A mechanism to pick the deserving winner[†]

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25th of June, 2013

Abstract

A group of individuals is choosing an individual (winner) among themselves. There exists a deserving winner, whose identity is common knowledge among the individuals. Each individual is selfish and always prefers to be chosen as a winner. But at the same time, if he is not chosen, he prefers the deserving winner to be selected. A simple mechanism of voting by veto is proposed as an alternative to the mechanism studied by Amorós (2011). Like Amorós'(2011), the suggested mechanism implements the socially desirable outcome (the deserving winner is chosen) in subgame perfect equilibria.

Keywords: Implementation, mechanism design, subgame perfect equilibrium, individuals choosing among themselves, voting by veto.
JEL classification: C72, D71, D78

[†] This paper is the final work to obtain the Master degree in Economics at the Universitat Rovira i Virgili. This paper has been benefited from comments and suggestions by my supervisor, Antonio Quesada.

1. Introduction

Voting is the most popular way of aggregating the individuals' preferences to reach a socially optimal goal. This paper considers the following problem. There are n voters in a committee who have to choose one candidate among themselves. Each voter has his own opinion about each candidate; he is biased by one of the candidates (himself) and ranks each candidate differently.

Think of a contest where the jury has to choose a winner among themselves. Each jury member knows who deserves to be the winner, but at the same time, he always wants to win the contest. Or, for instance, when there is a problem of choosing a delegate or a leader of some group, and the group members have to choose a winner among themselves by voting. Each member of the group knows who deserves to win, but at the same time each of them always wants to be selected as the winner (leader, etc).

Dealing with a problem of individual preferences aggregation always concerns the selfishness of the individuals. In most cases the individuals only care about their own private interests towards different outcomes, and each individual makes decisions to pursue his own individual objectives. Here the question arises: Is it possible to design a mechanism or institution (or when speaking on voting issues, a voting mechanism) so that no matter how selfish the individuals are, their actions will always lead to the outcome that is socially desirable? In other words, given the socially desirable outcome, is it possible to create the conditions according to which every (in some sense, optimal) action of the individuals results in it?

It is to handle this problem that the implementation theory or mechanism design intervenes. The issue of the implementation theory or mechanism design consists in designing a mechanism (or a game form) in which agents (individuals) interact. One can think of a mechanism design as a reverse game theory. A mechanism, or a game form, specifies the rules of a game. The players are the members of the society (agents, individuals), who interact according to the rules of this game. The interactions of agents (individuals) result in an outcome that the mechanism generates in equilibrium. The question is whether the equilibrium outcomes will be socially optimal. The problem is how to design a mechanism such that the equilibrium behaviour of the players will lead to socially desirable outcomes, no matter how selfish the individuals are. The socially desirable outcome is prescribed by a social choice rule. If, in each possible state of the world, the equilibrium outcome of the mechanism equals the set of optimal outcomes prescribed by the social choice rule, then this mechanism is said to implement the social choice rule.

The literature on implementation theory, like that on game theory, uses the game theoretic solution concepts which describe the agents' behaviour within the game: the notion of dominant strategies, Nash equilibrium (in case of complete information), Bayesian Nash equilibrium (in case of incomplete information), and subgame perfect equilibrium as a refinement of Nash equilibrium, the one with which this work is particularly concerned[1].

The general implementation framework is the following. There is a set of agents (individuals) and a set of feasible outcomes (or alternatives) A. Each agent i has his own preferences over the outcomes R_i. There is a social choice function $f(R)$ that associates an outcome with each profile of the agents' preferences. A mechanism, or a game form g, endows each agent with a strategy set S_i, and maps the vector of the strategies chosen by the agents into an outcome. Given a social choice function f, there exists a mechanism g, such that when the agents with preference profiles R play the corresponding game, the unique equilibrium outcome is $f(R)$.

Amorós (2011) studies the particular case where the set of agents (individuals) and a set of outcomes (alternatives) coincide. Specifically, a group of agents have to choose an outcome (a winner) among themselves. There exists a winner, called "the deserving winner" w, whose identity is common knowledge among all the individuals of the group. The socially optimal rule establishes that the deserving winner wins. However, each individual i is selfish: he always wants to be selected as a winner, no matter who the deserving winner is. But at the same time, if i is not selected, he prefers w to be chosen, i.e. i and w are, respectively, the most and the second most preferred outcomes for i.

