), and subgame perfect equilibrium as a refinement of Nash equilibrium, the one with which this chapter is particularly concerned [*].

This chapter is inspired by a related problem considered in Amorós [8]. Amorós [8] studies the problem of a group of agents choosing one winner among themselves in the presence of the deserving winner and all agents know him. Each agent always wants to be a winner, i.e. selfish. However, he is impartial towards the rest: if he cannot be chosen as the winner, he prefers the deserving winner to be chosen. The socially desirable outcome is that the deserving winner

[*] For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame (see Moore and Repullo [59]; Abreu and Sen [1]).

wins. To reach the socially desirable outcome Amorós [8] proposes a mechanism à la Maskin [51] that implements the social choice function in subgame perfect equilibria. For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame (see Moore and Repullo [59]; Abreu and Sen [1]). In spite of the criticism received by being unnatural (see Jackson [39]), these mechanisms can be applied to specific problems, as done in Amorós [8], who provides a simple and "natural" extensive form mechanism.

In the mechanism of Amorós [8] agents take turns announcing the winner. The announcement of the first agent is implemented only if he announces an individual different from himself. Otherwise, the turn passes to the next agent, and the process is repeated. The announcement of the last agent is implemented, even if he announces himself as the winner. The mechanism is such that truth-telling is an equilibrium, any subgame perfect equilibrium results in the deserving winner, and at least four individuals are necessary for the mechanism to work.

This chapter replicates Amorós [8] result by suggesting an alternative mechanism. The proposed mechanism can be considered as a reversal of the one by Amorós [8]. In particular, instead of announcing the individual whom they want to see as the winner, we allow the agents to announce the individual whom they do not want to see as the winner, i.e. *vetoing* an individual.

A mechanism of voting by veto (hereinafter, *veto mechanism*) also implements the desired social choice function in subgame perfect equilibria. Moreover, the proposed veto mechanism works for three individuals, improving upon Amorós's [8], whose mechanism needs at least four individuals to work and fails with three individuals.

The rest of the chapter is organized as follows. Section 3.2 provides the model. Section 3.3 describes the veto mechanism. Section 3.4 analyses the case of the implementation of the socially optimal rule in subgame perfect equilibria with n

= 3 individuals. Section 3.5 presents the general implementability result of the veto mechanism. Section 3.6 concludes the chapter.

3.2 Model

Let $N = \{1, 2, ..., n\}$ be a set of $n \geq 3$ individuals who must choose one individual (the winner) among them. All individuals know who deserves to win: the "deserving winner". The socially optimal outcome is that the deserving winner wins. However, each individual $i \in N$ is selfish: i always wants to be the winner. But at the same time, if i is not chosen as the winner, i prefers the deserving winner w to be chosen.

There is a fixed individual $w \in N$, interpreted as the deserving winner. The individuals have preferences defined over N, i.e. transitive and complete binary relations on N. A preference of individual i can be considered as i's ranking of all individuals in the group, including himself, from most to least preferred individual, with i being first in his preference profile and w being second. Let R_i denote i's preference and P_i denote the strict part of R_i.

Definition 3.1. A preference R_i of individual $i \in N$ is admissible if:

(i) for each $w \in N$ and each $j \in N$ such that $j \neq i$, iP_ij, and

(ii) for each $w \in N$ and each $j \in N$ such that $j \neq w$ and $j \neq i$, wP_ij.

Let Θ_i designate the set of admissible preferences for individual i. A social choice function with the deserving winner w is a function f_w: $\Pi_{i \in N}\Theta_i \to N$ that, for every admissible preference profile, selects the deserving winner w, i.e. for all $R \in \Pi_{i \in N}\Theta_i$, $f_w(R) = w$.

An extensive form mechanism, denoted by $\Gamma(M, g)$, consists of a set of (pure) strategies profiles of all individuals, $M = \Pi_{i \in N} M_i$, and the order, in which the individuals choose their strategies. An outcome function $g : M \to N$ associates an individual $g(m)$ with each profile m of messages and the order of the sequential game. For every profile $R \in \Pi_{i \in N} \Theta_i$, the pair (Γ, R) constitutes an extensive form game. It is a game of perfect information, as each individual, when playing his pure strategy, knows the previous history of the game and acts according to this history.

A subgame perfect Nash equilibrium (SPNE) of a perfect information game is a strategy profile that induces an equilibrium in every subgame of the game. The social choice function f_w is subgame perfect equilibria implementable if there exists a sequential mechanism Γ such that the set of SPNE outcomes of the game (Γ, R) has one element: $f_w(R)$, that is, w.

3.3 The mechanism

The veto mechanism. Given an arbitrary linear ordering $(1, 2, \ldots, n)$ of the $n \geq 3$ individuals, each individual from 1 to $n-1$ announces an individual to veto from those not having been vetoed before. Once individual $n-1$ has made his announcement, there only remains one individual, v. Let z be the first individual in the ordering $(1, 2, \ldots, n-1)$ that does not veto himself (i.e. the first individual that vetoes an individual different from himself), if such an individual exists. If no such z exists or if $z \neq v$, then the outcome of the mechanism is that v is chosen as a winner; if $z = v$, then the outcome of the mechanism is determined by letting n choose the winner between v and the individual v' vetoed by v: n chooses the most preferred individual, if there is one, and any of the two, if n is indifferent between v and v'.

3.4 The three individual case

This section analyses the mechanism when there are $n = 3$ individuals considering the different positions that w can occupy in the linear order. This analysis will demonstrate that all SPNE paths lead to the election of the deserving winner w as the final outcome.

Lemma 3.1. *For $n = 3$ the veto mechanism implements the social choice function f_w in subgame perfect equilibria.*

Proof. Suppose that the linear order is $(1, 2, 3)$. It will be demonstrated that f_w is implementable in subgame perfect equilibria by means of the veto mechanism. The mechanism starts with individual 1 announcing his veto. Individual 1 has three options: to veto 1, to veto 2 or to veto 3. Each option leads to a different path. The proof depends on the position that the deserving winner w occupies.

Case 1: $w = 1$

Path 1 (Fig. 3.1): 1 vetoes 1. Then 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, then $z = v = 2$ and, consequently, $n = 3$ chooses the winner between $v = 2$ and $v' = 3$. As $3P_32$, 3 chooses himself as the winner. Therefore, no matter whether 2 vetoes 2 or 3, 3 is the winner.

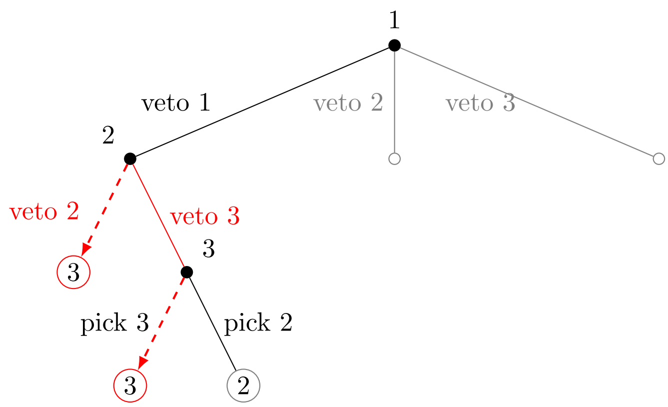


Figure 3.1: Path 1, $w = 1$

Path 2: (Fig. 3.2) 1 vetoes 2. Then 2 vetoes either 1 or 3. If 2 vetoes 1, then $z = 1$, $v = 3$, and as $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, then $z = v = 1$, and $n = 3$ chooses the winner between $v = 1$ and $v' = 2$. Since $1P_32$, 3 will choose $1 = w$ as the winner. At 2's node, given that $1P_23$, the best option for 2 is to veto 3, so that $1 = w$ is chosen as the winner.

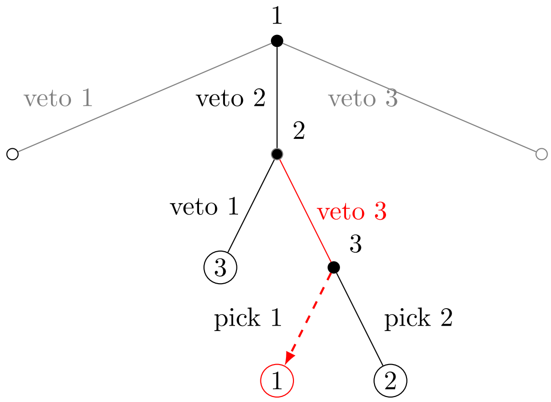


Figure 3.2: Path 2, $w = 1$

Path 3 (Fig. 3.3): 1 vetoes 3. Then 2 vetoes either 1 or 2. If 2 vetoes 1, $z = 1$ and $v = 2$. Since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$. Therefore, $n = 3$ picks the winner between $v = 1$ and $v' = 3$. Since $3P_31$, 3 will choose $v' = 3$ as the winner. As a result, at 2's node the best option for 2 is to veto 1.

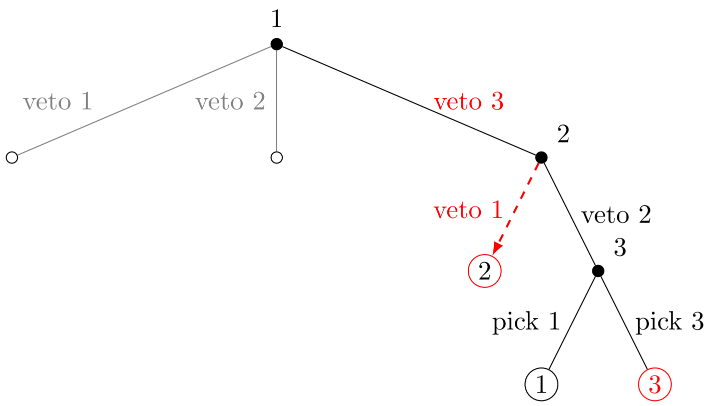


Figure 3.3: Path 3, $w = 1$

Given the outcomes of paths 1 - 3, at 1's node (see Fig. 3.4) the best option for 1 is to veto 2, as it is the only strategy that leads to the best outcome for

1: $1 = w$ is chosen as the winner. This proves that, when $w = 1$, all subgame perfect equilibria lead to the outcome when the deserving winner wins.

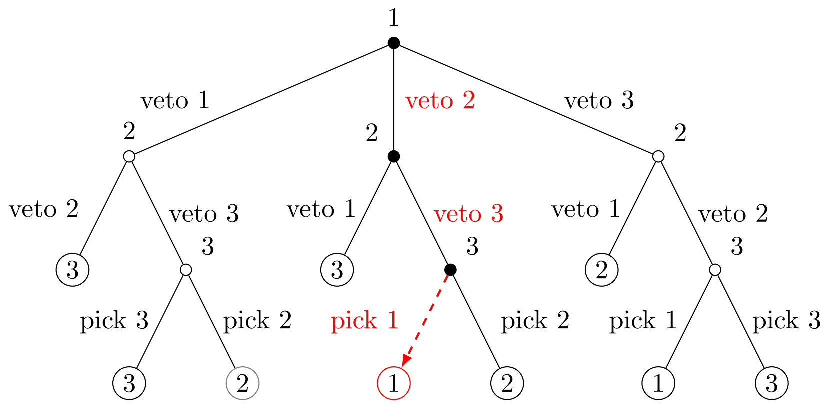


Figure 3.4: The SPNE outcomes when $w = 1$

Case 2: $w = 2$

Path 1 (Fig. 3.5): 1 vetoes 1. Then 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, then $z = v = 2$, and so $n = 3$ picks the winner between $v = 2$ and $v' = 3$. As $3P_32$, 3 chooses himself as the winner. Therefore, no matter if 2 vetoes 2 or 3, 3 is the winner.

