# Bribing parties in alternative parliament structures[†]

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Abstract

A parliament with $n$ seats and $m$ parties must decide whether to accept or reject a certain proposal (a bill or a motion). Each member of the parliament votes in favour or against. For a given $t$, if there are at least $t$ members in favour, the proposal is accepted; otherwise it is rejected. A non-member of the parliament, the briber, is interested in having the proposal accepted. To this end, he or she is willing to bribe members who vote against to induce them to vote in favour. Restricting attention to the cases $m \in \{2, 3, n\}$ and allowing any distribution of seats among parties, this paper determines, for given values of $n$ and $t$, the proportion of cases in which the briber needs to bribe some member of the parliament and the average number of seats that the briber has to bribe (with the average taken with respect to all the possible allocations of seats among the parties and also with respect to those allocations inducing the briber to bribe).

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## 1. Introduction

Voting is the typical procedure by means of which collective decisions are made. In democratic societies citizens express their opinion through the voting for the President, the members of the Parliament, and the local authorities. Many central banks are governed by boards that vote on monetary policy options.

One desirable feature of voting procedures is that voters reveal their opinions sincerely. If it is found to be easy to strategically influence voting outcomes, one cannot be confident of the reliability and legitimacy of political decisions. One of the most significant results in economic theory, the Gibbard (1973)-Satterthwaite (1975) theorem, asserts that, in essence, a voting procedure immune to strategic manipulation by the voters themselves must be dictatorial, namely, the decision power has to be concentrated on a single individual.

The internal manipulability of voting procedures (manipulability by voters) is also related to the external manipulability or pressure exercized on voters by non-voters interested on certain outcomes of the voting procedure. This pressure can be legal (lobbying in the United States) or illegal (bribery and corruption).

The lobbying literature attributes both a positive and a negative effect to lobbying activities; see, for instance, Grossman and Helpman (2001) and Dal Bo (2007). The positive view of lobbying relies on the idea that lobbying groups provide information to policymakers (though there is a clear incentive to furnish only the information that favours the interest of the lobby, so that the decision by policymakers may be biased). The negative side is just based on the fact that lobbying groups may bribe policymakers to decide in favour of the groups. This paper is motivated by this possibility.

A parallel line of research has paid attention to the question of how different electoral systems are connected with corruption. One of the first theoretical works on this topic was Myerson (1993), who investigated the connection between different electoral rules and corruption (which he defined as a given characteristic of parties). His findings about the effectiveness of the electoral rules in eliminating corruption from the parliament were contested by some empirical works (Persson et al. (2003), Rose-Ackerman (2005), and Birch (2007), for instance) and by the later work of Myerson (1999).

The studies on electoral systems and corruption has generally tended to focus on electoral rules when electing the parliaments, but little attention seems to have been paid to what happens inside the parliament once elected. For example, Charron (2011) studies empirically the connection between party systems and corruption. Taking the electoral formula as a proxy for the number of parties, he finds that multipartism in countries with dominance of single-member districts is associated with higher levels of corruption, while the party system's relationship with corruption plays no role in countries with proportional representation.

Much less attention seems to have been devoted to the connection between corruption and the structural characteristics of the parliaments, such as the number of seats (or size) of the parliament, the number of parties with representation, and the decision rules adopted by the parliament. These characteristics might have influence on the presence of political corruption inside the parliament.

One of the most important functions of any parliament is to represent various viewpoints of the inhabitants of the country. Based on this idea, the size of the parliament mostly depends on the population of the whole country or on the population of districts (in case of electoral districts' system being used). So, the parliament size is determined partly exogenously and changes sometimes. The Parliament of New Zealand can serve as a good example: since 1896 the size has changed 5 times (from 74 in 1896 to 120 in 1996). By establishing the number of seats of the parliament, the government may want the parliament to be large enough to fulfil completely its functions and at the same time be small enough not to increase its maintenance expenses. But what if we try to estimate the size of the parliament from the point of the cost of corrupting it? Maybe the size of the parliament can impact the cost of bribing its members and thereby encourage or discourage corruption.

An acceptance threshold is a number of votes enough to approve a decision inside the parliament. This threshold can also have a bearing on the cost of corruption. Usually it is 50% of the total number votes – simple majority rule. In case when the total number of seats is even, there is a tie 50/50, in which case a default answer should be defined. Increasing the threshold makes harder for the parliament to make a decision, but at the same time it might decrease the incentives to corrupt by raising the cost of corruption.

A huge and powerful corporation, to the extent that it has enough resources to bribe parties, can act as a briber. For example, the automobile producing company which is interested in that a law of increasing state duty in imported cars.

A model is presented where it is determined an average cost for a briber who intends to manipulate the outcome of a parliamentary voting procedure, expressed in terms of the number of seats in the parliament that have to be bribed to ensure that the voting outcome is the one the briber wants. We define a cost of corruption as a number of seats needed to be bribed to achieve the desirable result. The persistence of illegal procedures can be caused by a low cost of bribing. Under the assumption that each member votes according to the political position of the party he belongs to, the number of parties has to be taken into account when calculating the cost of corruption. And a basic question then is if a multi-party parliament makes corruption less costly and therefore increases possible corruption.

We focus on a single political decision to be voted for and offer a simple model of voting inside the parliament. The model allows us to provide a comparative analysis of different parliament systems in terms of the average cost of bribing, depending on structural characteristics of the parliament, such as total number of seats, number of parties, distribution seats between parliaments, and the threshold for a positive decision to be taken.

The comparison of bipartism with multipartism suggests the following intuition: under the assumption that members of the party are voting strongly according to the party line in average a briber needs to bribe more votes than it is exactly needed to achieve the desirable answer. So, the higher the number of parties in the parliament, the smaller are the parties, and in average the excess of seats to be bribed (difference between what a briber needs to buy and what he has to buy) in multiparty system is lower than in the 2-party system. It makes corruption less costly, therefore increases the demand for corruption and makes it persistent.

We show that two-party system may minimize the visibility (or the likelihood to observe) corruption. The less parties are in the parliament (at least 2) the higher the cost of corruption. We find that the cost of corruption in the parliament with 2 parties can be reproduced in the parliament with more parties by increasing the acceptance threshold.

The work is organized in the following way. In the section 2 we provide a description of the model and main parameters used in the model. In sections 3 to 6 we provide results by comparing the average cost of corruption in different parliaments. The full proofs of the results are presented in the Appendix. Our considerations and explanations about the results are given in section 7. In section 8 we propose future extensions of the model.

## 2. Model and definitions

**Definition 2.1.** An $(m, n)$-parliament is a parliament with $n$ seats and $m$ parties, where $n$ and $m$ are positive integers such that $m \leq n$.

**Definition 2.2.** A YES-NO decision problem for an $(m, n)$-parliament is a proposal over which each parliament member must vote in favour (vote Y) or against (vote N) and next some voting rule determines if the parliament accepts or rejects the proposal.

**Definition 2.3.** An acceptance threshold for a YES-NO decision problem for an $(m, n)$-parliament is a non-negative integer $t \leq n$ such that the proposal is accepted if the number of parliament members voting Y is at least $t$. If the threshold $t$ is reached, the parliament is then said to take the Y decision; otherwise, it is said to take the N decision.

**Definition 2.4.** There is party discipline in an $(m, n)$-parliament handling a YES-NO decision problem if, for each party $i$, all the holders of seats assigned to party $i$ must cast the same vote (the party's vote).

For the purposes of this paper, the case without party discipline can be identified with the party discipline case in which each member of the parliament is a party (the $m = n$ case). In view of this, party discipline will be assumed without notice letting $m = n$ implicitly represent the absence of party discipline. In addition, under party discipline, it can be interpreted that it is parties rather than parliament members who vote, since all the parliament members ascribed to the same party must cast the same vote.

**Definition 2.5.** The set of states of an $(m, n)$-parliament handling a YES-NO decision problem is defined as

$$\Omega_n^m = \left\{(n_1, n_2, \dots, n_m, d_1, d_2, \dots, d_m) \colon \sum_{i=1}^{m} n_i = n \text{ and, for all } i, d_i \in \{\text{Y}, \text{N}\} \text{ and } n_i \geq 1\right\}.$$

A state $(n_1, n_2, \dots, n_m, d_1, d_2, \dots, d_m) \in \Omega_n^m$ of an $(m, n)$-parliament handling a YES-NO decision problem represents the situation in which, for each $i \in \{1, \dots, m\}$, party $i$ is assigned $n_i$ seats in the parliament and the holders of those seats all vote $d_i$ (in favour if $d_i$ = Y and against if $d_i$ = N).

**Definition 2.6.** Given a YES-NO decision problem for an $(m, n)$-parliament, the briber is an agent, not having a seat in the parliament, who is interested in the Y decision and is willing to bribe parties voting N so that all the party members change their vote from N to Y.

**Definition 2.7.** Given the set of states $\Omega_n^m$ of an $(m, n)$-parliament handling a YES-NO decision problem and an acceptance threshold $t$, the proportion $p$ of states in which the briber has to bribe some party is the proportion of states in $\Omega_n^m$ where the parliament takes the N decision, that is, the number of states in $\Omega_n^m$ in which the sum of all the seats of parties voting Y is smaller than $t$, divided by the number of states in $\Omega_n^m$.

Is it preferable for the proportion $p$ to be large or small? The answer is unclear. Remember that, though $p$ captures the likelihood that the briber will have to bribe, $p$ represents the probability that the N decision is taken. There is a priori no obvious reason why the N decision is preferable over the Y decision or vice versa. Depending on the specific case, it would be desirable for $p$ to be high or to be low. Yet, from a purely subjective point of view, a low $p$ could be justified on the grounds that a high $p$ makes bribery a more likely event, so that a high $p$ contributes to making bribery and corruption a more visible phenomenon. If the aim is to convey the image of a parliament as an instituion which is difficult to corrupt, a low $p$ seems to be more attractive than a high $p$. The lower $p$, the lower the perception of corruption and bribery.

**Definition 2.8.** Given a set of states $\Omega_n^m$ of an $(m,n)$-parliament handling a YES-NO decision problem and an acceptance threshold $t$:

(a) the aggregate cost (for the briber) of bribing parties is the sum, over all states in $\Omega_n^m$ where the parliament takes the N decision, of the minimum number of seats of parties voting N that ensures that the parliament takes the Y decision when the vote of those parties is changed to Y;

(b) the ex-ante average cost $C_A$ of bribing parties (or ex-ante cost, for short) is the aggregate cost divided by the total number of states in $\Omega_n^m$; and

(c) the ex-post average cost $C_P$ of bribing parties (or ex-post cost, for short) is the aggregate cost divided by the number of states in $\Omega_n^m$ where the parliament takes the N decision.

The ex-ante cost $C_A$ can be viewed as a social cost of bribing, whereas the ex-post cost $C_P$ rather represents the private cost (for the briber) of bribing. The reason is that the same aggregate bribing cost (all the seats bribed in every possible state) are distributed among all the states in the ex-ante version, whereas it is distributed among a subset of the set of all states in the ex-post version, namely, those seats in which the briber has the need to bribe. Since, from a social point of view, all states should count as relevant, it seems natural for the ex-ante version to serve as a social (collective) measure of the bribing costs. And since the only relevant states for the briber are those in which he or she has actually to bribe, it also seems natural to regard the ex-post version as a private cost.

If the aim is to prevent bribery and corruption, it appears desirable to have at least a high ex-post cost instead of a low one: as with most commodities, the higher the price to pay to bribe seats, the smaller the number of seats bribed (and hence, the smaller the amount of corruption). On the other hand, when comparing different parliament structures (for example, of different countries), the appropriate cost measure seems to be the ex-ante one, so that one could deem less likely to be corrupt a parliament structure with a higher ex-ante cost.

## 3. The likelihood of having parties bribed

The purpose of this section is to determine, for any given set of states $\Omega_n^m$ of an $(m,n)$-parliament and any given acceptance threshold $t$, the proportion of cases in $\Omega_n^m$ where the briber is in need of bribing some party. This proportion can be interpreted as the probability that the need to bribe arises. To illustrate this task, the following example is suggested.

**Example 3.1.** Consider the (3, 100)-parliament consisting of $n = 100$ seats and $m = 3$ parties. Let the acceptance threshold be $t = 51$ (so at least 51 Y votes are necessary to accept the proposal). With party discipline, the possible variants of the parties' decisions $(d_1, d_2, d_3)$ are $2^3$. This eight possibilities can be grouped in four cases, depending on the number of parties voting N. Table 3.1. provides the details.

**Table 3.1. Proportion of states where bribing becomes necessary (*m* = 3, *n* = 100, *t* = 51)**

| *Cases* | *Possible variants of assignment* $(d_1, d_2, d_3)$ | *Proportion of the assignment* | *Condition establishing the need to bribe* | *Proportion of states where bribing is necessary* |
|---|---|---|---|---|
| Case 0 | (Y,Y,Y) | 1/8 | *no need to bribe* | $p_0 = 0$ |
| Case 1 | (Y,Y,N)<br>(Y,N,Y)<br>(N,Y,Y) | 3/8 | *seats of the two Y parties* < 51<br>⇔<br>*seats of the N party* > 49 | $p_1 = 0.2525$ |
| Case 2 | (N,N,Y)<br>(N,Y,N)<br>(Y,N,N) | 3/8 | *seats of the two N parties* > 49<br>⇔<br>*seats of the Y party* < 51 | $p_2 = 0.7575$ |
| Case 3 | (N,N,N) | 1/8 | *bribing always occurs* | $p_3 = 1$ |

In case 0 there is no need for the briber to bribe any party, so the associated proportion is $p_0 = 0$. In case 3, since all parties say N, some party has to be bribed and, accordingly, $p_3 = 1$. In case 1, let party 1 and party 2 be Y parties and party 3 the N party. The briber has to bribe only one party (party 3) if and only if $n_1 + n_2 < 51$ (or $n_3 > 49$). By symmetry, the same occurs if the N party is 1 and if the N party is 2. Case 2 is similarly handled, but now one or two parties may have to be bribed. The final proportion is:

$$p = \frac{1}{8}p_0 + \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{8}p_3 = \frac{1}{8}0 + \frac{3}{8}0.2525 + \frac{3}{8}0.7575 + \frac{1}{8}1 = 0.5037$$

This says that, when $m = 3$, $n = 100$, and $t = 51$, in 50.37 % of all the states the briber will have to bribe parties to achieve the desired decision.

**Proposition 3.1.** *For any given set of states $\Omega_n^m$ of an $(m, n)$-parliament handling a YES-NO decision problem and an acceptance threshold $t$, the proportion $p$ of states in which the briber has to bribe some party is defined by formula (1).*

$$p = \frac{1}{2^m \binom{n-1}{m-1}} \sum_{k=1}^{m} \left[ \binom{m}{k} \sum_{i=1}^{k} \binom{t-1}{m-i} \binom{n-t}{i-1} \right] \tag{1}$$

Table 3.2 next provides specific numerical values for the above formula.

