Essays on collective decision-making

Anna Moskalenko

Advisor: Antonio Quesada

Reus, 2019

UNIVERSITAT ROVIRA i VIRGILI

Outline

"We have an agreement in principle.
The question is, do we all have the same principles?"

- Introduction
- Chapter 2: Primaries on demand
- Chapter 3: A mechanism to pick the deserving winner
- Chapter 4: Does avoiding bad voting rules leads to good ones?
- Chapter 5: Dictatorship versus manipulability

Outline

"We have an agreement in principle.
The question is, do we all have the same principles?"

- **Introduction**
- Chapter 2: Primaries on demand
- Chapter 3: A mechanism to pick the deserving winner
- Chapter 4: Does avoiding bad voting rules leads to good ones?
- Chapter 5: Dictatorship versus manipulability

Introduction

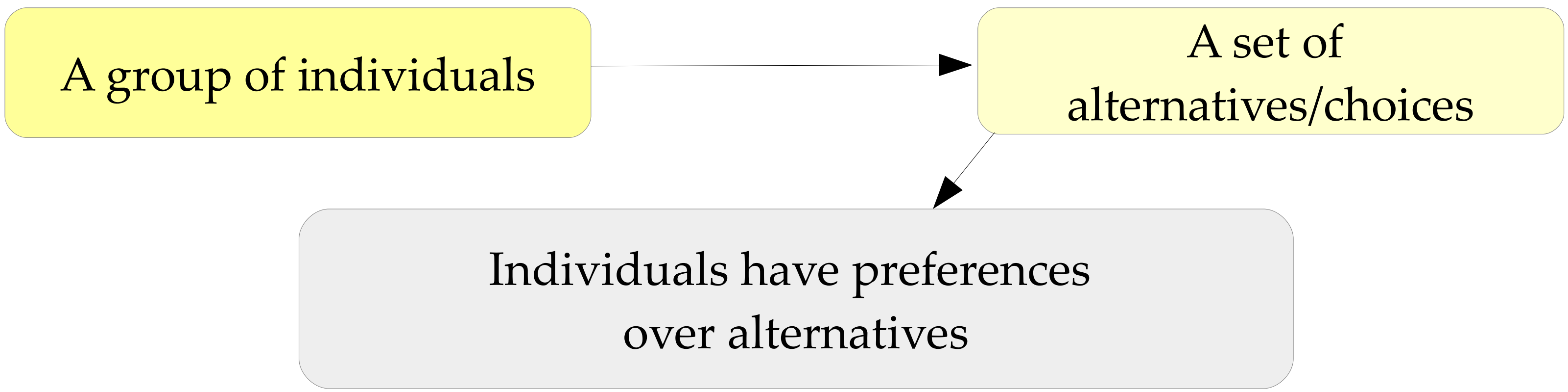
Typical setting of collective decision-making situation:

A group of individuals

A set of alternatives/choices

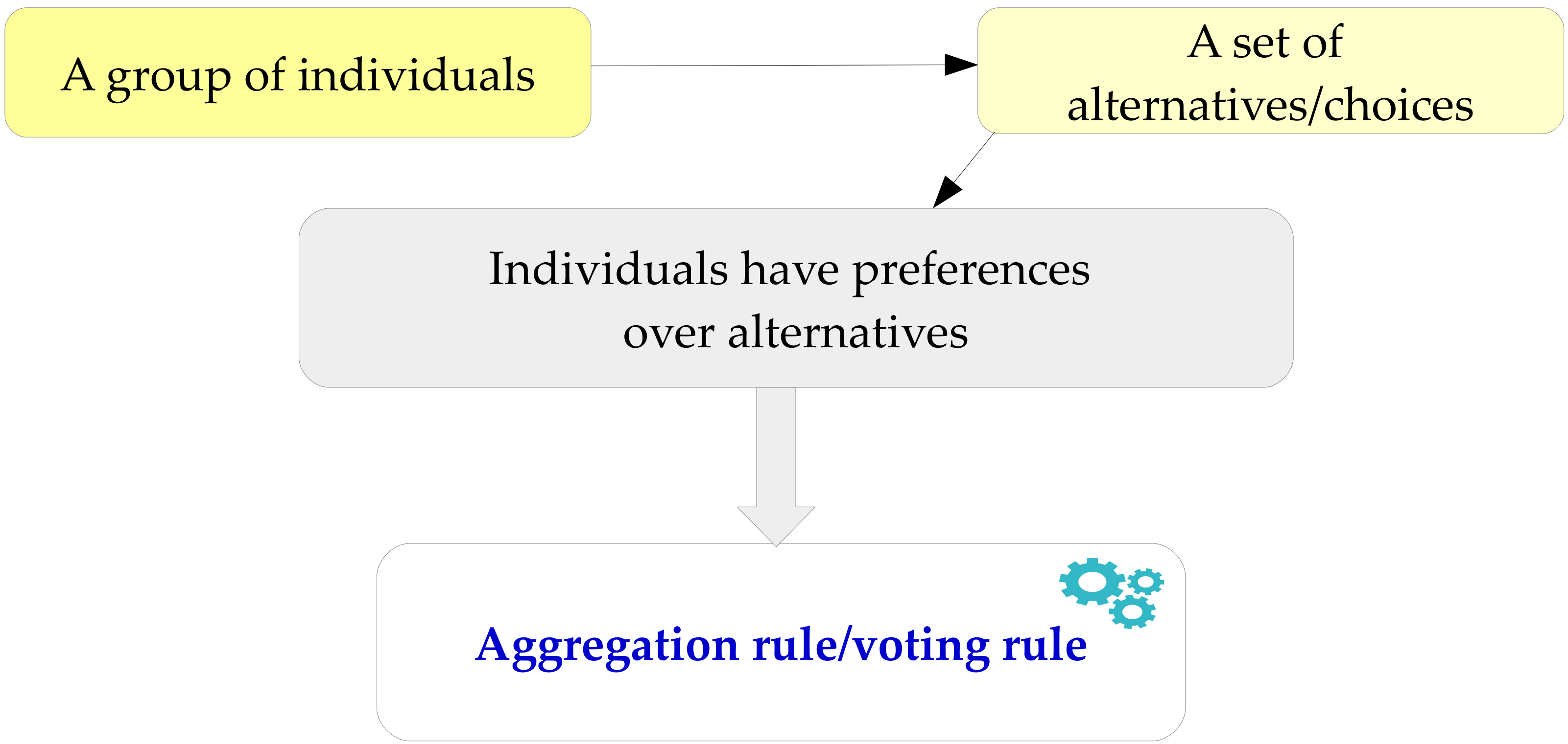
Introduction

Typical setting of collective decision-making situation:



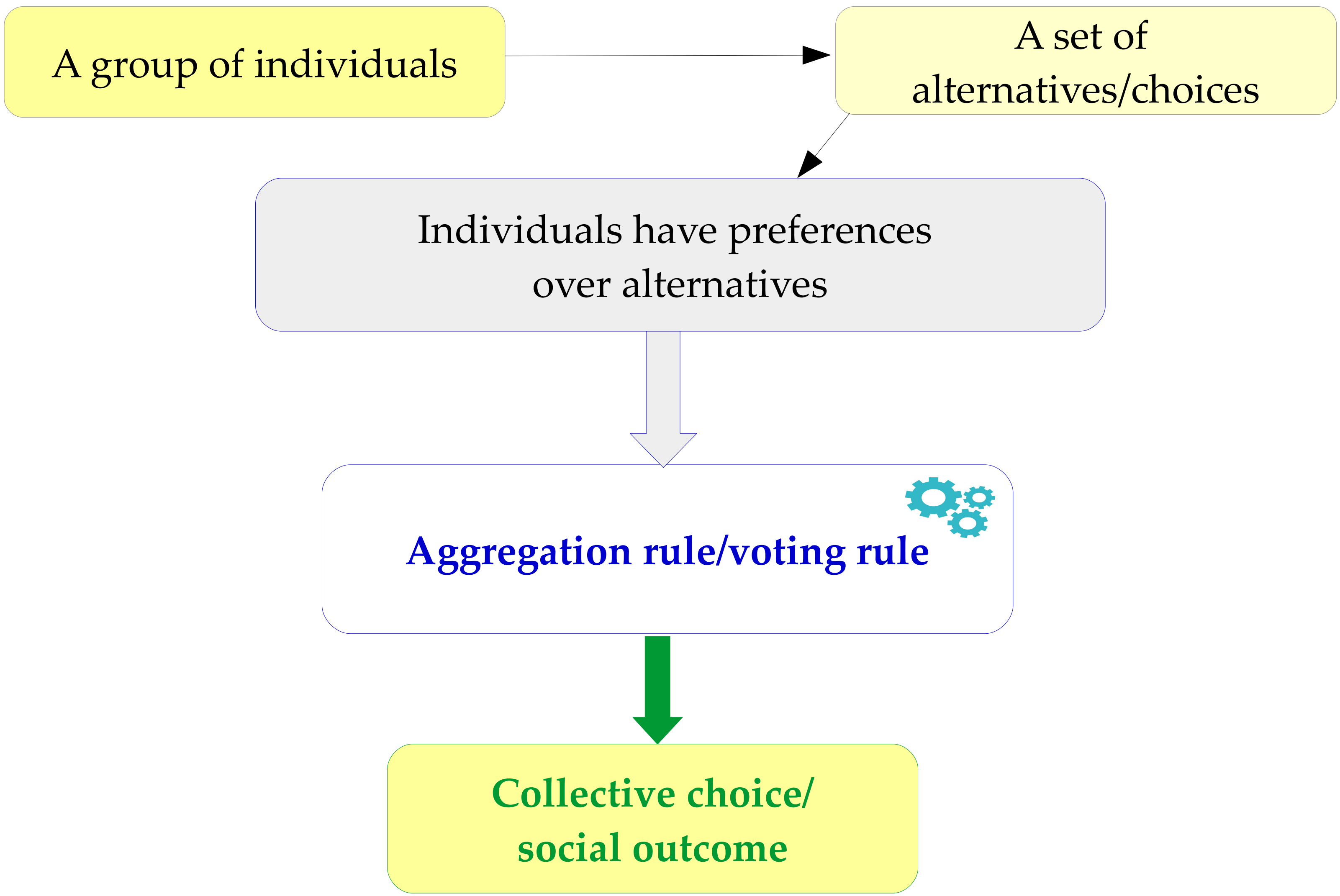
Introduction

Typical setting of collective decision-making situation:



Introduction

Typical setting of collective decision-making situation:



Introduction

Typical setting of collective decision-making situation:

A group of **different** individuals

A set of alternatives/choices

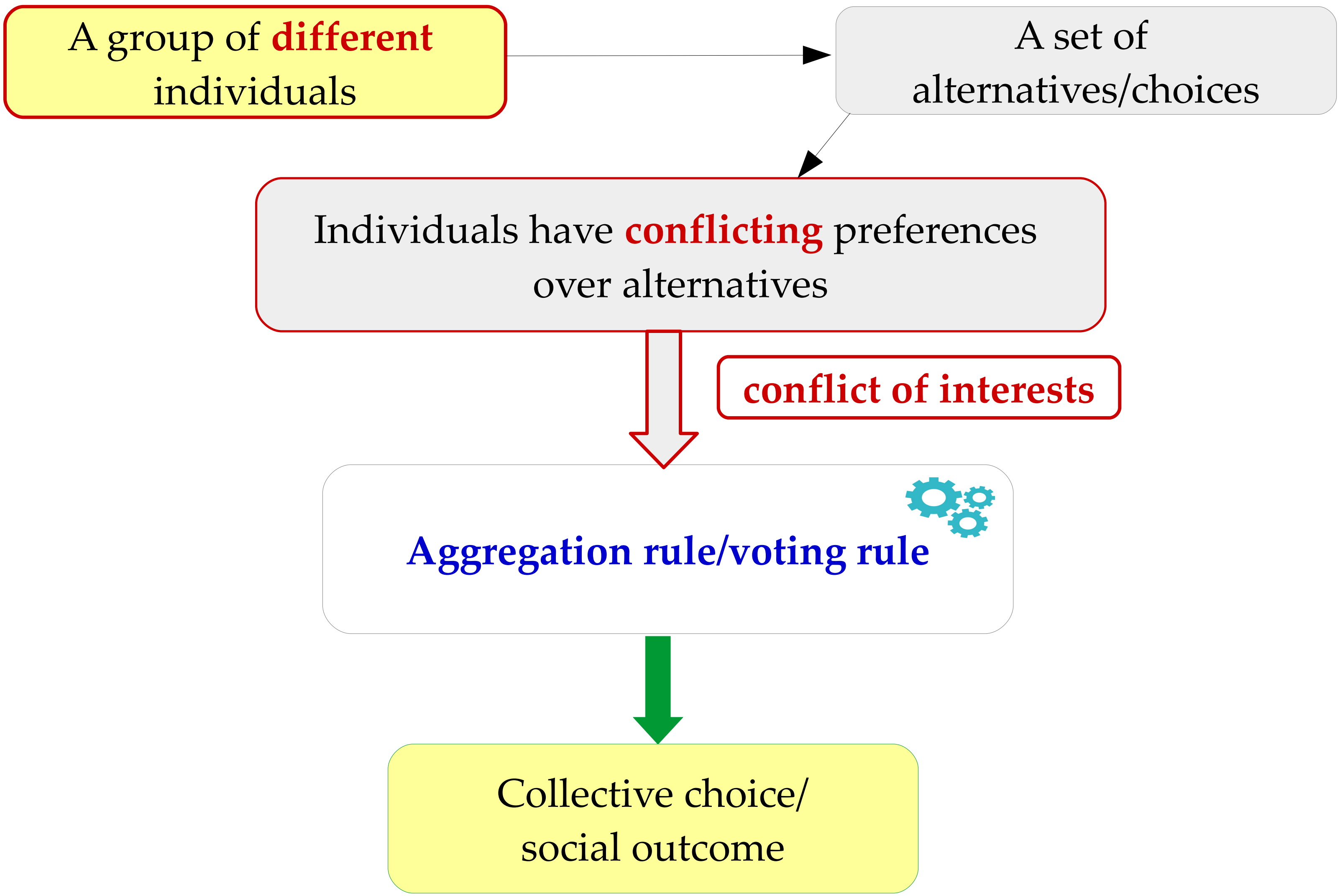
Individuals have **conflicting** preferences over alternatives