[1] For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame (see Moore and Repullo, 1998; Abreu and Sen, 1990).

To reach the socially desirable outcome (the deserving winner wins), Amorós (2011) proposes a mechanism à la Maskin (1999) that implements the socially desirable function in subgame perfect equilibria. Despite the fact, that these types of mechanisms have been criticised for being unnatural and quite abstract (see Jackson, 1992), these mechanisms characterize what can be implemented and they are able to handle a large number of situations (Serrano, 2004). When dealing with a specific situation, more detailed mechanism has to be designed. This is precisely what Amorós (2011) does in his work.

He proposes a simple and "natural" extensive form mechanism. In the mechanism agents take turns announcing the winner. The announcement of the first agent is implemented only if he announces an individual different from himself. Otherwise, the turn passes to the next agent, and the process is repeated. The announcement of the last agent is implemented, even if he announces himself as a winner. The mechanism is such that truth-telling is an equilibrium, and any subgame perfect equilibrium results in the deserving winner.

This paper replicates Amorós' (2011) result by suggesting an alternative mechanism. The proposed mechanism can be considered as a reversal of the one by Amorós. What if the individuals instead of voting for a most preferred candidate, are given the possibility reverse to voting, that is, vetoing a candidate? The question that motivates this paper is this: Would it be possible to reach the socially desirable outcome, the deserving winner election? The results of this paper provide the proof that it is.

A mechanism of voting by veto (hereinafter, veto mechanism) also implements the desired social choice function (the deserving winner wins) in subgame perfect equilibria. Moreover, the proposed veto mechanism works for three individuals, improving upon Amorós'(2011), whose mechanism needs at least four individuals to work and fails with three individuals.

The rest of the paper is organized as follows. Section 2 provides the model. Section 3 describes the veto mechanism. Section 4 analyses the case of the implementation of the socially optimal rule in subgame perfect equilibria with $n = 3$ individuals. Section 5 presents the general results of veto mechanism. Section 6 provides the conclusions.

2. The model

Let $N = \{1, 2, ... , n\}$ be a set of $n \geq 3$ individuals who must choose one individual (the winner) among them. All individuals know who deserves to win: the "deserving winner", denoted by $w \in N$. The socially optimal outcome is that the deserving winner wins. However, each individual is selfish: he always wants to win the election. But at the same time, if he is not chosen as a winner, he prefers the deserving winner to be chosen. The formal definitions follow.

The individuals have preferences defined over N. A preference of individual i can be considered as i's ranking of all individuals in the group, including himself, from most to less preferred individual. The set of preference profiles over N is denoted by $\boldsymbol{R}$. For a profile $R \in \boldsymbol{R}$, each individual $i = 1, \ldots , n$ has preference function R_i: $N \rightarrow \boldsymbol{R}$ which, given a deserving winner, $w \in N$, associates with each individual i in N a preference relation $R_i \in \boldsymbol{R}$. Let P_i denote the strict preference relation of R_i, that is, i strictly prefers i to j, $j \in N \setminus \{i\}$, $i\, P_i\, j$, if and only if $i > j$. This can be interpreted in the following way: if individual i is given the possibility to choose any individual j in the group, $j \in N \setminus \{i\}$, i will always choose himself, as he ranks himself as his most preferred alternative (individual).

Definition 1. For each individual $i \in N$ the preference function R_i: $N \rightarrow \boldsymbol{R}$ is admissible if:

1. for each $j \in N$ such that $j \neq i$, $i\, P_i\, j$
2. for each $j \in N$ such that $j \neq w$ and $j \neq i$, $w\, P_i\, j$

Denote by $\boldsymbol{R_i}$ the set of all preference functions that are admissible for individual i. A social choice function with deserving winner w is a function f_w: $\boldsymbol{R_i} \rightarrow N$ that, for every preference profile of individuals, selects the deserving winner w: for all $R \in \boldsymbol{R_i}$, $f_w(R) = w$.