**Table 3.2. Proportion of cases where bribing is necessary for several values of the parameters**

$n = 100$

| *Number of parties m* | *Acceptance threshold t* 1 | 33 | 49 | 50 | 51 | 66 | 75 | *n* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 2 | 0.25 | 0.4116 | 0.4924 | 0.4974 | 0.50253 | 0.57828 | 0.62374 | 0.75 |
| 3 | 0.125 | 0.36742 | 0.48864 | 0.49621 | 0.50379 | 0.61742 | 0.68561 | 0.875 |
| 4 | 0.0625 | 0.33536 | 0.48577 | 0.49526 | 0.50474 | 0.64609 | 0.72858 | 0.9375 |
| 5 | 0.03125 | 0.30936 | 0.48336 | 0.49445 | 0.50555 | 0.66953 | 0.76211 | 0.96875 |
| 6 | 0.01563 | 0.28721 | 0.48123 | 0.49374 | 0.50626 | 0.68965 | 0.78964 | 0.98438 |
| 10 | 0.00098 | 0.22053 | 0.47428 | 0.49142 | 0.50858 | 0.75136 | 0.86561 | 0.99902 |
| *n* | ~0 | 0.0002 | 0.38218 | 0.46021 | 0.5398 | 0.9991 | ~1 | ~1 |

$n = 101$

| *Number of parties m* | *Acceptance threshold t* 1 | 33 | 50 | 51 | 52 | 66 | 75 | *n* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 2 | 0.25 | 0.41 | 0.495 | 0.5 | 0.505 | 0.575 | 0.62 | 0.75 |
| 3 | 0.125 | 0.365 | 0.4925 | 0.5 | 0.5075 | 0.6125 | 0.68 | 0.875 |
| 4 | 0.0625 | 0.33241 | 0.49061 | 0.5 | 0.50939 | 0.64004 | 0.7219 | 0.9375 |
| 5 | 0.03125 | 0.30602 | 0.48902 | 0.5 | 0.51098 | 0.66261 | 0.75475 | 0.96875 |
| 6 | 0.01563 | 0.28357 | 0.48761 | 0.5 | 0.51239 | 0.68203 | 0.78184 | 0.98438 |
| 10 | 0.00098 | 0.2162 | 0.48303 | 0.5 | 0.51697 | 0.74189 | 0.85744 | 0.99902 |
| *n* | ~0 | 0.00015 | 0.42119 | 0.5 | 0.57881 | 0.99867 | ~1 | ~1 |

Table 3.2 shows that, as long as the threshold $t$ is not greater than $n/2$, the proportion of cases where the need to bribe arises increases as the number of parties declines. When $t$ is greater than $n/2$, the opposite occurs: the proportion decreases as the number of parties falls.

Are higher values of the proportion $p$ better than lower ones? In other words, is it preferable to have a parliament structure with a high $p$ or one with a low $p$? As already indicated, the answer does not seem to be obvious. On the one hand, a high $p$ sets the good ground for bribery and corruption: with a high $p$, the briber has more need to act and therefore corruption is more likely to be observed (or, at least, to happen). If the aim is to minimize the occurrence of bribery, a low $p$ appears then to be desirable.

On the other hand, if the Y decision is a proposal made by the briber to alter in his or her favour the status quo represented by N, then the higher $p$, the harder for the briber to get his or her proposal accepted. If it is unlikely that the briber's interests are in harmony with the interests of the people represented by the parliament, it seems desirable for $p$ to be large.

Figure 3.1 below may be helpful to illustrate the effects on $p$ of changes in the acceptance threshold $t$ and the number of parties $m$ when $n = 101$ (the Parliament of Estonia, for instance, has 101 seats and 4 parties). It is interesting to note that when $n$ is odd and $t = \frac{n+1}{2}$ the proportion $p$ does not depend on the number of parties.

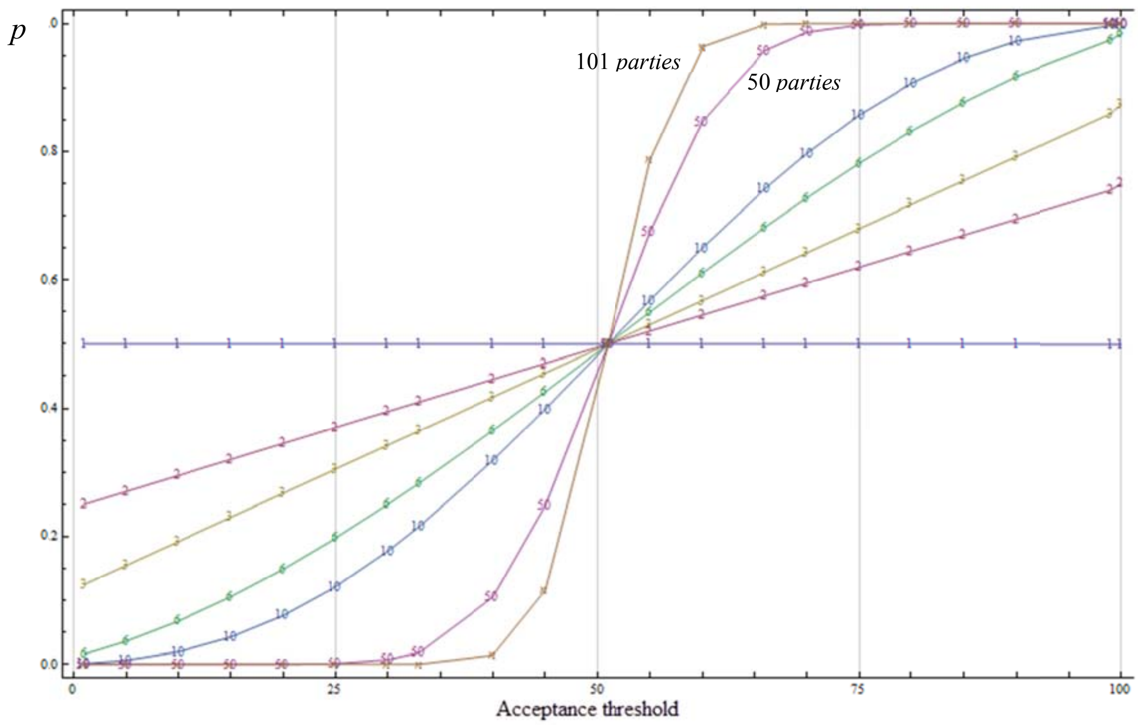

Figure 3.1. The proportion of states where bribing is necessary with $n = 101$ seats as a function of the acceptance threshold $t$ and the number of parties $m$ ($m$ shown on each curve)

## 4. The cost of bribing parties without party discipline

Consider the situation with no party discipline, so being a member of the same party does not mean casting the same vote. As already mentioned, this situation can be identified with the party discipline case in which every party has only one seat (that is, $m = n$). The following result determines two concepts of average bribing cost for the briber. The ex-ante cost $C_A$ is calculated assuming that the briber ignores if there is any need to bribe some party (the aggregate bribing cost is distributed among all the possible states). The ex-post cost $C_P$ is computed assuming that the briber knows that it is necessary to bribe some party (the aggregate bribing cost is distributed among the states in which the parliament takes the N decision). Table 4.1 shows the corresponding values for the case $m = n = 100$.

**Proposition 4.1.** *For any given set of states $\Omega_n^n$ of an $(n, n)$-parliament handling a YES-NO decision problem and an acceptance threshold $t$, the ex-ante cost $C_A$ and ex-post cost $C_P$ are*

$$C_A = \frac{1}{2^n}\sum_{i=0}^{t-1}(t-i)\binom{n}{i} \qquad \textit{and} \qquad C_P = \frac{\sum_{i=0}^{t-1}(t-i)\binom{n}{i}}{\sum_{i=0}^{t-1}\binom{n}{i}} \ .$$

**Table 4.1. Ex-ante and ex-post costs in the world without parties with n = 100**

| *Acceptance threshold t* | *Ex-ante cost* $C_A$ | $C_A(t)/C_A(t-1)$ | *Ex-post cost* $C_P$ | $C_P(t)/C_P(t-1)$ | *Ratio* $C_P/C_A$ |
|---|---|---|---|---|---|
| 1 | ~ 0 | | 1 | | 1.00E+30 |
| 33 | 0.0003 | | 1.767 | | 4893 |
| 49 | 1.529 | | 4.002 | | 2.62 |
| 50 | 1.99 | 1.3 | 4.32 | 1.08 | 2.17 |
| 51 | 2.53 | 1.27 | 4.68 | 1.08 | 1.85 |
| 52 | 3.15 | 1.24 | 5.09 | 1.08 | 1.62 |
| 53 | 3.84 | 1.22 | 5.55 | 1.09 | 1.45 |
| 54 | 4.6 | 1.19 | 6.06 | 1.09 | 1.32 |
| 55 | 5.41 | 1.17 | 6.63 | 1.09 | 1.23 |
| 56 | 6.28 | 1.16 | 7.26 | 1.09 | 1.16 |
| 57 | 7.18 | 1.14 | 7.95 | 1.09 | 1.11 |
| 58 | 8.11 | 1.13 | 8.69 | 1.09 | 1.07 |
| 59 | 9.07 | 1.12 | 9.49 | 1.09 | 1.05 |
| 60 | 10 | 1.11 | 10.3 | 1.09 | 1.03 |
| 66 | 16 | | 16.01 | | 1.01 |
| 75 | 25 | | 25.01 | | ~ 1 |
| *n* | 50 | | 50 | | ~ 1 |

*The ratio $C_P/C_A$ is the inverse of the proportion p of states where bribery occurs*

It is interesting to note that, as the third column in the Table 4.1 shows, the rate of growth of the ex-ante cost decreases with *t*: though a higher acceptance threshold *t* rises the ex-ante cost, the rise is each time smaller. On the other hand, the rate of growth of the ex-post cost follows a concave pattern: initially rises, reaches a maximum at *t* = 55, and then decreases. Figure 4.1 below plots ex-ante and ex-post costs on the whole range of the threshold *t*: the bigger *t*, the higher the costs; and the smaller *t*, the bigger the difference between ex-ante and ex-post costs.

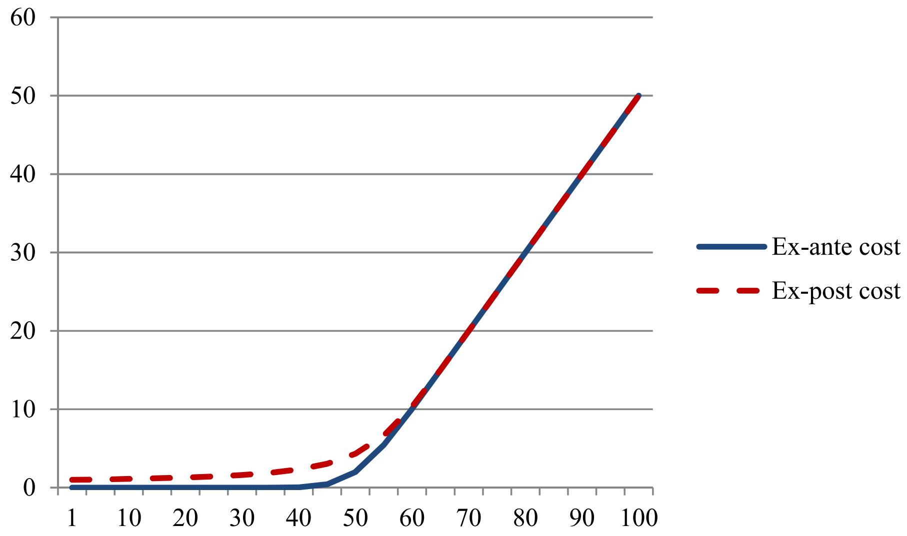


Figure 4.1. Ex-ante and ex-post costs with *m* = *n* parties

## 5. The cost of bribing parties with party discipline: 2 parties

The previous section considered one extreme case: maximum number of parties. This section deals with the other extreme: the two-party case (the one-party case can be seen as non-relevant). Table 5.1 shows all the cases needed to determine costs in the two-party case. Proposition 5.1 provides the formulas to compute the values of ex-ante and ex-post costs.

**Table 5.1. Total number of seats bribed with 2 parties**

| *Party decisions* ($d_1$, $d_2$) | *Total number of states* | *Number of states in which bribing occurs* | *Total number of seats bribed* | *Conditions* |
|---|---|---|---|---|
| (Y, Y) | $n-1$ | 0 | 0 | |
| (N, Y)<br>(Y, N) | $2(n-1)$ | $2(t-1)$ | $(t-1)(2n-t)$ | |
| (N, N) | $n-1$ | $n-1$ | $(t-1)(2n-t)$ | $t>\frac{n}{2}$ |
| | | | $\frac{3n^2}{4}-n$ | $t=\frac{n}{2}$ |
| | | | $2(t-1)(n-t)+\frac{n^2}{4}$ | $t<\frac{n}{2}$ *and n even* |
| | | | $2(t-1)(n-t)+\frac{n^2-1}{4}$ | $t<\frac{n}{2}$ *and n odd* |
| Total | $4(n-1)$ | $n+2t-3$ | | |

**Proposition 5.1.** *For any given set of states* $\Omega_n^2$ *of a* $(2,n)$*-parliament handling a YES-NO decision problem and an acceptance threshold* $t$*, the ex-ante cost* $C_A$ *and ex-post cost* $C_P$ *are*

$$C_A=\frac{(t-1)(2n-t)}{2(n-1)} \text{ and } C_P=\frac{2(t-1)(2n-t)}{n+2t-3} \qquad \text{if } t>\frac{n}{2}$$

$$C_A=\frac{(t-1)(2n-t)+\frac{3n^2}{4}-n}{4(n-1)} \text{ and } C_P=\frac{(t-1)(2n-t)+\frac{3n^2}{4}-n}{n+2t-3} \qquad \text{if } t=\frac{n}{2}$$

$$C_A=\frac{(t-1)(4n-3t)+\frac{n^2}{4}}{4(n-1)} \text{ and } C_P=\frac{(t-1)(4n-3t)+\frac{n^2}{4}}{n+2t-3} \qquad \text{if } t<\frac{n}{2} \text{ and } n \text{ even}$$

$$C_A=\frac{(t-1)(4n-3t)+\frac{n^2-1}{4}}{4(n-1)} \text{ and } C_P=\frac{(t-1)(4n-3t)+\frac{n^2-1}{4}}{n+2t-3} \qquad \text{if } t<\frac{n}{2} \text{ and } n \text{ odd}.$$

The following table exemplifies Proposition 5.1 for a parliament of size $n$ = 100 and several values of the acceptance threshold. It can be compared with Table 4.1.

**Table 5.2. Ex-ante and ex-post costs with 2 parties and *n* = 100 seats**

| *Acceptance threshold t* | *Ex-ante cost* $C_A$ | $C_A(t)/C_A(t-1)$ | *Ex-ante cost* $C_P$ | $C_P(t)/C_P(t-1)$ | *Ratio* $C_P/C_A$ |
|---|---|---|---|---|---|
| 1 | 6.31 | | 25.25 | | 4 |
| 25 | 26.01 | | 70.07 | | 2.69 |
| 33 | 30.64 | | 74.43 | | 2.43 |
| 49 | 36.98 | | 75.10 | | 2.03 |
| 50 | 37.25 | 1.0072 | 74.87 | 0.997 | 2.01 |
| 51 | 37.63 | 1.0102 | 74.87 | 1.000 | 1.99 |
| 52 | 38.12 | 1.0132 | 75.10 | 1.003 | 1.97 |
| 53 | 38.61 | 1.0127 | 75.31 | 1.003 | 1.95 |
| 54 | 39.08 | 1.0123 | 75.49 | 1.002 | 1.93 |
| 55 | 39.55 | 1.0119 | 75.65 | 1.002 | 1.91 |
| 56 | 40.00 | 1.0115 | 75.79 | 1.002 | 1.89 |
| 57 | 40.44 | 1.0111 | 75.91 | 1.002 | 1.88 |
| 58 | 40.88 | 1.0107 | 76.00 | 1.001 | 1.86 |
| 59 | 41.30 | 1.0104 | 76.07 | 1.001 | 1.84 |
| 60 | 41.72 | 1.0100 | 76.13 | 1.001 | 1.82 |
| 66 | 43.99 | | 76.07 | | 1.73 |
| 75 | 46.72 | | 74.90 | | 1.60 |
| 99 | 49.99 | | 67.11 | | 1.34 |
| 100 | 50.00 | | 66.67 | | 1.33 |

Figure 5.1 below (compare with Figure 4.1) plots ex-ante and ex-post costs on the total range of the threshold $t$. As in the $m = n$ case, the bigger $t$, the higher the costs. But now the ex-post cost does not exhibit a monotonic behaviour and is concave with a maximum at $t = 62$. The decrease of the function at $t = 51$ is connected with the growth rate of the likelihood of having parties bribed, as a $t = 50.5$ is an anti-saddle point for this function.