Aggregation rule/voting rule

Collective choice/ social outcome

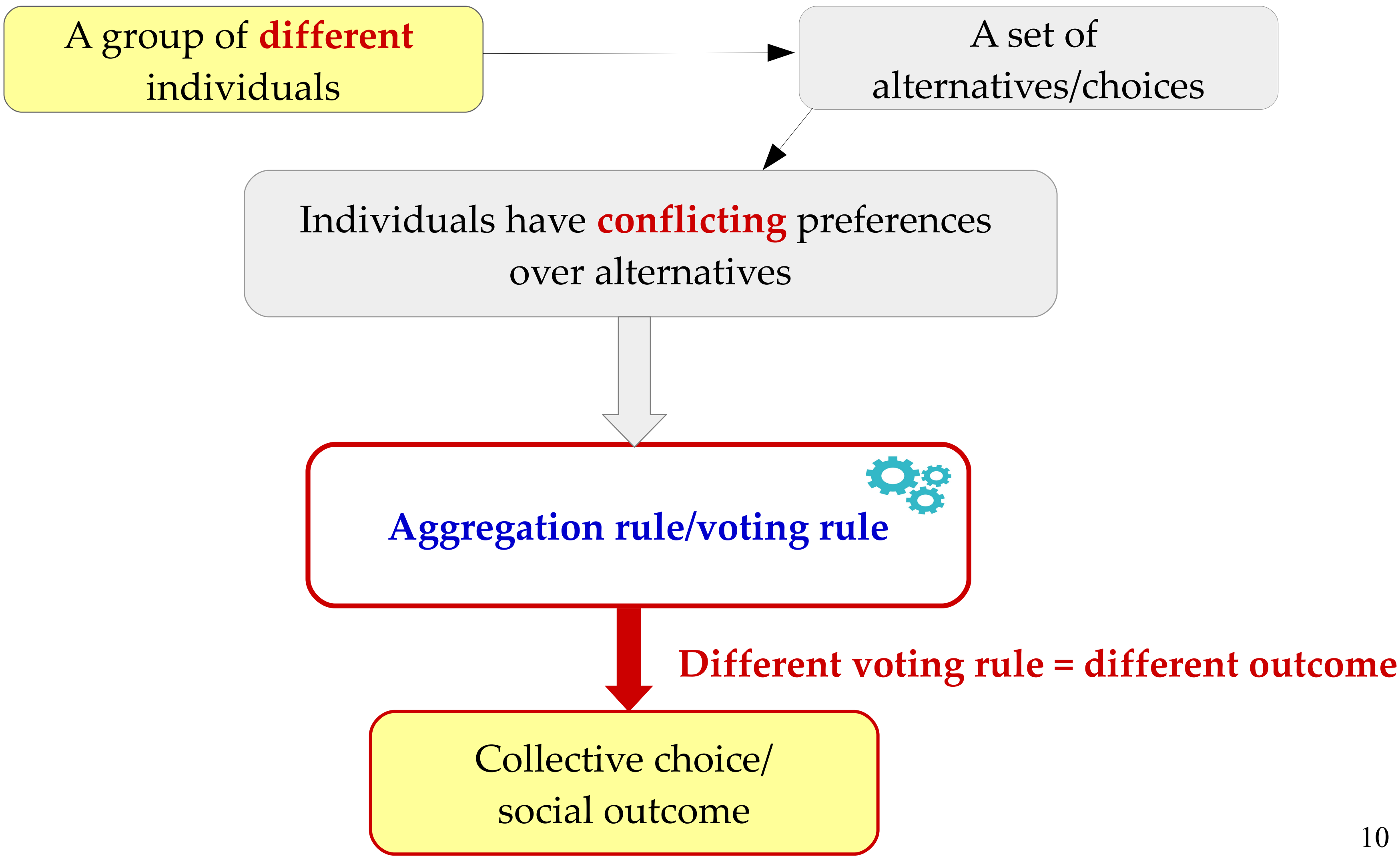
Introduction

Typical setting of collective decision-making situation:



Introduction

Typical setting of collective decision-making situation:



Introduction

Problematic nature of social choice/

collective decision-making:

1 Individuals' conflicting preferences → **conflict of interests** → strategic incentives

2 The choice of an **aggregation rule/ voting rule** to transform individual preferences or choices into collective preference or choice.

Introduction

Problematic nature of social choice/

collective decision-making:

1 Individuals' conflicting preferences → **conflict of interests** → strategic incentives

Chapter 2 + Chapter 3

2 The choice for an **aggregation rule/ voting rule** to transform individual preferences or choices into collective preference or choice

Introduction

Problematic nature of social choice/

collective decision-making:

1 Individuals' conflicting preferences → **conflict of interests** → strategic incentives

2 The choice for an **aggregation rule/ voting rule** to transform individual preferences or choices into collective preference or choice.

Chapter 4 + Chapter 5

Outline

"We have an agreement in principle. The question is, do we all have the same principles?"

- Introduction
- **Chapter 2: Primaries on demand**
- Chapter 3: A mechanism to pick the deserving winner
- Chapter 4: Does avoiding bad voting rules leads to good ones?
- Chapter 5: Dictatorship versus manipulability

Introduction

- Collective decision-making in a model of the intra-party politics.

- A political party, is composed of two factions: a party elite (leadership) and a dissenting faction (non-leadership).

- **Collective decision-making problem:** to choose the party's candidate.

- **Conflict of interests**: each faction wants its own faction's candidate to be the party's candidate.

Background

- This chapter was inspired by a similar problem in Hortalá-Vallve and Mueller, 2015 (HM hereinafter).

- They build a game-theoretical model as a strategic game between the elite faction and the dissenting faction.

- They show how the incorporation of internal democracy (**primaries**) can resolve the intra-party conflict.

- We build on their model but add some extensions.

HM model

- In HM model, the **party elite** is the **first-mover**.
- Elite decides on the institutional setup of the party (strategic top-down calculations).
- Dissenting faction is the last-mover.
- It has only two options: stay or exit the party.
- Two-stage game.
- The elite adopts primaries only under the credible exit threat.

HM model

- In HM model, the **party elite** is the **first-mover**.
- Elite decides on the institutional setup of the party (strategic top-down calculations).
- Dissenting faction is the last-mover.
- It has only two options: stay or exit the party.
- Two-stage game.
- The elite adopts primaries only under the credible exit threat.

Our model

- In our model, the **dissenting faction** is the **first-mover.**

HM model

- In HM model, the **party elite** is the **first-mover**.
- Elite decides on the institutional setup of the party (strategic top-down calculations).
- Dissenting faction is the last-mover.
- It has only two options: **stay** or exit the party.
- Two-stage game.
- The elite adopts primaries only under the credible exit threat.

Our model

- In our model, the dissenting faction is the first-mover.

Why?

HM model

- In HM model, the **party elite** is the **first-mover**.
- Elite decides on the institutional setup of the party (strategic top-down calculations).
- Dissenting faction is the last-mover.
- It has only two options: **stay** or exit the party.
- Two-stage game.
- The elite adopts primaries only under the credible exit threat.

Our model

- In our model, the dissenting faction is the first-mover.

  Why?

- We want to explicitly model the internal dissent.
- We add additional stage to the game, where dissenters can demand primaries (strategic bottom-up calculations).
- It has three options: stay loyal, demand primaries or exit the party.
- Three-stage game.

Our model

- New structure adds additional variable to the analysis:

  public cost of intra-party conflict, called the cost of party disunity.

- Divided parties lose election. Party unity is important for electoral success.

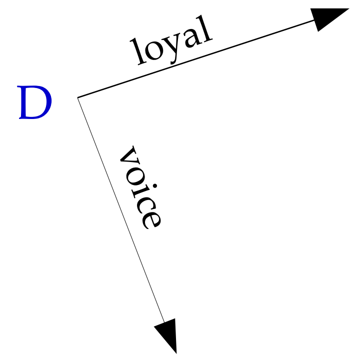
- We study in addition how the party (dis)unity influences the party's internal democratisation (primaries).

Model: key parameters

- Level of the intra-party conflict
- Electoral bonus (proportionality of electoral system)
- E's *relative strength* inside the party (whether E is in the majority or minority)
- Dimension of public cost of intra-party conflict = cost of party disunity

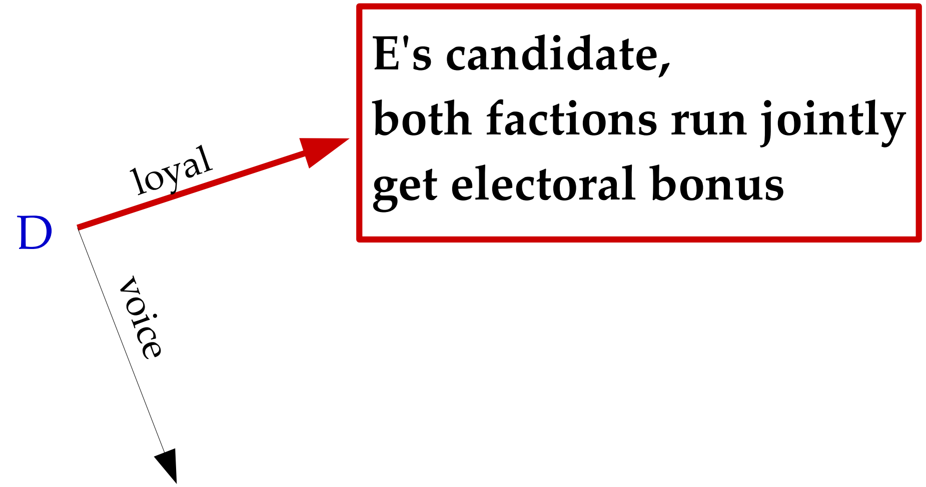
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries



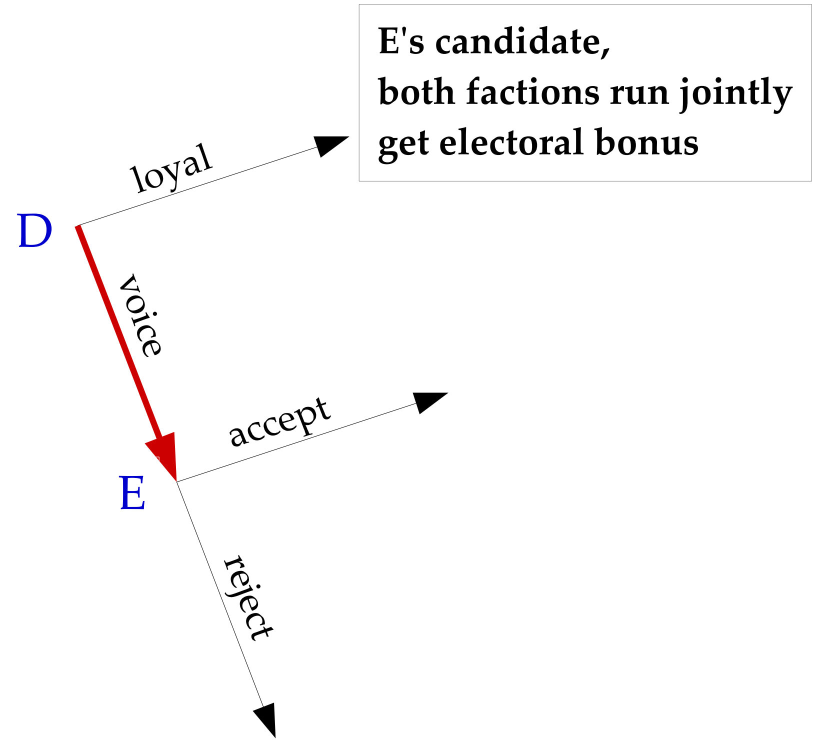
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses **loyal** → game ends, both factions run jointly



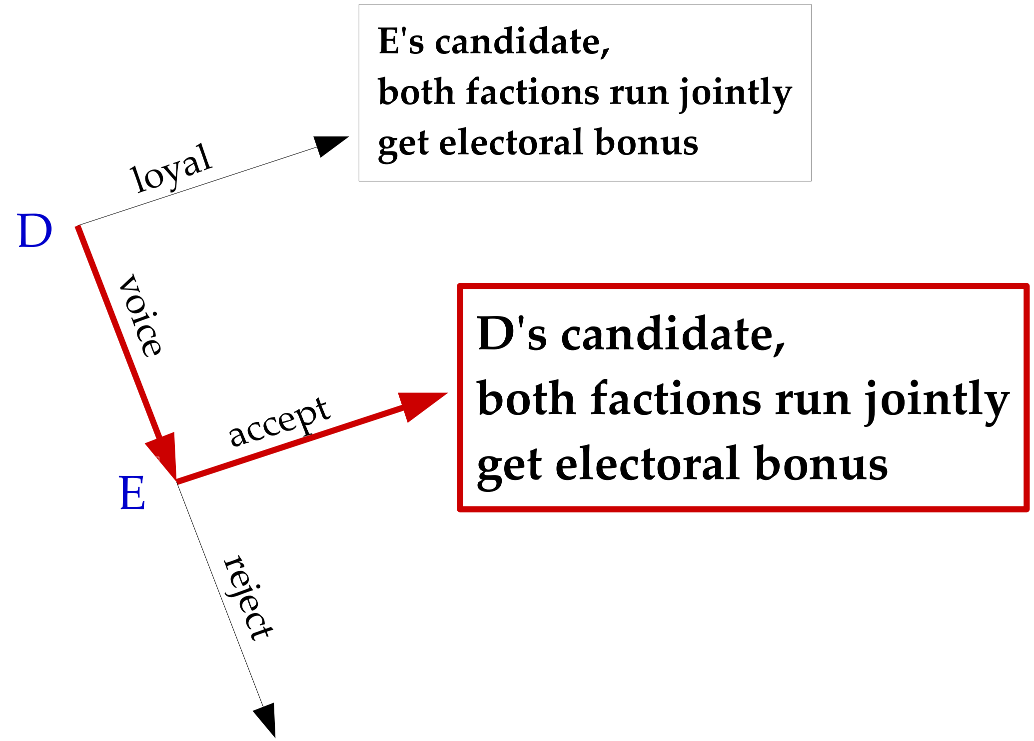
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses loyal → game ends, both factions run jointly
- If D chooses **voice** → next stage, where E chooses **accept** or **reject**