An extensive form mechanism, denoted by $\Gamma(M, g)$, consists of a set of (pure) strategies profiles of all individuals $M = \Pi_{i \in N} M_i$ and an outcome function $g\colon M \rightarrow N$ that associates an individual $g(m)$ with each profile m of messages. A pure strategy for individual i is a function that specifies the choice of individual i at every stage of the game. For every profile $R \in \boldsymbol{R_i}$, the pair (Γ, R) constitutes an extensive form game. The mechanism is sequential and specifies the order in which the individuals play their pure strategies. It is a game of perfect information, as each individual, when playing his pure strategy, knows the previous history of the game, and acts according to this history. A subgame perfect equilibrium (SPE) of a perfect information game is a strategy profile that induces an equilibrium in every subgame of the game. The social choice function f_w is subgame perfect equilibria implementable if there exists a sequential mechanism Γ such that SPE$(\Gamma, R) = f_w(R) = w$ for all $R \in \boldsymbol{R_i}$, i.e. for each preference profile R, the only subgame perfect equilibrium outcome is w.

The sequential extensive form mechanism is proposed in the next section.

3. The mechanism

The structure of the veto mechanism is the following. The individuals are arranged in any linear order $(1, 2, \ldots, n)$. Each individual takes turn to announce an individual to veto, starting with individual 1. When 1 has vetoed an individual, individual 2 then takes his turn to veto. Each individual can only be vetoed once, so no individual can be vetoed twice. Once individual $n - 1$ has made his announcement, there only remains one individual v that has not been vetoed. Let z be the first individual in the ordering $(1, 2, \ldots, n - 1)$ that does not veto himself, if such individual exists. If no such individual z exists or $v \neq z$, then v is chosen as a winner; if $v = z$, then n chooses between v and the individual v^* vetoed by v.

The described veto mechanism needs at least three individuals to work, improving upon Amorós' (2011), who requires at least four individuals.

4. **The three individual case**

This section analyses the mechanism when there are three individuals $n = 3$ considering the different positions that w can occupy in the linear order. This analysis will demonstrate that all SPE paths lead to the election of the deserving winner w as a final outcome, i.e. $SPE(\Gamma, R) = f_w(R) = w$.

Lemma 1. *For* $n = 3$ *the veto mechanism implements the social choice function* f_w *in subgame perfect equilibria*.

Proof. Suppose that the linear order is (1, 2, 3). It will be demonstrated that f_w is implementable in subgame perfect equilibria by means of a mechanism $\Gamma(M, g)$.

Case 1: $w = 1$. The game starts with individual 1 announcing his veto. Individual 1 has three options: to veto 1, to veto 2 or to veto 3. Each option leads to a different path.

Path 1 (Fig. 1). 1 vetoes 1. Then 2 chooses to veto between the remaining individuals: either 2 or 3.

If 2 vetoes 2, and as there does not exist z and $v = 3$, so $v = 3$ is chosen as a winner.

If 2 vetoes 3, $z = 2$ and $v = 2$, consequently, $v = z = 2$, so $n = 3$ chooses the winner between $v = 2$ and $v^* = 3$. As 3 P_3 2, 3 chooses himself as the winner.

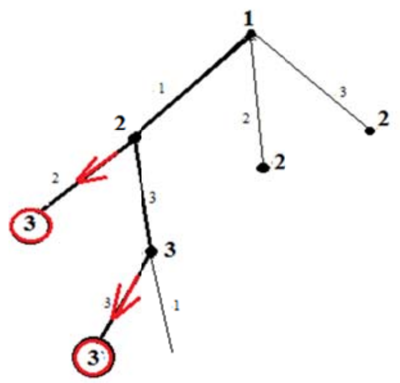

Fig.1. Path 1 when $w = 1$.

Path 2 (Fig. 2). 1 vetoes 2. In the next move, 2 vetoes the remaining individuals: either 1 or 3.

If 2 vetoes 1, and as $v = 3 \neq z$, $v = 3$ becomes the winner.

If 2 vetoes 3, as $v = z = 1$, $n = 3$ chooses the winner between $v = 1$ and $v^* = 2$. As 1 P_3 2, $1 = w$ is chosen as the winner.

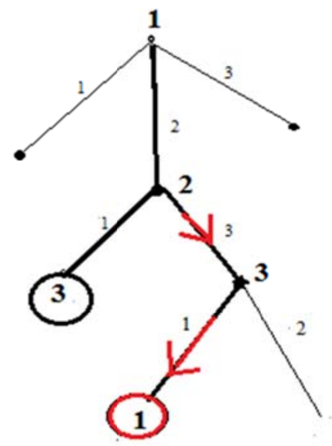

Fig.2. Path 2 when $w = 1$.

Observe 2's node: when 2 selects his move whom to veto, either 1 or 3, the outcome when vetoing 1 leads to the election of 3, while vetoing 3 leads to the election of 1. As 1 P_2 3, therefore, at 2's node the best option for 2 is to veto 3. As a result, 1 is the winner.