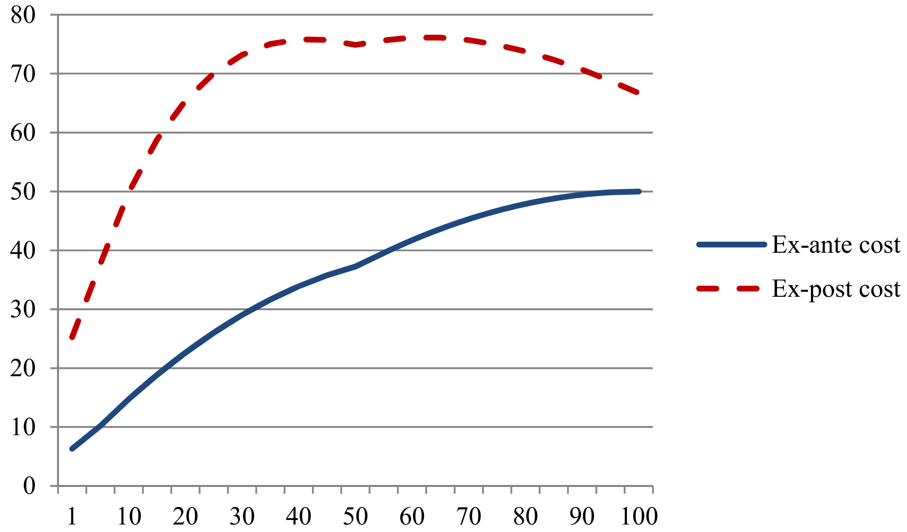


Figure 5.1. Ex-ante and ex-post costs with *m* = 2 parties

## 6. The cost of bribing parties with party discipline: 3 parties

Sections 4 and 5 provide ex-ante and ex-post costs without party discipline, or $m = n$ case, and with party discipline under just $m = 2$ parties. The conjecture is that the rest of cases (party discipline with $m$ between 3 and $n-1$) lie "between" these two cases. The cost computation for the simplest of those cases, $m = 3$, is not straightforward. Proposition 6.1 next deals with the special case with *t* between *n*/2 and 2*n*/3, which appears to be the most relevant interval in practical terms: for example, constitutional changes in Spain demand "procedimineto ordinario" – 3/5 of parliament to be approved, and for global changes of the constitution – "procedimiento agravado" with 2/3 of the parliament. When $t > 2n/3$ the situation when all parties have to be bribed to achieve the threshold can arise.

Partly the 3-party case can be sold inductively from the 2-party case by adding 1 more party. See the proofs in the Appendix.

**Proposition 6.1.** *For any given set of states* $\Omega_n^3$ *of a* $(3,n)$*-parliament handling a YES-NO decision problem and an acceptance threshold* $t$*, the ex-ante cost* $C_A$ *and ex-post cost* $C_P$ *are stated in Table 6.1.*

**Table 6.1. Total number of seats bribed with three parties for $\frac{n}{2} < t < \frac{2n}{3}$**

| *Party decisions* ($d_1$, $d_2$, $d_3$) | *Total number of states* | *Number of states where there is a need to bribe* | *Total number of seats bribed* | *Conditions* |
|---|---|---|---|---|
| (Y,Y,Y) | $\frac{1}{2}(n-1)(n-2)$ | 0 | 0 | |
| (N,Y,Y) (Y,Y,N) (Y,N,Y) | $\frac{3}{2}(n-1)(n-2)$ | $3\sum_{x=1}^{t-2}(t-1-x)$ | $3\sum_{x=1}^{t-2}\left[(t-x-1)\left(n-\frac{t+x}{2}\right)\right]$ | |
| (N,N,Y) (N,Y,N) (Y,N,N) | $\frac{3}{2}(n-1)(n-2)$ | $3\sum_{x=1}^{t-1}(n-x-1)$ | $3\sum_{x=1}^{2t-n-1}[(t-1-x)(2n-x-t)]$ | $x < 2t-n$ |
| | | | $3\left[\frac{3(2n-2t)^2}{4}-2n+2t\right]$ | $x = 2t-n$ |
| | | | $3\sum_{x=2t-n+1}^{t-1}\left[(t-1-x)(n-t)+\frac{(n-x)^2}{4}\right]$ | $x > 2t-n$ only for even *n–x* |
| | | | $3\sum_{x=2t-n+1}^{t-1}\left[(t-1-x)(n-t)+\frac{(n-x-1)^2}{4}\right]$ | $x > 2t-n$ only for odd *n–x* |
| (N,N,N) | $\frac{1}{2}(n-1)(n-2)$ | $\frac{1}{2}(n-1)(n-2)$ | $\sum_{k=0}^{n-t}(t+k)(3(n-t-k-1)+\mathrm{NS}_{2,t+k})$ | |
| | $2^2(n-1)(n-2)$ | | | |

*Variables k and x are positive integers.* $\mathrm{NS}_{2,t+k}$ *is counted according to Table V.1. in the Appendix*

For the case $t = \frac{n}{2}$ and assignment (N,N,N) it should be excluded from the sum the case $k = 0$, and to the aggregate cost it should be added $3(n-t-1)$.

## 7. Summary results

As the first step of the summary analysis we provide comparative table of costs and likelihood to bribe for $\frac{n}{2} \leq t < \frac{2n}{3}$, $n$ = 100, and $m = 2,\ 3,\ n$.

**Table 7.1. Summary results for a parliament with $n$ = 100 seats and 2, 3, or $n$ parties**

| *Acceptance threshold t* | *Bribery likelihood* | | | *Ex-ante cost* | | | *Ex-post cost* | | |
|---|---|---|---|---|---|---|---|---|---|
| | $m$ = 2 | $m$ = 3 | $m$ = $n$ | $m$ = 2 | $m$ = 3 | $m$ = $n$ | $m$ = 2 | $m$ = 3 | $m$ = $n$ |
| 50 | 0.497 | 0.496 | 0.460 | 37.25 | 28.61 | 1.99 | 74.87 | 57.66 | 4.32 |
| 51 | 0.503 | 0.504 | 0.540 | 37.63 | 29.13 | 2.53 | 74.87 | 57.82 | 4.69 |
| 52 | 0.508 | 0.511 | 0.618 | 38.12 | 29.66 | 3.15 | 75.10 | 58.00 | 5.09 |
| 53 | 0.513 | 0.519 | 0.691 | 38.61 | 30.19 | 3.84 | 75.31 | 58.18 | 5.55 |
| 54 | 0.518 | 0.527 | 0.758 | 39.08 | 30.75 | 4.60 | 75.49 | 58.39 | 6.06 |
| 55 | 0.523 | 0.534 | 0.816 | 39.55 | 31.30 | 5.41 | 75.65 | 58.60 | 6.63 |
| 56 | 0.528 | 0.542 | 0.864 | 40.00 | 31.87 | 6.28 | 75.79 | 58.83 | 7.26 |
| 57 | 0.533 | 0.549 | 0.903 | 40.44 | 32.43 | 7.18 | 75.91 | 59.05 | 7.95 |
| 58 | 0.538 | 0.557 | 0.933 | 40.88 | 33.01 | 8.11 | 76.00 | 59.29 | 8.69 |
| 59 | 0.543 | 0.564 | 0.956 | 41.30 | 33.59 | 9.07 | 76.07 | 59.51 | 9.49 |
| 60 | 0.548 | 0.572 | 0.972 | 41.72 | 34.17 | 10.04 | 76.13 | 59.74 | 10.33 |
| 61 | 0.553 | 0.580 | 0.982 | 42.12 | 34.75 | 11.02 | 76.16 | 59.96 | 11.22 |
| 62 | 0.558 | 0.587 | 0.990 | 42.52 | 35.33 | 12.01 | 76.18 | 60.17 | 12.14 |
| 63 | 0.563 | 0.595 | 0.994 | 42.90 | 35.90 | 13.01 | 76.18 | 60.37 | 13.09 |
| 64 | 0.568 | 0.602 | 0.997 | 43.27 | 36.48 | 14.00 | 76.16 | 60.57 | 14.05 |
| 65 | 0.573 | 0.610 | 0.998 | 43.64 | 37.04 | 15.00 | 76.12 | 60.74 | 15.03 |
| 66 | 0.578 | 0.617 | 0.999 | 43.99 | 37.60 | 16.00 | 76.07 | 60.91 | 16.02 |

As any commodity, demand for the corruption is likely to be regulated by its price: more corruption is likely to be observed the smaller the price paid by the briber. So to reduce corruption we have to create a parliament structure making the corruption to be highly costly.

The higher the likelihood to bribe, the more likely the corruption will be observed. So, comparing 3 different parliament systems we would prefer one with the lower values. For $t > n/2$ the 2-party system has lower values. For smaller $t$ the no-party world (the world where $m = n$) has lower values of the likelihood to bribe.

Ex-ante cost as a proxy of social view is desirable to be high. High price of corruption will deter bribers. From Table 7.1, we observe that 2-party system has higher ex-ante costs than the no-party and 3-party system. The ex-ante cost, which takes into account all possible states of the world, can be an instrument for comparing the parliament structures of different countries. The model predicts that if a country wants the corruption to be as costly as in the parliament with $m = 2$, $t = 51$ (37.25) but wants to have more than 2 parties, it has to increase the acceptance threshold to two thirds of the parliament, when the cost will be 37.6. The ex-ante cost in the parliament without parties can achieve value 37 by increasing the threshold up to 87. And the ex-ante cost will never exceed 50 with increasing the threshold.

Ex-post cost as a proxy for the private price of corruption paid by the briber is also desired to be high. For these thresholds the highest value has the cost in the parliament with 2 parties and $t = 62$. For $t = 51$ the ex-post cost in the world of no-parties is 15.9 times smaller than for 2-party system and 12.3 times smaller than for 3-party system. For $t = 66$ the ex-post cost in the world of no-parties is 4.7 times and 3.8 smaller than for 2-party 3-party and system, respectively.

The results are illustrated with Figure 7.1.

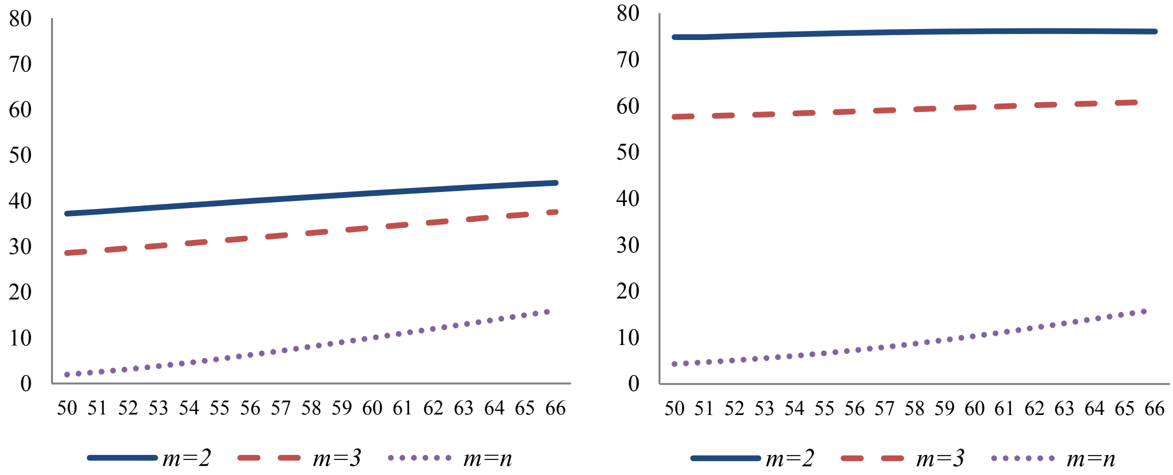


Figure 7.1. Ex-ante and ex-post cost for different parliament structures

For $\frac{n}{2} \leq t < \frac{2n}{3}$ the graphs predict that the 2-party system always has a higher cost of corruption than in parliaments with more parties. The corruption cost decreases as the number of parties increases.

To see the whole picture (for all possible values of $t$) consider the extreme cases $m = 2$ and $m = n$.

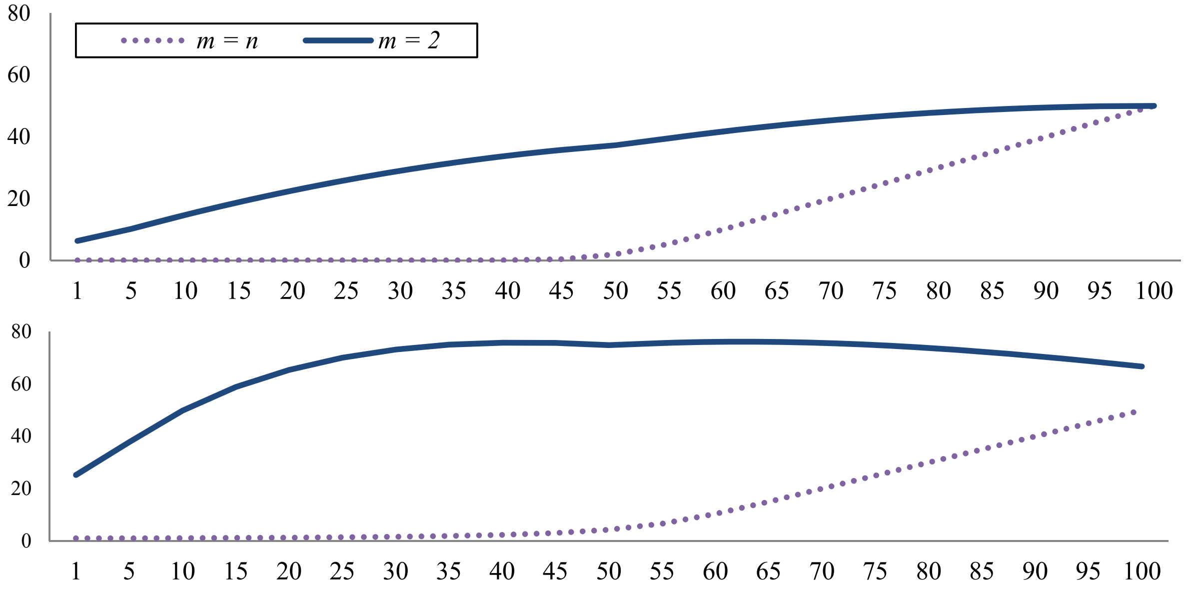


Figure 7.2. Ex-ante and ex-post costs for $m = 2$ and $m = n$

The function of costs being concave when $m = 2$ is becomes convex when $m = n$. It is obvious that functions for others $m$ are lying in the area between these extreme cases.

The comparative analysis predicts that the costs for 2-party system will be always higher than for more parties systems. The maximum ex-ante costs is reached when $m = 2$ and $t = 100$ (unanimity). The maximum ex-post cost is reached when $m = 2$ and $t = 62$.

## 8. Final remarks

For future research the proposed model can be extended to answer several more questions.

To be closer to the reality, the parties should differ between each other not only in the number of seats in the parliament, but also in the political views. Assume that we have a strong right party and a YES answer in favour of the left parties. Such a party is likely to be never bribed for YES decision, or, in other words, the price to be paid for each seat will be much higher than for a central party or a right party with more liberal views. Therefore, for a specific question we can define a level of corruption – a measure of the likelihood to accept bribing in favour of this question. Introducing such a measure may suggest that in some political situation in a country it is better to have more than 2 parties.

Widespread situation among the countries is to have a percentage threshold to enter the parliament. In the presence of the threshold the minimum number of seats in the parliament for each party changes from 1 to 7 (in case of 7 % threshold of entrance to the parliament). So, the model can provide comparison for parliament structures with different percentage threshold.

The same restriction can be also transformed into a measure of equality of distribution seats between parties. For instance, the restriction $n_i \geq 30$ when $n$ = 100 reflexes the situation with almost equal distribution of seats. Besides, changing this restriction can provide reasons to introduce the upper threshold to enter the parliament; for instance, no party can occupy more than 50 % of seats in the Parliament.

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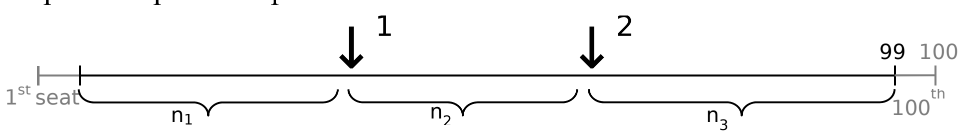
# Appendix

## A-I. Proportion of states where bribing becomes necessary: combinatoric approach

We need to determine the proportion of cases in $\Omega_n^m$ where the briber is in need of bribing some party for any given set of states $\Omega_n^m$ of an $(m,n)$-parliament and any given acceptance threshold t.