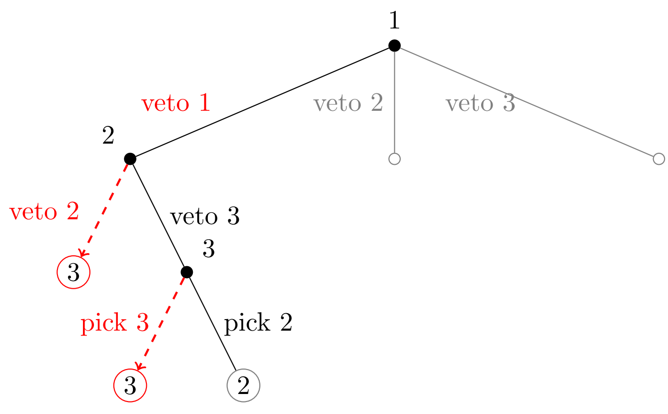


Figure 3.5: Path 1, $w = 2$

Path 2 (Fig. 3.6): 1 vetoes 2. 2 vetoes either 1 or 3. If 2 vetoes 1, then $z = 2$,

$v = 3$, and since $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, $z = v = 1$, then $n = 3$ chooses the winner between $v = 1$ and $v' = 2$. Since $2P_31$, 3 picks $2 = w$ as the winner. At 2's node, since $2P_23$, the best choice for 2 is to veto 3.

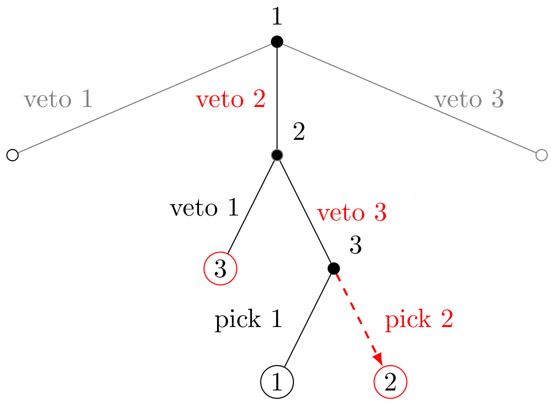


Figure 3.6: Path 2, $w = 2$

Path 3 (Fig. 3.7): 1 vetoes 3. 2 vetoes either 1 or 2. If 2 vetoes 1, then $z = 1$, $v = 2$, and since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$, and since $z = v$, $n = 3$ picks the winner between $v = 1$ and $v' = 3$. As $3P_31$, 3 will choose $v' = 3$ as the winner. At 2's node, as $2P_23$, the best choice for 2 is to veto 1. Therefore, $2 = w$ is chosen as the winner.

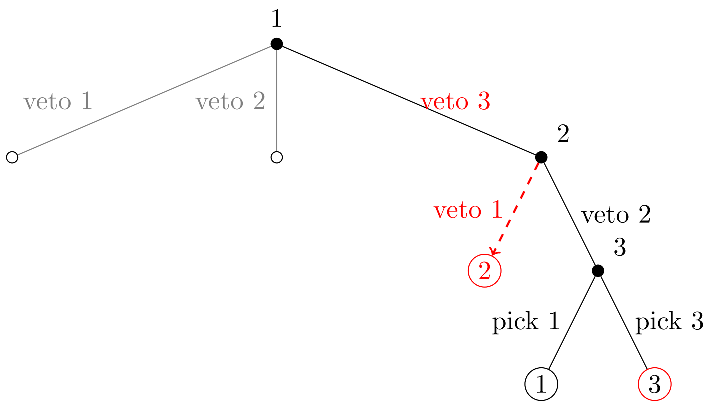


Figure 3.7: Path 3, $w = 2$

Given the outcomes of paths 1 - 3, at 1's node (Fig. 3.8) the best choice for 1 is to veto 2 or 3, as these paths both result in the best outcome for 1: $w = 2$ is chosen as the winner. Thus, it has been demonstrated that, when $w = 2$, all subgame perfect equilibria lead to the outcome that the deserving winner wins.

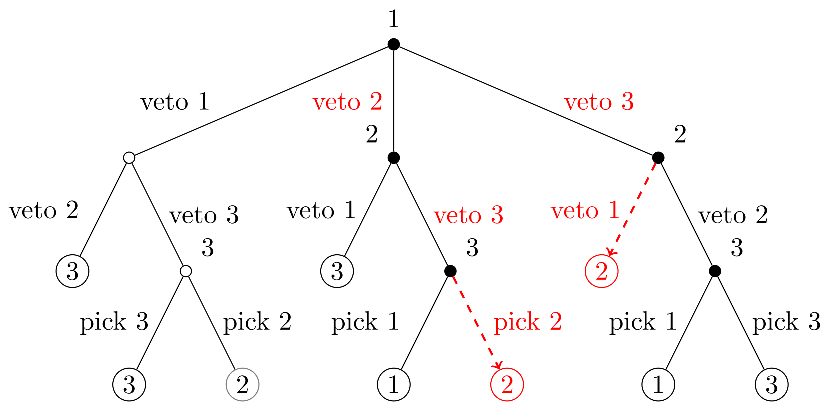


Figure 3.8: The SPNE outcomes when $w = 2$

Case 3: $w = 3$

Path 1 (Fig. 3.9): 1 vetoes 1. 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, $z = v = 2$, and, consequently, $n = 3$ chooses the winner between $v = 2$ and $v' = 3$. As $3P_3 2$, 3 chooses himself as the winner. Therefore, no matter whether 2 vetoes 2 or 3, $3 = w$ is the winner.

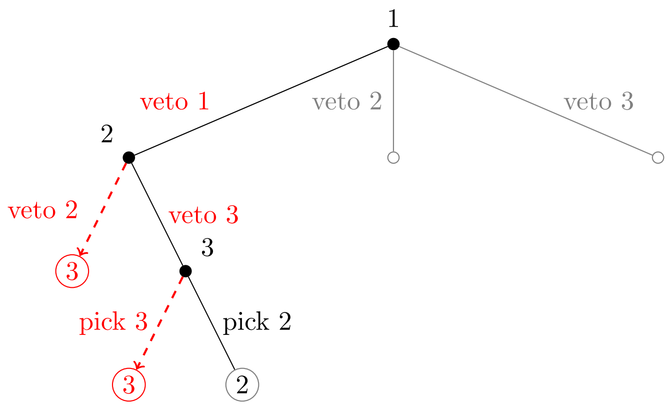
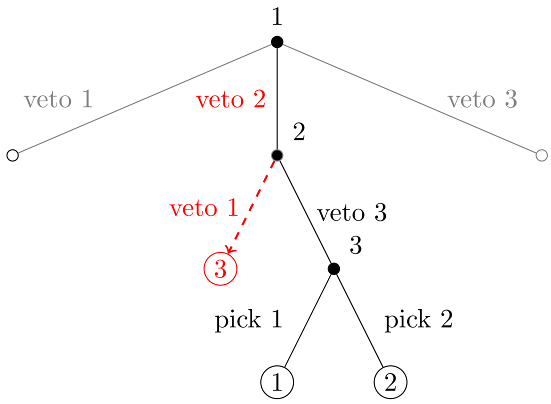


Figure 3.9: Path 1, $w = 3$

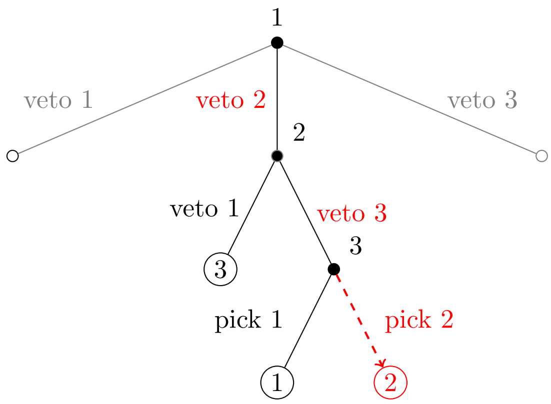
Path 2 (Fig. 3.10): 1 vetoes 2. 2 vetoes either 1 or 3. If 2 vetoes 1, $z = 1$ and $v = 3$, and since $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, as $v = z = 1$, at 3's node, $n = 3$ chooses the winner between $v = 1$ and $v' = 2$. Here consider two subcases: (i) if $1P_32$, 3 will choose 1 as the winner;

(ii) if $2P_31$, 3 will choose 2 as the winner.

At 2's node, in subcase (i), if $1P_32$, given that $3P_21$ when $w = 3$, the best choice of 2 is to veto 1, so that the outcome is $w = 3$. In subcase (ii), if $2P_31$, the best option for 2 is to veto 3, as 2 prefers himself to be the winner.



(a) Subcase (i)



(b) Subcase (ii).

Figure 3.10: Path 2, $w = 3$

Path 3 (Fig. 3.11): 1 vetoes 3. 2 vetoes either 1 or 2. If 2 vetoes 1, $z = 1$, $v = 2$, and since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$, and, therefore, $n = 3$ picks the winner between $v = 1$ and $v' = 3$. Given that $3P_31$, 3 will choose $v' = 3$ as the winner. At 2's node, since $2P_23$, the best option for 2 is to veto 1.

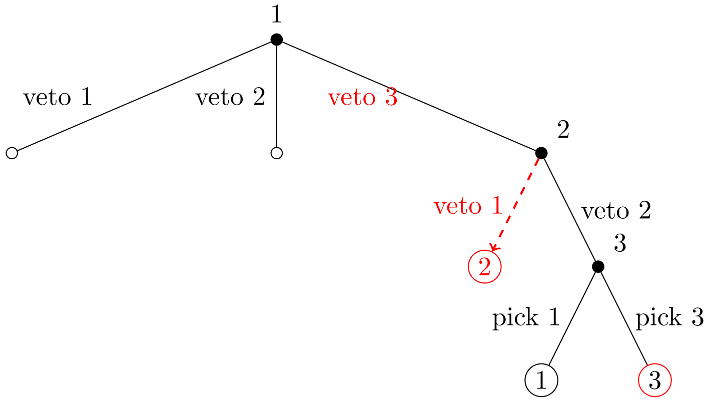


Figure 3.11: Path 3, $w = 3$

Given the outcomes of paths 1 - 3, at 1's node (Fig. 3.12 and Fig. 3.13) the best option for 1 is to veto 1 or 2 in subcase (i) (when $1P_32$), so that the winner is $w = 3$; and to veto 1 in subcase (ii) (when $2P_31$), thus, $w = 3$ is chosen as the winner. Therefore, all SPNE outcomes result in the election of w as the winner.

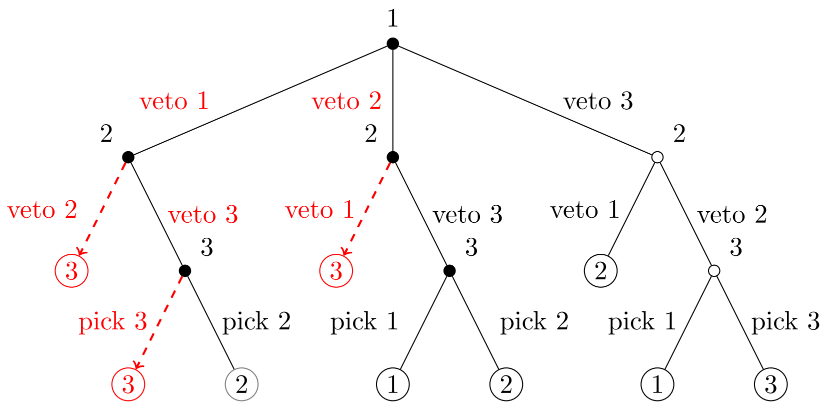


Figure 3.12: The SPNE outcomes of subcase(i) when $w = 3$

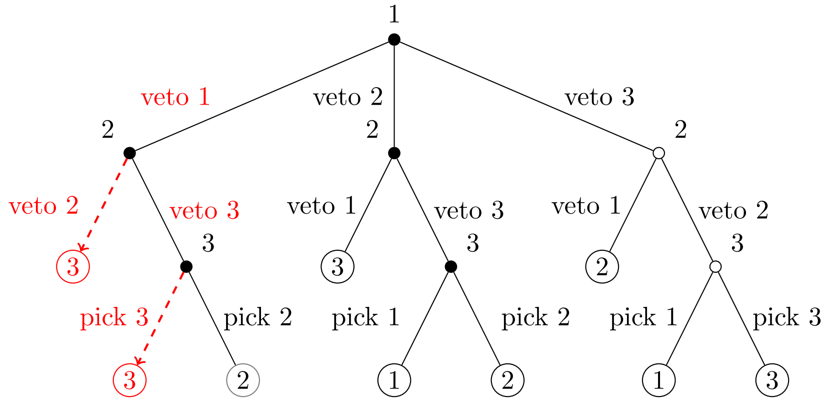


Figure 3.13: The SPNE outcomes of subcase(ii) when $w = 3$

□

3.5 Main result

Lemma 3.2. *For a given* $R \in \Pi_{i \in N} \Theta_i$*, let* p *be a path connecting the root of the game* (Γ, R) *with one of its outcomes. Let* r *be a decision node reached by the path such that: (i) the individual* i *assigned to* r *is vetoed by some predecessor* j *along* p*; and (ii) individual* j *is also vetoed by some predecessor along* p *(therefore,* $i \geq 3$*). Then no path starting at node* r *leads to outcome* i*.*

Proof. For i to be reached from r by another path p' (that coincides with p before r) under the conditions of Lemma 3.2 it is necessary (a) that the non-vetoed individual v along p' is i or (b) that the non-vetoed individual v along p' is the one that has vetoed i. By (i), i has already been vetoed before r is reached, so (a) cannot hold. By (ii), the individual j who has vetoed i has also been vetoed before r is reached, for which reason (b) cannot hold. □

Proposition 3.1. *If* $n \geq 3$*, then the veto mechanism implements the social choice function* f_w *in subgame perfect equilibria.*

Proof. Since Lemma 3.1 proves the result when $n = 3$, let $n \geq 4$ and assume the result is true for all $n' \in \{3, \ldots, n-1\}$.

