The multiplier effect in action

The AD function is assumed to only depend on C and I, so $AD = C + I$. Investment is given by the constant investment function $I = \bar{I}$. The value of consumption is determined by the consumption function $C = \hat{C} + c \cdot Y - \pi$, where the inflation rate π is a percentage, $\hat{C}$ (autonomous consumption) is the amount of consumption that does not depend on income Y (and is therefore determined by factors other than income and the inflation rate π), and c (a number between 0 and 1) is called the marginal propensity to consume (which fraction of an additional unit of income is consumed). For simplicity, define the AS function to be given by the linear function $Y = a \cdot \pi$, where a is a positive constant (the "Y" in the AS function is intended to represent aggregate production, that is, real GDP; since all the value of production is assigned to the agents that can contributed to generate that production, it also represents income). Specifically, assume the AD and AS functions to be as follows.

AD function	$AD = C + I = (4 + 0.8 \cdot Y - \pi) + 10 = 14 + 0.8 \cdot Y - \pi$
AS function	$Y = 30 \cdot \pi$

The macroeconomic equilibrium is obtained from the macroeconomic equilibrium condition.

Macroeconomic equilibrium condition $\quad Y = AD$

Therefore, in equilibrium, $Y = 14 + 0.8 \cdot Y - \pi$, where $Y = 30 \cdot \pi$. It then follows from $Y = 14 + 0.8 \cdot Y - \pi$ that $0.2 \cdot Y = 14 - \pi$. Since $Y = 30 \cdot \pi$, $0.2 \cdot 30 \cdot \pi = 14 - \pi$. That is, $6 \cdot \pi = 14 - \pi$, so $\pi^* = 2$ is the equilibrium inflation rate. Inserting this value into the AS function, the equilibrium aggregate production obtains: $Y^* = 30 \cdot 2 = 60$.

Initial macroeconomic equilibrium = $(\pi^*, Y^*) = (2, 60)$

Now, imagine that there is an exogenous boost to aggregate demand: the constant value $\bar{I}$ (autonomous investment) raises from 10 to 17, so $\Delta I = 7$. This change shifts the AD function to the right 7 units. Fig. 1 represents this change graphically (where it is made explicit the fact that each AD function is drawn for a given value of income Y: if income changes, the AD function also changes). The increase in I shifts the AD function from AD_0 to AD_1.

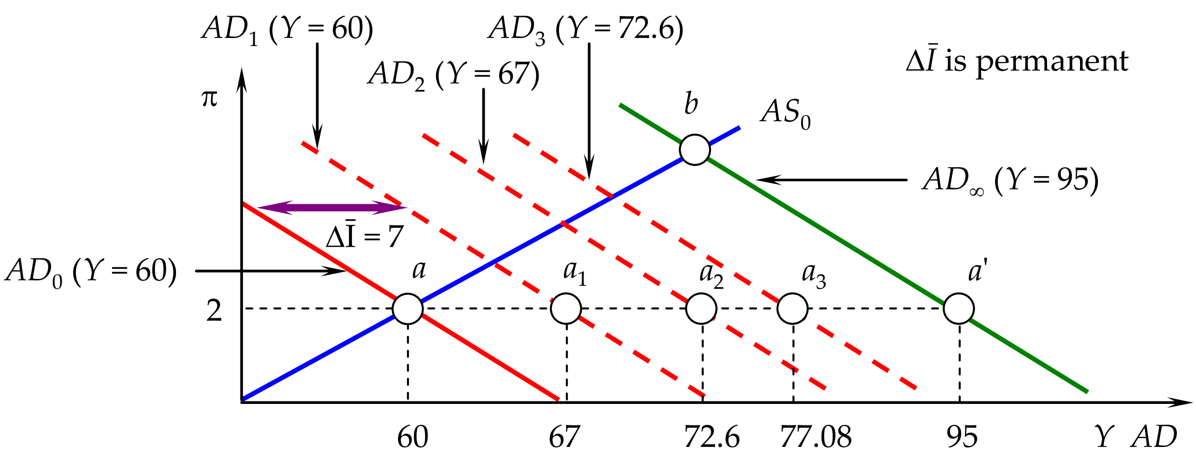

Fig. 1. Effect on income of a permanent change in autonomous spending (fixed inflation rate case)

permanent shock

time	Y	C	I	AD
0	60	$4 + 0.8 \cdot 60 - 2 = 50$	10	60
1	60	$4 + 0.8 \cdot 60 - 2 = 50$	**17**	$50 + 17 = 67$
2	67	$4 + 0.8 \cdot 67 - 2 = 55.6$	17	$55.6 + 17 = 72.6$
3	72.6	$4 + 0.8 \cdot 72.6 - 2 = 60.08$	17	$60.08 + 17 = 77.08$
4	77.08	$4 + 0.8 \cdot 77.08 - 2 = 63.664$	17	$63.664 + 17 = 80.664$
5	80.664	$4 + 0.8 \cdot 80.664 - 2 = 66.5312$	17	$66.5312 + 17 = 83.5312$
…	…	…	17	…
∞	95	$4 + 0.8 \cdot 95 - 2 = 78$	17	$78 + 17 = 95$