### Likelihood to bribe when $n = 100$, $m = 3$, and $t = 51$.

The quantity of all possible distributions of 100 seats between 3 parties can be represented as a simple combinatorial task: how many ways exist of choosing 2 numbers from the set of numbers? If we order all seats in a line (without making any difference between them) these two numbers represent the border between parties: all seats to the left of the first border are assigned to the party 1, between borders – to the party 2, to the right of the second border – to the party 3. There are 99 possible places to put a border.




To calculate the number of possible assignments we have to use the binomial coefficient. A k-combination of a set R is a subset of k distinct elements of R. If the set has r elements the number of k-combinations is equal to the binomial coefficient:

$$\binom{r}{k} = \frac{r(r-1)\dots(r-k+1)}{k(k-1)\dots 1} = \frac{r!}{k!\,(r-k)!}$$

In our case $k = m - 1 = 3 - 1 = 2$, and $r = n - 1 = 99$ (according to the restriction that at list 1 seat should be assigned to each party).

$$\binom{99}{2} = \frac{99!}{2!\,(99-2)!} = 4851$$

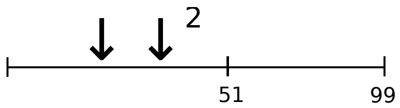
With party discipline, the possible variants of the parties' decisions $(d_1, d_2, d_3)$ are $2^3$. This eight possibilities can be grouped in four cases, depending on the number of parties voting N.

**Case 0: no party votes N**

In case 0 there is no need for the briber to bribe any party, so the associated proportion is $p_0 = 0$.

**Case 1: one party votes N**

Let party 1 and party 2 be Y parties and party 3 the N party. The briber has to bribe only one party (party 3) if and only if $n_1 + n_2 < 51 \Leftrightarrow n_3 > 49$.




$\downarrow^1\in[1,51)$, $\downarrow^2\in[1,51)$. The number of all combinations is the binomial coefficient:

$$\binom{51-1}{2} = \binom{50}{2} = \frac{50!}{2!\,(50-2)!} = 1225$$

And the probability of these combinations is:

$$p_1 = \frac{1225}{4851} = 0.2525$$

**Case 2: two parties vote N**

Consider the **Case 2**: $n_1 + n_2 > 49 \Leftrightarrow n_3 < 51$. We have two possible assignments of the borders, satisfying these conditions:

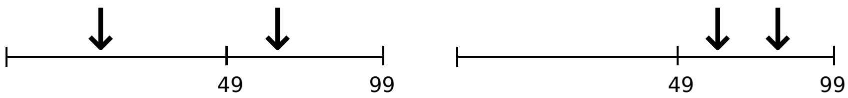



Figure 3 (a) (b)

a. Both border belong to the interval [1,49] The number of all possible combinations of such an assignment is number of choices of the first border (49) multiplied by the number of possible choices of the second border $(99 - 49)$. And the possibility of this assignment is this product divided by all possible combinations):

$$p_2(a) = \frac{49(99-49)}{4851} = 0.5050$$

b. One border belong to the interval (49,99], and the other border to (49,99]. The number of possible combinations is equal to binomial coefficient $\binom{99-49}{2}$. And the possibility of this assignment is this coefficient divided by all possible assignments:

$$\binom{99-49}{2} = \binom{50}{2} = \frac{50!}{2!\,(50-2)!} = 1225$$

$$p_2(b) = \frac{1225}{4851} = 0.2525$$

As events $a$ and $b$ are incompatible and we are interested either in $a$ or in $b$. So, to obtain the probability of the Case 2 we have to sum probabilities of $a$ and $b$:

$$p_2 = p_2(a) + p_2(b) = 0.5050 + 0.2525 = 0.7575$$

And now we can count the whole possibility of having incentives to corrupt:

$${}^{3}P = \frac{1}{8}p_0 + \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{8}p_3 = \frac{1}{8}0 + \frac{3}{8}0.2525 + \frac{3}{8}0.7575 + \frac{1}{8}1 = 0.5037$$

So with probability 0.5037 the briber has incentives to corrupt one or two parties in 3-party system.

## Likelihood to bribe: generalized formula

First we have to count the number of possible outcomes of voting. These outcomes can be represented as a typical combinatorial task of independent repeated trials of an experiment with two outcomes only (1 and 0 in our case). Such trials are called Bernoulli trials. In our case we have a binomial experiment, which consists of a fixed number m of statistically independent Bernoulli trials, in which 1 and 0 can appear with the same probability in each trial. Let consider that event $A$ is that 0 is assigned to a party (and event $\bar{A}$ is that 1 is assigned).

According to the Bernoulli formula the probability that the event A will appear exactly k times in m independent trials (m independent parties) is given by:

$$P_k = \binom{m}{k} p^k q^{m-k}$$

Where p is the probability of in each trial, and q is the probability of $\bar{A}$ and $q = 1 - p$. As only 1 or 0 can be assigned to a party, the $p = 1/2 = q$.

$$P_k = \binom{m}{k} p^k p^{m-k} = \binom{m}{k} p^m = \binom{m}{k} \frac{1}{2^m} = \frac{m!}{k!(m-k)!}\frac{1}{2^m}$$

So, the number of cases depends on the k – the number of zeros in the assignments:

k=0: $P_0 = \frac{m!}{0!(m-0)!}\frac{1}{2^m} = \frac{1}{2^m}$

k=1: $P_1 = \frac{m!}{1!(m-1)!}\frac{1}{2^m} = \frac{m!}{(m-1)!}\frac{1}{2^m}$

k=2: $P_2 = \frac{m!}{2!(m-2)!}\frac{1}{2^m}$

k=3: $P_3 = \frac{m!}{3!(m-3)!}\frac{1}{2^m}$

…

k=m: $\mathrm{P_m} = \frac{\mathrm{m!}}{\mathrm{m!(m-m)!}}\frac{1}{2^{\mathrm{m}}} = \frac{1}{2^{\mathrm{m}}}$

Further we need to count probability of need to corrupt in each case.

**Table I.1. Proportion of states where bribing becomes necessary (*m* = 3, *n* = 100, *t* = 51)**

| *Cases* | *Possible variants of assignment* $(d_1, d_2, d_3)$ | *Proportion of the assignment* | *Condition establishing the need to bribe* | *Proportion of states where bribing is necessary* |
|---|---|---|---|---|
| Case 0 | (Y,Y,Y) | 1/8 | *no need to bribe* | $p_0 = 0$ |

| | | | | |
|---|---|---|---|---|
| Case 1 | (Y,Y,N)<br>(Y,N,Y)<br>(N,Y,Y) | 3/8 | *seats of the two Y parties* < 51<br>$\Leftrightarrow$<br>*seats of the N party* > 49 | $p_1 = 0.2525$ |
| Case 2 | (N,N,Y)<br>(N,Y,N)<br>(Y,N,N) | 3/8 | *seats of the two N parties* > 49<br>$\Leftrightarrow$<br>*seats of the Y party* < 51 | $p_2 = 0.7575$ |
| Case 3 | (N,N,N) | 1/8 | *bribing always occurs* | $p_3 = 1$ |

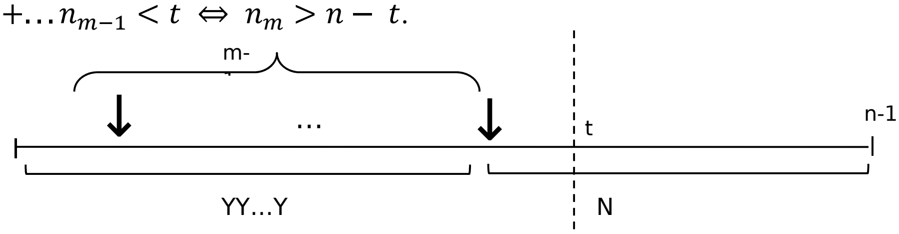
**Case 0**

The briber needs not to corrupt any party. The vector of assignment is (Y,Y,…Y). The $p_0 = p_0(Y, Y, \dots Y) = 0$.

**Case 1**

Consider all assignments with $m-1$ quantity of Y and only one N. The briber has to corrupt only 1 party, so the sum of the rest parties is less than t.

t: $n_1 + n_2 + \dots n_{m-1} < t \Leftrightarrow n_m > n - t$.




All $m-1$ borders belong to the interval $[1, t)$. The number of all combinations is the binomial coefficient:

$$\binom{t-1}{m-1} = \frac{(t-1)!}{(m-1)!\,(t-1-m+1)!} = \frac{(t-1)!}{(m-1)!\,(t-m)!}$$

All possible combinations is:

$$\binom{n-1}{m-1} = \frac{(n-1)!}{(m-1)!\,(n-1-m+1)!} = \frac{(n-1)!}{(m-1)!\,(n-m)!}$$

So, the probability of these combinations is:

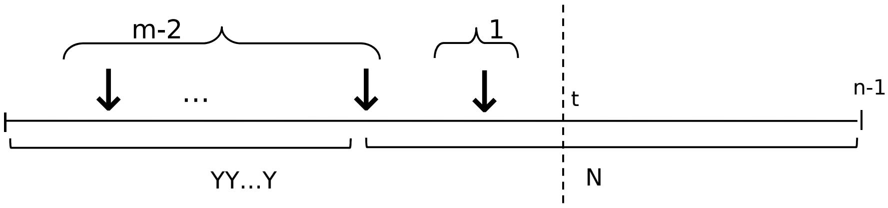
$$p_1 = \frac{\binom{t-1}{m-1}}{\binom{n-1}{m-1}} = \frac{\frac{(t-1)!}{(m-1)!(t-m)!}}{\frac{(n-1)!}{(m-1)!(n-m)!}} = \frac{(t-1)!\,(n-m)!}{(n-1)!\,(t-m)!}$$

**Case 2**

Consider the next Case 2: two parties (parties $n_m$ and $n_{m-1}$) are assigned to N, other parties ($m-2$ parties) – to Y. The briber has incentives to corrupt when $n_1 + n_2 + \cdots n_{m-2} < t \Leftrightarrow n_m + nm - 1 > n - t$.

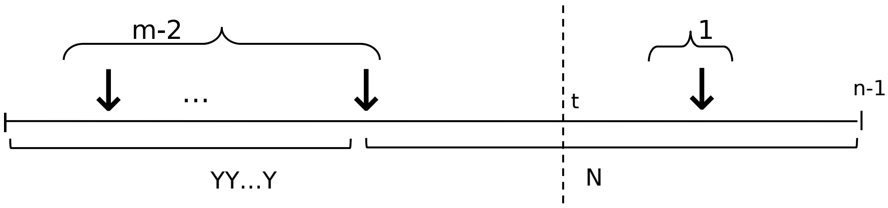
Here arises more complex case:

a. All borders belong to the interval $[1, t)$




$$p_2(a) = \frac{\binom{t-1}{m-1}\binom{n-1-t+1}{0}}{\binom{n-1}{m-1}} = \frac{\binom{t-1}{m-1}\binom{n-t}{0}}{\binom{n-1}{m-1}}$$

b. $m-2$ borders belong to the interval $[1, t)$ and 1 order belongs to the $[t, n-1]$ interval.



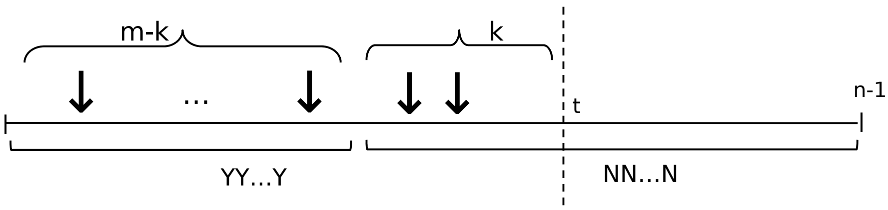

$$\mathrm{p}_2(\mathrm{b}) = \frac{\binom{\mathrm{t}-1}{\mathrm{m}-2}\binom{\mathrm{n}-\mathrm{t}}{1}}{\binom{\mathrm{n}-1}{\mathrm{m}-1}}$$

Total probability:

$$p_2 = p_2(a) + p_2(b) = \frac{\binom{t-1}{m-1}\binom{n-t}{0} + \binom{t-1}{m-2}\binom{n-t}{1}}{\binom{n-1}{m-1}}$$

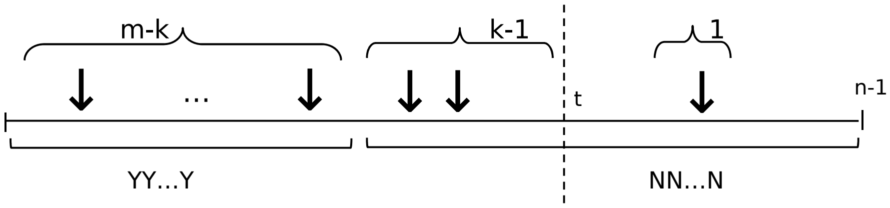
**Case k**

Consider the case k: all assignments when k parties vote NO and $m-k$ vote YES. There are $m+1$ subcases



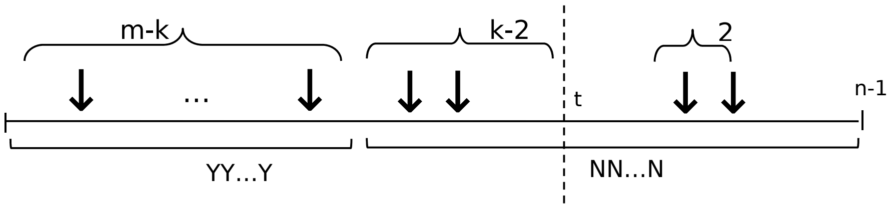

0.

$$p_k(k_0) = \frac{\binom{t-1}{(m-k)+(k-0)}\binom{n-t}{0}}{\binom{n-1}{m-1}} = \frac{\binom{t-1}{m-0}\binom{n-t}{0}}{\binom{n-1}{m-1}}$$



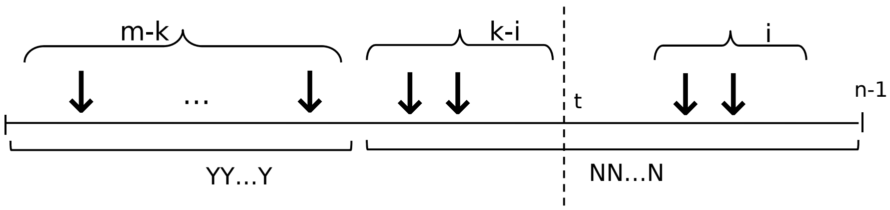

1.

$$p_k(k_1) = \frac{\binom{t-1}{(m-k)+(k-1)}\binom{n-1-t+1}{1}}{\binom{n-1}{m-1}} = \frac{\binom{t-1}{m-1}\binom{n-t}{1}}{\binom{n-1}{m-1}}$$




2.

$$p_k(k_2) = \frac{\binom{t-1}{(m-k)+(k-2)}\binom{n-1-t+1}{2}}{\binom{n-1}{m-1}} = \frac{\binom{t-1}{m-2}\binom{n-t}{2}}{\binom{n-1}{m-1}}$$




i.

$$p_k(k_i) = \frac{\binom{t-1}{(m-k)+(k-i)}\binom{n-1-t+1}{i-1}}{\binom{n-1}{m-1}} = \frac{\binom{t-1}{m-i}\binom{n-t}{i-1}}{\binom{n-1}{m-1}}$$

The total probability:

$$\mathrm{p_k} = \sum_{\mathrm{i=1}}^{\mathrm{k}} \frac{\binom{\mathrm{t-1}}{\mathrm{m-i}}\binom{\mathrm{n-t}}{\mathrm{i-1}}}{\binom{\mathrm{n-1}}{\mathrm{m-1}}} = \sum_{\mathrm{i=1}}^{\mathrm{k}} \frac{\frac{(\mathrm{t}-1)!}{(\mathrm{m}-\mathrm{i})!\,(\mathrm{t}-1-\mathrm{m}+\mathrm{i})!}\frac{(\mathrm{n}-\mathrm{t})!}{(\mathrm{i}-1)!\,(\mathrm{n}-\mathrm{t}-\mathrm{i}+1)!}}{\frac{(\mathrm{n}-1)!}{(\mathrm{m}-1)!\,(\mathrm{n}-\mathrm{m})!}}$$

$$= \sum_{i=1}^{k} \frac{(t-1)!\,(n-t)!\,(m-1)!\,(n-m)!}{(m-i)!\,(t-1-m+i)!\,(i-1)!\,(n-t-i+1)!\,(n-1)!}$$

The final formula:

$$P = \sum_{k=1}^{m}\left[\binom{m}{k}\frac{1}{2^m}\sum_{i=0}^{k}\frac{\binom{t-1}{m-i}\binom{n-t}{i-1}}{\binom{n-1}{m-1}}\right] = \frac{1}{2^m\binom{n-1}{m-1}}\sum_{k=1}^{m}\left[\binom{m}{k}\sum_{i=1}^{k}\binom{t-1}{m-i}\binom{n-t}{i-1}\right]$$

$$P = \sum_{k=1}^{m}\left[\frac{m!}{k!\,(m-k)!}\frac{1}{2^m}\sum_{i=1}^{k}\frac{(t-1)!\,(n-t)!\,(m-1)!\,(n-m)!}{(m-i)!\,(t-1-m+i)!\,(i-1)!\,(n-t-i+1)!\,(n-1)!}\right]$$

## A-II. The cost of bribing parties without party discipline

We examine all possible dividing $n$ seats between $m = n$ parties). Starting with the case, when all parties decide to vote YES we add 1 negative party until all parties vote NO. While there are at least *t* positive parties the briber do not need to bribe. When we have $t - 1$ positive parties, the briber has incentives to bribe 1 party, so, the cost of the bribe in this case is 1 seat. Considering possible cases step by step we obtain the following result.

