2 Overlapping generations model with private and public lending

1. Taxes

The analysis follows McCandless and Wallace (1991). Assume that a government is created that merely taxes endowments. A negative tax will be called "transfer". Individual i of generation t faces the tax scheme $\tau_t^i = \left(\tau_t^i(t), \tau_t^i(t+1)\right)$, where $\tau_t^i(s)$ is the tax that i pays (or receives) in period $s \in \{t, t+1\}$.

The budget constraint on the government when nothing is done with the taxes (taxes are just paid out as transfers) states that, for all $t \geq 1$,

$$\sum_{i\in N(t)} \tau_t^i(t) + \sum_{i\in N(t-1)} \tau_{t-1}^i(t) = 0.$$

To compute the GCE, consider the no tax case and replace $w_t^i(s)$ with $w_t^i(s) - \tau_t^i(s)$. The only additional condition to calculate GCE is the government budget constraint. In particular, the new lifetime budget constraint of consumer i is

$$c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) - \tau_t^i(t) + \frac{w_t^i(t+1) - \tau_t^i(t+1)}{R(t)}. \tag{1}$$

Define the savings $s^i(t)$ of (young) individual i as the part of i's disposable endowment $w_t^i(t) - \tau_t^i(t)$ that is not consumed. That is,

$$s^i(t) = w_t^i(t) - \tau_t^i(t) - c_t^i(t).$$

Accordingly, the equilibrium interest rate $R(t)$ in t is obtained (as in the no tax case) from the condition $\sum_{i\in N(t)} s^i(t) = 0$. The only difference with respect to the no tax case is that $s^i(t)$ has a different definition incorporating the taxes.

2. Government bonds

Assume that the government can issue one-period bonds. A bond is a (safe) promise (by the government) of delivering 1 unit of the good at $t+1$ in exchange for a (competitive) price $p(t) < 1$ paid to the government in period t. Hence, $p(t)$ is the price of the bond (when issued in period t) and the face (or nominal) value of the bond is 1. This definition of bonds means that they are issued at discount (price smaller than its face value).

The (implicit) rate of return of the bond is $\frac{1-p(t)}{p(t)}$. The gross rate of return of the bond is then $1+\frac{1-p(t)}{p(t)}=\frac{1}{p(t)}$: the investor gets 1 in $t+1$ by investing $p(t)$ in t. Though there is only one agent supplying bonds in the bond market (the government), it will nonetheless be assumed that the market is competitive: supply of bonds in t and demand for bonds in t determine the price $p(t)$ of bonds in t.

For $t \geq 1$, let $B(t)$ stand for the total number of bonds that the government issues in period t. Given that the face value of each bond is 1, $B(t)$ also represents the debt that the government must pay in period $t+1$.

The government budget constraint in period t holds that

$$\underbrace{B(t-1)}_{\textit{debt to be paid}} = \underbrace{\sum_{i\in N(t)} \tau_t^i(t)}_{\textit{taxes on the young}} + \underbrace{\sum_{i\in N(t-1)} \tau_{t-1}^i(t)}_{\textit{taxes on the old}} + \underbrace{p(t)B(t)}_{\textit{new bonds}}.$$

The constraint shows the three ways of redeeming in t bonds issued in $t-1$: tax the young in t; tax the old in t; issue new bonds in t (now $\sum_{i\in N(t)} \tau_t^i(t) + \sum_{i\in N(t-1)} \tau_{t-1}^i(t)$ need not be zero).

Since the old individuals never lend, the government can only borrow from (sell bonds to) the young individuals. Hence, only young individuals will buy bonds. A young individual i of generation t faces the budget constraint

$$c_t^i(t) + l^i(t) + \tau_t^i(t) + p(t)b^i(t) = w_t^i(t).$$

This says that there are four possible uses for i's wealth $w_t^i(t)$: it can be consumed, lent in the private loan market, given to the government (in the form of taxes), or lent to the government (by purchasing the amount $b^i(t)$ of bonds).

The budget constraint of an old individual i is

$$c_t^i(t+1) + \tau_t^i(t+1) = w_t^i(t+1) + R(t)l^i(t) + b^i(t).$$

Putting together the above two constraints (solve for $l^i(t)$ in the first equation and insert the result in the second), individual i's lifetime budget constraint is

$$c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) - \tau_t^i(t) + \frac{w_t^i(t+1) - \tau_t^i(t+1)}{R(t)} - b^i(t)\left[p(t) - \frac{1}{R(t)}\right]. \quad (2)$$

It will be argued next that, by arbitrage, it must that $p(t) = \frac{1}{R(t)}$ in equilibrium.