equilibrium

Table 1. Effects of a permanent change in autonomous spending with constant inflation rate

transitory shock

time	Y	C	I	AD
0	60	$4 + 0.8 \cdot 60 - 2 = 50$	10	60
1	60	$4 + 0.8 \cdot 60 - 2 = 50$	**17**	$50 + 17 = 67$
2	67	$4 + 0.8 \cdot 67 - 2 = 55.6$	10	$55.6 + 10 = 65.6$
3	65.6	$4 + 0.8 \cdot 65.6 - 2 = 54.48$	10	$54.48 + 10 = 64.48$
4	64.48	$4 + 0.8 \cdot 64.48 - 2 = 53.584$	10	$53.584 + 10 = 63.584$
5	63.584	$4 + 0.8 \cdot 63.584 - 2 = 52.8672$	10	$52.8672 + 10 = 62.8672$
…	…	…	10	…
∞	60	$4 + 0.8 \cdot 60 - 2 = 50$	10	$50 + 10 = 60$

equilibrium

Table 2. Effects of a transitory change in autonomous spending with constant inflation rate

Table 1 shows the <u>multiplicative effect that the initial increase in investment has on income</u> when the inflation rate is assumed fixed (so it is as if the AS function were horizontal at value $\pi = 2$). Time 0 represents the initial equilibrium state (point *a* in Fig. 1). The rise in investment causes the shift from AD_0 to AD_1. In time 1, production remains at $Y_1 = Y_0 = 60$, but AD goes up to 67 (these are the additional 7 units of investment). Assuming that <u>firms adjust production to demand</u>, in time 2 production Y_2 equals aggregate demand AD_1 from the previous period, so $Y_2 = 67$. This is an important change because the increase in outcome (from Y_1 = 60 to Y_2 = 67) shifts the AD function from AD_1 to AD_2 (observe that AD_2 corresponds to an income value of 67). When income is 67, consumption is 55.6, so aggregate demand for time 2 is AD_2 = 72.6. By the assumption that firms completely adjust to demand, AD_2 is the value for aggregate production in the next period: Y_3 = AD_2 = 72.6. This increase in income expands again the AD function, from AD_2 to AD_3: for the latter, Y_3 = 72.6 means that consumption is 60.08. Given that I is always 17, AD_3 = 77.08. And so on. The following sequence describes the changes arising from the initial shock.

$$\Delta\bar{I}_1 \Rightarrow \Delta AD_1 \Rightarrow \Delta Y_2 \Rightarrow \Delta C_2 \Rightarrow \Delta AD_2 \Rightarrow \Delta Y_3 \Rightarrow \Delta C_3 \Rightarrow \Delta AD_3 \Rightarrow \Delta Y_4 \ldots \quad (1)$$

The initial increase in investment gives the economy a push. <u>That push creates enough inertia for the economy to move by itself without needing more shoves</u>. Yet, the expansion cannot go on forever: as time passes, <u>the economy slows down</u>. From time 1 to 2, $\Delta Y = 7$; for 2 to 3, $\Delta Y = 5.6$; from 3 to 4, 4.48; from 4 to 5, 3.584. Eventually, $\Delta Y = 0$.

The last row in Table 1 establishes the limiting values: in the limit, Y converges to 95, consumption to 78, investment to 17, and aggregate demand to 95. <u>In the limit, equilibrium is reached</u> (because production coincides with demand). Those values can be obtained algebraically as follows.

AD function	$AD = C + I = (4 + 0.8\cdot Y - \pi) + 17 = 21 + 0.8\cdot Y - \pi$
Constant inflation rate	$\pi = 2$ (this condition replaces the AS function)
Macroeconomic equilibrium	$Y = AD$

By inserting $\pi = 2$ into the AD function, $AD = 21 + 0.8\cdot Y - 2 = 19 + 0.8\cdot Y$. Combining this equation with the equilibrium condition $Y = AD$, the following equation obtains: $Y = 19 + 0.8\cdot Y$. Solving for Y yields $Y = 95$. The corresponding pair $(\pi, Y) = (2, 95)$ is point a' in Fig. 1, where the AD function passing through a' is defined for the equilibrium income $Y = 95$.