For a given $R \in \Pi_{i \in N} \Theta_i$, consider the game (Γ, R) induced by the veto mechanism when the deserving winner is a given $w \in \{1, \ldots, n\}$ and the preferences of the individuals are the admissible preferences with deserving winner w. It must be shown that w is the only subgame perfect equilibrium of the game.

Case 1: $w = 1$. Let p be the path that results when, for all $k \in \{1, \ldots, n-1\}$, k vetoes $k+1$. Observe that, along path p: (i) no one vetoes w and, hence, $v = w$; and (ii) the first individual not vetoing himself is 1, that is, $z = 1$. Since $w = 1$, the outcome is given by the choice of individual n between $w = 1$ and the individual 2 vetoed by 1. Given that $n \geq 4$ prefers w to 2, n chooses w. To sum up, path p leads to outcome w.

By Lemma 3.2, no individual $k \geq 3$ has an incentive to deviate from p, because by deviating from p no such individual can obtain the only outcome more preferred than w: outcome k. Obviously, being 1 the deserving winner w, 1 has neither an incentive to deviate. Finally, as 2 has been vetoed by 1, the only reason why 2 could deviate from p is that $v = z = 1$, in which case individual n chooses from 1 and 2 (the individual vetoed by 1). Yet, being 1 the deserving winner, $n \neq 2$ prefers 1 to 2, on account of which no deviation by 2 from p makes it possible for 2 to obtain a better outcome than w.

The final conclusion is that no individual has an incentive to deviate from p. This makes p lead to a subgame perfect equilibrium outcome (the deserving winner) and no other subgame perfect equilibrium outcome can be different from

w.

Case 2: $w = n$. Let now the path p be the one that results when, for all $k \in \{1, \ldots, n-2\}$, k vetoes $k+1$ and $n-1$ vetoes 1. In this case, z (the first individual not vetoing himself along p) is 1, whereas the non-vetoed individual is the deserving winner w. Given that $v \neq z$, the outcome that corresponds to p is $v = w$.

As in case 1, by Lemma 3.2, no individual $k \geq 3$ has an incentive to deviate from p. As regards $k = 2$, the only reason that could justify a deviation from p by 2 is that some deviation leads to outcome 2, the only outcome more preferred by 2 to w. But even if there existed a subgame perfect equilibrium path starting at 2's node leading to outcome 2, this would not constitute a subgame perfect equilibrium of the whole game because, by vetoing himself, 1 can force the occurrence of outcome w. In fact, when 1 vetoes 1, the subgame that starts at 2's node is the game induced by the veto mechanism when the deserving winner is a given $w \in \{2, \ldots, n\}$ and the preferences of the $n-1$ individuals are the admissible preferences with deserving winner w and, by the induction hypothesis, the only subgame perfect equilibrium outcome of this game is w.

Finally, it rests to be shown that no subgame perfect equilibrium leads to outcome 1 when 1 vetoes $x \neq 1$, for in that case p would lead to the subgame perfect equilibrium outcome w and no other such outcome would exist.

To this end, notice that, as just shown, there is no subgame perfect equilibrium in which 2 is the outcome. Consequently, 2's best prospect is to make w the winner. The claim is that 2 can ensure that w is the winner by vetoing 1 whenever 1 vetoes $x \neq 1$. Observe that the game obtained when, for all $x \neq 1$, 2 chooses to veto 1 is like the game in which individual 1 has been removed and 2 vetoes x. The induction hypothesis ensures that the only subgame perfect equilibrium of this game is the deserving winner w. In view of this, individual 1 cannot do better than trying to get w and this is ensured by path p.

Case 3: $1 < w < n$. Let now the path p be the one that results when, for all $k \in \{1, \ldots, n-1\} \backslash \{w-1, w\}$, k vetoes $k+1$, $w-1$ vetoes $w+1$ and w vetoes 1. Similarly to Case 2, z (the first individual not vetoing himself along p) is 1, while the non-vetoed individual v is the deserving winner w. Given that $z \neq v$, the outcome that p results in is $v = w$.

As in all previous cases, by Lemma 3.2, no individual $k \geq 3$ has an incentive to deviate from p if $k = w$, this follows from the fact that w is his most preferred individual. To complete the proof, first consider individual $k = 2$. If $2 \neq w$, then the only outcome 2 prefers more than w is when 2 becomes the winner. But 2 has been vetoed by 1, so the only possibility for 2 to be chosen as a winner is when 1 is not vetoed by anyone along the path, so that $z = v = 1$, and n picks between 1 and 2. If it happens that n prefers 2 more than 1, n could choose 2. But this outcome would not constitute a subgame perfect equilibrium of the whole game because, by vetoing himself, 1 can force the occurrence of outcome w.

If $2 = w$, then $w + 1$ has been vetoed by $w - 1$, namely 1. If 2 vetoes 1, then notice that this is the same game as, when 1 is removed and 2 vetoes the individual that 1 has vetoed, namely $w+1$ (subsequent individual in the ordering after $w = 2$). Now we are in Case 1, when w is the first individual in the ordering, vetoing the subsequent individual. As it has been previously proved, this path results in the outcome w.

Finally, what is left to show is that no subgame perfect equilibrium leads to outcome 1 when 1 vetoes $x \neq 1$. In that case p would lead to the subgame perfect equilibrium outcome w and no other such outcome would exist.

As it has just been shown, there is no subgame perfect equilibrium in which 2 is the outcome, unless 2 is the deserving winner himself. Consequently, if $2 \neq w$, the best option for 2 is to make w the winner. As in the previous claim of Case 2, individual 2 can ensure that w is the winner by vetoing 1 whenever 1 vetoes $x \neq 1$. The game obtained when, for all $x \neq 1$, 2 chooses to veto 1 is like the game in which individual 1 has been removed and 2 vetoes x. By the induction hypothesis, the only subgame perfect equilibrium of this game is the deserving winner w. Consequently, individual 1 cannot do better than trying to get the outcome w and this is ensured by path p. $\square$

3.6 Conclusion

In this chapter we have analyzed the problem of choosing a winner among the individuals when the identity of the deserving winner is common knowledge. We have proved the proposed veto mechanism implements the socially desirable outcome (that the deserving winner wins) in subgame perfect equilibria.

One contribution is that the veto mechanism conceptualizes a counterpart to Amorós [8] mechanism: in his mechanism the individuals choose, whereas in the veto mechanism the individuals reject. In practice, it seems easier to reject a bad (or worse) option, than to pick the best option (or sufficiently good option).

The other contribution is that the veto mechanism works when there are at least three individuals, improving upon Amorós [8], which requires a minimum of four individuals.

4

Does avoiding bad voting rules lead to good ones?

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Abstract

Distance rationalization of voting rules is based on the minimization of the distance to some plausible criterion, such as unanimity or the Condorcet criterion. We propose a new alternative: the optimization of the distance to undesirable voting rules, namely, the dictatorial voting rules. Applying a plausible metric between social choice functions, we obtain two results: (i) the plurality rule minimizes the sum of the distances to the dictatorial rules and can be regarded in some sense as a compromise lying between all dictatorial rules; (ii) the reverse-plurality rule maximizes the distance to the closest dictator.

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4.1 Introduction

When a group of individuals is collectively trying to choose among several alternatives, the most common way to reach a collective decision is to vote. In this case, the preference aggregation procedure is called a voting rule. The voting rule or a social choice function takes as an input the preferences of all individuals, usually in the form of a ranking (a ballot) and outputs the collective outcome. The voting rule is supposed to output the best outcome taking the preferences of all individuals involved into the voting process. The expression *social choice* itself suggests that what is primarily important is to choose a procedure that will select the alternative that will reflect the "will of the people". There exists a multitude of different preference aggregation procedures or voting rules. A natural question is then which voting rule is the *best*. One can answer this question by imposing certain desirable properties or axioms that the voting rule should satisfy - applying a *normative* (*axiomatic*) approach.

However, as proved by Arrow [11] in his famous *General Possibility Theorem* or *Impossibility Theorem*, there is no voting method that fairly chooses a winner that involves three or more alternatives while satisfying certain desirable properties, which are unrestricted domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship. Stated differently, the only voting method that satisfies the desirable properties is *dictatorship*. As it may seem obvious, having a dictator violates the essence of the social choice, that is, the "will of the people". Another important property that any reasonable voting method should satisfy is that of being immune to manipulation by any voter. However, we get another negative result also in this respect. It was proved independently by Gibbard [33] and Satterthwaite [73] known as the *Gibbard-Satterthwaite theorem* that the only voting method for at least three alternatives that is non-manipulable or strategy-proof is *dictatorship*. Therefore, by considering which voting method to use, we are left with a dilemma between manipulability and dictatorship, as given by the two cornerstone theorems of the social choice theory.

Due to the negativity results of the normative approach, since then many authors have tried to circumvent those impossibility results by applying different ways of evaluating (characterising) voting rules. One such approach is a *metric* or *distance-based* approach which consists in viewing a voting rule as an objective function. Almost all voting rules can be characterized in terms of a goal state and a metric used in measuring the distance between the preference ranking and the goal state (Baigent [14]; Meskanen and Nurmi [56], Eckert and Klamer [28]; Elkind et al. [29]).

The concept of distance rationalization of voting rules entails explaining voting rules in terms of consensus and distances. Given a notion of consensus and a metric (distance function), a voting rule that is rationalizable chooses the alternative that is closest to being a consensus winner. The seminal work was initiated by Farkas and Nitzan [30], who derived the Borda count as the solution of an optimization problem on the set of social choice functions by minimizing the distance from the unanimity principle. Taking other metrics, Nitzan [63] obtained the plurality rule among other rules. The approach of minimizing the distance from a set of profiles with a clear winner such as the unanimous winner, the majority winner, or the Condorcet winner has been developed further by Lerer and Nitzan [48], Elkind et al. [29], Andjiga et al. [9], Mahajne et al. [50], and Zwicker [86] among others.

All previous works have dealt with the distance rationalizability based on the minimization of the distance to some plausible criterion, such as unanimity or the Condorcet criterion. In contrast, we propose a new alternative, namely, the optimization of the distance to the undesirable dictatorial voting rules, motivated by the classical impossibility results of Arrow [11] and Gibbard–Satterthwaite ([33]; [73]), roughly stating that every voting rule satisfying a subset of reasonable properties leads to dictatorship. In particular, we ask the following question: will we obtain a “good” voting rule if we want to get as close as possible to all dictatorial voting rules or if we get away from the closest dictatorial rule? We investigate this question by employing a quite simple and natural distance function between social choice functions.

By getting as close as possible to all dictatorial rules, we are searching for the rules that minimize the sum of the distances to the dictatorial rules, which is identical to the set of rules choosing a top alternative of a voter in as many cases as possible. We call these rules balanced since they represent a kind of compromise between all dictatorial rules. Using this terminology, we find that the plurality rule and the balanced rule are the same. Therefore, we consider this as a positive result since the plurality rule is the most frequently applied one.

By getting away from the closest dictatorial rule, we are searching for the rules that maximize the distance to the closest dictatorial rule. We refer to these rules as the least dictatorial rules since in some sense they are the furthest from dictatorship, which emerges if the collective outcome is determined by a dictatorial rule. In particular, any other rule in the space of voting rules lies closer to at least one of the dictatorial rules than any of the least dictatorial rules.