Path 3 (Fig. 3). 1 vetoes 3. In the next move, 2 chooses between the remaining individuals to veto: either 1 or 2.

If 2 vetoes 1, $v = 2$ and $z = 1$, as $z \neq v$, $v = 2$ is chosen as the winner.

If 2 vetoes 2, $z = v = 1$, therefore, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. Since 3 P_3 1, 3 will choose $v^* = 3$ as the winner.

Observe 2's node: when making the choice whom to veto, as vetoing 1 will lead to the outcome when 2 is chosen as the winner, while when vetoing 2 leads to the outcome when 3 wins, and as 2 P_2 3, 2 will choose the strategy to veto 1. In view of this, at 2's node the best option for 2 is to veto 1. This makes 2 the winner.

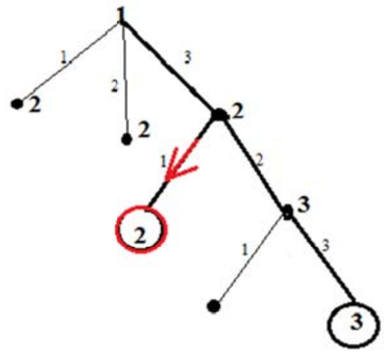

Fig.3. Path 3 when $w = 1$.

Going backwards to 1's node (see Fig. 4), when it is 1's choice to veto an individual among 1, 2, 3, the best strategy for 1 is to veto 2, as it is the only strategy that leads to the best outcome for 1: $1 = w$ is chosen as the winner.

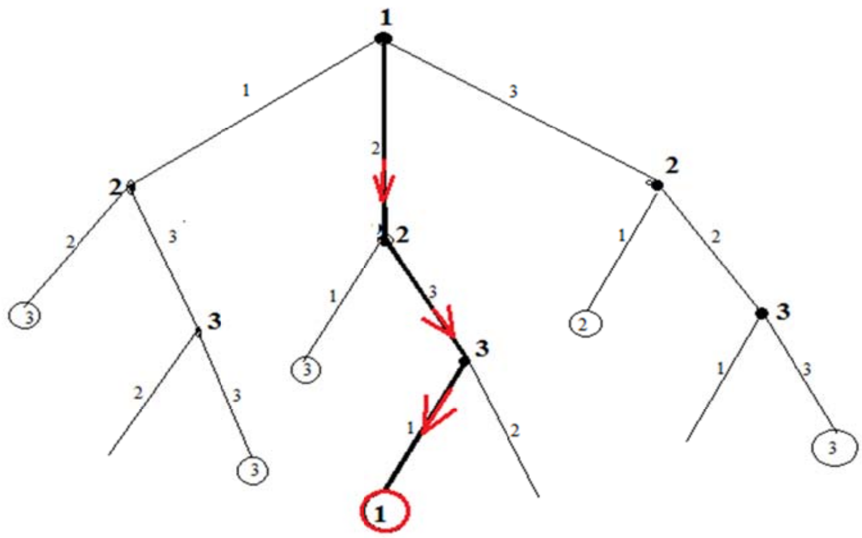

Fig. 4. SPE outcome when $w = 1$.

It has been demonstrated that when $w = 1$, all subgame perfect equilibria lead to the outcome that the deserving winner wins.

Case 2: $w = 2$.

The game starts from the individual 1 making his choice to veto an individual. Individual 1 has three options: to veto 1, to veto 2 or to veto 3. Each option leads to different paths.

Path 1 (Fig. 5). 1 vetoes 1. Then 2 chooses between the remaining individuals to veto: either 2 or 3.

If 2 vetoes 2, as no z exists and $v = 3$, $v = 3$ is chosen as the winner.

If 2 vetoes 3, as $v = z = 2$, $n = 3$ then picks the winner between $v = 2$ and $v^* = 3$. As 3 P_3 2, 3 choses himself as the winner.

Going backwards to 2's node: 2 is indifferent between vetoing 2 or 3, as both strategies lead to the outcome, when 3 is chosen as the winner.

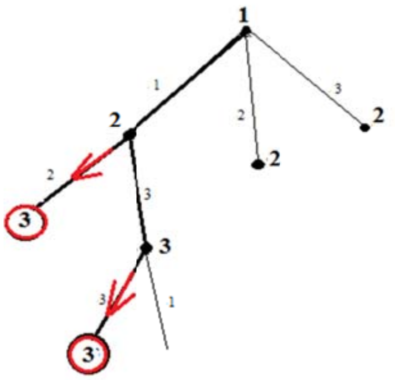

Fig. 5. Path 1 when $w = 2$.