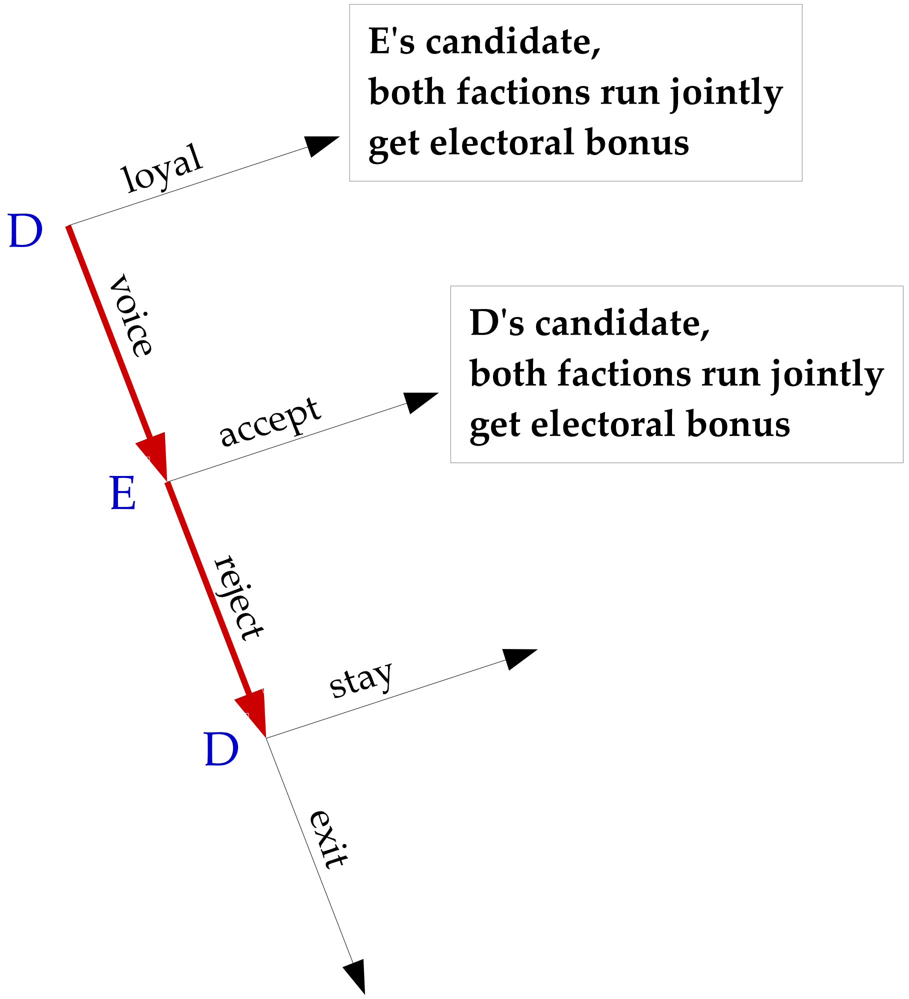
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses loyal → game ends, both factions run jointly
- If D chooses voice → next stage, where E chooses accept or reject
- If E **accepts** → primaries, D wins



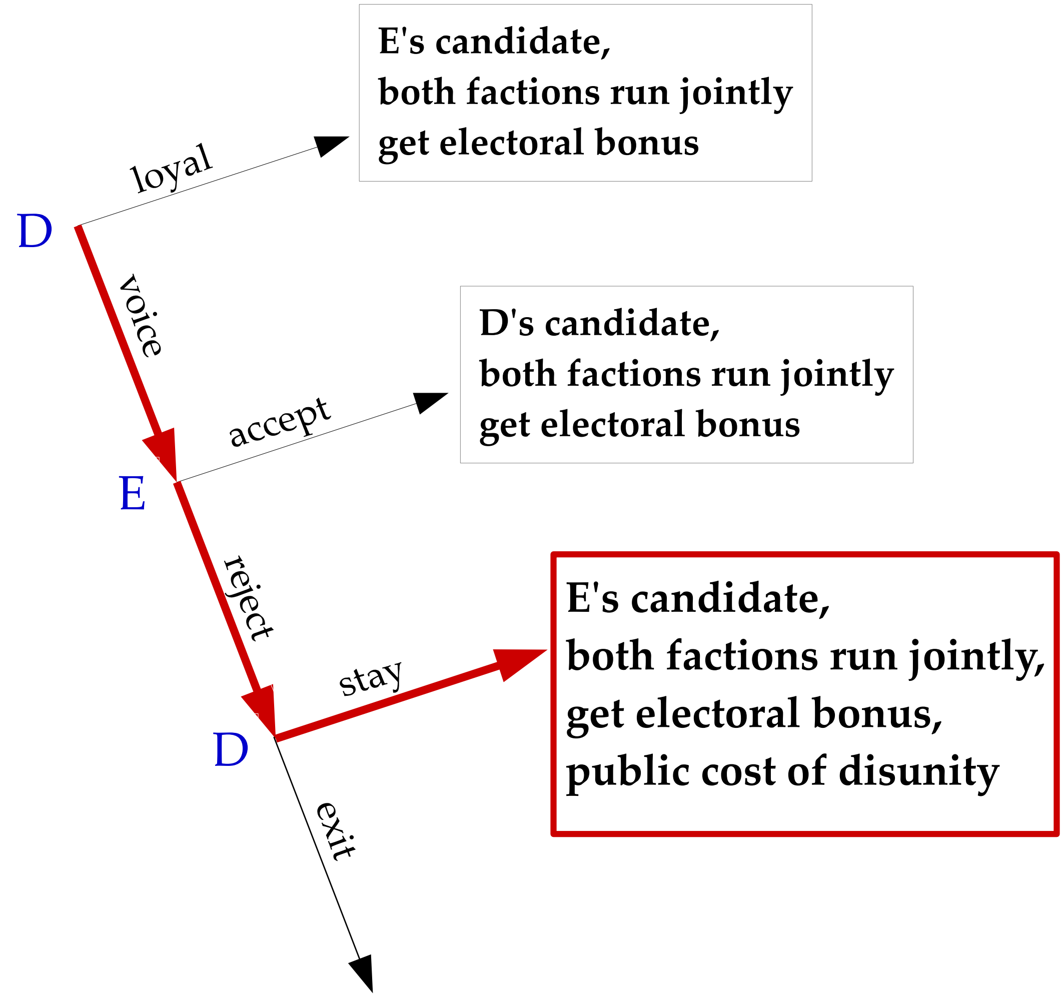
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses loyal → game ends, both factions run jointly
- If D chooses voice → next stage, where E chooses accept or reject
- If E accepts → primaries, D wins
- If E **rejects** → next stage, D chooses **stay** or **exit**



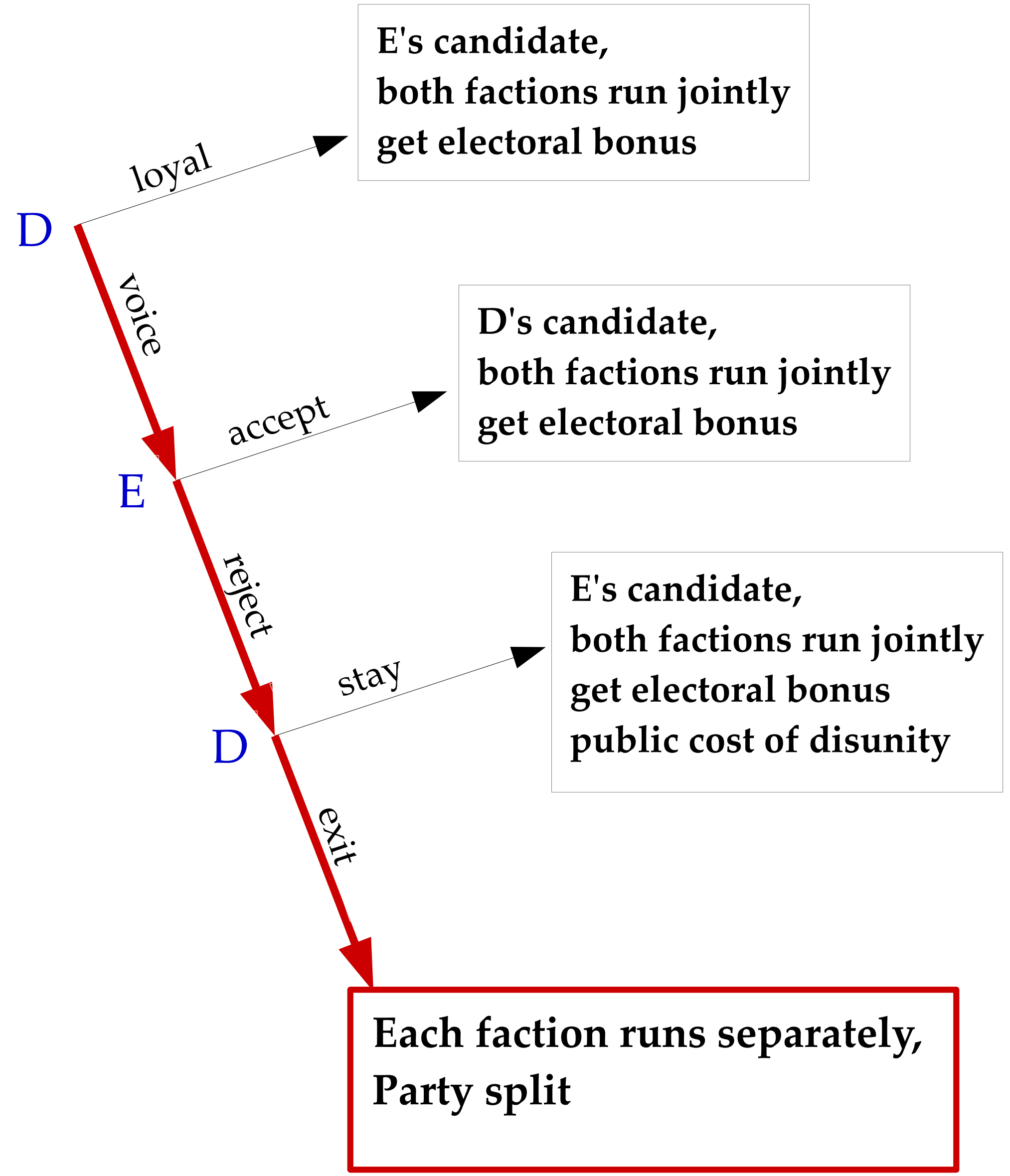
Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses loyal → game ends, both factions run jointly
- If D chooses voice → next stage, where E chooses accept or reject
- If E accepts → primaries, D wins
- If E rejects → next stage, D chooses stay or exit
- If D chooses **stay** → public **cost** of unresolved conflict



Intra-party game

- By default, the party's candidate belongs to E
- D can either agree or voice discontent and demand primaries
- If D chooses loyal → game ends, both factions run jointly
- If D chooses voice → next stage, where E chooses accept or reject
- If E accepts → primaries, D wins
- If E rejects → next stage, D chooses stay or exit
- If D chooses stay → public cost of unresolved conflict
- If D **exits**, the **party splits**



Results

The solution concept is SPNE.

All our results depend on the relative values of the four key parameters:

- level of the intra-party conflict
- electoral bonus
- relative strength of E
- cost of party disunity

We find two equilibria when the primaries are adopted.