To this end, consider an individual that would like to save $p(t)$ units of the good in period t. The individual has two options.

• **Option 1.** To become a lender in the government bond market. Since $p(t)$ is the price of one bond in t, the outcome (profit) of this saving decision is 1 unit of the good in the next period $t + 1$.

• **Option 2.** To become a lender in the private loan market. By lending $p(t)$ units of the good in the private loan market (assumed competitive), and given the (gross) interest rate $R(t)$ in t, it follows that the individual obtains $p(t)R(t)$ units of the good in the next period $t + 1$.

By arbitrage, it is to be expected that both options will yield the same result; that is, $1 = p(t)R(t)$: lending to the government should generate the same profit as lending to individuals.

For if $1 > p(t)R(t)$, then public lending would be more profitable than private lending. By borrowing $p(t)$ in the private loan market to purchase one bond, in $t + 1$ the bond pays 1, whereas the refund of the loan requires $p(t)R(t)$. A sure profit of $1 - p(t)R(t)$ is made. But in equilibrium sure profits cannot arise. A growing demand for both loans and bonds cause a rise in both $R(t)$ and $p(t)$.

Similarly, arbitrage opportunities also occur if $1 < p(t)R(t)$ (public lending is less profitable than private lending). Consequently, for both markets to exist (for lenders to be willing to participate in both markets) the corresponding returns should be the same: $1 = p(t)R(t)$ implies $\frac{1}{p(t)} = R(t)$, where $\frac{1}{p(t)}$ represents the (gross) interest rate of the bond and $R(t)$ is the (gross) interest rate of a (private) loan. Clearly, $\frac{1}{p(t)} = R(t)$ entails $p(t) - \frac{1}{R(t)}$. Accordingly, the term $b^i(t)\left[p(t) - \frac{1}{R(t)}\right]$ in (2) is zero. To sum up, the lifetime budget constraint of each young individual i is identical to the one from the no bond case; see (1).

$$c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) - \tau_t^i(t) + \frac{w_t^i(t+1) - \tau_t^i(t+1)}{R(t)}$$

3. General equilibrium with bonds

There are <u>three markets</u> in the economy: the market for the (consumption of the) <u>good</u>; the (private) <u>loan</u> market; and the (government) <u>bond</u> market. In a general equilibrium all three markets must clear (must be in equilibrium). By Walras' law equilibrium in two markets guarantees equilibrium in the third one.

The summation of the budget constraints of all the young individuals at t yields

$$\sum_{i\in N(t)} c_t^i(t) + \sum_{i\in N(t)} l^i(t) + \sum_{i\in N(t)} \tau_t^i(t) + p(t)\sum_{i\in N(t)} b^i(t) = \sum_{i\in N(t)} w_t^i(t).$$

Equilibrium in the loan market requires $\sum_{i\in N(t)} l^i(t) = 0$. Rearranging,

$$\sum_{i\in N(t)}\left[w_t^i(t) - c_t^i(t) - \tau_t^i(t)\right] = p(t)\sum_{i\in N(t)} b^i(t).$$

That is, $\sum_{i\in N(t)} s^i(t) = p(t)B(t)$.

Recall that the aggregate savings function S_t, derived from maximization of the consumers' utility subject to their lifetime budget constraints, was a function of the interest rate $R(t)$ and the consumers' endowments. To simplify notation, and given that endowments are held fixed, it will be only emphasized that savings depend on $R(t)$ by writing $S_t(R(t))$.

In sum, defining $S_t(R(t)) = \sum_{i\in N(t)} s^i(t)$ to be the <u>aggregate savings function</u> in period t, the condition to determine the equilibrium interest rate in t is given by (3).

$$S_t(R(t)) = p(t)B(t) \qquad (3)$$

Equation (3) ensures equilibrium in both the private and the public loan (bond) market (it should be verified that this condition implies that the good market is in equilibrium, too). In fact, (3) expresses equilibrium in the bond market: whereas $p(t)B(t)$ represents the government demand for funds (in good units), $S_t(R(t))$ is the total supply available to finance the government when the (equilibrium) interest rate in the (private) loan market is $R(t)$.