We find that our goal results in a quite unpleasant rule, which we call the reverse-plurality rule, violating properties like unanimity or monotonicity. Therefore, we consider our second main result as a negative one in the sense that we obtain an undesirable rule. However, based on our result, from a philosophical point of view, one could argue that eliminating the 'dictatorial ingredient' from voting rules completely should not be our goal.

Furthermore, we investigate the relationship between minimizing (maximizing) the sum of distances and minimizing (maximizing) the minimum of distances in our objective function.

The plan of the chapter is as follows. Section 4.2 introduces our framework, Section 4.3 describes our main results, and, finally, Section 4.4 provides conclusion and mentions possible future research directions.

4.2 Framework

Let $A = \{1, \ldots, m\}$ be the set of alternatives and $N = \{1, \ldots, n\}$ be the set of voters. We shall denote by $\mathcal{P}$ the set of all linear orderings (irreflexive, transitive and total binary relations) on A and by $\mathcal{P}^n$ the set of all preference profiles. If $\succ \in \mathcal{P}^n$ and $i \in N$, then $\succ_i$ is the preference ordering of voter i over A.

Definition 4.1. A mapping $f : \mathcal{P}^n \to A$ that selects the winning alternative is called a *social choice function*, henceforth, SCF.

Note that our definition of an SCF does not allow for possible ties, in which case a fixed tie-breaking rule will be employed. A tie-breaking rule $\tau : \mathcal{P}^n \to \mathcal{P}$ maps preference profiles to linear orderings on A, which will be only employed when a formula does not determine a unique winner. If there are more alternatives chosen by a formula 'almost' specifying an SCF, then the highest ranked alternative is selected, based on the given tie-breaking rule among tied alternatives. In particular, anonymous tie-breaking rules will play a central role in our analysis.

We will also allow for domain restrictions, since for some preference profiles we may prescribe certain outcomes, which are plausible. Let $\mathcal{S} \subseteq \mathcal{P}^n$ be a subdomain on which the outcome is already prescribed by some externally chosen principle. Then the values of a SCF have to be specified only on $\overline{\mathcal{S}}$, where $\overline{\mathcal{S}} = \mathcal{P}^n \setminus \mathcal{S}$, and therefore we only need to consider SCFs restricted to $\overline{\mathcal{S}}$. For instance, for profiles with a Condorcet winner denoted by $\mathcal{S}_c$, we may only consider Condorcet consistent SCFs; or for profiles with a majority supported alternative, denoted by $\mathcal{S}_m$, we may require that the majority winner should be chosen. We consider the following type of domain restriction.

Definition 4.2. A domain restriction $\mathcal{S} \subseteq \mathcal{P}^n$ is called *anonymous* if for any bijection $\sigma : N \to N$ we have for all $(\succ_1, \ldots, \succ_n) \in \mathcal{P}^n$ that $(\succ_1, \ldots, \succ_n) \in \mathcal{S}$ implies $(\succ_{\sigma^{-1}(1)}, \ldots, \succ_{\sigma^{-1}(n)}) \in \mathcal{S}$.

It can be verified that if $\mathcal{S}$ is anonymous, then also $\overline{\mathcal{S}}$ is anonymous. If $\mathcal{S} = \emptyset$, we have the case of an unrestricted domain. It is easy to see that $\mathcal{S}_c$ and $\mathcal{S}_m$ are anonymous. The introduction of domain restrictions results in a more general framework.

Let $\mathcal{F} = A^{\mathcal{P}^n}$ be the set of SCFs and $\mathcal{F}^{an} \subset \mathcal{F}$ be the set of anonymous voting rules. The subset of $\mathcal{F}$ consisting of the dictatorial rules will be denoted by $\mathcal{D} = \{d_1, \ldots, d_n\}$, where d_i is the dictatorial rule with voter i as the dictator. In order to define several optimization problems related to dictatorial rules we will employ the following distance function between SCFs:

$$\rho_{\mathcal{S}}(f, g) = \#\{\succ \in \overline{\mathcal{S}} \mid f(\succ) \neq g(\succ)\}, \tag{4.1}$$

where f, g are SCFs and $\rho_S(f, g)$ stands for the number of profiles on which f and g choose different alternatives within $\overline{\mathcal{S}}$. It can be checked that ρ_S specifies a metric over the set of SCFs restricted to $\overline{\mathcal{S}}$. If $S = \emptyset$, we simply write $\rho(f, g)$. Since in case of SCFs we only care about the chosen outcome (and not about a social ranking), and we do not assume any kind of structure on the set of alternatives A, it appears natural that we count the number of profiles on which f and g differ. We discuss some possible extensions in Section 5.4.

We specify the set of least dictatorial rules by those ones which are the furthest away from the closest dictatorial rule, which means that we are maximizing the minimum of the distances to the dictators.

Definition 4.3. We define the set of *least dictatorial rules* for domain restriction $\mathcal{S}$ by

$$\mathcal{F}_{ld}(\mathcal{S}) = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \min_{i \in N} \rho_{\mathcal{S}}(f, d_i) \geq \min_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

in general and by

$$\mathcal{F}^{an}_{ld}(\mathcal{S}) = \left\{ f \in \mathcal{F}^{an} \mid \forall f' \in \mathcal{F}^{an} : \min_{i \in N} \rho_{\mathcal{S}}(f, d_i) \geq \min_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

over the set of anonymous voting rules.

When defining least dictatorial rules based on the distance function ρ_S, we could have taken the average distance, or equivalently the sum of the distances from the dictators. However, we feel that if we would like to be 'least dictatorial', we should be more concerned about the closest dictatorial rule. Nevertheless, we will consider the other possibility at the end of this section and for anonymous SCFs it will turn out that we will obtain the same rules.

An alternative approach to getting as far away from the closest dictator as possible would be getting as close as possible to all dictators at the same time, which could be considered as a kind of neutral or balanced solution with respect to all dictators and, in this sense, as a kind of desirable solution. For simplicity reasons, we will minimize the sum of the distances to the n dictators.

Definition 4.4. We define the set of *balanced rules* for domain restriction $\mathcal{S}$ by

$$\mathcal{F}_b(\mathcal{S}) = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

in general and by

$$\mathcal{F}_b^{an}(\mathcal{S}) = \left\{ f \in \mathcal{F}^{an} \mid \forall f' \in \mathcal{F}^{an} : \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

over the set of anonymous voting rules.

An equivalent formulation of balanced rules, stating that these rules maximize the number of cases in which a top alternative of a voter is chosen, is derived at the beginning of Section 3.

Instead of looking for the rules which are the furthest away from the closest dictatorial rule we could consider the rules which are the closest ones to the furthest dictatorial rule, which means that we are minimizing the maximum of the distances to the dictators.

Definition 4.5. We define the set of *minmax* rules for domain restriction $\mathcal{S}$ by

$$\mathcal{F}_{\min\max}(\mathcal{S}) = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \max_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \max_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

in general and by

$$\mathcal{F}^{an}_{\min\max}(\mathcal{S}) = \left\{ f \in \mathcal{F}^{an} \mid \forall f' \in \mathcal{F}^{an} : \max_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \max_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

over the set of anonymous voting rules.

In relation to the definition of balanced rules, we obtain the reverse-balanced rules by getting furthest from all dictators at the same time. In particular, we maximize the sum of the distances to the n dictators.

Definition 4.6. We define the set of *reverse-balanced rules* for domain restriction $\mathcal{S}$ by

$$\mathcal{F}_{rb}(\mathcal{S}) = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \geq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

in general an by

$$\mathcal{F}^{an}_{rb}(\mathcal{S}) = \left\{ f \in \mathcal{F}^{an} \mid \forall f' \in \mathcal{F}^{an} : \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \geq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\}$$

over the set of anonymous voting rules.

Clearly, there are an infinite number of possibilities to define a set of voting rules based on the distances from individual dictators (for instance, any generalized mean of the individual distances could have been considered). However, we believe that we have chosen the simplest and most natural ones as far as the distance of two alternatives from a set which has no internal structure and the aggregation of individual distances are concerned.

4.3 Results

First, we start by providing a different interpretation of balanced rules. When defining $\mathcal{F}_{ld}(\mathcal{S})$, we are looking for SCFs which are in some sense the least dictatorial ones. From another perspective, a SCF that chooses top alternatives of voters in as many cases as possible could result in a desirable SCF. Having this goal in mind, the measure

$$\mu_{\mathcal{S}}(f, \mathcal{D}) = \sum_{\succ \in \overline{\mathcal{S}}} \# \{i \in N \mid f(\succ) = d_i(\succ)\},$$

appears as a natural candidate, which we call the measure of conformity.

Introducing the notation $\mu_{\mathcal{S}}(f, g) = \sum_{\succ \in \overline{\mathcal{S}}} \mathbf{1}_{f(\succ)=g(\succ)}$, where $\mathbf{1}_{f(\succ)=g(\succ)}$ indicates whether the two chosen alternatives equal, we can obtain the following relationship between $\mu_{\mathcal{S}}$ and $\rho_{\mathcal{S}}$:

$$\mu_{\mathcal{S}}(f, \mathcal{D}) = \sum_{\succ \in \overline{\mathcal{S}}} \sum_{i \in N} \mathbf{1}_{f(\succ)=d_i(\succ)} = \sum_{i \in N} \mu_{\mathcal{S}}(f, d_i) = n \cdot \# \overline{\mathcal{S}} - \sum_{i \in N} \rho_S(f, d_i).$$

Therefore,

$$\{f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \mu_{\mathcal{S}}(f, \mathcal{D}) \geq \mu_{\mathcal{S}}(f', \mathcal{D})\} = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \right\},$$

which means that the set of rules which maximize the number of cases in which a top alternative of a voter is chosen is identical to the set of balanced rules.

The following rule will play a special role:

Definition 4.7. The plurality rule $\tilde{f}_\tau$, where τ is an arbitrary tie-breaking rule, is defined in the following way: If there is a unique alternative, ranked first most often, then that alternative is the chosen one. If not, disregard those alternatives that are not ranked first most often, and select the chosen alternative based on the given tie-breaking rule.

Hence, we have defined a plurality rule with an associated tie-breaking rule. The next proposition shows how the plurality rule relates to the set of balanced rules and the set of minmax rules.

Proposition 4.1. *Assume that $\mathcal{S}$ is an anonymous subdomain of $\mathcal{P}^n$. Then $\tilde{f}_\tau \in \mathcal{F}_b(\mathcal{S})$ and if τ is anonymous, then $\tilde{f}_\tau \in \mathcal{F}_{\min\max}(\mathcal{S})$ is also true. For any $f \in \mathcal{F}_b^{an}(\mathcal{S})$ and any $g \in \mathcal{F}_{\min\max}^{an}(\mathcal{S})$, there exist tie-breaking rules τ and φ, respectively, such that $f = \tilde{f}_\tau$ and $g = \tilde{f}_\varphi$ on $\overline{\mathcal{S}}$.*

Proof. By the definition of $\tilde{f}_\tau$ we have

$$\forall \succ \in \mathcal{P}^n : \# \left\{ i \in N \mid \tilde{f}_\tau(\succ) = d_i(\succ) \right\} \geq \# \left\{ i \in N \mid f(\succ) = d_i(\succ) \right\} \tag{4.2}$$

for any $f \in \mathcal{F}$. Now summing (4.2) over $\overline{\mathcal{S}}$, we get

$$\mu_S(\tilde{f}_\tau, \mathcal{D}) \geq \mu_S(f, \mathcal{D}), \tag{4.3}$$

from which it follows that $\tilde{f}_\tau \in \mathcal{F}_b$. From now on we assume that τ is anonymous. Note that (4.3) is equivalent with

$$\sum_{i \in N} \rho_{\mathcal{S}}(\tilde{f}_\tau, d_i) \leq \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i),$$

and therefore for any $j \in N$

$$\rho_{\mathcal{S}}(\tilde{f}_\tau, d_j) = \frac{1}{n} \sum_{i \in N} \rho_{\mathcal{S}}(\tilde{f}_\tau, d_i) \leq \frac{1}{n} \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \leq \max_{i \in N} \rho_{\mathcal{S}}(f, d_i) \tag{4.4}$$

since τ and $\mathcal{S}$ are anonymous and the average is smaller than the maximum; meaning that $\tilde{f}_\tau \in \mathcal{F}_{\min\max}(\mathcal{S})$.