Primaries

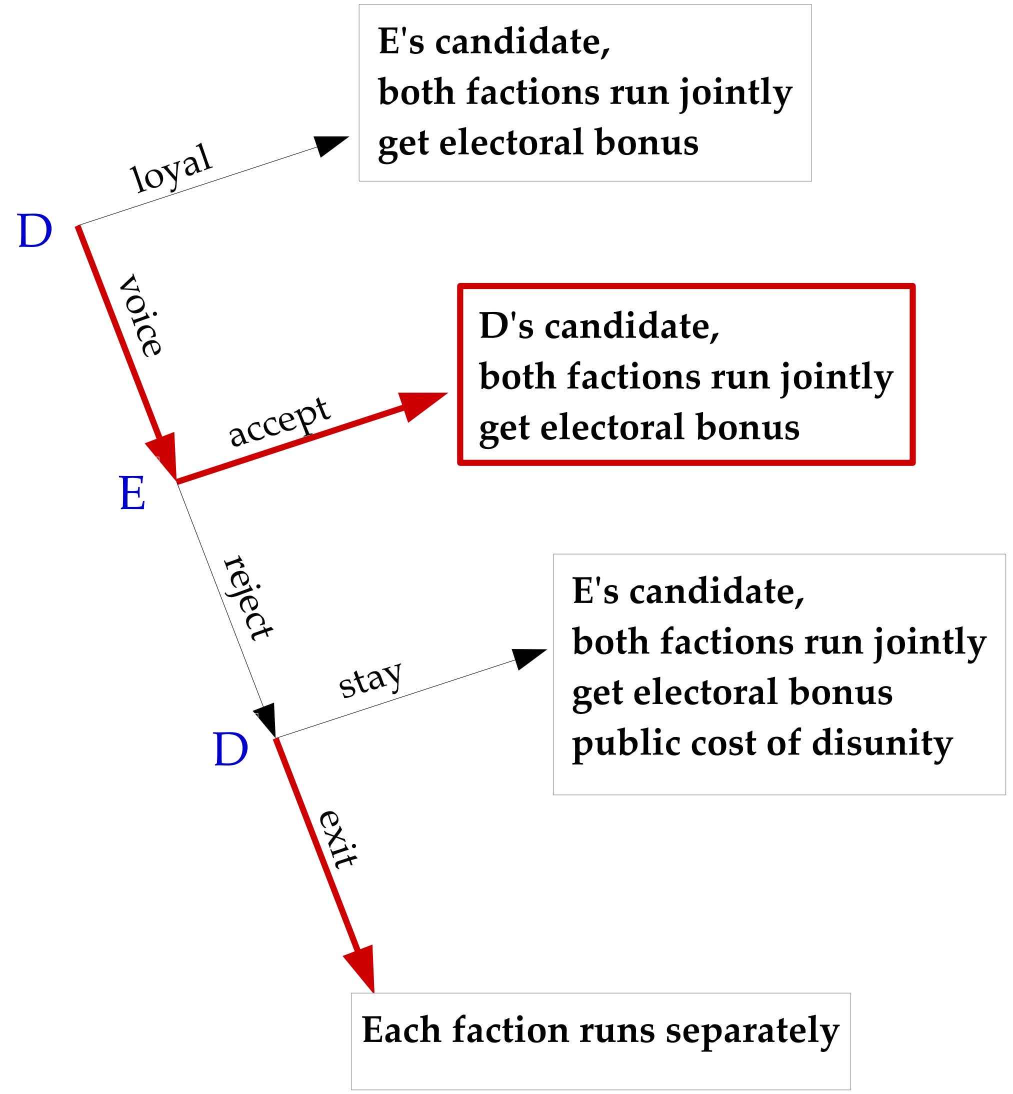
- Two types of primaries:

  - **Primaries with threat**

  Conditions:

  - Credible exit threat from the dissidents (**internal pressure**)

  - High public **cost of intra-party conflict**, high cost of disunity (**external pressure**)



Primaries

- Two types of primaries:

  - **Primaries no threat**

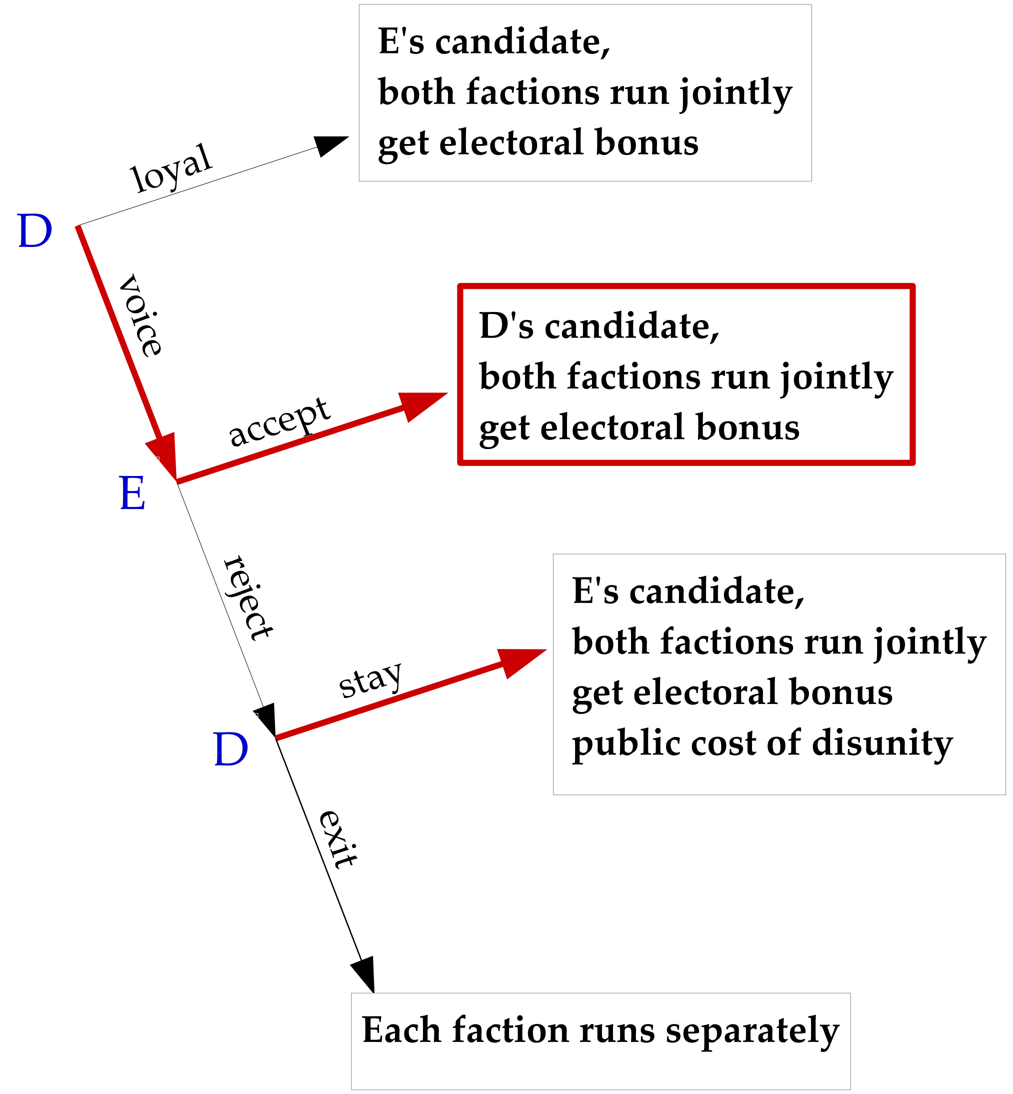
  Conditions**:**

  - No internal nor external pressure

  - Voluntary adoption of primaries by the party elite

  - Requires **high ideological cohesion** between both factions

  - **Low cost of disunity**



Final remarks

- Primaries are adopted in two cases:

1 There is an internal and external pressure to adopt primaries.

- Internal pressure: **D's threat to exit the party.**
- External pressure: **public cost of intra-party conflict.**

- As the cost of disunity **decreases**, the likelihood of this type of primaries increases.

Final remarks

- Primaries are adopted in two cases:

2 E's initiative to adopt primaries when both factions are close ideologically.

New results in comparison with HM model:

- Primaries occur when there is **no exit threat** from the dissidents.

- Primaries are more likely when the elite and the dissenting faction are **more ideologically closer**.

- The cost of disunity needs to be sufficiently low.

Final remarks

- Additional factor influencing the adoption of primaries – **cost of party disunity**.

- The cost of party disunity is inversely related to the proportionality of electoral system.

Final remarks

- In highly **disproportional (majoritarian) electoral systems** , electoral bonus of running jointly is the highest (equivalently, the public cost of intra-party conflict is high)

↓

Party elite is willing to adopt primaries in order to conceal factional divisions from the public.

Final remarks

- In **proportional electoral systems**, electoral bonus is minimal (equivalently, public cost of intra-party conflict is small)

Party elite is willing to adopt primaries if there is a high ideological cohesion between both factions.

Outline

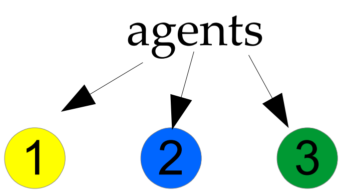
"We have an agreement in principle. The question is, do we all have the same principles?"

- Introduction
- Chapter 2: Primaries on demand
- **Chapter 3: A mechanism to pick the deserving winner**
- Chapter 4: Does avoiding bad voting rules leads to good ones?
- Chapter 5: Dictatorship versus manipulability

Motivation

- Voting = strategic game.
- Some individuals may be tempted to manipulate the final outcome.
- Which can lead to a **suboptimal decision for the group**.
- The goal: to avoid this kind of situations.

Problem



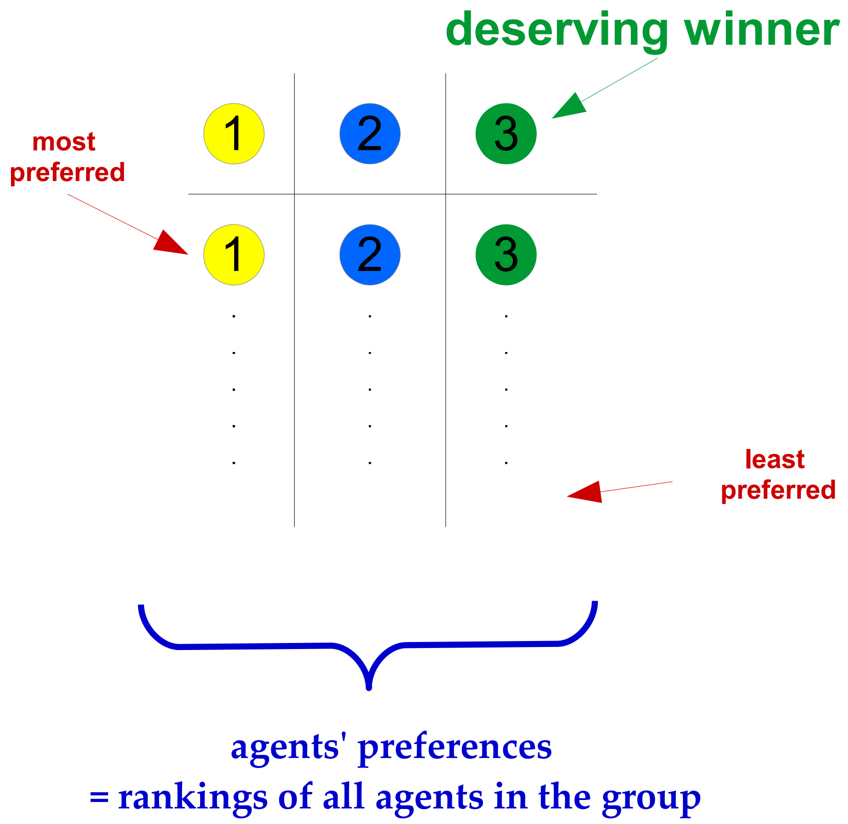
- A group of agents choosing a winner among themselves
- Voters = candidates

Problem

- A group of agents choosing a winner among themselves
- Voters = candidates
- There exist a **deserving winner** = desirable outcome

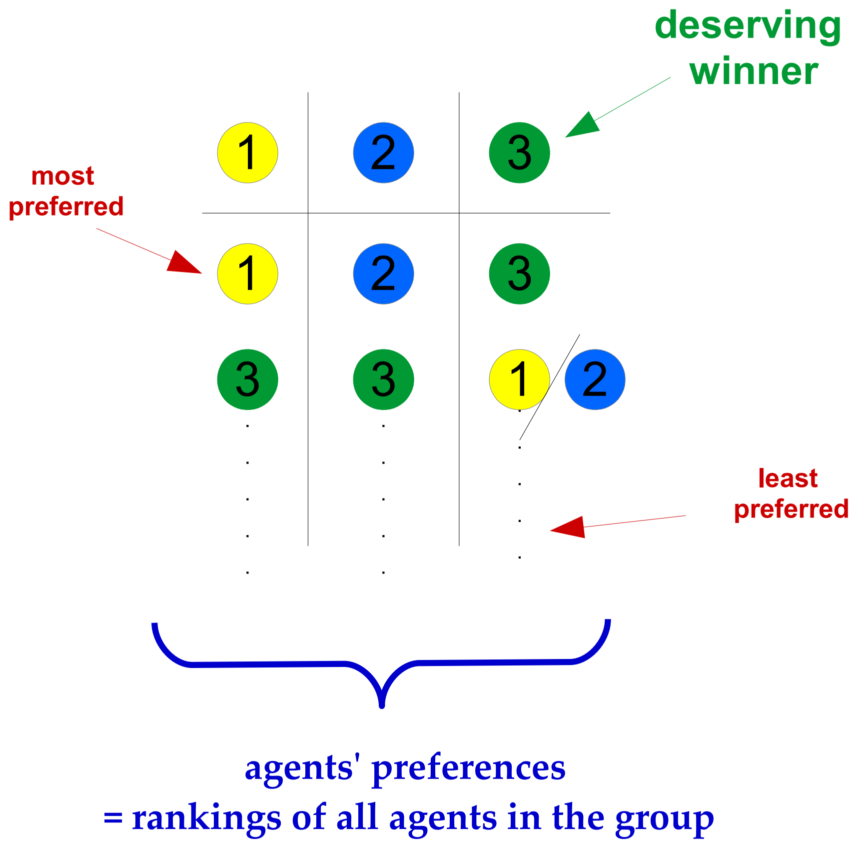
Problem

- A group of agents choosing a winner among themselves
- Voters = candidates
- There exist a **deserving winner** = desirable outcome
- Each agent is **selfish**: he always wants to be the winner



Problem

- A group of agents choosing a winner among themselves
- Voters = candidates
- There exist a **deserving winner** = desirable outcome
- Each agent is **selfish**: he always wants to be the winner
- If an agent is not chosen, he prefers the deserving winner to be chosen (impartiality)



The goal

To design a **voting mechanism** (a game form)

that always chooses the **deserving winner**

We apply a mechanism design approach

Background

- This chapter was inspired by the work of Amorós (2011)
- A sequential mechanism where agents take turns to ***announce*** an individual to be the winner
- The winner is always the **deserving winner**
- Needs at least **four** individuals to work

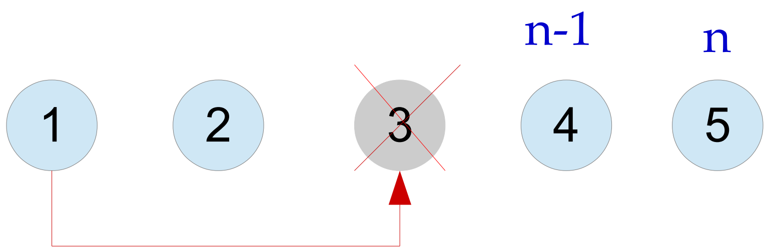
Background

- This chapter was inspired by the work of Amorós (2011)
- A sequential mechanism where agents take turns to ***announce*** an individual to be the winner
- The winner is always the **deserving winner**
- Needs at least **four** individuals to work