**Table II.1 Total number of seats bribed with no party discipline**

| *Number of yes-parties* | *Number of no-parties* | *Total number of seats bribed* | *Number of states in which bribing occurs* | *Aggregate cost* |
|---|---|---|---|---|
| $n$ | 0 | 0 | $\binom{n}{n}$ | 0 |
| ... | | | | ... |
| $t$ | $n-t$ | 0 | $\binom{n}{t}$ | 0 |
| $t-1$ | $n-t+1$ | 1 | $\binom{n}{t-1}$ | $1\binom{n}{t-1}$ |
| $t-2$ | $n-t+2$ | | $\binom{n}{t-2}$ | $2\binom{n}{t-2}$ |
| ... | | | | ... |
| 1 | $n-1$ | $t-1$ | $\binom{n}{1}$ | $(t-1)\binom{n}{1}$ |
| 0 | $n$ | $t$ | $\binom{n}{0}$ | $t\binom{n}{0}$ |

Total number of states is $2^n$. Total number of states in which corruption occurs is:

$$\sum_{i=0}^{t-1}\binom{n}{i}$$

The probability to have incentives to bribe seats is given by:

$$P = \frac{1}{2^n}\sum_{i=1}^{t}\binom{n}{t-i} = \frac{1}{2^n}\sum_{i=0}^{t-1}\binom{n}{i} = \frac{1}{2^n}\sum_{i=0}^{t-1}\frac{n!}{i!\,(n-i)!}$$

Therefore, the ex-ante and ex-post costs are aggregate cost divided by total number of states and total number of states when corruption occurs, respectively:

$$C_A = \frac{1}{2^n}\sum_{i=0}^{t-1}(t-i)\binom{n}{i} \qquad and \qquad C_P = \frac{\sum_{i=0}^{t-1}(t-i)\binom{n}{i}}{\sum_{i=0}^{t-1}\binom{n}{i}}\ .$$

## A-III. The cost of bribing parties with party discipline: 2 parties

$\Omega^2_n(n_1, d_1, d_2)$ – all possible states of the world with 2 parties. It is not necessary to define the number of the seats of the second party, as this number is defined as $n_2 = n - n_1$.

There are 3 possible assignments of the world:

$\Omega_0(n_1, Y, Y)$

$\Omega_1(n_1, N, Y) = \Omega_1(n_1, Y, N)$

$\Omega_2(n_1, N, N)$

**Table III.1. Total number of seats bribed with 2 parties**

| *Party decisions* $(d_1, d_2)$ | *Total number of states* | *Number of states in which bribing occurs* | *Total number of seats bribed* | *Conditions* |
|---|---|---|---|---|
| (Y, Y) | $n-1$ | 0 | 0 | |
| (N, Y)<br>(Y, N) | $2(n-1)$ | $2(t-1)$ | $(t-1)(2n-t)$ | |
| (N, N) | $n-1$ | $n-1$ | $(t-1)(2n-t)$ | $t > \frac{n}{2}$ |
| | | | $\frac{3n^2}{4} - n$ | $t = \frac{n}{2}$ |
| | | | $2(t-1)(n-t) + \frac{n^2}{4}$ | $t < \frac{n}{2}$ *and n even* |
| | | | $2(t-1)(n-t) + \frac{n^2-1}{4}$ | $t < \frac{n}{2}$ *and n odd* |
| Total | $4(n-1)$ | $n+2t-3$ | | |

### $\boldsymbol{\Omega_0(n_1, Y, Y)}$

Total number of cases is quantity of possible dividing n seats between two parties:

$$TNS_{11} = \binom{n-1}{1} = \frac{(n-1)!}{1!\,(n-2)!} = n-1$$

Or in the other way, given the number of seats of one party the number of seats of the other party is determined only in 1 way. So the total number of cases is the number of values which can take the number of seats of one party. There is no need to bribe. There are 0 seats to bribe.

### $\boldsymbol{\Omega_1(n_1, N, Y) = \Omega_1(n_1, Y, N)}$

There are 2 symmetric cases: $\Omega_1(n_1, 0, 1)$=$\Omega_1(n_1, 1, 0)$. Total number of states is:

$$TNS(\mathrm{n_1, Y, N}) = \binom{n-1}{1} = (n-1) = TNS(\mathrm{n_1, N, Y})$$

$$TNS_{10} = 2TNS(\mathrm{n_1, Y, N}) = 2(n-1)$$

Consider the case $d_1 = Y,\ d_2 = N$. There is a need to buy the party 1. When $n_1 \in [t,\ n-1]$ there is no need to bribe. When $n_1 \in [1,\ t-1]$ it is necessary to bribe the second party.

**Table III.2. Total number of seats bribed with 2 parties, $d_1 = Y, d_2 = N$**

| $n_1$ *(Y)* | $n_2$ *(N)* | *Seats to bribe* | | *States when need to bribe* | |
|---|---|---|---|---|---|
| 1 | $n-1$ | $n-1$ | $n(t-1) - \frac{t(t-1)}{2}$ | 1 | $t-1$ |
| 2 | $n-2$ | $n-2$ | | 1 | |
| … | | … | | … | |
| $t-1$ | $n-(t-1)$ | $n-(t-1)$ | | 1 | |
| $t$ | $n-t$ | 0 | 0 | 0 | 0 |
| $t+1$ | $n-(t+1)$ | 0 | | 0 | |
| … | | … | | … | |
| $n-2$ | 2 | 0 | | 0 | |
| $n-1$ | 1 | 0 | | 0 | |

Total seats to be bribed is:

$$n(t-1) - \frac{t(t-1)}{2} = (t-1)(n - \frac{t}{2})$$

$\Omega_2(\boldsymbol{n_1}, \boldsymbol{N}, \boldsymbol{N})$

In all possible subsets of this case it is necessary to bribe one party or both parties.

$$TNC_{00} = n - 1$$

Consider subcases when $n$ is even and $t > \frac{n}{2}$ $(t > n - t)$, or when $n$ is odd and $t \geq \frac{n+1}{2}$ $(t \geq n - t + 1)$

| $n_1$ *(Y)* | $n_2$ *(N)* | *Seats to bribe* | | | *States when need to bribe* | |
|---|---|---|---|---|---|---|
| 1 | $n-1$ | $n_2$ | $n-1$ | $\frac{n(n-1)}{2} - \frac{t(t-1)}{2}$ | 1 | $n-t$ |
| 2 | $n-2$ | $n_2$ | $n-2$ | | 1 | |
| … | | $n_2$ | … | | … | |
| $n-t-1$ | $t+1$ | $n_2$ | $t+1$ | | | |
| $n-t$ | $t$ | $n_2$ | $t$ | | 1 | |
| $n-t+1$ | $t-1$ | $n_1+n_2$ | $n$ | $n(2t-n-1)$ | 1 | $2t-n-1$ |
| … | | | | | … | |
| $t-1$ | $n-t+1$ | $n_1+n_2$ | $n$ | | 1 | |
| $t$ | $n-t$ | $n_1$ | $t$ | $\frac{n(n-1)}{2} - \frac{t(t-1)}{2}$ | 1 | $n-t$ |
| $t+1$ | $n-(t+1)$ | $n_1$ | $t+1$ | | 1 | |
| … | | | … | | … | |
| $n-2$ | 2 | $n_1$ | $n-2$ | | 1 | |
| $n-1$ | 1 | $n_1$ | $n-1$ | | 1 | |

Total seats to be bribed is:

$$n(n-1) - t(t-1) + n(2t-n-1) = (t-1)(2n-t)$$

Next consider subcases when n is even and $t = \frac{n}{2}$.

| $n_1$ *(Y)* | $n_2$ *(N)* | *Seats to bribe* | | | *States when need to bribe* | |
|---|---|---|---|---|---|---|
| 1 | $n-1$ | $n_2$ | $n-1$ | $n(t-1)-\frac{t(t-1)}{2}$ | 1 | $t-1$ |
| 2 | $n-2$ | $n_2$ | $n-2$ | | 1 | |
| … | | | | | … | |
| $t-1$ | $n-(t-1)$ | $n_2$ | $n-(t-1)$ | | 1 | |
| $t$ | $n-t$ | $n_1$ or $n_2$ | t | t | 1 | 1 |
| $t+1$ | $n-(t+1)$ | $n_1$ | $t+1$ | $\frac{n(n-1)}{2}-\frac{t(t+1)}{2}$ | 1 | $n-(t+1)$ |
| … | | | … | | … | |
| $n-2$ | 2 | $n_1$ | $n-2$ | | 1 | |
| $n-1$ | 1 | $n_1$ | $n-1$ | | 1 | |

Total seats to be bribed is:

$$n(t-1)-\frac{t(t-1)}{2}+t+\frac{n(n-1)}{2}-\frac{t(t+1)}{2}=\frac{3n^2}{4}-n$$

Consider cases when n is even and $t < \frac{n}{2}\,(t < n-t)$.

| $n_1$ *(Y)* | $n_2$ *(N)* | *Seats to bribe* | | | *States when need to bribe* | |
|---|---|---|---|---|---|---|
| 1 | $n-1$ | $n_2$ | $n-1$ | $n(t-1)-\frac{t(t-1)}{2}$ | 1 | $t-1$ |
| 2 | $n-2$ | $n_2$ | $n-2$ | | 1 | |
| … | | | … | | … | |
| $t-1$ | $n-(t-1)$ | $n_2$ | $n-(t-1)$ | | 1 | |
| $t$ | $n-t$ | $n_1$ | $t$ | $\frac{n(n-2)}{8}-\frac{t(t-1)}{2}$ | 1 | $n/2-t$ |
| | | | | | … | |
| $n/2-1$ | $n/2+1$ | $n_1$ | $n/2-1$ | | 1 | |
| $n/2$ | $n/2$ | $n_1$ or $n_2$ | $n/2$ | $n/2$ | 1 | 1 |
| $n/2+1$ | $n/2-1$ | $n_2$ | $n/2-1$ | $\frac{n(n-2)}{8}-\frac{t(t-1)}{2}$ | 1 | $n/2-t$ |
| | | | | | … | |
| $n-t$ | $t$ | $n_2$ | t | | 1 | |
| $n-t+1$ | $t-1$ | $n_1$ | $n-t+1$ | $n(t-1)-\frac{t(t-1)}{2}$ | 1 | $t-1$ |
| | | | | | … | |
| $n-1$ | 1 | $n_1$ | n-1 | | 1 | |

Total seats to be bribed is:

$$2n(t-1)-t(t-1)+\frac{n(n-2)}{4}-t(t-1)+\frac{n}{2}=2(t-1)(n-t)+\left(\frac{n}{2}\right)^{2}$$

Consider subcases when n is odd and $t < \frac{n}{2}\,(t < n - t)$.

| $n_1$ *(Y)* | $n_2$ *(N)* | *Seats to bribe* | | | *States when need to bribe* | |
|---|---|---|---|---|---|---|
| 1 | $n-1$ | $n_2$ | $n-1$ | $n(t-1) - \frac{t(t-1)}{2}$ | 1 | $t-1$ |
| 2 | $n-2$ | $n_2$ | $n-2$ | | 1 | |
| … | | | … | | … | |
| $t-1$ | $n-t+1$ | $n_2$ | $n-(t-1)$ | | 1 | |
| t | $n-t$ | $n_1$ | $t$ | $\frac{(n-1)(n+1)}{8} - \frac{t(t-1)}{2}$ | 1 | $\frac{n-1}{2} - t + 1$ |
| | | | | | … | |
| $\frac{n-1}{2}$ | $\frac{n+1}{2}$ | $n_1$ | $\frac{n-1}{2}$ | | 1 | |
| $\frac{n+1}{2}$ | $\frac{n-1}{2}$ | $n_2$ | $\frac{n-1}{2}$ | $\frac{(n-1)(n+1)}{8} - \frac{t(t-1)}{2}$ | 1 | $n - t - \frac{n+1}{2} + 1$[1] |
| | | | | | … | |
| $n-t$ | $t$ | $n_2$ | $t$ | | 1 | |
| $n-t+1$ | $t-1$ | $n_1$ | $n-t+1$ | $n(t-1) - \frac{t(t-1)}{2}$ | 1 | $t-1$ |
| | | | | | … | |
| $n-1$ | 1 | $n_1$ | $n-1$ | | 1 | |

Total seats to be bribed is:

$$2n(t-1) - t(t-1) + \frac{(n-1)(n+1)}{4} - t(t-1) = 2(t-1)(n-t) + \frac{n^2-1}{4}$$

We aggregate for any given set of states $\Omega_n^m$ of a $(2, n)$-parliament all seats and finally obtain the ex-ante sand ex-post costs:

$$C_A = \frac{(t-1)(2n-t)}{2(n-1)} \text{ and } C_P = \frac{2(t-1)(2n-t)}{n+2t-3} \qquad \text{if } t > \frac{n}{2}$$

$$C_A = \frac{(t-1)(2n-t) + \frac{3n^2}{4} - n}{4(n-1)} \text{ and } C_P = \frac{(t-1)(2n-t) + \frac{3n^2}{4} - n}{n+2t-3} \qquad \text{if } t = \frac{n}{2}$$

$$C_A = \frac{(t-1)(4n-3t) + \frac{n^2}{4}}{4(n-1)} \text{ and } C_P = \frac{(t-1)(4n-3t) + \frac{n^2}{4}}{n+2t-3} \qquad \text{if } t < \frac{n}{2} \text{ and } n \text{ even}$$

$$C_A = \frac{(t-1)(4n-3t) + \frac{n^2-1}{4}}{4(n-1)} \text{ and } C_P = \frac{(t-1)(4n-3t) + \frac{n^2-1}{4}}{n+2t-3} \qquad \text{if } t < \frac{n}{2} \text{ and } n \text{ odd}.$$

## A-IV. The cost of bribing parties with party discipline: 3 parties

**Proposition 6.1.** *For any given set of states* $\Omega_n^m$ *of a* $(3, n)$*-parliament handling a YES-NO decision problem and an acceptance threshold* $t$*, the ex-ante cost* $C_A$ *and ex-post cost* $C_P$ *are stated in following Table.*

[1] $\frac{n-1}{2} - t + 1 = \frac{n+1}{2} - t = n - t - \frac{n+1}{2} + 1$

**Table IV.1. Total number of seats bribed with three parties for $\frac{n}{2} < t < \frac{2n}{3}$**

| *Party decisions* $(d_1, d_2, d_3)$ | *Total number of states* | *Number of states where there is a need to bribe* | *Total number of seats bribed* | *Conditions* |
|---|---|---|---|---|
| (Y,Y,Y) | $\frac{1}{2}(n-1)(n-2)$ | 0 | 0 | |
| **(N,Y,Y)** (Y,Y,N) (Y,N,Y) | $\frac{3}{2}(n-1)(n-2)$ | $3\sum_{x=1}^{t-2}(t-1-x)$ | $3\sum_{x=1}^{t-2}\left[(t-x-1)\left(n-\frac{t+x}{2}\right)\right]$ | |
| **(N,N,Y)** (N,Y,N) (Y,N,N) | $\frac{3}{2}(n-1)(n-2)$ | $3\sum_{x=1}^{t-1}(n-x-1)$ | $3\sum_{x=1}^{2t-n-1}[(t-1-x)(2n-x-t)]$ | $x < 2t-n$ |
| | | | $3\left[\frac{3(2n-2t)^2}{4}-2n+2t\right]$ | $x = 2t-n$ |
| | | | $3\sum_{x=2t-n+1}^{t-1}\left[2(t-1-x)(n-t)+\frac{(n-x)^2}{4}\right]$ | $x > 2t-n$ only for even $n-x$ |
| | | | $3\sum_{x=2t-n+1}^{t-1}\left[2(t-1-x)(n-t)+\frac{(n-x-1)^2}{4}\right]$ | $x > 2t-n$ only for odd $n-x$ |
| (N,N,N) | $\frac{1}{2}(n-1)(n-2)$ | $\frac{1}{2}(n-1)(n-2)$ | $\sum_{k=0}^{n-t}(t+k)(3(n-t-k-1)+\mathrm{NC}_{2,t+k})$ | |
| | $2^2(n-1)(n-2)$ | | | |

*Where k, x are positive integers.*
$\mathrm{NC}_{2,t+k}$ *is counted according to the Table V.1. in the Appendix*

We are forced to divide the proof of this proposition in two parts due to the different used approaches. Proofs for all but (N,N,N) cases are provided in the current section (part a). The case when all parties vote NO (part b of the proof) is presented in the next section of the Appendix.