For the second statement observe that if f selects for at least one profile in $\overline{\mathcal{S}}$ an alternative that is not the most times on the top, then the inequality in (4.3) will be strict, and therefore also the inequality in (4.4) will be strict. The tie-breaking rule τ can be selected in line with f. $\square$

Since the set of anonymous plurality rules equals both $\mathcal{F}_b^{an}(\mathcal{S})$ and $\mathcal{F}_{\min\max}^{an}(\mathcal{S})$ by Proposition 4.1 we obtain the following corollary.

Corollary 4.1. $\mathcal{F}_b^{an}(\mathcal{S}) = \mathcal{F}_{\min\max}^{an}(\mathcal{S})$.

The following remark clarifies the relationship between $\mathcal{F}_{\min\max}(\mathcal{S})$ and $\mathcal{F}_b(\mathcal{S})$.

Remark 4.1. $\mathcal{F}_{\min\max}(\mathcal{S}) \subseteq \mathcal{F}_b(\mathcal{S})$.

Proof. By Proposition 4.1 we know that $\tilde{f}_\tau \in \mathcal{F}_b(\mathcal{S}) \cap \mathcal{F}_{\min\max}(\mathcal{S})$ if τ is anonymous. Assume that $f' \in \mathcal{F}_{\min\max}(\mathcal{S})$. Then for any $f \in \mathcal{F}(\mathcal{S})$ and any $j \in N$ we have

$$\rho_{\mathcal{S}}(\tilde{f}_\tau, d_j) = \max_{i \in N} \rho_{\mathcal{S}}(\tilde{f}_\tau, d_i) = \max_{i \in N} \rho_{\mathcal{S}}(f', d_i) \leq \max_{i \in N} \rho_{\mathcal{S}}(f, d_i), \tag{4.5}$$

where the first equality follows from the anonymity of τ. By $\tilde{f}_\tau \in \mathcal{F}_b(\mathcal{S})$

$$n \cdot \rho_{\mathcal{S}}(\tilde{f}_\tau, d_j) = \sum_{i \in N} \rho_{\mathcal{S}}(\tilde{f}_\tau, d_i) \leq \sum_{i \in N} \rho_{\mathcal{S}}(f', d_i) \tag{4.6}$$

for any $j \in N$. Combining (4.5) and (4.6), we get

$$\rho_{\mathcal{S}}(\tilde{f}_\tau, d_i) = \rho_{\mathcal{S}}(f', d_j)$$

for any $i, j \in N$, which in turn implies $f' \in \mathcal{F}_b(\mathcal{S})$. $\square$

The next remark points out that we have a proper inclusion in Remark 4.1.

Remark 4.2. $\mathcal{F}_{\min\max}(\mathcal{S}) \neq \mathcal{F}_b(\mathcal{S})$.

Proof. Consider the plurality rule, which breaks ties by selecting the most favored alternative of voter 1 from the set of tied alternatives. It can be verified that this rule minimizes the sum of the distances to the dictatorial rules, while it does not minimize the maximum distance to the dictatorial rules. In particular, replacing an anonymous plurality rule with a non-anonymous one does not change the sum of the distances, but may change the maximum of the distances. □

Turning to the reverse-balanced rules, the following rules play a central role:

Definition 4.8. The *reverse-plurality rule* f_τ^*, where τ is an arbitrary tie-breaking rule, is defined in the following way: If there is a single alternative, ranked first least often, then that alternative is the chosen one. If not, disregard those alternatives that are not ranked first least often, and select the chosen alternative based on the given tie-breaking rule.

Clearly, the above specified rule can also be just taken on a subset of profiles $\overline{\mathcal{S}}$ in case of a domain restriction $\mathcal{S}$ and any other known rule can be employed on $\mathcal{S}$. It is worth noting that the reverse-plurality rule differs from the anti-plurality rule known in the literature. Though both select the alternatives receiving the fewest number of votes, the former one requests the voters to vote for their most preferred alternative, while the latter one requires that they vote for their least preferred one.

The next proposition shows how the reverse-plurality rules relate to the set of reverse-balanced rules and the set of least dictatorial rules.

Proposition 4.2. *Assume that $\mathcal{S}$ is an anonymous subdomain of $\mathcal{P}^n$. Then $f_\tau^* \in \mathcal{F}_{rb}(\mathcal{S})$ and if τ is anonymous, then $f_\tau^* \in \mathcal{F}_{ld}(\mathcal{S})$ is also true. For any anonymous*

$f \in \mathcal{F}_{ld}(\mathcal{S})$ and any anonymous $g \in \mathcal{F}_{rb}(\mathcal{S})$, there exist tie-breaking rules τ and φ, respectively, such that $f = f^*_\tau$ and $g = f^*_\varphi$ on $\overline{\mathcal{S}}$.

Proof. First, observe that

$$\begin{aligned} \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) &= \sum_{i \in N} \# \left\{ \succ \in \overline{\mathcal{S}} \mid f(\succ) \neq d_i(\succ) \right\} \\ &= \# \left\{ (i, \succ) \in N \times \overline{\mathcal{S}} \mid f(\succ) \neq d_i(\succ) \right\} \\ &= \sum_{\succ \in \overline{\mathcal{S}}} \# \left\{ i \in N \mid f(\succ) \neq d_i(\succ) \right\} \end{aligned} \tag{4.7}$$

for any SCF f.

By the definition of f^*_τ for any f we have

$$\forall \succ \in \mathcal{P}^n : \# \left\{ i \in N \mid f^*_\tau(\succ) \neq d_i(\succ) \right\} \geq \# \left\{ i \in N \mid f(\succ) \neq d_i(\succ) \right\}. \tag{4.8}$$

Now taking the sums over $\overline{\mathcal{S}}$ on both the left hand side and the right hand side of equation (4.8) and then combining it with (4.7), we get

$$\sum_{i \in N} \rho_{\mathcal{S}}(f^*_\tau, d_i) \geq \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i), \tag{4.9}$$

from which it follows that $f^*_\tau \in \mathcal{F}_{rb}(\mathcal{S})$. From now on we assume that τ is anonymous. Furthermore, (4.9) implies for any $j \in N$ that

$$\rho_{\mathcal{S}}(f^*_\tau, d_j) = \frac{1}{n} \sum_{i \in N} \rho_{\mathcal{S}}(f^*_\tau, d_i) \geq \frac{1}{n} \sum_{i \in N} \rho_{\mathcal{S}}(f, d_i) \geq \min_{i \in N} \rho_{\mathcal{S}}(f, d_i) \tag{4.10}$$

since f^*_τ and $\mathcal{S}$ are anonymous and the average is larger than the minimum; meaning that $f^*_\tau \in \mathcal{F}_{ld}(\mathcal{S})$.

For the second statement observe that if f selects for at least one profile in $\overline{\mathcal{S}}$ an alternative that is not the fewest times on the top, then the inequality in (4.9), and therefore also the inequality in (4.10) will be strict. Finally, an anonymous tie-breaking rule can be chosen in line with f. $\square$

Since the set of anonymous reverse-plurality rules equals both $\mathcal{F}_{ld}^{an}(\mathcal{S})$ and $\mathcal{F}_{rb}^{an}(\mathcal{S})$ by Proposition 4.2 we obtain a similar result to Corollary 4.1.

Corollary 4.2. $\mathcal{F}_{ld}^{an}(\mathcal{S}) = \mathcal{F}_{rb}^{an}(\mathcal{S})$.

The following two remarks can be established in an analogous way to Remarks 4.1 and 4.2.

Remark 4.3. $\mathcal{F}_{ld}(\mathcal{S}) \subseteq \mathcal{F}_{rb}(\mathcal{S})$.

Remark 4.4. $\mathcal{F}_{ld}(\mathcal{S}) \neq \mathcal{F}_{rb}(\mathcal{S})$.

Though f_τ^* performs well according to our specification of a least dictatorial rule, as it can be easily verified, over the universal domain it can select a Pareto dominated alternative, never selects a unanimous winner, and violates monotonicity among many other desirable properties. Therefore, we have introduced anonymous domain restrictions so that, for instance, on profiles with a unanimous winner, the unanimous winner should be selected, and we are searching for the least dictatorial rules only over the set of profiles which do not have a unanimous winner. However, Proposition 4.2 shows that even if we fix our choices over an anonymous subset $\mathcal{S}$ of profiles, f_τ^* has to be employed over $\overline{\mathcal{S}}$, if we would like to be anonymous and least dictatorial according to our definition.

4.4 Conclusions

In this chapter, we aimed to get away from the undesirable dictatorial rule, searching for the voting rule balancing between all dictatorial rules and for the one getting away from the closest dictatorial rule. To this end, we defined a simple and natural metric between social choice functions, thus introducing a new alternative to the distance rationalizability literature. Instead of minimizing the distance to some plausible criterion, we are optimizing the distance to the "bad" dictatorial rule.

First, we searched for the voting rules that minimize the distance to all dictatorial rules (or equivalently, that may allow the voters to feel themselves to be a dictator in as many cases as possible), which we call the balanced ones. We find that the plurality rule and the balanced rule are the same. This implies that the plurality rule is a kind of compromise between individual dictators.

Second, we call the furthest from the closest dictatorial rule the "least" dictatorial one. We obtain that the "reverse-plurality" voting rule is the "least" dictatorial one according to our specifications. It still violates some desirable properties, such as unanimity and monotonicity, and can select a Pareto dominated alternative.

To get partially rid of the unwanted behavior of the least dictatorial rules, we employ domain restrictions such as only investigating the set of those profiles on which there is no 'consensus winner' (e.g. no unanimous winner, no majoritarian winner, or no Condorcet winner).

Third, we also investigate the relationship between minimizing (maximizing) the sum of the distances and minimizing (maximizing) the minimum of the distances in our objective function. Our results show that being away from a "bad" rule is not necessarily a sensible property, as we end up with an undesirable voting rule. Moreover, since we consider a "collective dictatorship" by defining the sum, the minimum and the maximum of the distances from the dictatorial rules across all individuals, it is not obvious anymore whether it is straightforward to say the dictatorial voting rules are "bad".

Too see this, consider a profile in which there is a unanimously preferred alternative. We may expect that any desirable voting rule should select that alternative. However, since such a choice makes all individuals dictators, in terms of the contribution to the distances from the dictatorial voting rules defined in the chapter, if we try to be as far as possible from the bad dictatorial rules, then the unanimously-preferred alternative should not be chosen. Hence, creating collectively as many dictators as possible is not necessarily a "bad" thing.

Finally, we state the directions for future research. In Section 4.3 we consider a metric, which does not take the distribution of preferences in a profile into consideration. A possible extension of the metric given by (4.1), which can be then considered as the special uniform case, may lead to the metric of the functional form specified below:

$$\rho_{\mathcal{S},w}(f,g) = \sum_{\succ \in \overline{\mathcal{S}}} w(\succ)\mathbf{1}_{f(\succ)\neq g(\succ)}, \tag{4.11}$$

where the weight function w could take into account the homogeneity of profile $\succ$, for instance, in case of identical preferences it would be the most disturbing that the alternatives chosen by f and g differ (heavy weight), while in case of 'very heterogeneous' profiles this might seem more natural (light weight), and $\mathbf{1}_{f(\succ)\neq g(\succ)}$ indicates whether the two chosen alternatives differ. Of course, other metrics over the set of SCFs are possible, e.g. $\rho(f,g) = \sum_{\succ \in \mathcal{P}^n} \sum_{i=1}^{n} |bs(\succ_i, f(\succ)) - bs(\succ_i, g(\succ))|$, where the Borda score is denoted by bs.

We could get a more refined picture if we consider social choice rules instead of SCFs, that is, we care about the whole social ranking and not only about the socially best alternative. We plan to address the investigation of metrics given by (4.11) and the case of social choice rules in future research.

5

Dictatorship versus manipulability

This chapter has been written in collaboration with Dezső Bednay and Attila Tasnádi, Department of Mathematics, Corvinus University of Budapest

Abstract

The Gibbard–Satterthwaite theorem roughly states that we have to accept dictatorship or manipulability in case of at least three alternatives. A large strand of the literature estimates the degree of manipulability of social choice functions (e.g. Aleskerov and Kurbanov [3]; Favardin et al. [31], and Aleskerov et al. [6]), most of them employing the Nitzan-Kelly index of manipulability. We take a different approach and introduce a non-dictatorship index based on our recent work (Bednay et al. [13]), where we have analysed social choice functions based on their distances to the dictatorial rules. By employing computer simulations, we investigate the relationship between the manipulability and non-dictatorship indices of some prominent social choice functions, putting them into a common framework.