To recap, the <u>general equilibrium condition</u> (3) holds that total private (net) savings by the young in t must equal the total value of the goverment debt <u>in t</u>.

Insofar as $R(t) = 1/p(t)$, (3) can be equivalently expressed as

$$S_t(R(t)) = \frac{B(t)}{R(t)},$$

so aggregate savings in t equal the present value of the debt $B(t)$ in $t+1$ (the debt in $t+1$ corresponds to the bonds issued in t: one bond issued in t involves a debt of one unit of the good in $t+1$).

4. An example of general equilibrium with bonds and taxes

Example 1. For all t and i, $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$, $N(t) = 100$, individuals in each generation are numbered from 1 to 100, $w_t^i = (2, 0)$ if i is odd, and $w_t^i = (1, 1)$ if i is even. The government wishes to borrow 25 units of the good in $t = 1$, transfer them to the old in $t = 1$, and pay off the debt by taxing the young individuals in $t = 2$: each such individual pays the amount τ in taxes. Find $R(1)$, $B(1)$, $R(2)$, $R(3)$, and τ.

It should not be difficult to verify that, when no tax is paid in t, the savings function is $s^i(t) = 1$ if i odd and $s^i(t) = \frac{1}{2} - \frac{1}{2R(t)}$ if i even.

The aggregate savings function is $S_t = 50(1) + 50\left(\frac{1}{2} - \frac{1}{2R(t)}\right) = 75 - \frac{25}{R(t)}$.

General equilibrium in t requires $S_t = p(t)B(t)$. For $t = 1$, $p(1)B(1) = 25$. This is because the government would like to raise 25 units of the good by selling the amount $B(1)$ of bonds. Therefore, in equilibrium in period 1, $S_1 = 25$. Since $S_1 = 75 - \frac{25}{R(1)}$, it follows that $75 - \frac{25}{R(t)} = 25$ and, hence, $R(1) = \frac{1}{2}$.

Savings in $t = 1$ are $s^i(1) = 1$ for i odd and $s^i(1) = -\frac{1}{2}$ for i even. In words: in period 1 odd-numbered individuals lend 50 units in total, while even-numbered individuals borrow 25 units in total. The difference (25 units) is what the government borrows.

Using $S_1 = B(1)/R(1)$, with $S_1 = 25$ and $R(1) = 1/2$, the conclusion is that $B(1) = 12.5$. This is the amount of bonds issued in $t = 1$ and the total amount of taxes that the young individuals in $t = 2$ will have to pay. Thus, $\tau = 0.125$.

Consider now period 2. As young individuals must pay the tax, the lifetime budget constraint of a young individual i is $c_2^i(2)+\frac{c_2^i(3)}{R(2)}=2-\tau$ if i is odd and $c_2^i(2)+\frac{c_2^i(3)}{R(2)}=1-\tau+\frac{1}{R(2)}$ if i is even.

For i odd the demand function for the good (when i is young) is $c_2^i(2)=1-\frac{\tau}{2}$. As a result, his savings function is $s^i(2)=2-\tau-c_2^i(2)=1-\frac{\tau}{2}$. In consequence, i pays the tax τ by reducing consumption and savings in the same amount: $\frac{\tau}{2}$. It can be interpreted that a half of the tax is financed by reducing consumption and the other half by reducing savings.

For i even the demand function for the good (when i is young) is $c_2^i(2)=\frac{1}{2}+\frac{1}{2R(2)}-\frac{\tau}{2}$. In view of this, his savings function is $s^i(2)=1-\tau-c_2^i(2)=\frac{1}{2}-\frac{\tau}{2}-\frac{1}{2R(2)}$. Just like an odd i, an even i pays the tax τ by reducing consumption and savings in the same amount: $\frac{\tau}{2}$.

The aggregate savings function is $S_2=50\left(1-\frac{\tau}{2}\right)+50\left(\frac{1}{2}-\frac{\tau}{2}-\frac{1}{2R(2)}\right)=75-50\tau-\frac{25}{R(2)}$. Given that $\tau=0.125$, $50\tau=6.25$. Summing up, $S_2=58.75-\frac{25}{R(2)}$.

Presuming that $B(2)=0$ (the government has no need to borrow in period 2), the equilibrium condition turns out to be $S_2=0$. Therefore, $R(2)=\frac{25}{58.75}=\frac{1}{2.35}\approx 0.4255$.