## Proof of Proposition 6.1. (part a):

For the 3-Prty system total number of cases is:

$$TNC = 2^3\binom{n-1}{2} = \frac{2^3(n-1)!}{2!\,(n-3)!} = \frac{2^3(n-1)(n-2)}{2} = 2^2(n-1)(n-2)$$

Partly the 3-party case can be sold inductively from the 2-party case by adding 1 more party:

### Lemma IV

*If the added party votes YES we can apply calculations from 2-party case with new conditions $n' = n-x$ and $t' = t-x$, where $x$ is the size of added party ($x \in [1,\ t-1]$):*

$$\Omega^2_{(n-x),(t-x)}\{n_1, d_1, d_2\}$$

When $x$ takes values more or equal to $t$, there is no need to bribe. When $x$ takes other values, there can appear cases when need and cases when no need to bribe, depending on $d_1$ and $d_2$.

There are $2^3 = 8$ possible assignments, which can be divided in 4 groups according to the vector $(\mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3)$:

${}^0\Omega$: (Y,Y,Y) – no party to be bribed;

${}^1\Omega$: (N,Y,Y), (Y,Y,N), (Y,N,Y) – maximum 1 party to be bribed;

${}^2\Omega$: (N,N,Y), (N,Y,N), (Y,N,N) – maximum 2 parties to be bribed;

${}^3\Omega$: (N,N,N) – maximum 3 parties to be bribed.

Step by step we need to consider each group of cases.

**${}^0\Omega$**

There is no party to be bribed, therefore there are no cases when need to bribe and no seats to be bribed.

**${}^1\Omega$**

We have three symmetric cases. Consider the case (N,Y,Y) and that the added party is the party 3 ($d_3 = 1$). So, according to the Lemma IV this situation can be presented as if we have 2 parties, changed number of seats $n' = n - x$ and the changed threshold $t' = t - x$, where $x$ takes values from 1 to $t - 2$. As only $x$ takes $t - 1$ value, there is no need to bribe as we have at least 1 seat of the YES party.

So seats to bribe are calculated for each $x$ as in state $\Omega^2_{n',t'}(n_1, N, Y)$.

$$3\sum_{x=1}^{t-2}\left[(t-x-1)\left(n-x-\frac{t-x}{2}\right)\right] = 3\sum_{x=1}^{t-2}\left[(t-x-1)\left(n-\frac{t+x}{2}\right)\right]$$

And for each $x$ the number of states when need to bribe is $(t - 1 - x)$. The total number of states when need to bribe is:

$$3\sum_{x=1}^{t-2}(t-1-x)$$

**${}^2\Omega$**

We again have three symmetric cases. Consider the case NNY: $d_3$=1. The changed $n$ is $n' = n - x$ and the changed threshold is $t' = t - x$, where $x$ can take values from 1 to $t - 1$. So, seats to bribe are calculated for each x as in case $\Omega^2_{n',t'}(n_1, N, N)$:

*a.* If $t - x > \frac{n-x}{2} \Leftrightarrow x < 2t - n$

$$3\sum_{x=1}^{2t-n-1}[(t-1-x)(2n-2x-t+x)] = 3\sum_{x=1}^{2t-n-1}[(t-1-x)(2n-x-t)]$$

*b.* If $t = \frac{n}{2} \Leftrightarrow x = 2t - n$

$$3\left[\frac{3(n-x)^2}{4} - n + x\right] = 3\left[\frac{3(n-2t-n)^2}{4} - n + 2t - n\right] = 3\left[\frac{3(2n-2t)^2}{4} - 2n + 2t\right]$$

*c.* If $t < \frac{n}{2}$ and $n - x$ is even:

$$3 \sum_{x=2t-n+1}^{t-1} \left[ 2(t-x-1)(n-t) + \frac{(n-x)^2}{4} \right] \; for\ only\ even\ n-x$$

*d.* If $t < \frac{n}{2} \Leftrightarrow x > 2t - n$ and $n - x$ is odd:

$$3 \sum_{x=2t-n+1}^{t-1} \left[ 2(t-x-1)(n-t) + \frac{(n-x)^2 - 1}{4} \right] \; for\ only\ odd\ n-x$$

States when need to bribe for each x is $n - x - 1$. Total number of states when need to bribe is:

$$3 \sum_{x=1}^{t-1} (n - x - 1)$$

**$^3\Omega$**

The most complicated case, the only case which cannot be solved by induction from the case of 2-party. In the next section of the Appendix we provide proofs for the (3, 100)-parliament and (N,N,N) case.

## A-V. The cost of bribing parties with party discipline: 3 NO-parties

We are in the situation, when all parties are against: $d_1 = d_2 = d_3 = N$. This situation may be considered as a part of previous section.

## Proof Proposition 6.1. (part b)

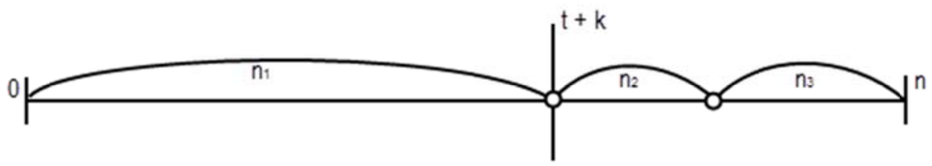
The states of the world are:

$$\Omega_{100}^3\{(n_1, n_2, n_3)(N, N, N)\}, \qquad \frac{n}{2} \le t < \frac{2n}{3}$$

Let us consider the situation in general when we have to bribe $t + k$ seats, where $k \in [0, n - t]$. There are two possible cases we need to bribe exactly $t + k$ seats: one party is equal to $t + k$ or two parties are equal to $t + k$.

**First case**

Let party 1 to be the one party to be bribed. If $n_1 = t + k$, the less costly is to bribe exactly $n_1$ $(t + k > n_2 + n_3)$.



The number of states is the number of possible dividing the rest seats between 2 parties. Due to the symmetry of the cases we have to multiply by 3:

$$\mathrm{NS}_{1,t+k} = 3\binom{n-1-t-k}{1} = 3(n-t-k-1)$$

Total number of seats to be bribed (aggregate cost for the particular case):

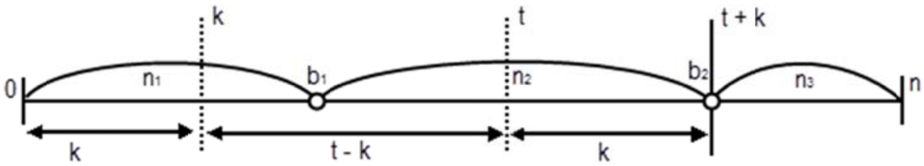
$$3(t+k)(n-t-k-1)$$

And conditions are: $t+k \leq n-2$

**Second case**

Let party $n_1$ and $n_2$ be the two parties to be bribed. For more convenience consider border between party 1 and party 2 as $b_1$, and border between party 2 and party 3 as $b_2$. All seats to the left of $b_1$ belong to the party 1. All seats between $b_1$ and $b_2$ belong to party 2. All seats to the right from $b_2$ belong to party 3. So, $b_1 \in [1, b_2]$ and $b_2 \in [b_1, n\text{-}1]$. Fixing $n_1+n_2$ means that we fix $b_2 = t+k$. To easy the notation consider $n_3 = x = n-t-k$.

On the scheme it can be drawn in the following way:




Sector 1 $(0,k]$, Sector 2 $(k,t)$, Sector 3 $[t,t+k)$ Sector 4 $[t+k,n)$

And now our aim is to define subsets of states when to bribe $n_1+n_2$ is the best choice (meaning that all parties are smaller than *t*, and sums $n_1+n_3$, $n_3+n_2$greater or equal to $t+k$, or strongly smaller than *t*).

In all states when $b_1 \in [1, k]$ it is better to bribe $n_2$, when $b_1 \in [t, t+k-1]$ it is better to bribe $n_1$. So we are interested in all states when $b_1 \in [k+1, t-1]$. In these states the cost depends on the proportion of $t$, $k$, $x$ and $n$ to decide what is the best option to bribe.

Measure out the $x$ from the $b_1$ to the left ($b_{3L}$) and to the left ($b_{3R}$) allows to compare $n_1+n_3$, and $n_3+n_2$sums with $t$, $t+k$ and $n_1+n_2$.

If $b_{3R}$ belongs to $[t, t+k-1]$ it is better to bribe $n_1+n_3$. If $b_{3L}$ belongs to $[2, k]$ it is cheaper to bribe$n_3+n_2$. We are not interested in these states, as these states will be counted in case of $t+k'$.

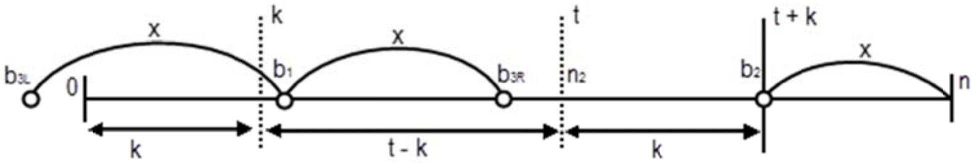
Let us go over all possible options when to bribe$n_1+n_2$is the best (but not always the only best) choice.

**Remark.** As we consider only the case $n_1+n_2$we have to think how should we extend it on the $n_1+n_3$, and $n_2+n_3$ cases. From the first point of view it seems to be easily multiplied by 3. But it is much more complicated if we think about the case $(n_1, n_2, n_3)$=(33,33,34). Multiplying by 3 predicts that we make different between 2 sets (33, 33, 34) and (33, 33, 34). With such a set when we have two parties with the same number of seats we should be accurate. To compare:

| | | |
|---|---|---|
| (33, 33, 34) | (31, 32, 36) | (32, 31, 36) |
| (33, 34, 33) | (31, 36, 32) | (36, 31, 32) |
| (34, 33, 33) | (32, 36,31) | (36, 32, 31) |
| Totally 3 cases | Totally 6 cases. | |

So, depending on whether $t+k$ is odd or even we have to multiply by different coefficient.

**<u>First option</u>.** $b_{3R}$ falls into the Sector 2. And $b_{3L}$ falls outside.




Such a location of borders states that $n_2+n_3>n_1+n_2$ ($n_2+n_3>t+k$) and $n_1+n_3<t$. So the best choice is to bribe $n_1+n_2$.

Conditions:

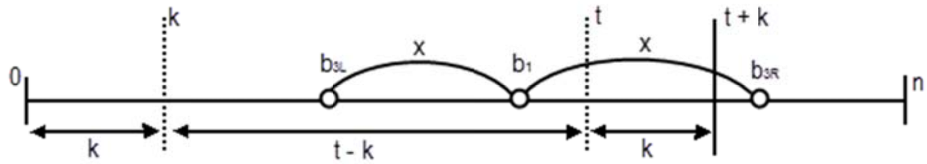
$$\begin{cases} x \geq k+2 \\ x \leq t-k-2 \end{cases} \leftrightarrow \begin{cases} n \geq t+2k+2 \\ n \leq 2t-2 \end{cases}$$

To count the number of states we have to know extreme position of $b_1$. And depending on its' values arise 2 possibilities:

$$NS_{2,t+k} = \begin{cases} x-1-[k+1]+1, if\ t-1-x \geq x-1 \\ t-1-x-[k+1]+1, if\ t-1-x < x-1 \end{cases} = \begin{cases} n-2k-t-1, if\ 2n \leq 3t+2k \\ 2t-n-1, if\ 2n > 3t+2k \end{cases}$$

To count the symmetric states we have to consider first the next option.

**<u>Second option</u>.** $b_{3L}$ belongs to the Sector 2, $b_{3R}$ belongs to the Sector 4.




Such a location of borders states that $n_1+n_3>n_1+n_2$ ($n_1+n_3>t+k$) and $n_2+n_3<t$. So the best choice is to bribe $n_1+n_2$.