The classic result of Gibbard [33] and Satterthwaite [73] theorem states that for at least three alternatives every universal and resolute social choice function is either manipulable or dictatorial. There is a large literature on how to escape from the negative implications of the Gibbard-Satterthwaite theorem by restricting the set of possible preference profiles, most of them related to single-picked preferences and their generalizations (e.g. Balck [18]; Moulin [60]; Barberá et al. [12]; and Nehring and Puppe [61], [62], just to name a few). Since the normative approach does not give us the ultimate answer for choosing between social choice functions, another strand of the literature tries to estimate to which extent different voting rules are susceptible to manipulation and to compare the common voting rules according to their 'degree of manipulability'.

There is no universally accepted way to measure the degree of manipulability, but one of the most common approaches is to consider the ratio of preference profiles where manipulation is possible to the total number of profiles, which is called the Nitzan-Kelly's index (NKI, hereinafter) of manipulability, since it was first introduced in Nitzan [64] and Kelly [41] (for other indices of manipulability, see Smith [79]). A voting rule is thought to be less manipulable if it is manipulable at fewer preference profiles, or equivalently, if it has a smaller NKI (clearly, the dictatorial voting rule is the least manipulable one). There are a number of studies investigating voting rules under this approach. Kelly [41] found the minimal number of manipulable profiles for social choice rules which are unanimous and non-dictatorial. This research direction is continued in Fristrup and Keiding [32] and a series of studies in Maus et al. [52, 53, 54, 55].

Kelly [42] compares the manipulability of the Borda rule with the manipulability of different classes of social choice procedures by developing computational results. Aleskerov and Kurbanov [3] continue this line of research. The authors study the degree of manipulability of several social choice rules via computational experiments, considering the NKI and in addition introducing some new indices, which are further elaborated in Aleskerov et al. [4], [5], [6]. Peters et al. [67] study both theoretically and using simulations the manipulability of approval voting rule and a family of k-approval rules.

In this chapter we follow a different route and formulate indices in relation to the dictatorial voting rule, thus picking dictatorship as a reference point instead of manipulability, when looking at the two incompatible properties appearing in the Gibbard-Satterthwaite theorem. In Bednay, Moskalenko and Tasnádi [13] we have derived the plurality rule as the most balanced one in the sense that it minimizes the sum of the distances to all dictatorial rules, and we have obtained the reverse-plurality rule by maximizing the distance to the closest dictatorial rule. Based on this approach we introduce the non-dictatorship index (NDI). When employing manipulability indices on the set of commonly used social choice functions, the literature strives for the rules with the lowest manipulability index by assuming that a relatively less manipulable rule is deemed to be more desirable. In an analogous way, we are looking for the social choice function with the highest NDI, i.e., the social choice function with low degree of dictatorship.

Our research is also motivated by the fact that regarding manipulability and non-dictatorship there is a kind of weak agreement on the 'most extreme' social choice functions. In particular, the reverse-plurality rule, which is the most extreme social choice function in the sense that it lies the furthest away from the closest dictatorial rule, is also group manipulable at every preference profile. Moreover, the reverse-dictatorial social choice function, which always chooses the worst alternative of a fixed voter, is individually manipulable at each profile. Trivially, any dictatorial voting rule is non-manipulable at any preference profile.

The aim of this chapter is to investigate the relationship between NDIs and NKIs and to put them into a common framework. By employing computer simulations, we estimate the NDIs of some well-known social choice functions (some scoring rules and Condorcet consistent rules). We calculate NDIs for 3, 4 and 5 alternatives and up to 100 voters by generating 1000 random preference profiles, where each profile is selected with the same probability, i.e. we assume an impartial culture. We find that among the prominent social choice functions the plurality rule has the smallest NDI, the Borda count, the Black rule and the Copeland method follow with approximately identical NDIs, while k-approval voting (for $k = 2$ or $k = 3$) has the highest NDI among the most common social choice functions. In measuring manipulability we restrict ourselves to NKI, which measures the strategy-proofness by counting the number of profiles on which a social choice function is manipulable. While for determining the values for NDI we have written our own program, for determining NKI we have downloaded the results available at Aleskerov et al. [7],* where we employ the alphabetical tie-breaking rule. Thereafter, we compare our results with Aleskerov et al. [5].

We find that, when unifying the NDIs and NKIs for our social choice functions under study, both indices move in the opposite directions, which is a plausible sign for our non-dictatorship index. Next we look at both NDI and NKI of the social choice functions. From our findings we would like to highlight that basically the plurality rule performs the worst in terms of both its NDI and NKI with exception of the case of 4 alternatives for which the 3-approval voting rule has even higher NKI. However, there is no such uniquely best performing rule based on the two indices.

The structure of the chapter is as follows. Section 5.1 introduces the basic notations and the indices to measure the degree of dictatorship of social choice functions. Section 5.2 presents the social choice rules under study. Section 5.3 explains the computational scheme, presents and discusses the results. Finally, Section 5.4 concludes.

5.1 The framework

Let $A = \{1, \ldots, m\}$ be the set of alternatives, where $m \geq 2$, and $N = \{1, \ldots, n\}$ be the set of voters. We shall denote by $\mathcal{P}$ the set of all linear orderings (irreflexive, transitive and total binary relations) on A and by $\mathcal{P}^n$ the set of all preference profiles. If $\succ \in \mathcal{P}^n$ and $i \in N$, then $\succ_i$ is the preference ordering of voter i over A.

Definition 5.1. A mapping $f : \mathcal{P}^n \to A$ that selects the winning alternative is called a *social choice function*, henceforth, SCF.

*http://manip.hse.ru/index.html

As our definition of an SCF does not allow for possible ties, in this event a fixed (anonymous) tie-breaking rule will be employed. A tie-breaking rule $\tau : \mathcal{P}^n \to \mathcal{P}$ maps preference profiles to linear orderings on A, which will be only employed when a formula does not determine a unique winner. If there are more alternatives chosen by a formula 'almost' specifying an SCF, then the highest ranked alternative is selected, based on the given tie-breaking rule among tied alternatives.

Let $\mathcal{F} = A^{\mathcal{P}^n}$ be the set of SCFs and $\mathcal{F}^{an} \subset \mathcal{F}$ be the set of anonymous voting rules. The subset of $\mathcal{F}$ consisting of the dictatorial rules will be denoted by $\mathcal{D} = \{d_1, \ldots, d_n\}$, where d_i is the dictatorial rule with voter i as the dictator. By counting the number of profiles, on which f and g choose different alternatives we define a metric:

$$\rho(f, g) = \#\{\succ \in \mathcal{P}^n | f(\succ) \neq g(\succ)\} \tag{5.1}$$

on $\mathcal{F} = A^{\mathcal{P}^n}$.

We define our non-dictatorship index (NDI) by taking the distance to the closest dictator.

Definition 5.2. The *non-dictatorship index* (NDI) is given by

$$NDI(f) = \min_{i \in N} \rho(f, d_i)$$

We specify the set of least dictatorial rules by those ones which are the furthest away from the closest dictatorial rule, which means that we are maximizing the minimum of the distances to the dictators.

Definition 5.3. We define the set of *least dictatorial rules* by

$$\mathcal{F}_{ld} = \arg\max_{f \in \mathcal{F}} \min_{i \in N} \rho(f, d_i) = \arg\max_{f \in \mathcal{F}} NDI(f)$$

in general and by

$$\mathcal{F}_{ld}^{an} = \arg\max_{f \in \mathcal{F}^{an}} \min_{i \in N} \rho(f, d_i) = \arg\max_{f \in \mathcal{F}^{an}} NDI(f)$$

over the set of anonymous voting rules.

In Bednay, Moskalenko and Tasnádi [13] we have established that $\mathcal{F}_{ld}^{an}$ equals the set of reverse-plurality rules with anonymous tie-breaking rules, where the *reverse-plurality rule* f_τ^* select the alternative being the fewest times on the top and in case of ties, a fixed anonymous tie-breaking rule τ is employed.

5.2 Voting rules

We will need some additional notations. Let q be the cardinality of A and let $s : \{1, ..., q\} \to \mathbf{R}$ satisfy $s(1) \geq s(2) \geq ... \geq s(q)$ and $s(1) > s(q)$. Moreover, let $rk[a, \succ]$ denote the *rank* of alternative a in the ordering $\succ \in \mathcal{P}$ (i.e. $rk[a, \succ] = 1$ if a is the top alternative in the ranking $\succ$, $rk[a, \succ] = 2$ if a is second-best, and so on). We consider the following five common voting rules.

1. *Plurality Rule:* A voting rule PL is the *plurality rule* if for all $(\succ_i)_{i=1}^n \in \mathcal{P}^n$

$$PL\left((\succ_i)_{i=1}^n\right) = \arg\max_{a \in A} \ \# \left\{ i \in N \mid rk[a, \succ_i] = 1 \right\}.$$

PL chooses the alternative that is ranked first by the maximum number of voters.

2. *Borda Count:* We shall denote the *Borda score* of alternative $a \in A$ according to ordering $\succ$ by $bs\,[a, \succ] = q - rk[a, \succ]$. A voting rule BC is the *Borda count* if for all $(\succ_i)_{i=1}^n \in \mathcal{P}^n$

$$BC\left((\succ_i)_{i=1}^n\right) = \arg\max_{a \in A} \sum_{i=1}^{n} bs[a, \succ_i].$$

BC chooses an alternative with the maximum *Borda score bs*.

3. *k-Approval Rule:* A voting rule $k{-}AV$ is the *k-approval voting rule* if for all $(\succ_i)_{i=1}^n \in \mathcal{P}^n$

$$k{-}AV\left((\succ_i)_{i=1}^n\right) = \arg\max_{a \in A} \#\left\{i \in N \mid rk[a, \succ_i] \le k\right\}.$$

$k - AV$ chooses the alternatives which are admitted to be among the k best by the highest number of voters. We will consider $k - AV$ for $k = 2, 3$.

4. *Copeland Method:* For a given profile $(\succ_i)_{i=1}^n \in \mathcal{P}^n$ we say that alternative $a \in A$ *beats* alternative $x \in A$ if $\#\{i \in N \mid a \succ_i x\} > \#\{i \in N \mid x \succ_i a\}$, i.e. a wins over x by pairwise comparison. We shall denote by $l[a, (\succ_i)_{i=1}^n]$ the number of alternatives beaten by alternative $a \in A$ for a given profile $(\succ_i)_{i=1}^n$. Then a voting rule CM is the *Copeland method* if for all $(\succ_i)_{i=1}^n \in \mathcal{P}^n$

$$CM\left((\succ_i)_{i=1}^n\right) = \arg\max_{a \in A} \; l[a, (\succ_i)_{i=1}^n].$$

5. *Black's procedure:* Let μ be a majority relation for a given profile $(\succ_i)_{i=1}^n \in \mathcal{P}^n$, then $a\mu x$ if $\#\{i \in N \mid a \succ_i x\} > \#\{i \in N \mid x \succ_i a\}$. Condorcet winner CW in a profile $(\succ_i)_{i=1}^n$ is an element undominated in the majority relation μ (constructed according to the profile), i.e.

$$CW((\succ_i)_{i=1}^n) = \left\{a \in A \mid \text{for all } x \in A \setminus \{a\} : a\mu x\right\}.$$

Black's rule chooses a Condorcet winner if it exists, otherwise it chooses the alternative with the highest *Borda score*.

5.3 Computation scheme and results

The calculation of indices is performed for 3, 4 and 5 alternatives and up to 100 voters. We generate 1000 random preference profiles, where each profile occurs with equal probability, i.e. under the Impartial Culture assumption.

Tables 5.1, 5.2 and 5.3 present the results of the NDIs of the voting rules for the case of 3, 4 and 5 alternatives and for 3, 4, 5, 20 and 100 voters. We look at the non-dictatorship property of the 6 voting rules with taking into account the reverse-plurality rule, which serves as a benchmark (clearly, it has the highest NDI). It can be readily seen that among the common voting rules the plurality rule performs the worst, having the lowest NDI. The k - approval voting ($k = 3$ or $k = 2$) performs the best, with the highest NDI. The Borda count, Black's rule and Copeland method lie between the plurality and the k-approval rules without clear difference between them.