As regards $R(3)$, it is as if the government disappeared in period 3: no tax and no bond market. The aggregate savings function is as in period 1: $S_3=75-\frac{25}{R(3)}$. With the equilibrium condition now being $S_3=0$, it follows that $R(2)=\frac{1}{3}$. The same result holds for the rest of periods, as long as the government does not issue more debt or does not introduce taxes.

Hence, if the situation in period 3 is supposed to represent the initial situation in period 0, Example 1 suggests that <u>the interest rate may go up because of a rise in the government debt or an increase in the taxes paid by individuals</u>.

As rising taxes is not a popular economic policy measure, the following section considers the possibility that the government pays off bonds by, instead of rising taxes, issuing more bonds.

5. Rolling over debt

A goverment rolls over debt when debt is paid off with new debt. To illustrate this policy, suppose that in Example 1 the young in $t = 2$ are not taxed: new bonds are issued in $t = 2$ to pay off the bonds issued in $t = 1$, $B(1) = 12.5$.

Now, in equilibrium, $S_2 = B(2)/R(2)$ and $S_2 = B(1)$. Therefore, $R(2) = 0.4$ and $B(2) = 5$. If the same policy is followed in period $t = 3$, $S_3 = B(3)/R(3)$ and $S_3 = B(2)$. Accordingly, $R(3) = 0.35$ and $B(3) = 1.78$.

The acumulation of bonds and the dynamics of the interest rate are determined by the formulae

$$R(t) = \frac{25}{75 - B(t-1)} \text{ and } B(t) = \frac{25B(t-1)}{75 - B(t-1)}. \quad (4)$$

Define a steady state as a state in which equilibrium variables take the same value in every t. In particular, a steady state would require $B(t-1) = B(t)$. This occurs in two cases: (i) $B = 50$ and $R = 1$; and (ii) $B = 0$ and $R = 1/3$ ($R = 1/3$ is the equilibrium rate in absence of government intervention).

The formulae in (4) hold when the government borrows initially at most 50, so $S(1) \leq 50$. If $S(1) < 50$, $B(t)$ goes to 0 and $R(t)$ converges to 1/3; see Figs. 1 and 2. If $S(1) > 50$, borrowing becomes unfeasible for some t: a bubble eventually arises, which means that the price of the bonds follows an unsustainable path; see Figs. 3 and 4.

t	$S(t)$	$R(t)$	$B(t)$	$\%$ change in $B(t)$
1	49.99	0.9996	49.97001	–
2	49.97001	0.998802	49.91014	-0.11981
3	49.91014	0.996419	49.7314	-0.35814
4	49.7314	0.98937	49.20276	-1.06299
5	49.20276	0.969096	47.68218	-3.09042
6	47.68218	0.915154	43.63652	-8.48464
7	43.63652	0.797105	34.78291	-20.2895
8	34.78291	0.621626	21.62197	-37.8374
9	21.62197	0.468358	10.12681	-53.1642
10	10.12681	0.385367	3.902542	-61.4633
11	3.902542	0.35163	1.372251	-64.837
12	1.372251	0.339546	0.465942	-66.0454
13	0.465942	0.335417	0.156285	-66.4583
14	0.156285	0.334029	0.052204	-66.5971
15	0.052204	0.333566	0.017413	-66.6434