The important conclusion is that <u>the seven additional units of investment have generated an income increase of 35 units</u> (from $Y_0 = 60$ to $Y_\infty = 95$). This is the (expenditure) <u>multiplier effect</u>: by means of the process (1), 7 have been transformed into 35. The value of the multiplier is then 5 (35 divided by 7). It turns out that is values is $1/(1 - c)$, where c is the marginal propensity to consume (hence, the larger c, the higher the multiplier).

(1) is reminiscent of the process associated with the <u>money multiplier</u>:

$$\Delta \text{ deposits} \Rightarrow \Delta \text{ loans} \Rightarrow \Delta \text{ spending} \Rightarrow \Delta \text{ revenues} \Rightarrow \Delta \text{ deposits} \Rightarrow \ldots \quad (2)$$

Processes (1) and (2) are strongly related. When consumption increases in (1), sales rise, so sellers receive money. Part of this money is deposited on banks, which, through loans, fuels consumption. Hence, the process that creates production (physical wealth) also creates money (financial wealth) and vice versa. So both (1) and (2) go hand in hand. Naturally, the two processes also work in the reverse. Table 2 illustrates this. In Table 2, the initial positive shock to investment is assumed transitory: it only lasts a period. The values in the table show that, even though the initial stimulus raises income, the disappearance of the stimulus condemns the economy to return to its initial equilibrium: from $Y = 67$ in time 1, income monotonically decreases to reach $Y = 60$. Despite this, nobody can take always the good times… In sum, point a' in Fig. 1 establish how far the economy can go (the maximum amount of income it can generate) with just 7 additional units of investment applied in point *a*.

Though valuable to establish a benchmark case, the assumption that a demand expansion does not affect the inflation rate is not entirely reasonable. It would then be interesting to know <u>what would happen if the inflation rate can vary</u>. In that case, the economy would reach point *b* in Fig. 2 rather than the point *a* reached when the inflation rate is stuck at $\pi = 2$.

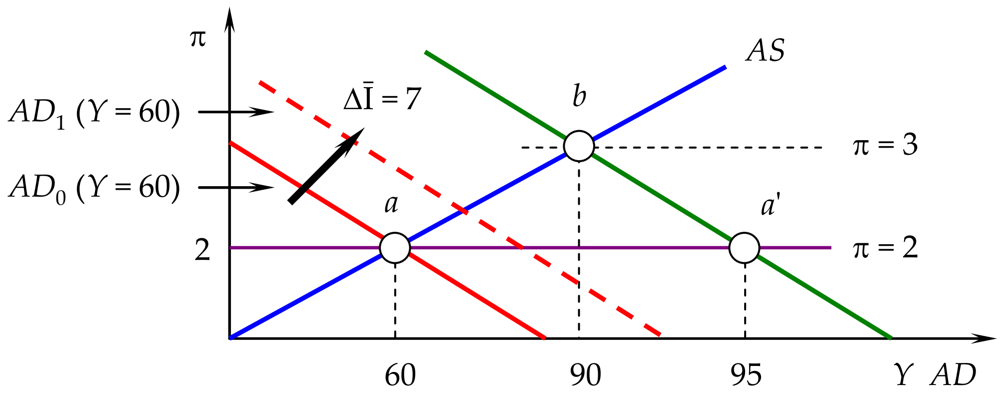

Fig. 2. Effect on income of a permanent change in autonomous spending (variable inflation rate case)

The macroeconomic equilibrium point *b* is obtained from the following system of equations.

AD function	$AD = C + I = (4 + 0.8 \cdot Y - \pi) + 17 = 21 + 0.8 \cdot Y - \pi$
AS function	$Y = 30 \cdot \pi$
Macroeconomic equilibrium condition	$Y = AD$

By the equilibrium condition, $Y = 21 + 0.8 \cdot Y - \pi$. Therefore, $0.2 \cdot Y = 21 - \pi$. Using the AS function, $0.2 \cdot 30 \cdot \pi = 21 - \pi$. Accordingly, $6 \cdot \pi = 21 - \pi$. As a result, $\pi = 3\%$. Given $\pi = 3$, by the AS function, $Y = 90$.

Hence, when the economy is allowed to adjust prices in response to demand conditions, the increase in income generated by $\Delta \bar{I} = 7$ is 30 (Y goes from 60 to 90) instead of 35. The additional point in inflation (π goes from 2 to 3) is the price to be paid for the 30 additional units of income (the cost for the economy of mobilizing and reorganizing resources to produce the additional amount of goods). In fact, just observe that consumption depends negatively on π, so more inflation cuts spending that could otherwise generate more income. To better understand how this cost is created, try to solve the following exercise

Exercise. Compute again the values in Table 1 when the economy reacts to demand shocks by adjusting both aggregate production and the inflation rate.

PS. (1) and (2) are macroeconomic instances of the ubiquitous <u>snowball effect</u>. Here is another, perhaps more familiar example.

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