Conditions:

$$\begin{cases} x \geq k+2 \\ x \leq t-k-2 \end{cases} \leftrightarrow \begin{cases} n \geq t+2k+2 \\ n \leq 2t-2 \end{cases}$$

To count the number of states we have to know extreme position of $b_1$. And depending on its' values arise 2 possibilities:

$$NS_{2,t+k} = \begin{cases} x-1-[k+1]+1, if\ t-1-x \geq x-1 \\ t-1-x-[k+1]+1, if\ t-1-x < x-1 \end{cases} = \begin{cases} n-2k-t-1, if\ 2n \leq 3t+2k \\ 2t-n-1, if\ 2n > 3t+2 \end{cases}$$

This option is the symmetric to the first one. The conditions and the number of cases are the same. To find out the coefficient let us give an example:

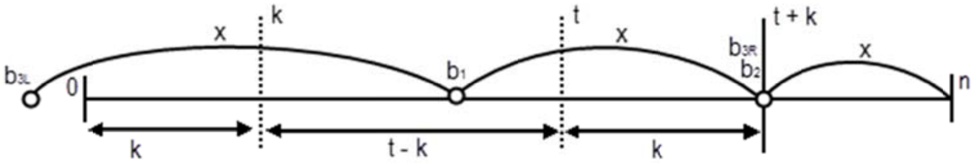
**Example.** $n=12$, $t=7$, $k=1$. *The only possible vector of parties which satisfy the first option is (2, 6, 4). These dividing can be assigned to parties in 6 ways (2, 4, 6), (4, 2, 6) and so on. Notice that there would be 3 ways of assignment if two parties had the same number of seats, and only 1 way, if all parties had the same number of seats. For the second option is (6, 2, 4).*

*The same assignments of seats were already considered in the first option. So if we multiply the first option by 6, all assignments of the second option will enter into the first option.*

So, the number of seats to bribe should be multiplied by 6:

$$6(t+k)NS_{2,t+k}$$

**Third option.** $b_{3R}$=$b_2$ is equal to $t+k$ and $b_{3L}$ falls outside.

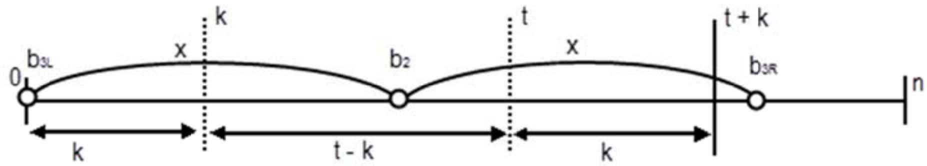



Conditions:

$$\begin{cases} x \geq k+1 \\ 2x \geq t+k+1 \end{cases} \leftrightarrow \begin{cases} n \geq t+2k+1 \\ 2n \geq 3t+3k+1 \end{cases}$$

$b_{3R}$=$b_2$ means that $n_2 = n_3$. So we have (a, x, x) type of vector. There is only 3 ways of assigning this vector to the parties (as only two parties can have the same number of seats). To count the symmetric cases we have to consider first the next situation

**Fourth option**. $b_{3R}$ belongs to the Sector 4, $b_{3L}$ =0.



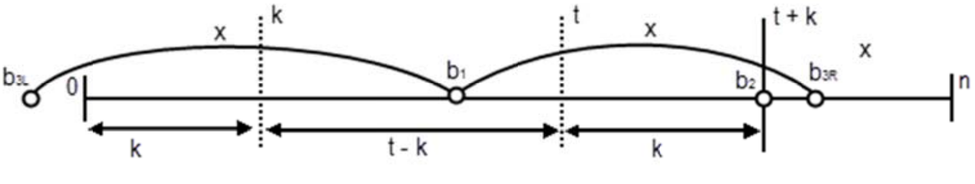

Conditions:

$$\begin{cases} x \geq k+1 \\ 2x \geq t+k+1 \end{cases} \leftrightarrow \begin{cases} n \geq t+2k+1 \\ 2n \geq 3t+3k+1 \end{cases}$$

$b_{3L}$ =0 means that $n_1 = n_3$and we are indifferent between bribing $n_1 + n_2$ or $n_2 + n_3$. So we have (x, a, x) type of vector. Notice, that $a = t + k - x$, which exactly replicates the $a$ in the previous options. So, we have to count this situation as a part of the previous one with taking into account all possible assignments of the vector to the parties. So the number of seats to bribe in both situations is:

$$3(t+k)$$

**Fifth option.** $b_{3R}$ belongs to the Sector 4. And $b_{3L}$ falls outside.



Conditions:

$$\begin{cases} x \geq k+2 \\ 2x \geq t+k+2 \end{cases} \leftrightarrow \begin{cases} n \geq t+2k+2 \\ 2n \geq 3t+3k+2 \end{cases}$$

$$NC_{2,t+k} = x - 1 - [t+k+1-x] + 1 = 2n - 3t - 3k - 1$$

Such a location states that $n_1 + n_3 > n_1 + n_2$ ($n_1 + n_3 > t + k$) and $n_2 + n_3 > n_1 + n_2$ ($n_2 + n_3 > t + k$). The best choice is $n_1$+$n_2$.

At the first glance it seems that due to the symmetry we have to multiply by 6 when calculating number of seats to bribe. So for each possible location of the $b_1$ we have 3 possible assignments seats to parties. But if $t + k$ is even, there is one location of $b_1$, when there are not 6, but 3 possible assignments: when $n_1 = n_2$. Besides, all possible assignments when $b_1$ falls on the right of the middle are included already in all possible assignments when $b_1$ falls on the left of this middle point. So the assignments vector (a, b, x) at the left points is repeated by the vector (b, a, x) at the right points.

So, we have to multiply number of cases only by 6/2 and to add 1 case when $n_1 = n_2$ (if $t + k$ is even), also multiplied by 3.

$$S_{2,t+k} = 3(t+k)(2n - 3t - 3k - 1)$$

***Proof:*** *We have to recalculate the number of symmetric cases (which we have to multiply by 6):*

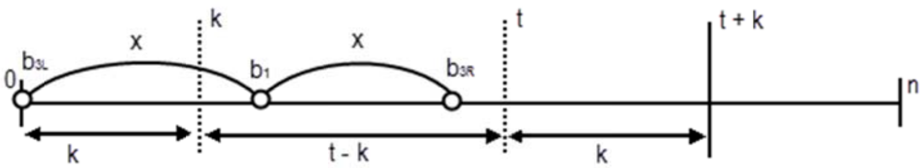
$$NS_{2,t+k} = \begin{cases} \dfrac{t+k-1}{2} - [t+k+1-x] + 1 = n - \dfrac{3t+3k+1}{2}, when\ t+k\ is\ odd \\ \dfrac{t+k}{2} - 1 - [t+k+1-x] + 1 = n - \dfrac{3t+3k+2}{2}, when\ t+k\ is\ even \end{cases}$$

*And one more case when $n_1 = n_2$.So we have to calculate seats to bribe as following:*

$$S_{2,t+k} = \begin{cases} 6(t+k)\left(n - \dfrac{3t+3k+1}{2}\right), when\ t+k\ is\ odd \\ 6(t+k)\left(n - \dfrac{3t+3k+2}{2}\right) + 3(t+k), when\ t+k\ is\ even \end{cases}$$

$$S_{2,t+k} = \begin{cases} 3(t+k)(2n-3t-3k-1), when\ t+k\ is\ odd \\ 3(t+k)(2n-3t-3k-1), when\ t+k\ is\ even \end{cases}$$

**Sixth option.** $b_{3R}$ belongs to the sector 2, $b_{3L}$ =0.

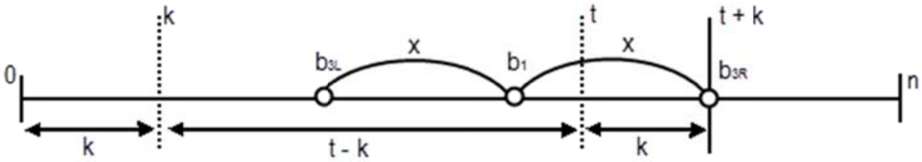



Conditions:

$$\begin{cases} 2x \leq t-1 \\ x \geq k+1 \\ x \leq t-k-2 \end{cases} \leftrightarrow \begin{cases} 2n \leq 3t+3k-1 \\ n \geq 2k+t+1 \\ n \leq 2t-2 \end{cases}$$

When $b_{3L}$=0 $n_1$=$n_3$. It means that we are indifferent between bribing $n_1 + n_2$or $n_2 + n_3$. There is only 1 case. To count the number of seats to bribe we have to consider first the next option.

**Seventh option**. $b_{3R}=t+k$, $b_{3L}$ belongs to the Sector 2.



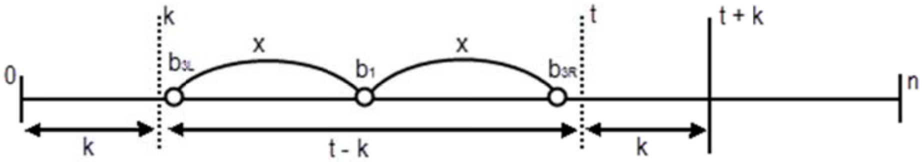

Conditions:

$$\begin{cases} 2x \le t-1 \\ x \le t-k-2 \\ x \ge k+1 \end{cases} \leftrightarrow \begin{cases} 2n \le 3t+2k-1 \\ n \le 2t-2 \\ n \ge t+2k+1 \end{cases}$$

$b_{3R}=t+k$ imposes that $n_2 = n_3$ So we are indifferent in bribing $n_1+n_2$ or $n_1+n_3$. There is only 1 case. By analogy with the Third and Fourth options the number of seats to bribe:

$$\mathrm{S}_{2,t+k} = 3(\mathrm{t}+\mathrm{k})$$

**Eighth option**. $b_{3L}$, $b_{3R}$ belong to the Sector 2.




Conditions:

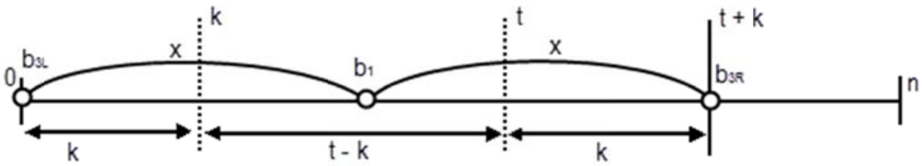
$$2x \le t-k-2 \leftrightarrow 2n \le 3t+k-2$$

$$NS_{2,t+k} = t-1-x-[k+1+x]+1 = 3t-2n+k-1$$

Such a location shows as that $n_1+n_3 < t$, $n_2+n_3 < t$. So, the $n_1+n_2$ is the best choice. By analogy with the Fifth option:

$$S_{2,t+k} = 3(t+k)(3t-2n+k-1)$$

**Ninth option**. $b_{3R}=t+k$, $b_{3L}$ =0.




Conditions:

$$\begin{cases} 2x = t+k \\ n \vdots 3 \end{cases} \leftrightarrow \begin{cases} t+k = \frac{2}{3}n \\ n \vdots 3 \end{cases}$$

$b_{3R}=t+k$, $b_{3L}=0$ mean that $n_1 = n_2 = n_3$ and we are indifferent between bribing $n_1+n_2$ or $n_1+n_3$or $n_2+n_3$. There is only1 case.

$$S_{2,t+k} = (t+k)$$

**Aggregated result**

The final aggregate cost for all states with (N,N,N) assignment is:

$$C_A(N,N,N) = \sum_{k=0}^{k=n-t} (t+k)(NS_{1,t+k} + NS_{2,t+k})$$

Where:

$$NS_{1,t+k} = 3(n-t-k-1), \qquad t+k <= n-2$$

And the value of $NS_{2,t+k}$ takes value depending on parameters $n$, $t$, $k$ according to the following Table.

**Table V.1. Number of seats to be bribed when need to buy 2 parties**

| *Options* | *Number of states taking into account symmetry cases* | *Conditions* |
|---|---|---|
| 1 and 2 | $6(n-2k-t-1)$ | $\begin{cases} n \geq t+2k+2 \\ n \leq 2t-2 \\ 2n \leq 3t+2k \end{cases}$ |
| | $6(2t-n-1)$ | $\begin{cases} n \geq t+2k+2 \\ n \leq 2t-2 \\ 2n > 3t+2k \end{cases}$ |
| 3 and 4 | 3 | $\begin{cases} n \geq t+2k+1 \\ 2n \geq 3t+3k+1 \end{cases}$ |
| 5 | $3(2n-3t-3k-1)$ | $\begin{cases} n \geq t+2k+2 \\ 2n \geq 3t+3k+2 \end{cases}$ |
| 6 and 7 | 3 | $\begin{cases} 2n \leq 3t+3k-1 \\ n \geq 2k+t+1 \\ n \leq 2t-2 \end{cases}$ |
| 8 | $3(3t-2n+k-1)$ | $\begin{cases} 2n \leq 3t+k-2 \\ k < n-t \end{cases}$ |
| 9 | 1 | $\begin{cases} t+k = \frac{2}{3}n \\ n \vdots 3 \end{cases}$ |

**Remark.** For the case $t = n/2$ appears a subset of states when the third party is enough to bribe. So, the k=0 value should be excluded from the sum. And especially for the case k=0 the cost of bribing the only the third party should appear:

$$C_A(N,N,N) = 3(t+k)(n-t-1) + \sum_{k=1}^{n-t} (t+k)(NS_{1,t+k} + NS_{2,t+k})$$

## A-VI. Simplex approach to the 3-party system, when all parties votes NO

All possible assignments of the seats form an equilateral triangle, consisting of points. Each point represents 1 possible state of the world $\Omega_n^3\{(n_1, n_2, n_3)(0,0,0)\}$.

For each $i, j, k \in \{1, 2, 3\}$ and $i \neq j \neq k$ it is true the following statements:

1. Each party begins from one side of the triangle. For all the points belong to the side *i* it is true the following statement: $n_{\mathrm{i}} = 1$.

2. Intersection of side $n_i$ and $n_j$ ($i \neq j$) form an apex *ij* of the triangle with the following assignment of seats to the parties: $\mathrm{n_i} = \mathrm{n_j} = 1$, $\mathrm{n_k} = n - 2$.

3. An altitude from apex *ij* divides the triangle in two equivalent areas $a_i$ and $a_j$, where index *i* represents that the side *i* belongs to the area. For area $a_i$ it is true that all belonging to it points are the states of the world, such that the party *i* has less seats than the party *j*. Following Figure provides illustration:

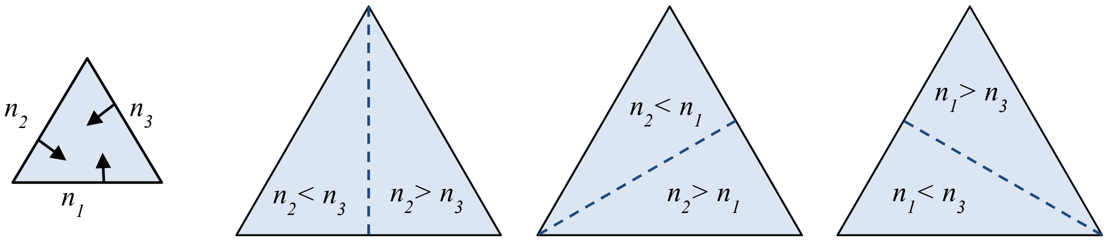



Figure 1. Defining conditions for possible assignments

With the arrows the direction of increase is denoted.

4. Assume, that when having a choice to bribe 1 party or to bribe 2 parties (the number of seats is equal, ex.: $n_1 + n_2 = n_3$) the briber prefers to bribe one party.

5. And counting values of parties supposed to bribe gives the aggregated cost for each area. Counting these numbers should be careful with the borders for not to count twice.

6. For each $l \in [1, n-2]$ the line $n_i = l$, is a line, such that all belonging to it points are the states of the world with the following assignments: $n_i = l,\ n_j + n_k = n - l$ (Ex.: for the side *i* the line $l = 1$ is exactly this side). Therefore, the line $n_i = l$ intersects the side *j* in the point with the assignment $n_i = l,\ n_j = 1,\ n_k = n - l - 1$.

7. For each $l \in [2, n-1]$ the line $n_i + n_j = l$. is a line, such that all belonging to it points are the states of the world with the following assignments: $n_i + n_j = l,\ n_k = n - l$. Therefore, the line $n_i + n_j = l$ intersects the side *i* in the point with the assignment $n_i = l - 1,\ n_j = l,\ n_k = n - l$.

Dealing with the Parliament of 3 parties we have an equilateral triangle and 6 crucial lines $n_i = t$ and $n_j + n_k = t$. These lines divide the triangle in areas. For each area the briber has to define the best (the least costly) choice of the party/ies to bribe.

The location of these lines and the number of areas depends on the t. There are 4 possibilities.