Path 2 (Fig. 6). 1 vetoes 2. In the next move, 2 chooses between the remaining individuals to veto: either 1 or 3.

If 2 vetoes 1, as $z = 1$, $v = 3$, and as $z \neq v$, $v = 3$ is chosen as the winner.

If 2 vetoes 3, in the next move as $v = z = 1$, $n = 3$ chooses the winner between $v = 1$ and $v^*= 2$. As 1 P_3 2, $2 = w$ is chosen as the winner.

Going backwards to 2's node: when 2 chooses his move whom to veto, either 1 or 3, and as the outcome when vetoing 1 leads to the election of 3, while vetoing 3 leads to the election of 2, and as 2 P_2 3, the best choice of 2 is to veto 3.

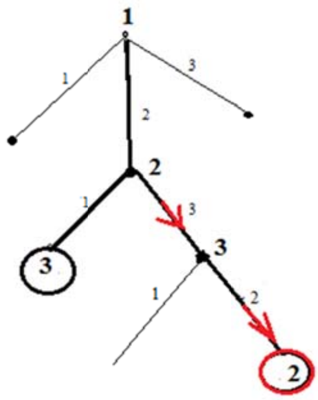

Fig. 6. Path 2 when $w = 2$.

Path 3 (Fig. 7). 1 vetoes 3. In the next move, 2 chooses between the remaining individuals to veto: either 1 or 2.

If 2 vetoes 1, $z = 1$ and $v = 2$, and as $z \neq v$, $v = 2$ is chosen as the winner.

If 2 vetoes 2, $z = v = 1$, and since $z = v$, therefore, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. As 3 P_3 1, 3 will choose $v^* = 3$ as the winner.

Going backwards to 2's node (see Fig. 7): when making the choice whom to veto, as vetoing 1 will lead to the outcome when 2 is chosen as the winner, while when vetoing 2 leads to the outcome when 3 is chosen as the winner, and since 2 P_2 3, the best choice for 2 is to veto 1. Therefore, $2 = w$ is chosen as the winner.

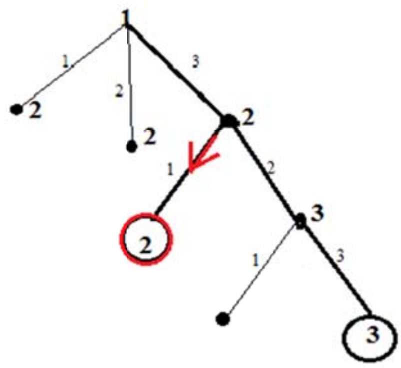

Fig. 7. Path 3 when $w = 2$.

Going backwards to 1's node (see Fig. 8): since 2 P_1 3, the best choice for 1 is to veto 2 and 3, as both these paths result in the best outcome for 1: $w = 2$ is chosen as the winner.

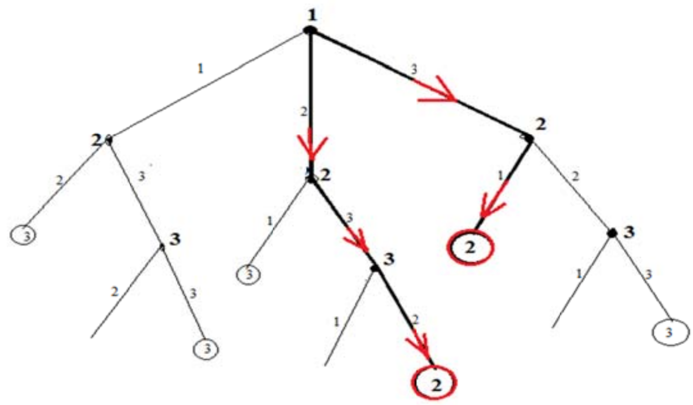

Fig. 8. The SPE outcomes when $w = 2$.

It has been demonstrated that when $w = 2$, all subgame perfect equilibria lead to the outcome that the deserving winner wins.

Case 3: $w = 3$.

The game starts from the individual 1 making his choice to veto. Individual 1 has three options: to veto 1, to veto 2 or to veto 3. Each option leads to different paths.