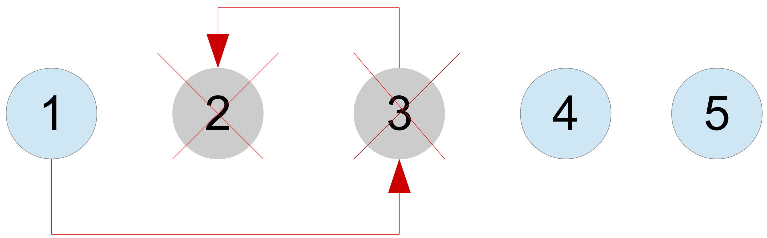
- We propose an *alternative* mechanism
- A sequential mechanism where agents take turns to ***veto*** an individual *not to be* the winner
- The winner is always the **deserving winner**
- Needs at least **three** individuals to work

The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1



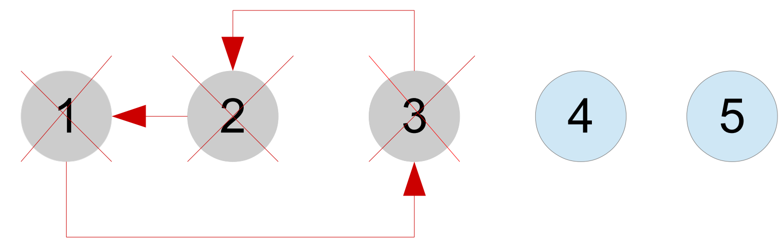
The veto mechanism



- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1

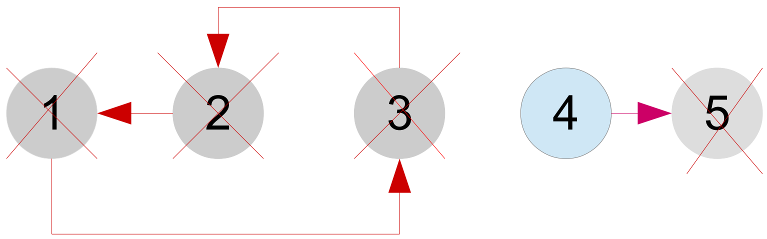
The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1
- Each one can only be vetoed once



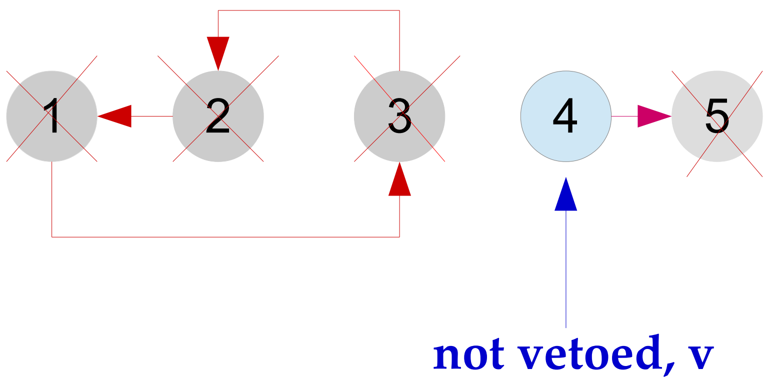
The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1
- Each one can only be vetoed once



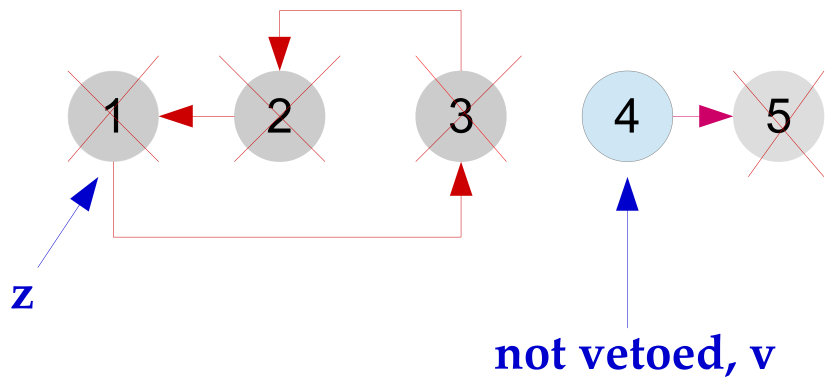
The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1
- Each one can only be vetoed once
- After n-1 has made his veto, there only remains one not vetoed agent, **v**



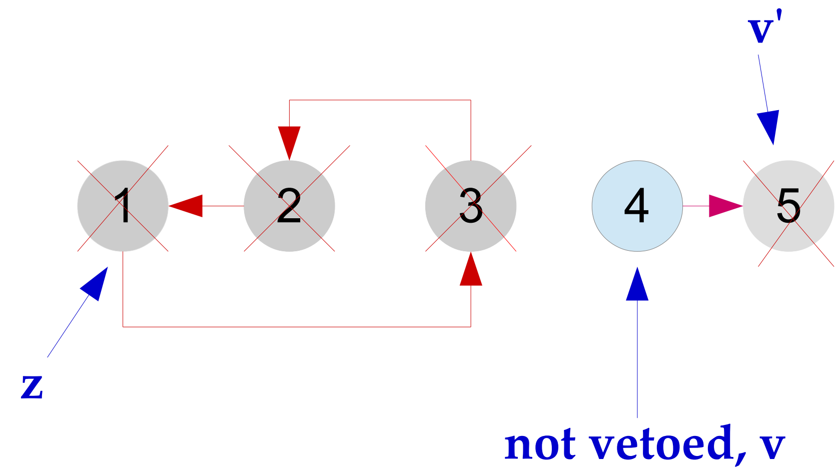
The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1
- Each one can only be vetoed once
- After n-1 has made his veto, there only remains one not vetoed agent, **v**
- Call **z** a first agent who <u>does not veto himself</u>, if such exists (the first to veto different agent than himself)

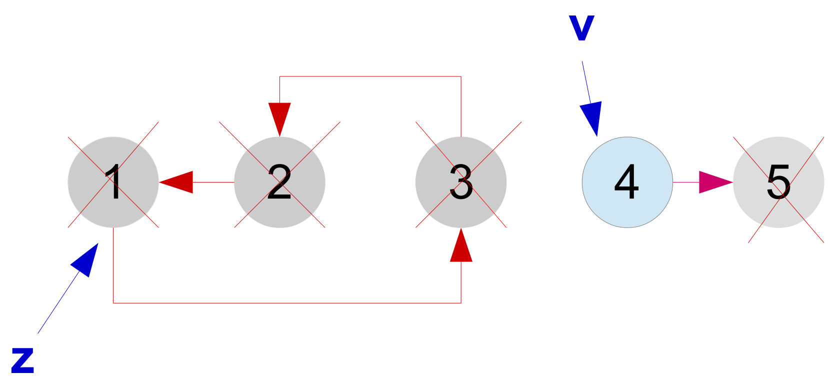


The veto mechanism

- There are n agents, who are placed in an arbitrary linear ordering from 1 to n
- Take turns to veto an agent from 1 till n-1
- Each one can only be vetoed once
- After n-1 has made his veto, there only remains one not vetoed agent, **v**
- Call **z** a first agent who <u>does not veto himself</u>, if such exists (the first to veto different agent than himself)
- Let v' be an agent vetoed by **v**



The veto mechanism



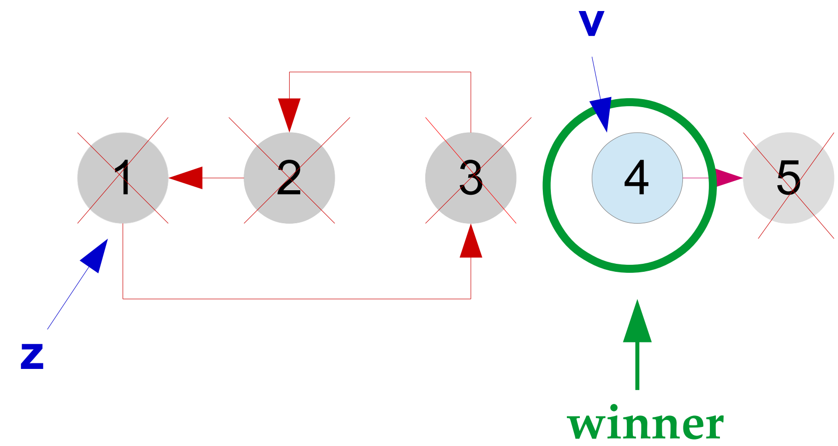
Picking rules:

- If no **z** exists or if **z** ≠ **v,** then **v** is chosen as the winner.

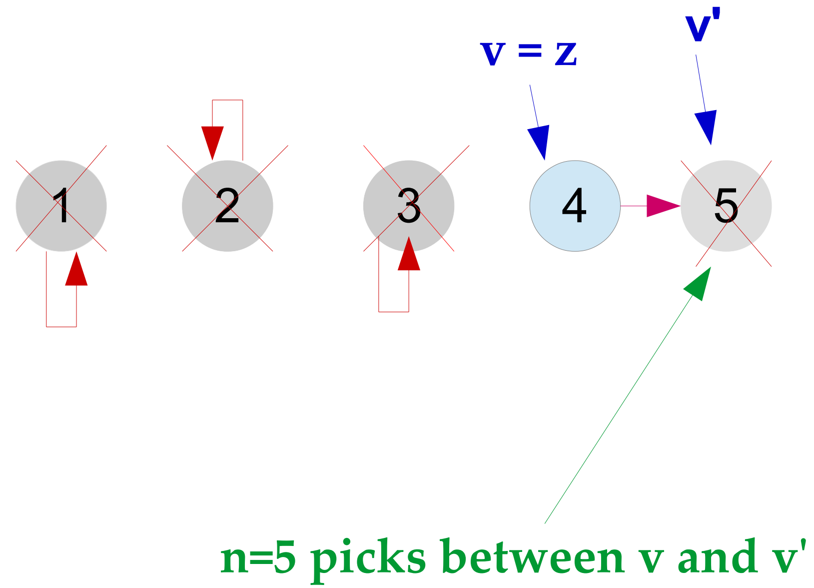
The veto mechanism

Picking rules:

- If no $\mathbf{z}$ exists or if $\mathbf{z} \neq \mathbf{v}$, then $\mathbf{v}$ is chosen as the winner.



The veto mechanism



Picking rules:

- If no **z** exists or if **z** ≠ **v**, then *v* is chosen as the winner

- If v = z, then the last individual n picks between v and v'.

Results

- The veto mechanism always chooses the deserving winner.

- Even if he has been vetoed before.

Observations

- The veto mechanism asks agents from 1 to n-1 to cast a veto.

- The last n agent does not veto anyone.

- However, n has his role to choose the winner, which happens if some agent vetoes the deserving winner.

Observations

- If some agent vetoes the deserving winner, all subsequent agents do not veto this agent (he is not vetoed, v)

- Then $z = v$ and so the last agent n picks the winner between v and v' (the deserving winner)

Final remarks

- Works with at least three agents.
- Uses **veto rule**, allows the agents to express **negative** preferences.

Outline

"We have an agreement in principle.
The question is, do we all have the same principles?"

- Introduction
- Chapter 2: Primaries on demand
- Chapter 3: A mechanism to pick the deserving winner
- **Chapter 4: Does avoiding bad voting rules leads to good ones?**

  joint with D. Bednay and A. Tasnádi, Corvinus University of Budapest

- Chapter 5: Dictatorship versus manipulability

Motivation

- Voting – the most common way to reach a decision.
- Aggregation rule is a voting rule.
- There are a lot of voting rules. Which rule is the ***best***?
- Important: to select a voting rule that will reflect the "will of the people".
- **Axiomatic approach**: evaluate voting rules according to a set of certain desirable properties (axioms).

Motivation

- Negative results from the two cornerstone theorems of social choice theory:

  - Arrow's Impossibility Theorem (Arrow, 1951)

  - Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975)

Motivation

- Negative results from the two cornerstone theorems of social choice theory:

    - Arrow's Impossibility Theorem (Arrow, 1951)

    No voting procedure that fairly chooses a winner for more than three alternatives and satisfying unrestricted domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.

Motivation

- Negative results from the two cornerstone theorems of social choice theory:

  - Arrow's Impossibility Theorem (Arrow, 1951)

No voting procedure that fairly chooses a winner for more than three alternatives and satisfying unrestricted domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.

The only voting method satisfying certain desirable properties = **dictatorship**.

Motivation

- Negative results from the two cornerstone theorems of social choice theory:

  - Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975)

  The only voting rule for at least three alternatives that is strategy-proof (immune to manipulation) is ***dictatorship***.

Motivation

- Negative results from the two cornerstone theorems of social choice theory:

  - Arrow's Impossibility Theorem (Arrow, 1951)

  - Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975)

dictatorship

Motivation

- **Dictatorial voting rule → bad.**

Motivation

- **Dictatorial voting rule → bad.**

If we get away from a *bad* voting rule will we obtain a *good* one?

Motivation

- **Dictatorial voting rule → bad.**

  If we get away from a *bad* voting rule will we obtain a *good* one?

- This chapter tries to answer this question.

Motivation

- Our goal: to get away from “bad” dictatorial voting rule.

- We search for *least-dictatorial* voting rules.

- We construct a **distance function (a metric)** between Social Choice Functions (SCF).

Background

- *Distance-based approach*: to explain voting rules in terms of the distance function.

Background

- *Distance-based approach*: to explain voting rules in terms of the distance function.

A voting rule can be characterised in terms of a goal state (e.g. unanimity, Condorcet winner) and a metric used in measuring the distance between the observed state and the goal state.