Voting rules	Number of voters					
	3	4	5	10	20	100
Plurality	0.360	0.392	0.427	0.484	0.533	0.583
Borda	0.363	0.442	0.450	0.486	0.527	0.587
k-Approval $k = 2$	0.488	0.550	0.544	0.571	0.570	0.605
k-Approval $k = 3$	0.642	0.661	0.656	0.648	0.644	0.627
Copeland	0.348	0.438	0.439	0.491	0.527	0.581
Black	0.348	0.442	0.436	0.486	0.519	0.585
Reverse-plurality	0.918	0.881	0.867	0.797	0.746	0.682

Table 5.1: Non-dictatorship indices in case of three alternatives

Voting rules	Number of voters					
	3	4	5	10	20	100
Plurality	0.437	0.448	0.485	0.548	0.602	0.655
Borda	0.455	0.501	0.531	0.575	0.628	0.677
k-Approval $k = 2$	0.591	0.580	0.613	0.629	0.649	0.688
k-Approval $k = 3$	0.617	0.662	0.669	0.695	0.685	0.701
Copeland	0.437	0.493	0.506	0.565	0.629	0.673
Black	0.435	0.499	0.510	0.571	0.628	0.676
Reverse-plurality	1	0.969	0.950	0.882	0.842	0.770

Table 5.2: Non-dictatorship indices in case of four alternatives

Voting rules	Number of voters					
	3	4	5	15	20	100
Plurality	0.449	0.497	0.524	0.618	0.651	0.706
Borda	0.517	0.556	0.581	0.673	0.685	0.735
k-Approval $k = 2$	0.601	0.623	0.641	0.699	0.705	0.725
k-Approval $k = 3$	0.676	0.692	0.704	0.733	0.716	0.744
Copeland	0.465	0.552	0.567	0.660	0.684	0.727
Black	0.473	0.554	0.572	0.662	0.684	0.734
Reverse-plurality	1	1	0.991	0.906	0.898	0.824

Table 5.3: Non-dictatorship indices in case of five alternatives

The graphical representations of the NDIs for 3, 4 and 5 alternatives case and up to 100 voters are shown in Figures 5.1, 5.2 and 5.3. On the X-axis we define the number of voters and on the Y-axis we define the values of NDIs. From these figures we observe that the reverse-plurality rule has the highest NDI, while the plurality rule has the lowest NDI. The NDIs of the other voting rules under study lie between the NDIs of these two rules.

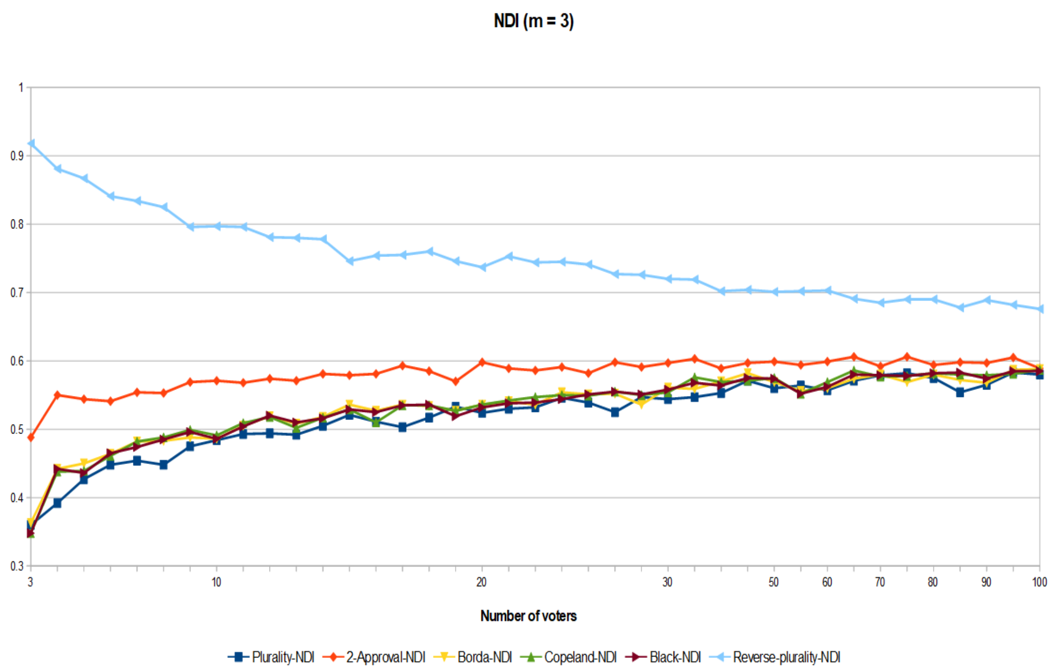

Figure 5.1: Non-dictatorship indices in case of three alternatives

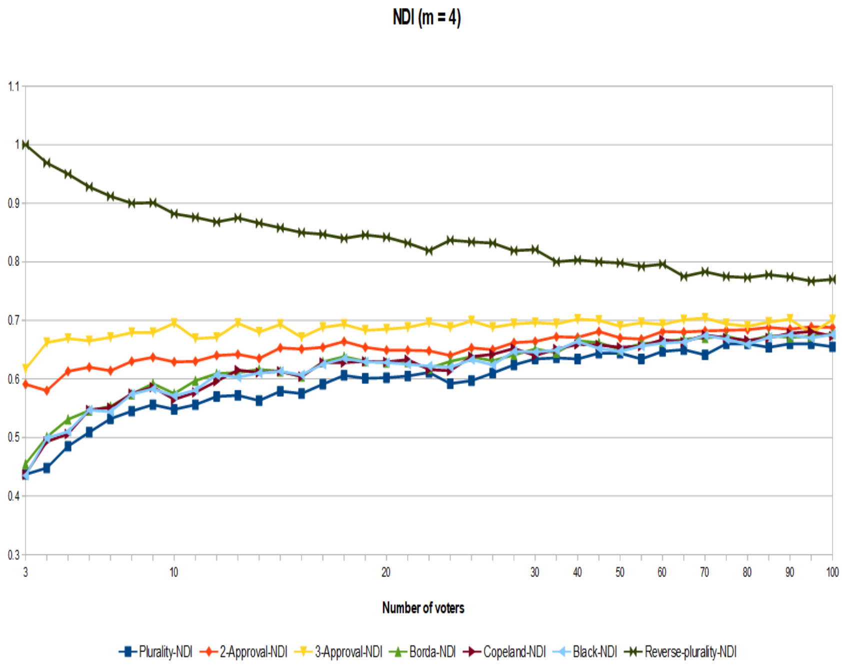

Figure 5.2: Non-dictatorship indices in case of four alternatives

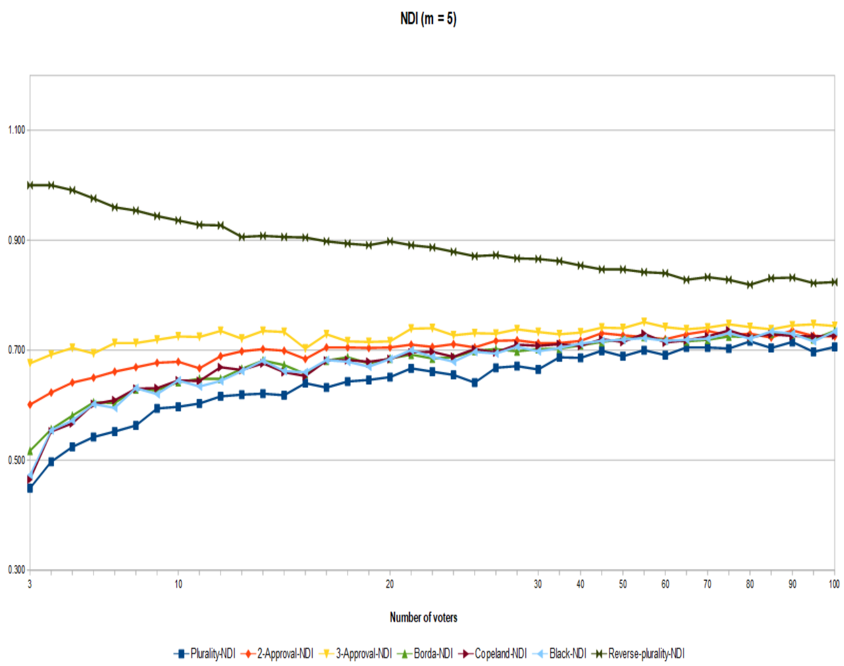

Figure 5.3: Non-dictatorship indices in case of five alternatives

If we also add the degree of manipulability, we expect that while the NDIs of common voting rules are increasing, their NKIs are decreasing in the number of voters. To check this, we put both the NDIs and NKIs in the same tables and figures. Tables 5.4, 5.5 and 5.6 show the unified results for the NKIs and NDIs of the voting rules under study for the case of 3 alternatives, and 3, 4 and 10 voters. High NDI means a low degree of dictatorship, while low NKI means a low degree of manipulability. Thus, the voting rule which has high NDI and low NKI will perform the best.

Voting rules	$n = 3$		$n = 4$		$n = 10$		$n = 100$	
	NDI	NKI	NDI	NKI	NDI	NKI	NDI	NKI
Plurality	0.360	0.167	0.392	0.185	0.484	0.284	0.583	0.139
Borda	0.363	0.236	0.442	0.310	0.486	0.241	0.587	0.083
k-Approval $k = 2$	0.488	0.264	0.550	0.275	0.571	0.278	0.605	0.128
Copeland	0.348	0.111	0.438	0.296	0.491	0.194	0.581	0.056
Black	0.348	0.111	0.442	0.144	0.486	0.147	0.585	0.065

Table 5.4: NDIs and NKIs for the case of three alternatives

Voting rules	$n = 3$		$n = 4$		$n = 10$		$n = 10$	
	NDI	NKI	NDI	NKI	NDI	NKI	NDI	NKI
Plurality	0.437	0.294	0.448	0.325	0.548	0.421	0.655	0.187
Borda	0.455	0.512	0.501	0.500	0.575	0.419	0.677	0.153
k-Approval $k = 2$	0.591	0.394	0.580	0.426	0.629	0.419	0.688	0.170
k-Approval $k = 3$	0.617	0.500	0.662	0.525	0.695	0.490	0.701	0.198
Copeland	0.437	0.227	0.493	0.453	0.565	0.343	0.673	0.135
Black	0.435	0.276	0.499	0.263	0.571	0.275	0.676	0.127

Table 5.5: NDIs and NKIs for the case of four alternatives

Voting rules	$n = 3$		$n = 4$		$n = 15$		$n = 100$	
	NDI	NKI	NDI	NKI	NDI	NKI	NDI	NKI
Plurality	0.449	0.389	0.497	0.426	0.618	0.469	0.706	0.227
Borda	0.517	0.691	0.556	0.639	0.673	0.465	0.735	0.206
k-Approval $k = 2$	0.517	0.691	0.556	0.639	0.673	0.465	0.735	0.206
k-Approval $k = 3$	0.676	0.576	0.692	0.582	0.733	0.464	0.744	0.201
Copeland	0.465	0.329	0.552	0.561	0.660	0.382	0.727	0.202
Black	0.473	0.409	0.554	0.357	0.662	0.419	0.734	0.182

Table 5.6: NDIs and NKIs for the case of 5 alternatives

Figures 5.4, 5.5 and 5.6 show graphically the results for both NDIs and NKIs. We can see that both NDIs and NKIs are moving in the opposite directions, which is plausible and a positive sign for our non-dictatorship index. In case of three alternatives ($m = 3$), the plurality rule performs the worst from the point of view of both dictatorship and manipulability.