Path 1 (Fig. 9). 1 vetoes 1. In the next move, 2 chooses between the remaining individuals to veto: either 2 or 3.

If 2 vetoes 2, as no z exists and $v = 3$, $v = 3$ is chosen as the winner.

If 2 vetoes 3, $z = v = 2$, therefore, $n = 3$ chooses the winner between $v = 2$ and $v^* = 3$. As $3\ P_3\ 2$, $3 = w$ is chosen as the winner.

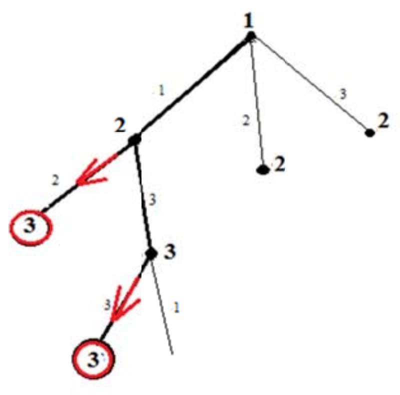

Fig. 9. Path 1 when $w = 3$.

Path 2 (Fig. 10). 1 vetoes 2. In the next move, 2 chooses the remaining individuals to veto: either 1 or 3.

If 2 vetoes 1, as $z = 1$, $v = 3$, and since $z \neq v$, $v = 3$ is chosen as the winner.

If 2 vetoes 3, in the next move as $v = z = 1$, $n = 3$ chooses the winner between $v = 1$ and $v^* = 2$. Here we have to consider two cases:

1) if 1 P_3 2, 3 will choose 1 as the winner;
2) if 2 P_3 1, 3 will choose 2 as the winner.

Going backwards to 2's node and taking these two cases into account: when 2 chooses an individual to veto, either 1 or 3, two cases have to be considered. In the first case, when 1 P_3 2, the best choice of 2 is to veto 1, since 2 prefers 3 to be chosen as the winner (in the case when $w = 3$) rather than 1. In the second case, when 2 P_3 1, the best option for 2 is to veto 3, as among 2 and 3 to be the winner, 2 prefers himself as the winner.

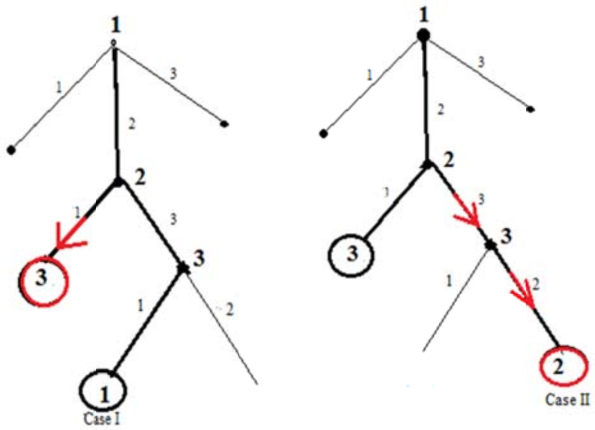

Fig 10. Path 2 when $w = 3$ (two cases)

Path 3 (Fig. 11). 1 vetoes 3. In the next move, 2 chooses between the remaining individuals to veto: either 1 or 2.

If 2 vetoes 1, $v = 2$, $z = 1$, and since $z \neq v$, $v = 2$ is chosen as the winner.

If 2 vetoes 2, $z = v = 1$, therefore, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. As 3 P_3 1, 3 will choose $v^* = 3$ as the winner.

Going backwards to 2's node: when making the choice whom to veto, as vetoing 1 will lead to the outcome when 2 is chosen as the winner, while when vetoing 2 leads to the outcome when 3 is chosen to be the winner, and since 2 P_2 3, the best option for 2 is to veto 1. But there is no SPE.

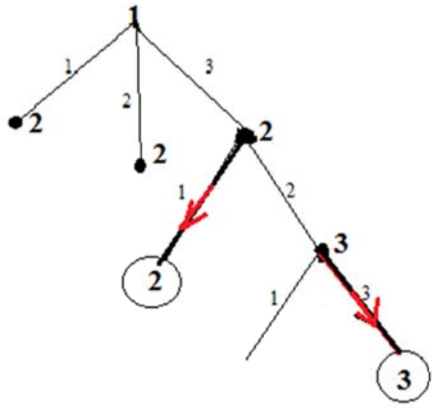

Fig. 11. Path 3 when w = 3.