↓

Distance rationalization of voting rules.

Notations and notions

- A set of voters $N = \{1, \ldots, n\}$.
- A set of alternatives $A = \{1, \ldots, m\}$.
- A preference $>_i$ as a linear order in P (the set of all preference relations) of voter i in N.
- A voting rule (a SCF) for n voters is a function $f: P^n \rightarrow A$.
- Ties are broken by an anonymous tie-breaking rule.

Notations and notions

- $F = A^{P^n}$ is the set of all SCFs (Borda, plurality, etc)

- $D = \{d_1, \ldots d_n\}$ is a set of dictatorial voting rules and d_i is the dictatorial rule with voter i as a dictator.

- D is a subset of F

Distance function

Definition:

Let f and g be two distinct SFCs.

The **distance function** counts the number of preference profiles on which f and g choose different alternatives.

Formally,

$$\rho(f, g) = \#\{\succ \in \mathcal{P}^n | f(\succ) \neq g(\succ)\}$$

Distance function: example

- Consider the preference profile >:

$>_1$	$>_2$	$>_3$	$>_4$
a	a	b	c
b	b	c	b
c	c	a	a

tie-breaking rule: b>a>c

Let f be plurality rule.

Let g be a Borda count.

Distance function: example

- Consider the preference profile >:

$>_1$	$>_2$	$>_3$	$>_4$
a	a	b	c
b	b	c	b
c	c	a	a

tie-breaking rule: b>a>c

Let *f* be plurality rule.
Then *a* is the plurality winner.

Let *g* be a Borda count.

Distance function: example

- Consider the preference profile >:

$>_1$	$>_2$	$>_3$	$>_4$
a	a	b	c
b	b	c	b
c	c	a	a

tie-breaking rule: b>a>c

Let f be plurality rule.
Then a is the plurality winner.

Let g be a Borda count.
Then b is the Borda winner.

Distance function: example

- Consider the preference profile >:

$>_1$	$>_2$	$>_3$	$>_4$
a	a	b	c
b	b	c	b
c	c	a	a

tie-breaking rule: b>a>c

Let *f* be plurality rule.
Then *a* is the plurality winner.

Let *g* be a Borda count.
Then b is the Borda winner.

$\varrho(f, g) = 1$, since $f(>) \neq g(>)$

Distance function between f and g counts 1 on this preference profile, since the two SCFs f and g choose different alternatives.

Least-dictatorial voting rules

Definition:

The set of least-dictatorial voting rules are the rules for which the distance function is the greatest for the closest dictatorial rule.

- Formally,

$$\mathcal{F}_{ld} = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \min_{i \in N} \rho(f, d_i) \geq \min_{i \in N} \rho(f', d_i) \right\}$$

Least-dictatorial voting rules

- Consider the following preference profile >:

dictator

$>_1$	$>_2$	$>_3$
b	a	a
c	b	b
b	c	c

Let f be plurality rule

Let g be the Borda rule

Let d_2 be dictatorial rule

tie-breaking rule: a>b>c

Least-dictatorial voting rules

- Consider the following preference profile $>$:

dictator

$>_1$	$>_2$	$>_3$
b	a	a
c	b	b
b	c	c

Let f be plurality rule

Let g be the Borda rule

Let d_2 be dictatorial rule

tie-breaking rule: a>b>c

$f(>) = a$ is the plurality winner

$g(>) = b$ is the Borda winner

$d_2(>) = a$

Least-dictatorial voting rules

- Consider the following preference profile >:

dictator

$>_1$	$>_2$	$>_3$
b	a	a
c	b	b
b	c	c

Let f be plurality rule

Let g be the Borda rule

Let d_2 be dictatorial rule

tie-breaking rule: a>b>c

f(>) = a is the plurality winner

g(>) = b is the Borda winner

$\mathbf{d_2(>) = a}$

$\mathbf{f(>) = d_2(>)}$ $\varrho(f,d_2) = 0$

$\boxed{\mathbf{g(>) \neq d_2(>)} \quad \varrho(g,d_2) = 1}$

Least-dictatorial voting rules

- Consider the following preference profile >:

dictator

$>_1$	$>_2$	$>_3$
b	a	a
c	b	b
b	c	c

Let f be plurality rule

Let g be the Borda rule

Let d_2 be dictatorial rule

f(>) = a is the plurality winner

g(>) = b is the Borda winner

$\mathbf{d_2(>) = a}$

$\mathbf{f(>) = d_2(>)}$ $\varrho(f,d_2) = 0$

$\boxed{\mathbf{g(>) \neq d_2(>)} \quad \boldsymbol{\varrho(g,d_2) = 1}}$

What is left to see is what happens on all preference profiles and calculate the distances.

Least-dictatorial voting rule

- The **reverse-plurality** rule is the least-dictatorial voting rule.

- The algorithm to find it:

  Step 1: if there is a unique alternative being the fewest times on the top (incl. 0 cases), then choose it.

  Step 2: If not, disregard those alternatives that are not the fewest times on the top, and select the chosen alternative based on the given tie-breaking rule.

Balanced voting rules

- *Alternative*: to get as close as possible to **all** dictators at the same time.
- The balanced solution with respect to all dictators.
- We minimize the sum of the distances to all n dictators.

Balanced rules

Definition:

The set of balanced rules are rules for which the distance measure is the smallest.

$$\mathcal{F}_b = \left\{ f \in \mathcal{F} \mid \forall f' \in \mathcal{F} : \sum_{i \in N} \rho(f, d_i) \leq \sum_{i \in N} \rho(f', d_i) \right\}$$

Equivalent formulation of the balanced rules → they maximize the number of cases in which a top alternative of a voter is chosen.

Balanced rule

- The **plurality rule** = the balanced rule.
- The plurality rule can be considered as a kind of *compromise* between all dictatorial rules.

Final remarks

- We were motivated by the negative results from the two cornerstone theorems in social choice theory, both of which point to **dictatorship.**

- We asked: what rule will we obtain if we get away from dictatorial rule.

- We searched for the *least-dictatorial* rules.

Final remarks

- We found that the rule that is furthest away from the closest dictatorial rule is **reverse-plurality rule** (the least-dictatorial rule).

- It still violates many desirable properties.

- This questions the necessity to completely eliminate *dictatorial component* of a voting rule.

Final remarks

- **Opposite approach**: to look for the rule balancing between all dictators → **balanced rules**.

- We were maximizing the sum of the distances to all dictators → "collective" dictatorship.

- We found that the **plurality rule** and the balanced rule are the same.

- Plurality rule *minimizes* collectively the distances from the dictatorial rules.

Open questions

- Consider other metrics.

$$\rho_w(f,g) = \sum_{\succ \in \mathcal{P}} \underbrace{w(\succ)}_{\text{weight function}} \overbrace{1_{f(\succ)\neq g(\succ)}}^{\text{whether two alternatives differ}}$$

- Consider all distribution of preference profiles, not just the top alternatives.

Outline

"We have an agreement in principle.
The question is, do we all have the same principles?"

- Introduction
- Chapter 2: Primaries on demand
- Chapter 3: A mechanism to pick the deserving winner
- Chapter 4: Does avoiding bad voting rules leads to good ones?
- **Chapter 5: Dictatorship versus manipulability**

joint with D. Bednay and A. Tasnádi, Corvinus University of Budapest

Motivation

Gibbard-Satterthwaite Theorem (Gibbard, 1973; Satterthwaite, 1975):

for at feast three alternatives, every universal and resolute social choice function is either **dictatorial** or **manipulable**.

When choosing a voting rule → dilemma between **dictatorship** and **manipulability**.

Motivation

Two incompatible properties: dictatorship and manipulability.

1) Can we know to what degree a voting rule is manipulable?

2) And to what degree a voting rule is dictatorial?

Motivation

Positive answer to the first question:

- Strategy-proofness can be measured by counting the number of profiles on which SCF is manipulable.

- Nitzan-Kelly index of manipulability, **NKI** (Nitzan 1985; Kelly 1993).

- A voting rule is less manipulable for which NKI is the smallest.

- For more see Aleskerov and Kurbanov, 1999; Aleskerov et al. 2011, 2012 among others.

Motivation

- In this chapter we try to answer to the second question.

- Based on Bednay, Moskalenko and Tasnádi (2017) we can define non-dictatorship index.

Notations (from Chapter 4)

Non-dictatorship index (NDI):

counts the number of profiles for which a SCF f chooses different alternative than the closest dictatorial voting rule d_i.

- Formally,

$$NDI(f) = \min_{i \in N} \rho(f, d_i)$$

Our goal

To explore the relationship between manipulability and non-dictatorship indices, NKI and NDI.

For the following voting rules:

- Plurality
- Borda count
- Copeland
- Black's procedure
- k-Approval voting rule (k = 2 and k = 3)

Our goal

To explore the relationship between manipulability and non-dictatorship indices, NKI and NDI.

For the following voting rules:

- Plurality → chooses alternative ranked first by max number of voters
- Borda count → chooses alternative with the highest Borda score
- Copeland → chooses alternative that beats other alternatives by pairwise comparison
- Black's procedure → chooses a Condorcet winner if exists, otherwise chooses a Borda winner
- k-Approval voting rule (k = 2 and k = 3) → chooses alternative admitted to be among k best by a max number of voters

Our goal

- Less manipulable voting rule has the smallest NKI.

- Similarly, less dictatorial voting rule has the highest NDI.

- Ideal combination = small NKI and high NDI.

Computation scheme

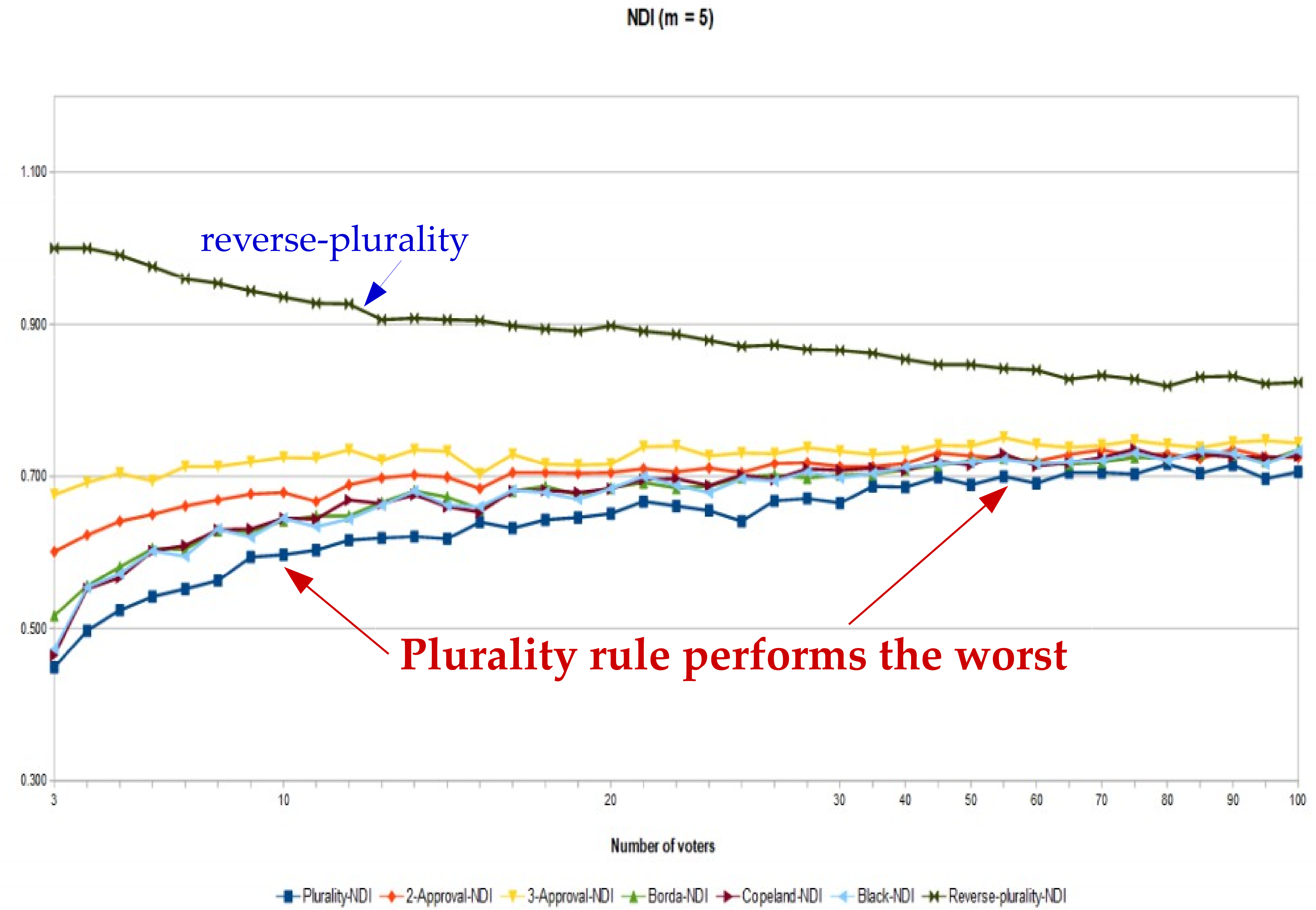
- Information about NKIs is taken from http://manip.hse.ru/index.html (created by F. Aleskerov et al.).

- For NDI we write our own program.