Fig. 1. Dynamics when the government borrows at most 50 in $t = 1$

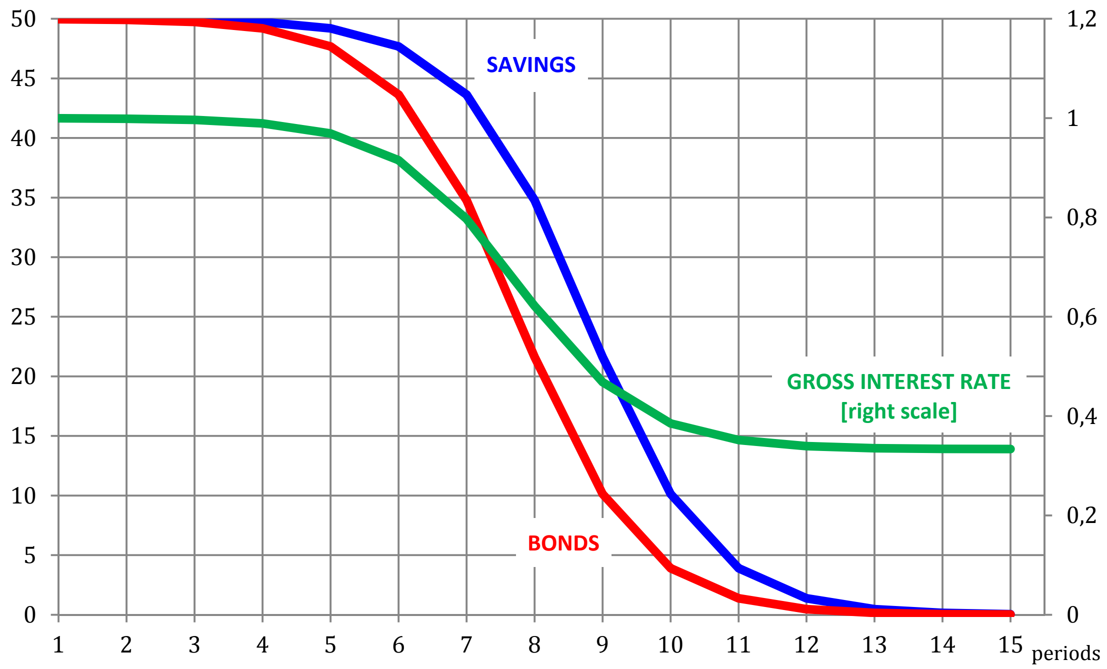

Fig. 2. Dynamics when the government borrows at most 50 in $t = 1$

t	$S(t)$	$R(t)$	$B(t)$	*% change in* $B(t)$
1	50.00001	1	50.00003	–
2	50.00003	1.000001	50.00009	0.00012
3	50.00009	1.000004	50.00027	0.00036
4	50.00027	1.000011	50.00081	0.00108
5	50.00081	1.000032	50.00243	0.00324
6	50.00243	1.000097	50.00729	0.009721
7	50.00729	1.000292	50.02188	0.029173
8	50.02188	1.000876	50.0657	0.087595
9	50.0657	1.002635	50.19761	0.263477
10	50.19761	1.007967	50.59755	0.796729
11	50.59755	1.024487	51.83654	2.448716
12	51.83654	1.079286	55.94645	7.928594
13	55.94645	1.312091	73.40684	31.20912
14	73.40684	15.69205	1151.904	1469.205
15	1151.904	-0.02321	-26.7411	-102.321

Impossible

Fig. 3. Dynamics when the government borrows more than 50 in $t = 1$

According to the values in Fig. 3, in period $t = 14$ the goverment asks for more units of the good (1151) than are available in that period (200). To make this part of an equilibrium, a negative gross interest rate R would be required. Yet a negative R cannot arise in equilibrium, because $R < 0$ means that, after lending one unit of the good in t, rather than receive good, you must still pay more good in $t+1$. A negative R is like a tax on lending. When $R<0$ it is plain that consuming is better than lending. In fact, the following cases may arise:

• if $r(t)>0$ ($R(t)>1$), then, by lending L at t, you get more than L at $t+1$;

• if $-1\leq r(t)\leq 0$ ($0\leq R(t)\leq 1$), by lending L at t, you get less than L at $t+1$;

• if $r(t)<-1$ ($R(t)<0$), by lending L at t, you have to pay at $t+1$. In this case, in equilibrium, no one lends: the sacrifice of current consumption that represents lending yields no future benefit, so a utility maximizer consumer will consume everything that is available (actually, everybody would like to borrow).

To put it in a nutshell, in period 14 the debt bubble bursts. Fig. 4 next depicts the data from Fig. 3.