Going backwards to 1's node (see Fig.12): the best option for 1 is to veto 1 in the first case (when 1 P_3 2), so that the winner is w = 1; and to veto 1 and 2 in the second case (when 2 P_3 1), thus w = 1 is chosen as the winner. Therefore, all SPE outcomes result in the election of w as the winner.

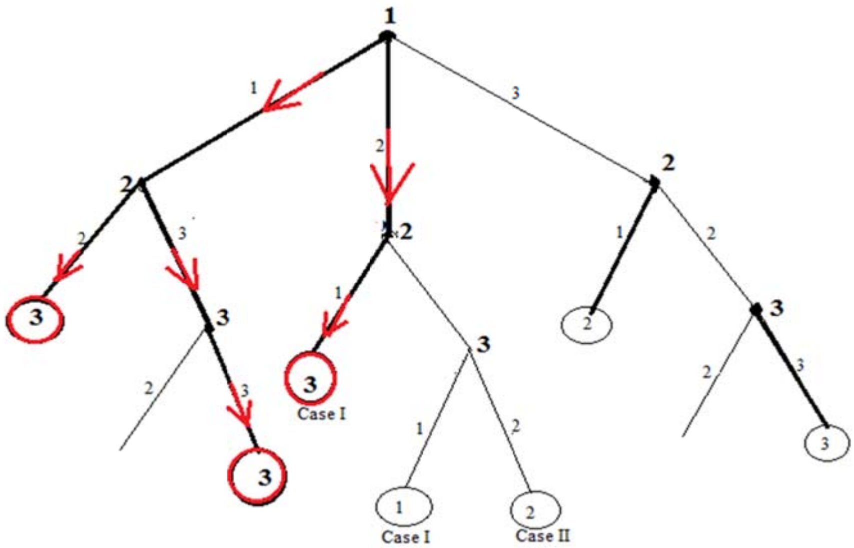

Fig. 12. The SPE outcomes when w = 3.

It has been demonstrated that when w = 3, all subgame perfect equilibria lead to the outcome when the deserving winner wins.

Lemma 1 has thus been proved. ■

5. General results

This section presents the general proof that the veto mechanism implements the social choice rule f_w in subgame perfect equilibria when there are $n \geq 3$ individuals.

Veto mechanism. Given an arbitrary linear ordering (1, 2, … , n) of the $n \geq 3$ individuals, each individual announces an individual to veto. This individual is chosen among those not having been previously vetoed by some individual in the ordering. Once individual $n - 1$ has made his announcement, there only remains one individual, v. Let z be the first individual in the ordering (1, 2, … , $n - 1$) that does not veto himself (i.e. the first individual that vetoes an individual different from himself), if such individual exists. If no such z exists or if $v \neq z$, then v is chosen as a winner; if $v = z$, then n chooses the winner between v and the individual vetoed by v.

Proof. Let (1, 2, … , n) be the ranking of the individuals according to the veto mechanism and w the deserving winner. Since Lemma 1 has already proved the result for $n = 3$, let $n \geq 4$. Taking Lemma 1 as the base case of an induction argument, suppose the result true for $n' < n$: the veto mechanism implements the deserving winner whenever $n' < n$.

Consider the path p generated when, for each $k \in \{1, \dots, n - 1\}$, k vetoes $k + 1$ if $k + 1 \neq w$ and vetoes $k + 2$ if $k + 1 = w$ (if the deserving winner is n, then $n + 1$ is 1). The only individual not vetoed along this path is w. Hence, $v = w$. Moreover, $z = 1$. Therefore, if $w \neq 1$, then w is chosen. If $w = 1$, then n chooses between w and 2; since $n \geq 3$, $n \neq 2$ and, given that n prefers w to 2, n picks w. This proves that path p leads to w.

The aim is now to prove that the part of p that starts at individual 2's node is a subgame perfect equilibrium (SPE) path: once 1 has vetoed 2, no individual along p has an incentive to depart from p. That is, no $k \in \{2, \dots, n - 1\}$ can obtain a better outcome by departing from p, given that the previous individuals remain along p.