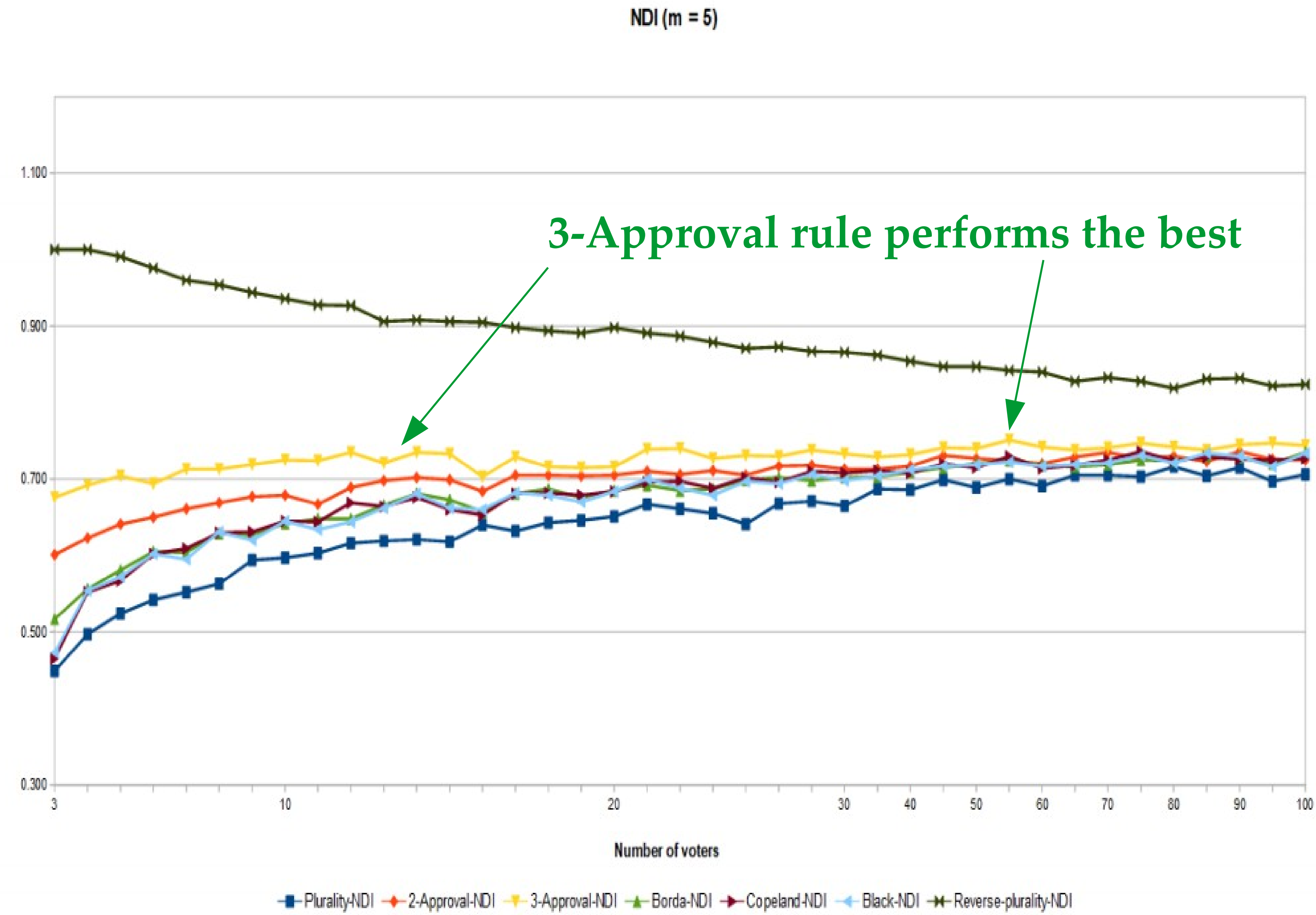
- We calculate NDIs for three, four and five alternatives.

- Up to 100 voters, by generating 1000 random preference profiles , where each profile is selected with the same probability.