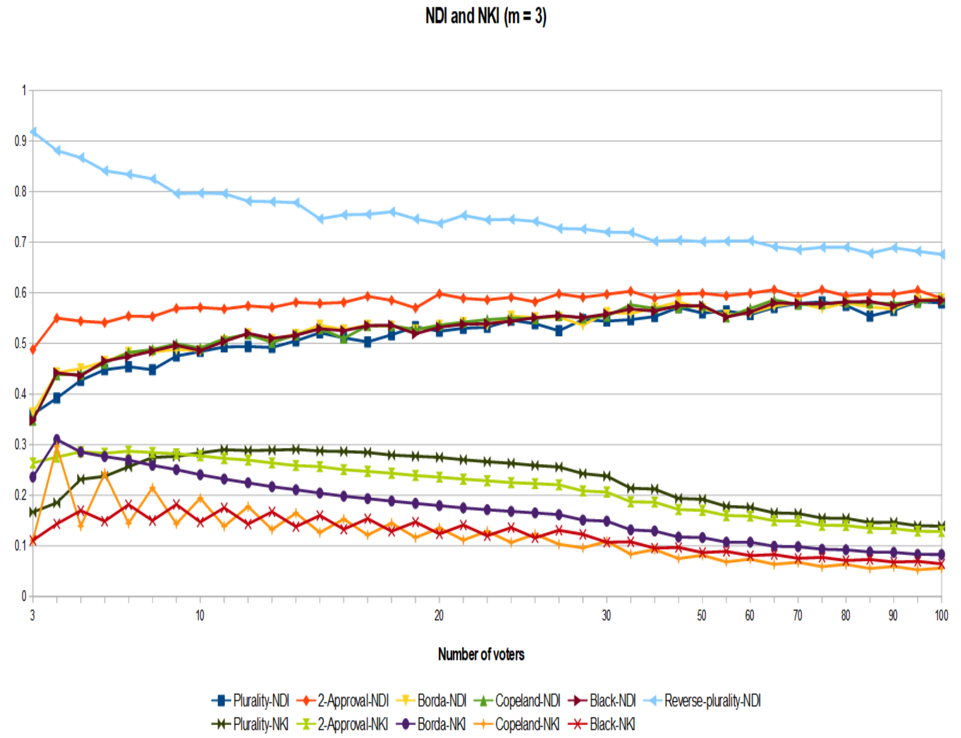

Figure 5.4: Non-dictatorship and Nitzan-Kelly indices in case of 3 alternatives

However, when there are four alternatives ($m = 4$), the rule that performs the worst from the point of view of manipulability is k-approval rule with $k = 3$, and the plurality rule performs the worst from the point of view of dictatorship (has the lowest NDI), while k-approval rule with $k = 3$ has the highest NDI, but of course, without taking into account the reverse-plurality rule's NDI.

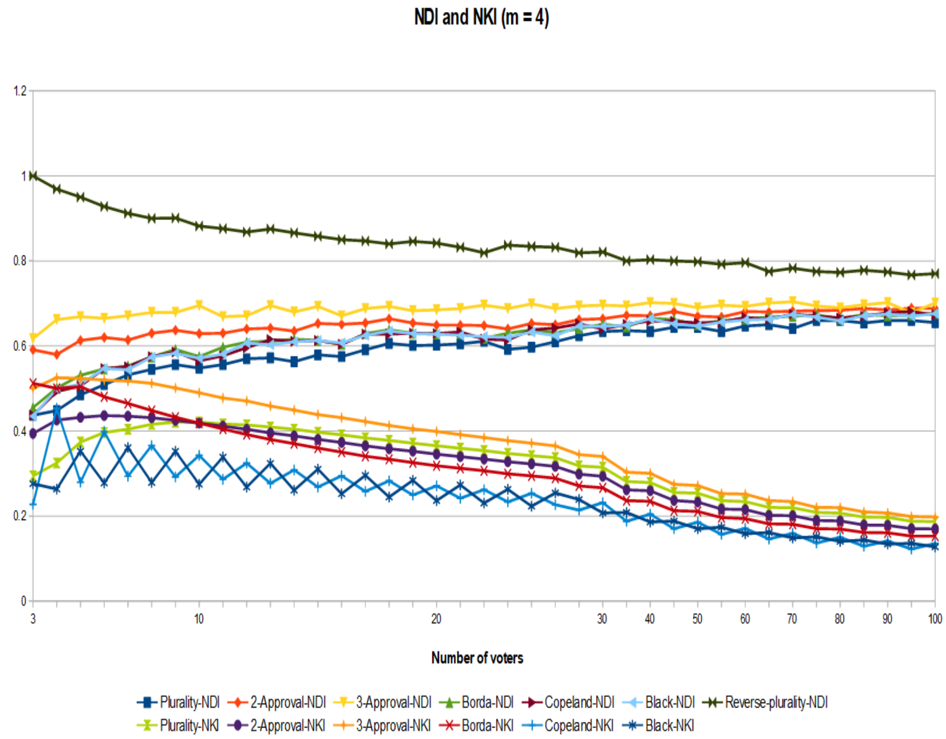

Figure 5.5: Non-dictatorship and Nitzan-Kelly indices in case of 3 alternatives

Considering the case of five alternatives ($m = 5$), for a small number of voters ($n < 15$), the Borda count has the highest degree of manipulability (NKI is high), while from the point of view of dictatorship its NDI is identical to Black's rule and lies between the plurality and k-approval rule with $k = 3$. As the number of voters increases ($n \geq 15$), again the only voting rule that performs the worst from the point of view of both manipulability and dictatorship is the plurality rule (with the exception of the 3-approval voting rule, which has a higher NKI for the case of four alternatives).

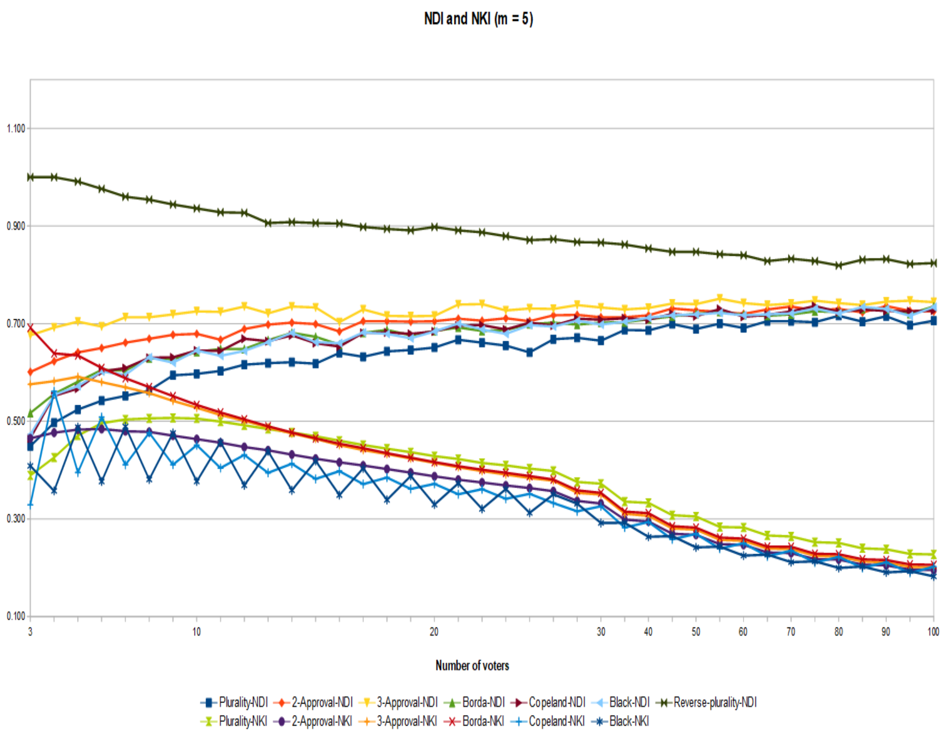

Figure 5.6: Non-dictatorship and Nitzan-Kelly indices in case of 3 alternatives

5.4 Conclusion

Based on the classic Gibbard-Satterthwaite's impossibility theorem the properties of strategy-proofness and non-dictatorship are incompatible if there are at least three alternatives, any preference profile is possible and the social choice function has to be onto. Therefore, whenever a decision has to be made of which social choice function should be employed, we always face the dilemma of choosing between a degree of dictatorship and a degree of manipulability.

Both incompatible properties are undesirable. However, when picking a voting rule it could be helpful and informative to know about its degree of manipulability and its distance from dictatorship. Concerning non-manipulability indices, we have selected the Nitzan-Kelly index, which counts the number of manipulable profiles in the total number of profiles, from the several non-manipulability indices already employed in the literature. We have introduced a non-dictatorship index, which measures the dictatorial component of voting rules, and we have ranked common voting rules based on this index.

Clearly, we could think about different ways of measuring the dictatorial component of a voting rule. We have chosen a fairly straightforward distance based approach for defining our non-dictatorship index. In this chapter we have explored the relationship between the Nitzan-Kelly index and our non-dictatorship index. Though these two approaches differ substantially we nevertheless arrived to the same conclusion that among the prominent voting rules basically the plurality rule performs the worst from the point of view of dictatorship and manipulability.

Finally, we would like to mention that since by having less manipulable profiles we are getting closer to the dictatorial rule it is not at all clear whether minimizing the number of manipulable profiles should be the right goal. We have a similar situation in case of our non-dictatorship index since by maximizing our non-dictatorship index we are getting closer to the undesirable reverse-plurality rule. Despite challenging these approaches we believe that they shed some light on the evergreen problem of choosing an appropriate voting rule.

6

Final summary

This thesis has considered several collective decision-making problems. The common issue that unites all the chapters is the analysis of group decision-making in the presence of conflict. In the first two chapters the conflict is explicit and modelled as a strategic interplay among the agents. In the last two chapters we analyse the aggregation procedures (namely, voting rules), however, there is an implicit conflict behind the analysis. In particular, we analyse the two conflicting properties of voting rules, those of dictatorship and manipulability.

In Chapter 2 we have seen how intra-party conflict could lead to different outcomes for the party's fortune, and how the incorporation of the party's internal democracy can save the party from splitting. A high level of intra-party conflict is unsustainable for the party's continuation and may lead to the party split. Nevertheless, the intra-party conflict, when ranging from low to moderate levels, can be resolved by incorporating the intra-party democracy. We have also seen how the public perception of party unity may increase the power of the dissenting faction, influencing the party elite to incorporate democracy into the party's internal life.

In Chapter 3 we have analysed the problem of individuals with selfish (and, therefore, conflicting) preferences, who must choose a winner among themselves. There exists a deserving winner, whom all agent know. Our goal is to construct a voting mechanism so that the outcome always results in the deserving winner being chosen. To this end, we have designed a veto mechanism that implements the socially desirable outcome. This mechanism involves a veto rule, allowing the agents express their negative preferences.

In Chapter 4 we have questioned the assumption of the harmfulness of the undesirable dictatorial voting rule. We aimed to get away from the undesirable dictatorial voting rule, searching for the rule that is the furthest away from the closest dictatorial rule and for the rules balancing between all dictatorial rules. As our first result, we have found the reverse-plurality rule as the furthest from the closest dictatorial voting rule. Unfortunately, this rule violates some apparently desirable properties. Therefore, if we eliminate completely the dictatorial ingredient, we still end up with a very undesirable rule. As our second result, we have derived the plurality rule as the one that balances the agreement between all dictatorial voting rules, and so this rule can be considered as a kind of a compromise rule among all individual dictators.

In Chapter 5 we have aimed to shed some light on the evergreen problem of choosing an appropriate voting rule. To this end, we have investigated the relationship between the two incompatible properties of dictatorship and manipulability that lie at the core of two cornerstone social choice theorems. Whenever we are to choose some voting procedure, there is always a dilemma between dictatorship and manipulability. Therefore, it could be helpful to know to which degree a voting rule is manipulable and to which degree it is dictatorial. We can measure the degree of manipulability by the index of manipulability already present in the literature. In this final chapter we have introduced an index of non-dictatorship. We have put both indices of manipulability and non-dictatorship into a common framework for some common social choice rules. We have found that the plurality rule performs the worst in terms of both indices, and that, there is no voting rule that performs the best based on the two indices.

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FAIG CONSTAR que aquest treball, titulat **Essays on Collective Decision-Making**, que presenta n'**Anna MOSKALENKO** per a l'obtenció del títol de Doctor, ha estat realitzat sota la meva direcció al Departament d'Economia d'aquesta universitat.

HAGO CONSTAR que el presente trabajo, titulado **Essays on Collective Decision-Making**, que presenta **Anna MOSKALENKO** para la obtención del título de Doctor, ha sido realizado bajo mi dirección en el Departamento de Economía de esta universidad.

I STATE that the present study, entitled **Essays on Collective Decision-Making**, presented by **Anna MOSKALENKO** for the award of the degree of Doctor, has been carried out under my supervision at the Department of Economics of this university.

Reus, 26 d'octubre de 2018 / Reus, 26 de octubre de 2018 / Reus, 26th of October, 2018

El/s director/s de la tesi doctoral
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