1. **States when $\frac{n}{2} \leq t \leq \frac{2n}{3}$**

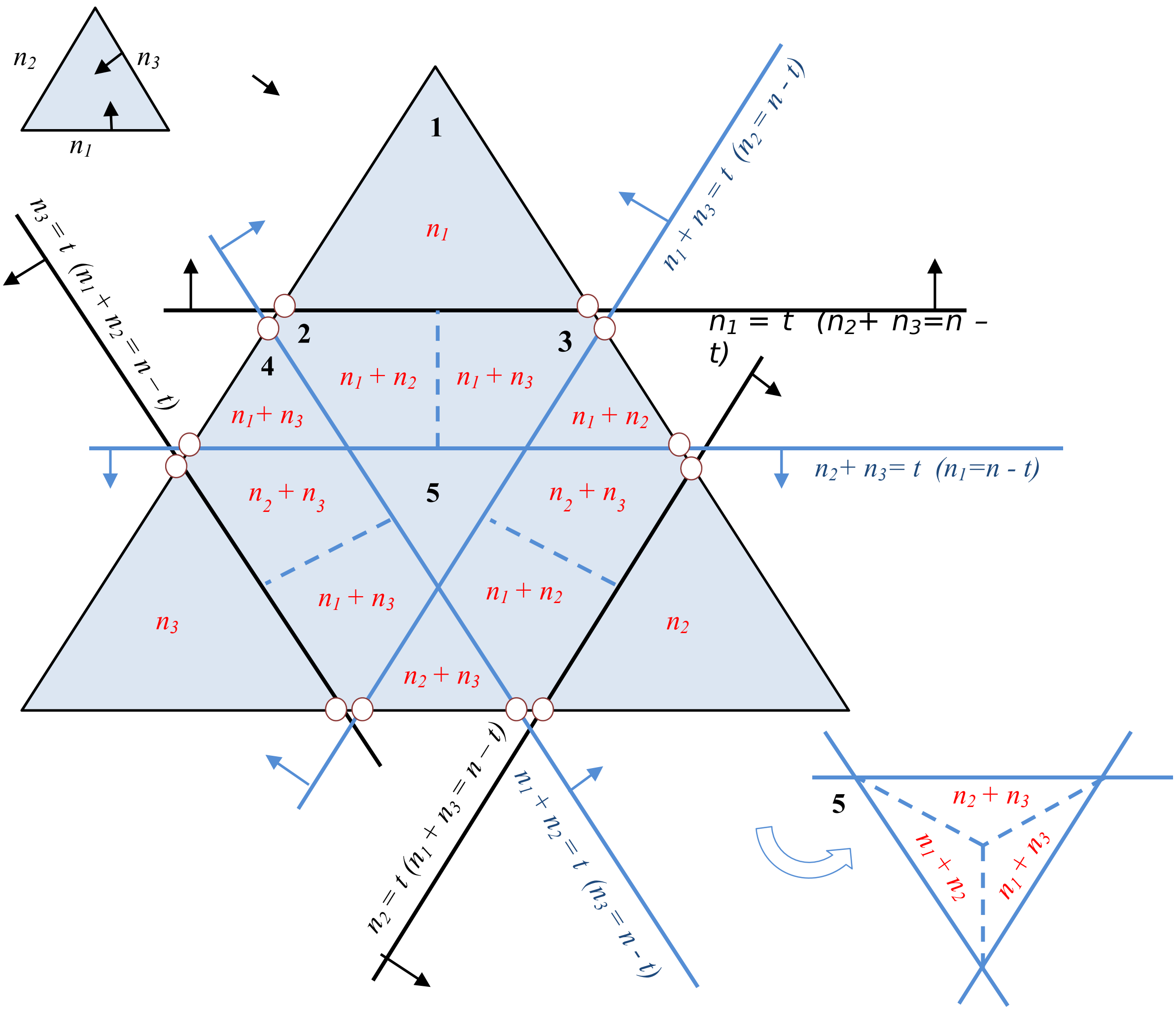



Figure 2. Areas of choice which party to bribe when $\frac{n}{2} \leq t \leq \frac{2n}{3}$

| 1 area | | 2 area | | 3 area | |
|---|---|---|---|---|---|
| $n_1 \geq t$ | $n_1 + n_2 > t$ | $n_1 < t$ | $n_1 + n_2 > t$ | $n_1 < t$ | $n_1 + n_2 > t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ |
| $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 < t$ |
| The best choice: $n_1$ | | According to the statement 3 the best choice is $n_1 + n_2$ | | According to the statement 3 the best choice is $n_1 + n_3$ | |

| 4 area | | 5 area | |
|---|---|---|---|
| $n_1 < t$ | $n_1 + n_2 < t$ | $n_1 < t$ | $n_1 + n_2 > t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ |
| $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 > t$ |
| The best choice: $n_1 + n_3$ | | According to the statement 3 the area is divided in 3 subareas | |

## 2. States when $t > \frac{2n}{3}$

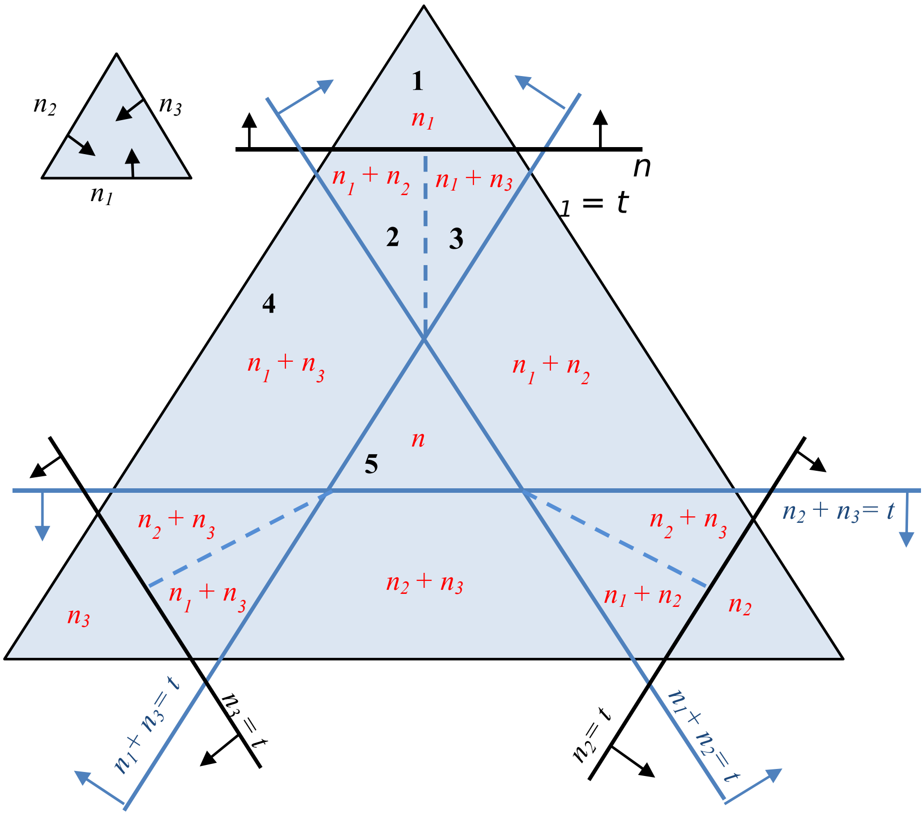



Figure 3. Areas of choice which party to bribe when $t > \frac{2n}{3}$

When $t > \frac{2n}{3}$ there are the following areas:

| 1 area | | 2 area | | 3 area | |
|---|---|---|---|---|---|
| $n_1 \geq t$ | $n_1 + n_2 > t$ | $n_1 < t$ | $n_1 + n_2 > t$ | $n_1 \geq t$ | $n_1 + n_2 > t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ |
| $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 \geq t$ | $n_2 + n_3 > t$ |
| The best choice: $n_1$ | | According to the statement 3 the best choice is $n_1 + n_2$ | | According to the statement 3 the best choice is $n_1 + n_3$ | |

| 4 area | | 5 area | |
|---|---|---|---|
| $n_1 < t$ | $n_1 + n_2 < t$ | $n_1 < t$ | $n_1 + n_2 < t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 < t$ |
| $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 < t$ |
| The best choice is $n_1 + n_3$ | | The only possible choice is n | |

### 3. States when $\frac{\boldsymbol{n}}{\mathbf{3}} < \boldsymbol{t} < \frac{\boldsymbol{n}}{\mathbf{2}}$

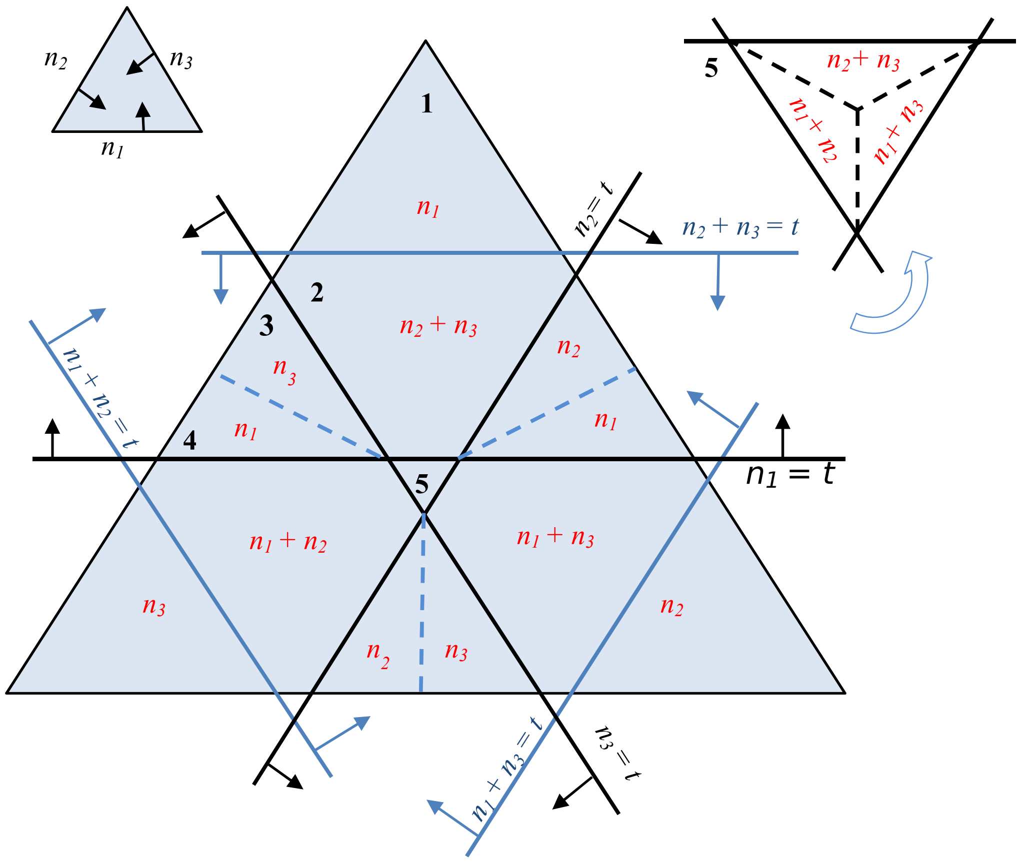



Figure 4. Areas of choice which party to bribe when $\frac{\boldsymbol{n}}{\mathbf{3}} < \boldsymbol{t} < \frac{\boldsymbol{n}}{\mathbf{2}}$

When $\frac{\boldsymbol{n}}{\mathbf{3}} < \boldsymbol{t} < \frac{\boldsymbol{n}}{\mathbf{2}}$ there are the following areas:

| 1 area | 2 area | 3 area |
|---|---|---|
| $n_1 \geq t$ $\quad n_1 + n_2 > t$<br>$n_2 < t$ $\quad n_1 + n_3 > t$<br>$n_3 < t$ $\quad n_2 + n_3 < t$<br>The best choice: $n_1$ | $n_1 \geq t$ $\quad n_1 + n_2 > t$<br>$n_2 < t$ $\quad n_1 + n_3 > t$<br>$n_3 < t$ $\quad n_2 + n_3 > t$<br>As $n_2 + n_3 < n_1$, the best choice is $n_2 + n_3$ | $n_1 \geq t$ $\quad n_1 + n_2 > t$<br>$n_2 < t$ $\quad n_1 + n_3 > t$<br>$n_3 \geq t$ $\quad n_2 + n_3 > t$<br>According to the statement 3 the best choice is $n_3$ |

| 4 area | 5 area |
|---|---|
| $n_1 \geq t$ $\quad n_1 + n_2 > t$<br>$n_2 < t$ $\quad n_1 + n_3 > t$<br>$n_3 \geq t$ $\quad n_2 + n_3 > t$<br>By analogy with the area 3. The best choice is $n_1$ | $n_1 < t$ $\quad n_1 + n_2 > t$<br>$n_2 < t$ $\quad n_1 + n_3 > t$<br>$n_3 < t$ $\quad n_2 + n_3 > t$<br>According to the statement 3 the area is divided in 3 subareas |

### 4. States when $t < \frac{n}{3}$

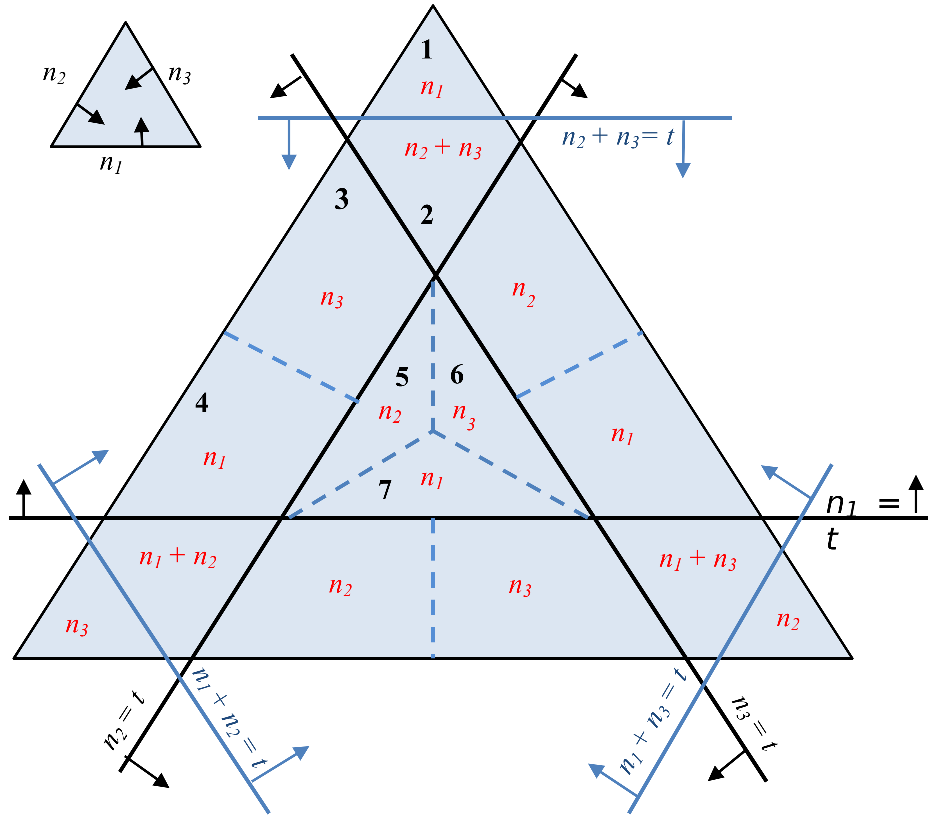



Figure.5. Areas of choice which party to bribe when $t < \frac{n}{3}$

When $t < \frac{n}{3}$ there are the following areas:

| 1 area | | 2 area | | 3 area | |
|---|---|---|---|---|---|
| $n_1 \geq t$ | $n_1 + n_2 > t$ | $n_1 < t$ | $n_1 + n_2 > t$ | $n_1 \geq t$ | $n_1 + n_2 > t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 < t$ | $n_1 + n_3 > t$ |
| $n_3 < t$ | $n_2 + n_3 < t$ | $n_3 < t$ | $n_2 + n_3 > t$ | $n_3 \geq t$ | $n_2 + n_3 > t$ |
| Best choice: $n_1$ | | As $n_1 > n_2$, and $n_1 > n_3$ the best choice is $n_2 + n_3$ | | As $n_3 < n_1$ the best choice is $n_3$ | |

| 4 area | | 5 area | | 6 area | |
|---|---|---|---|---|---|
| $n_1 \geq t$ | $n_1 + n_2 > t$ | $n_1 > t$ | $n_1 + n_2 > t$ | $n_1 > t$ | $n_1 + n_2 > t$ |
| $n_2 < t$ | $n_1 + n_3 > t$ | $n_2 > t$ | $n_1 + n_3 > t$ | $n_2 > t$ | $n_1 + n_3 > t$ |
| $n_3 \geq t$ | $n_2 + n_3 > t$ | $n_3 > t$ | $n_2 + n_3 > t$ | $n_3 > t$ | $n_2 + n_3 > t$ |
| By analogy with the area 3. The best choice is $n_1$ | | According to the statement 3 $n_2$ is the smallest party | | According to the statement 3 $n_3$ is the smallest party | |

| 7 area | |
|---|---|
| $n_1 > t$ | $n_1 + n_2 > t$ |
| $n_2 > t$ | $n_1 + n_3 > t$ |
| $n_3 > t$ | $n_2 + n_3 > t$ |
| According to the statement 3 $n_3$ is the smallest party | |