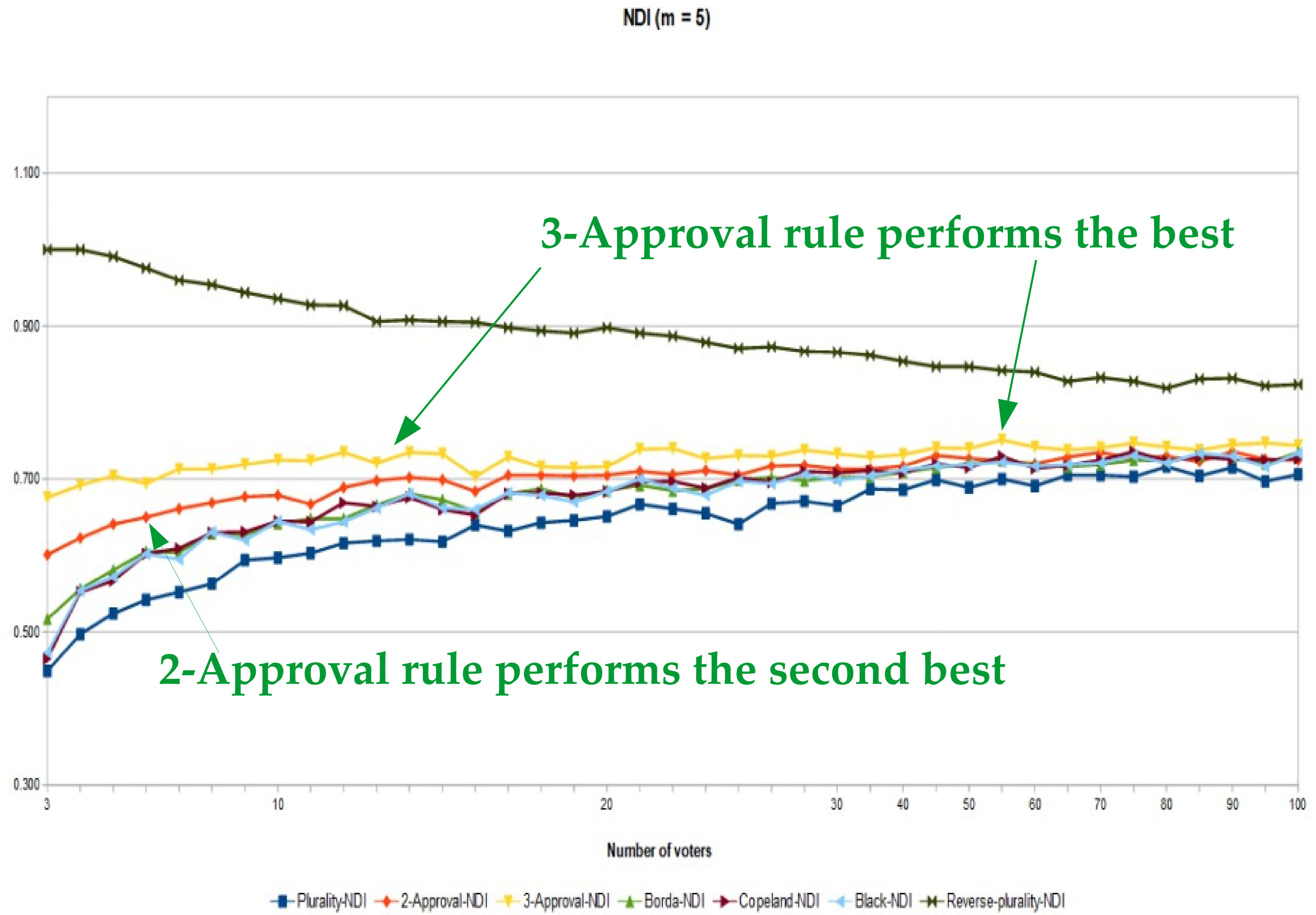
NDI for five alternatives



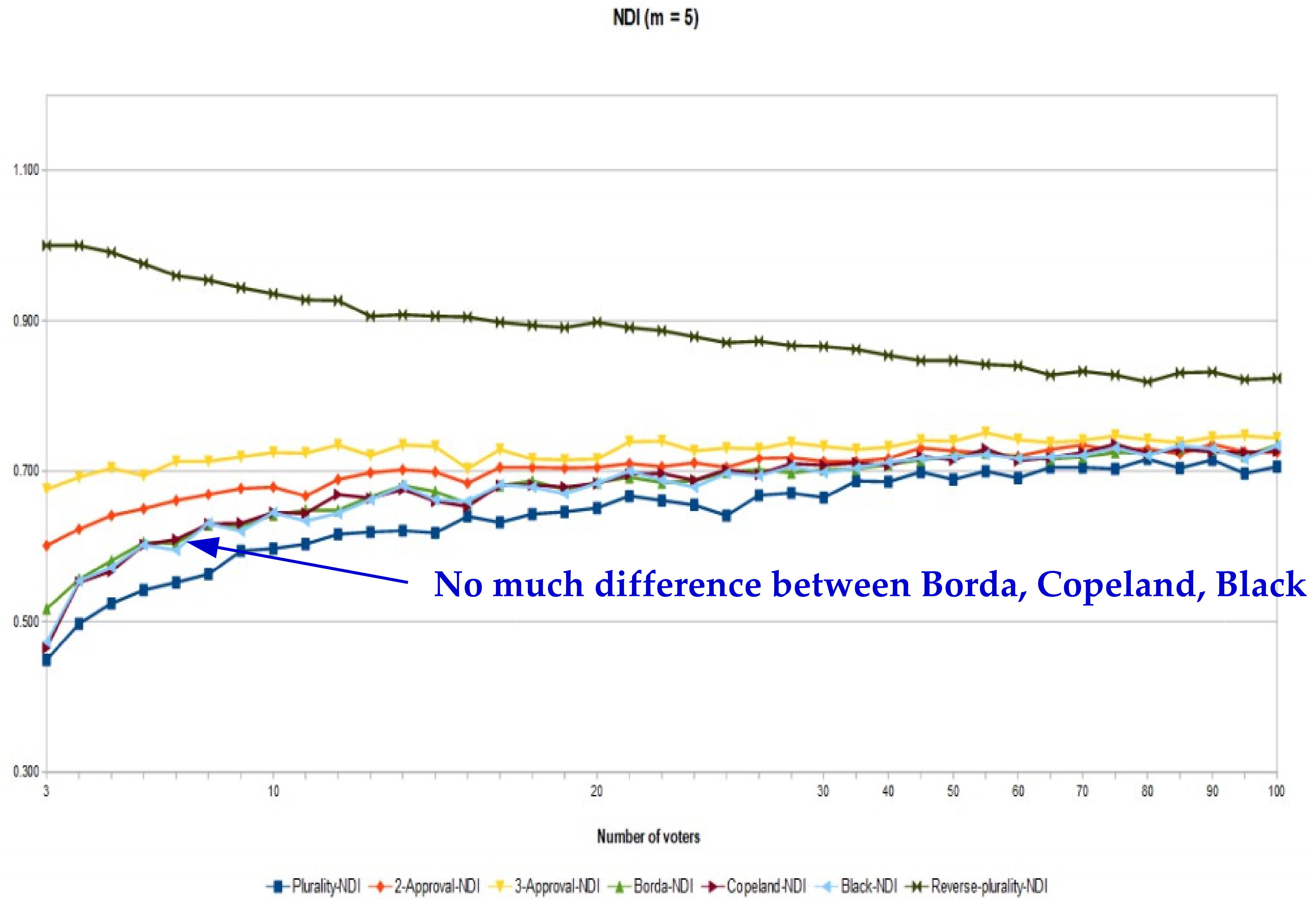
NDI for five alternatives



NDI for five alternatives



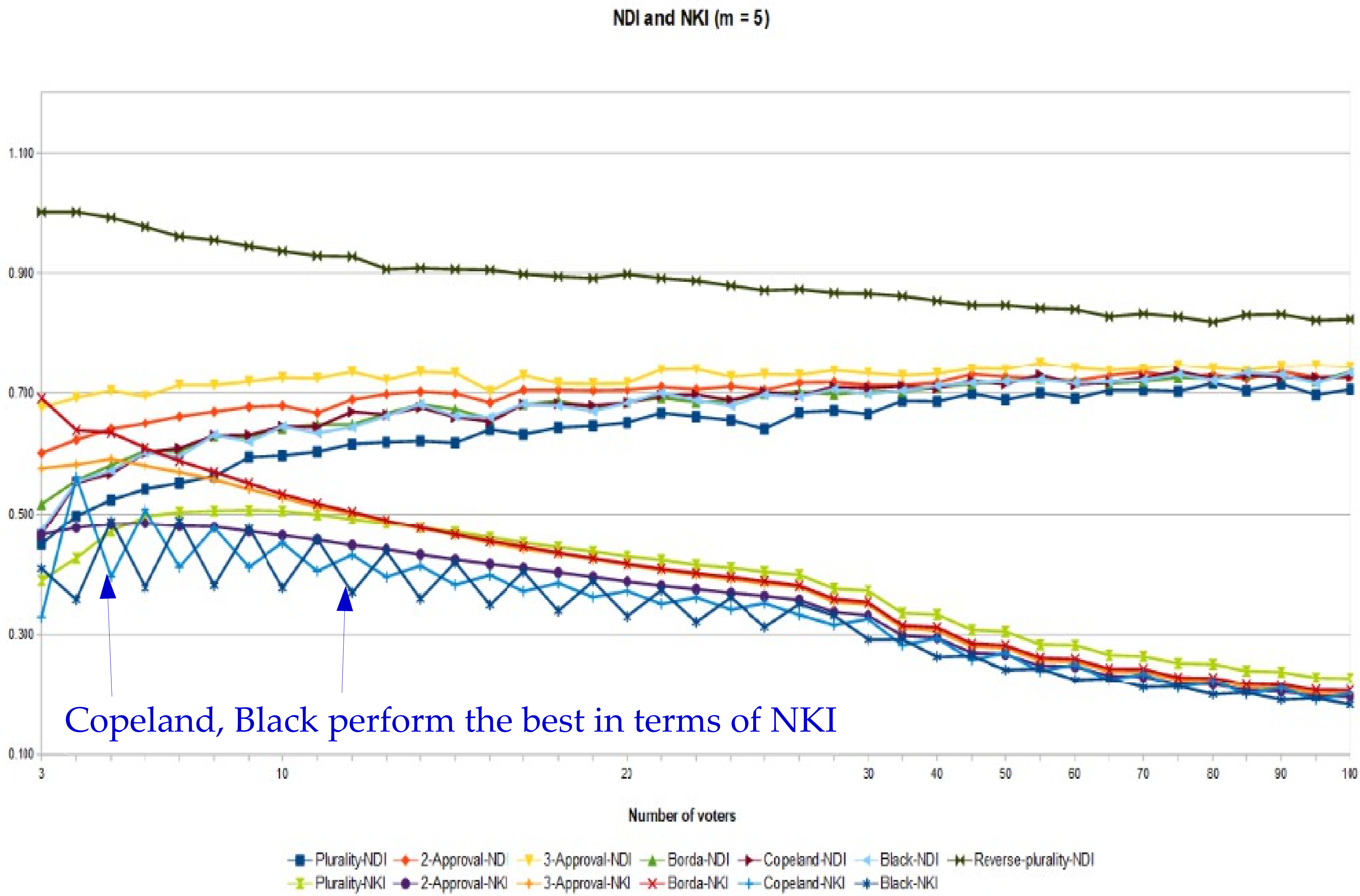
NDI for five alternatives



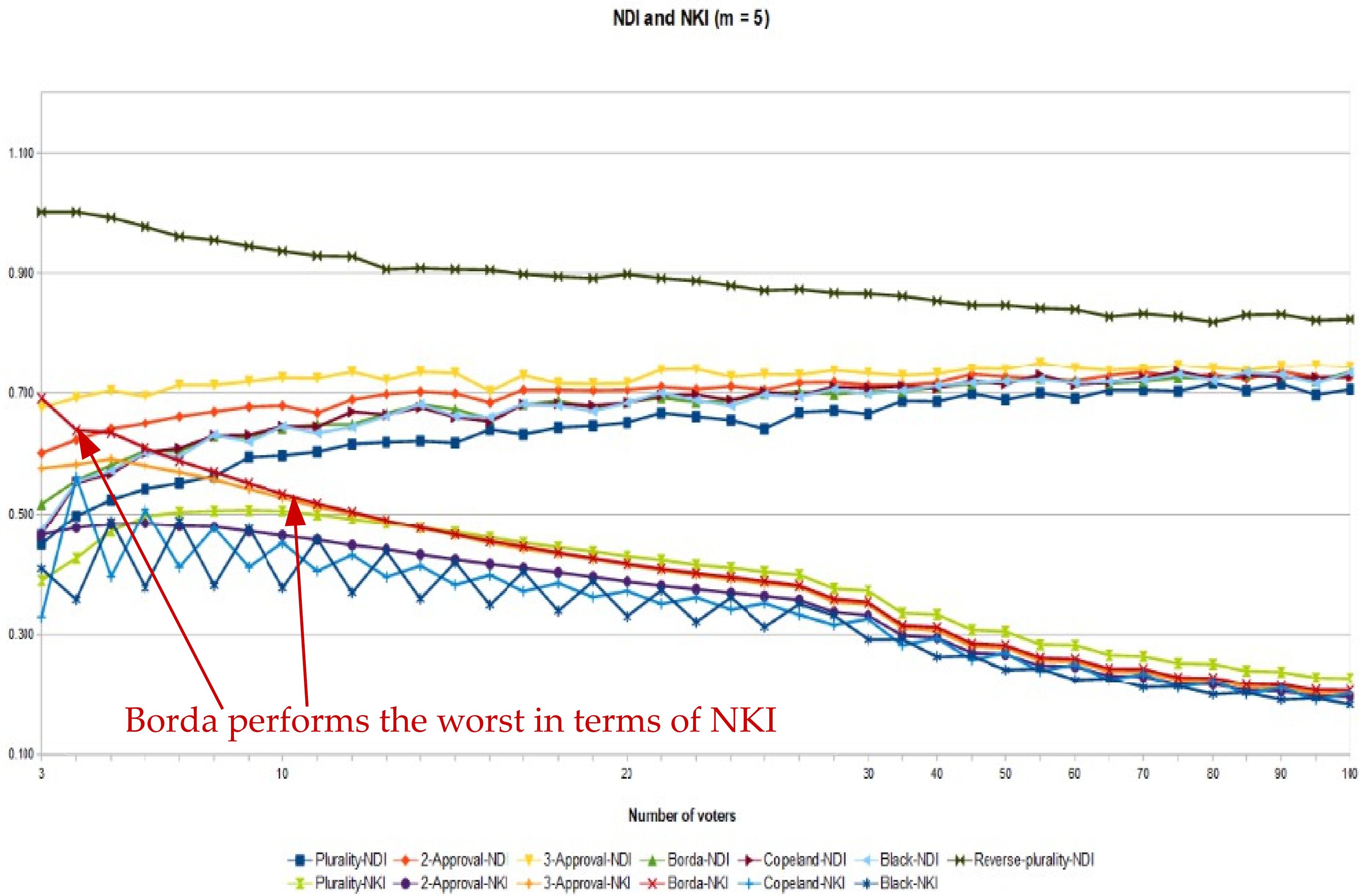
Observations

- Reverse-plurality serves as a benchmark (though it is outperformed by reverse dictatorial, which is not anonymous)
- Plurality rule performs the worst
- 3-approval voting rule is the best from the investigated voting rules
- Borda, Black and Copeland lie between plurality and 3-approval voting rules without clear difference between them
- If we add now NKI, do they converge to the same limit?

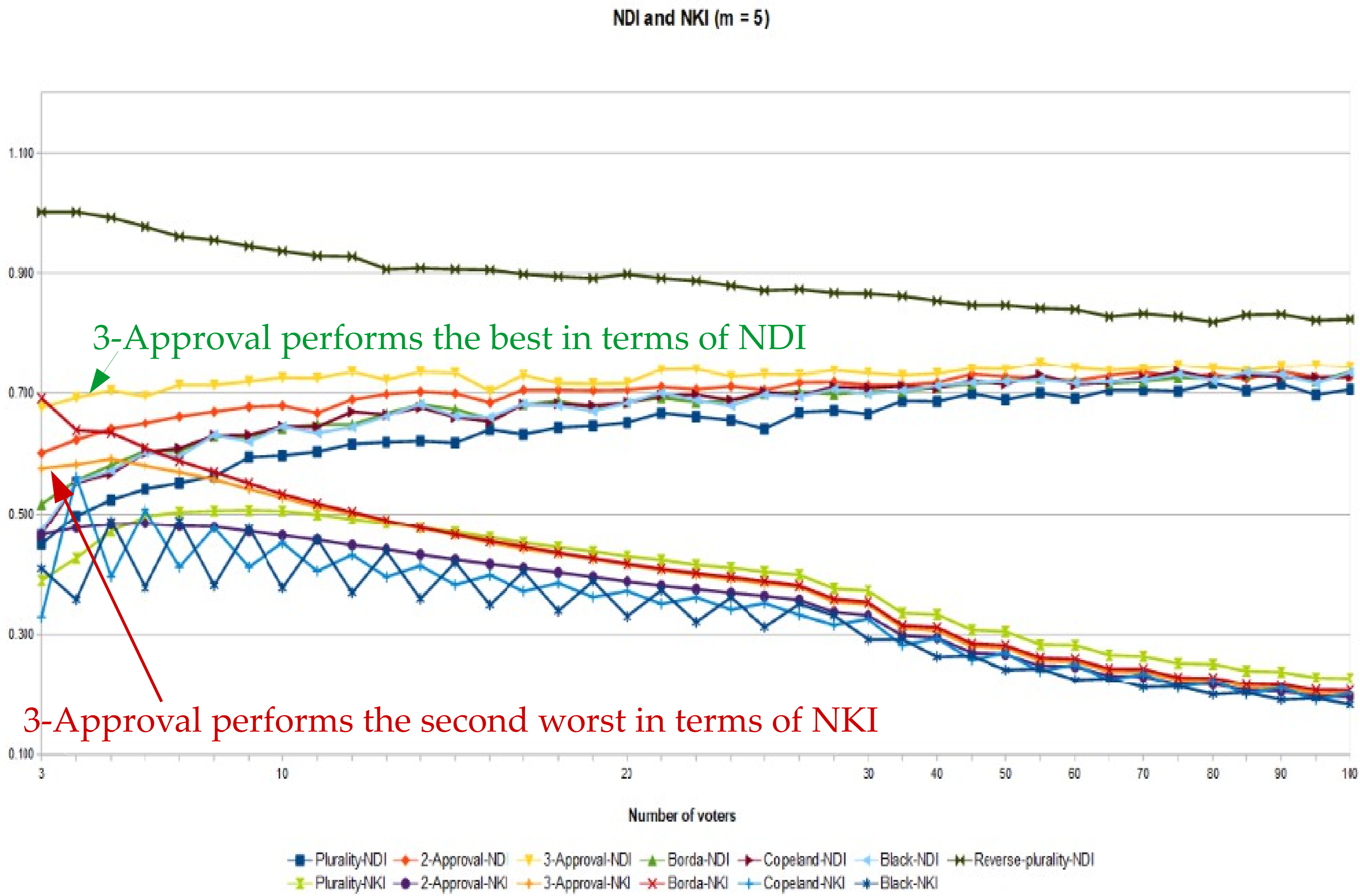
NDI and NKI for five alternatives



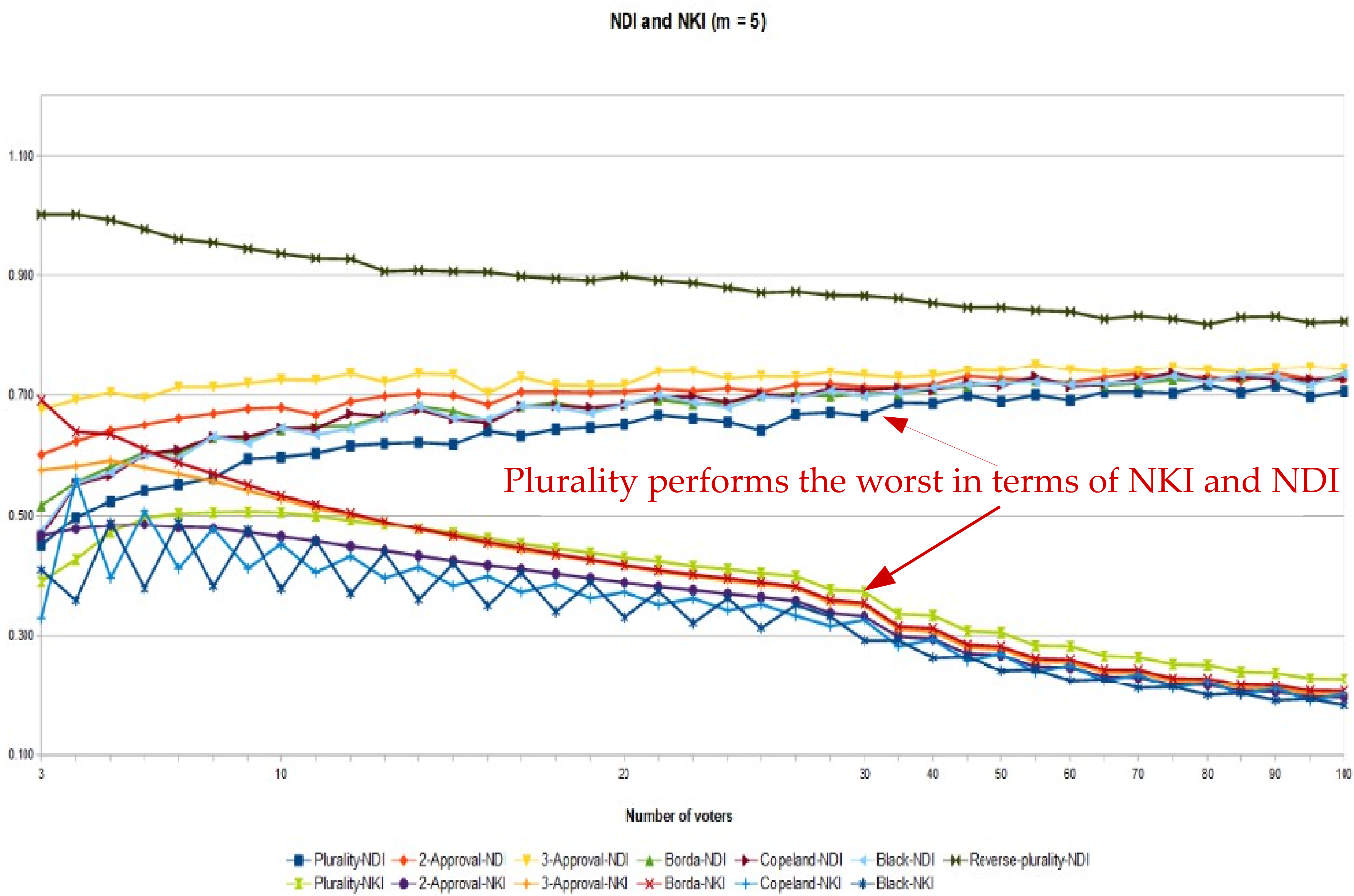
NDI and NKI for five alternatives



NDI and NKI for four alternatives



NDI and NKI for four alternatives



Observations and remarks

- NDI and NKI move in different directions → plausible and positive sign of our non-dictatorship index.

- There is no voting rule which performs the best in terms of both indices.

- In both cases, plurality rule performs the worst.

- 3-Approval voting performs the best in terms of NDI, however, it is the second worst in terms of NKI.

Conclusions

- Both undesirable properties of dictatorship and manipulability are incompatible.

- It could be helpful and informative to classify the voting rules in terms of their degree of manipulability and distance to dictatorship.

- There could be different ways of measuring the dictatorial component of a voting rule.

- We have chosen a straightforward distance based approach.

- Chapter 3 is published as “A mechanism to pick the deserving winner” in Economics Bulletin, 2015.

- Chapter 4 is published as “Does avoiding bad voting rules lead to good ones” in Operations Research Letters, 2017.

- Chapter 2 and Chapter 5 are submitted and under review.

Thank you