Choose first n - 1, who is the last individual that vetoes along the path. When it is (n - 1)'s turn, the set of vetoed individuals includes himself, because n - 2 has vetoed n - 1, and does not include w. If n - 1 remains in the path by vetoing the required individual (that is, n if $n \neq w$ and 1 if $n = w$), the outcome is, as has been shown, w. The only reason for n - 1 to veto a different individual is that he (n - 1) becomes the chosen individual. If $n \neq w$ and n - 1 does not veto n, then $v = n$ and $z = 1$, so n is the chosen individual. This outcome is less preferred by n - 1 than w. If $n = w$ and n - 1 does not veto 1, then $v = 1 = z$. In this case, n chooses between 1 and 2. As $n \geq 4$, $2 \neq n - 1$, so neither 1 nor 2 is preferred by n - 1 to w. In sum, n - 1 has no incentive to leave p when the preceding individuals have chosen to remain in p.

Taking this result as the base of an induction argument, choose $k \in \{2, \dots, n - 2\}$ and suppose: (i) that all the individuals playing before k have chosen to remain in p; and (ii) if k chooses to remain in p, the outcome is w. It has to be shown that, by abandoning p, k cannot force an outcome which is better for him than w.

Since the only such outcome is k himself, it has to be shown that, by vetoing an individual r different from the one that k has to veto along p, k is not the outcome of a SPE. When it is k's turn to choose, he has been vetoed by k - 1. In view of this, $v \neq k$. Accordingly, the only way k could be chosen by the veto mechanism is to have $v = z = k$ - 1. In that case, n would have to choose between k - 1 and what k - 1 vetoed, namely, k. If $k \geq 3$, then k - 1 has been vetoed by k - 2, so $v = k$ is impossible. If $k = 2$ and it finally turns out that $v = z = k - 1 = 1$ and n prefers 2 to 1, then it could be that 2 is finally chosen. Notice that this requires two conditions. First, that $w \neq 1$: if n has to choose between w and 2, n would choose w. And second, that 1 vetoes 2. To discard that having 2 chosen is part of a SPE, it is enough to show that 1 has some other veto leading to an outcome that he prefers more than 2. Consider 1's option of vetoing himself. If 1 vetoes himself, then, since $w \neq 1$, what results is just the problem of implementing the deserving winner by means of the veto mechanism with n - 1 individuals. By the induction hypothesis according to which the veto mechanism implements the deserving winner whenever $n' < n$, the only SPE outcome in that case is w. Accordingly, by vetoing himself, 1 ensures outcome w. Since 1 prefers w to 2, choosing to stay along path p when this path leads to outcome 2 is not a SPE. Summing up, no $k \in \{2, \dots, n - 2\}$ can force an outcome which is better for him than w.

The proof concludes by showing that this is also true for individual 1. If $w = 1$, then, by remaining in p, 1 ensures that the outcome is w. In this case, w is the only SPE outcome. If $w \neq 1$, then it has to be shown that 1 cannot obtain an outcome more preferred than w by leaving the path p, that is, 1 cannot make himself elected by vetoing an individual different from 2 (which is the individual that 1 has to veto in p). As it has just been shown, w is the only SPE outcome when 1 vetoes himself. Now, suppose that, by vetoing $x \notin \{1, 2\}$, there is a SPE path that leads to outcome 1. To reach a contradiction, it is enough to show that, when 1 vetoes x, there is a SPE path that starts at 2's node and leads to outcome w. Consider the following path p^* that starts at 2's node when 1 vetoes $x \notin \{1, 2\}$: for each $k \in \{2, \dots, n - 1\}$, k vetoes $k + 1$ except if $k + 1 \in \{w, x\}$, in which case k vetoes 1 if $k + 1 = x$ and vetoes 2 if $k + 1 = w$.

A reasoning analogous to that used for path p shows that p^* leads to outcome w and that, starting from 3's node, p^* is a SPE path. To complete the proof it has to be shown that p^* is a SPE path starting at 2's node. If this is not the case, then 2 has a veto that leads to outcome 2 in a SPE path that starts at 2's node. But in this case, 1's best response is not to veto x: by vetoing x, 2 is the outcome, whereas, as shown, by vetoing himself 1 ensures outcome w, which is preferred by 1 to outcome 2.■

6. Conclusion

The problem of choosing a winner among the individuals when the identity of the deserving winner is common knowledge has been analyzed. It has been proved that the proposed veto mechanism implements the socially desirable outcome (that the deserving winner wins) in subgame perfect equilibria. The considered veto mechanism works when there are at least three individuals, improving upon Amorós' (2011), which requires minimum four individuals. Open problem that is left for further research is which class of social choice functions the veto mechanism implements.

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