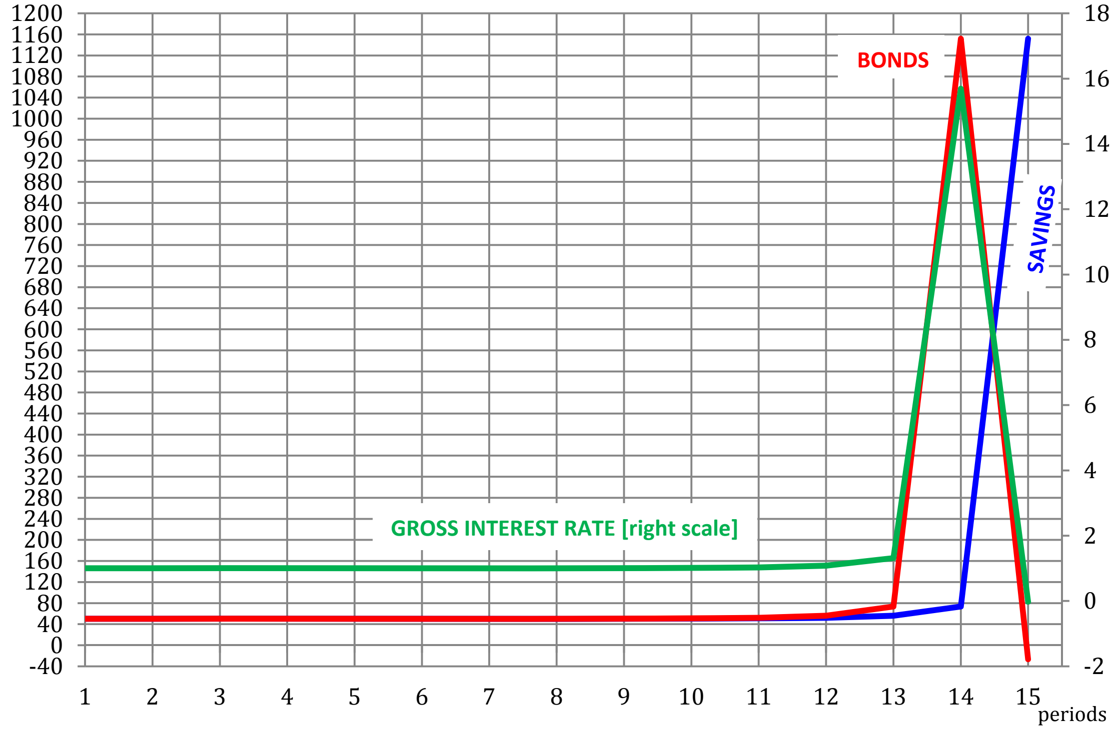

Fig. 4. Dynamics when the government borrows more than 50 in $t=1$

To sustain a growing government debt, population must grow or endowments must grow.

Example 2. Consider the economy from Example 1 without taxes and where endowments double each period.

Then $S_t = \left(75 - \frac{25}{R(t)}\right) 2^{t-1}$, $R(t) = \frac{25}{75 - \frac{B(t-1)}{2^{t-1}}}$, and $B(t) = \frac{25B(t-1)}{75 - \frac{B(t-1)}{2^{t-1}}}$. As a consequence, the government can now borrow initially 62 but not 63; see Fig. 5.

t	$S(t)$	$R(t)$	$B(t)$	$S(t)$	$R(t)$	$B(t)$
1	62	1,92	119,2	63	2,08	131,2
2	119,2	1,62	193,7	131,25	2,66	350
3	193,7	0,94	182,3	350	-2	-700
4	182,3	0,47	87,3	-700	0,15	-107,6
5	87,3	0,35	31,3	-107,6	0,3	-32,9
6	31,3	0,337	10,6	-32,9	0,32	-10,8
7	10,6	0,334	3,5	-10,8	0,33	-3,6

Fig. 4. Dynamics when the government borrows 62 or 63 in $t = 1$ in the economy of Example 2 (the yellow colour indicates an impossible situation)

6. On the equivalence between bonds and taxes

Apparently, financing government debt issuing new debt does not seem to be the same thing as financing by increasing taxes. Individuals may dislike having to pay more taxes because that (as illustrated in Example 1) will shrink their consumption possibilities. But (as suggested by also Example 1) replacing taxes by bond issue will lead to an increase in the interest rate, which harms borrowers. Moreover, the strategy of rolling over debt may lead to explosive situations in which the (private) loan market collapses (so both lenders and borrowers end up being worse off).

The next result indicates that, at least with respect to equilibrium consumption allocations, what can be achieved through bonds can be replicated using taxes.

Proposition 1. *Let C be an equilibrium consumption allocation with bonds. Then, for some tax-transfer scheme (without bonds) that balances the government's budget in each period t (total taxes in t equal total transfers in t), C is also an equilibrium consumption allocation.*

To show this, observe that, with bonds, the equilibrium interest rate $\hat{R}(t)$ in t solves $S_t\left(\hat{R}(t)\right) = B(t)/\hat{R}(t)$. Given $\hat{R}(t)$ and the bond holdings $b^i(t)$, the same equilibrium consumption allocation can be obtained with taxes (but without bonds) by setting $\tau_t^i(t) = b^i(t)$ and $\tau_t^i(t+1) = -\hat{R}(t) \cdot b^i(t)$.

The so-called Ricardian equivalence proposition (attributed to David Ricardo, $1772-1823$) is the result according to which the method of financing government spending (bonds or taxes) does not affect the consumers' decisions. The result relies on the presumption that consumers internalize the government's budget constraint when making consumption decisions. The following is a rough way of formulating this equivalence.

Proposition 2. *Consumption allocations & interest rates do not change if the government borrows now & ("appropriately") taxes later instead of just taxing now.*

The proof of the propisition amounts to showing that moving from one policy (borrow now and tax later) to the other (tax now) does not alter the consumer's present value of endowments. The equivalence may fail if, for instance, one policy is to borrow from generation 1 and tax generation 2 while the other is to tax generation 1. As different generations are involved, the budget constraints of some individuals may be different, in which case their consumption decisions may be altered.

Example 3. Members of each generation t are all identical, with $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$. This implies that there is no private borrowing. In view of this, the price $p(t)$ of bonds can be normalized to 1 in each t. There are two policies.

- Policy 1: set tax $\tau_t^i(t) = m$.
- Policy 2: borrow m from generation t in t and tax generation t in $t+1$ to pay off the bonds issued in t.

Under policy 1, the consumption basket is $c_t^i = \left(w_t^i(t) - m, w_t^i(t+1)\right)$. Under policy 2, $c_t^i = \left(w_t^i(t) - b^i(t), w_t^i(t+1) + R(t)b^i(t) - \tau_t^i(t+1)\right)$. Since taxes must only pay off the bonds, $\tau_t^i(t+1) = R(t)b^i(t)$. But $b^i(t) = m$, so $\tau_t^i(t+1)$ has present value m ($m = \tau_t^i(t+1)/R(t)$). This means that the present value of i's tax liability is not altered: it is m (in t) under policy 1 and $m \cdot R(t)$ (in $t+1$) under policy 2. The consumption basket is the same under the two policies.

Moreover, in equilibrium, $R(t) = MRS_t^i$. As $MRS_t^i = \frac{c_t^i(t+1)}{c_t^i(t)}$, the MRS does not change. Accordingly, the interest rate is the same under both policies.

7. Exercises

Exercici 1. Equilibri amb bons. La funció d'utilitat de cada consumidor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació està formada per 100 membres, 80 amb dotació $(1,0)$ y els altres 20 amb dotació $(2,0)$. El govern pretén aplegar 10 unitats amb l'emissió de bons amb venciment d'un període. Al venciment, els bons es paguen amb l'emissió de més bons, també amb venciment d'un període. I així successivament.

(i) Calcula la taxa d'interès d'equilibrio, el preu dels bons i la quantitat de bons emesa en els períodes 1, 2 i 3.

(ii) Respon a (i) si la dotació dels 20 és $(2,1)$ en comptes de $(2,0)$.

(iii) Al cas (ii), troba un import inicial a aplegar que provoqui que el refinançament continuat del deute faci que el volum de bons emès cada període sigui el mateix.

(iv) Al cas (ii), indica un import inicial que faci eventualment insostenible el refinançament del deute.

(v) Respon a l'apartat (i) amb les dades del (ii) si, en el període 2, traspassen la meitat dels consumidors joves amb dotació $(1,0)$

Exercici 2. Equivalència ricardiana. La funció d'utilitat de cada consumidor i és $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$. Cada generació està formada per 100 membres, 80 amb dotació $(1,0)$ y els altres 20 amb dotació $(2,0)$. El govern pretén finançar una despesa de 10 feta en $t = 1$. Verifica si la taxa d'interès d'equilibri és la mateixa amb cada política.

(i) Política 1: en $t = 1$, el govern estableix un impost igual a $1/8$ que han de pagar el joves amb dotació $(1,0)$. Política 2: en $t = 1$, el govern emet bons, que es paguen en $t = 2$ amb imposts que paguen els joves de $t = 2$ amb dotació $(1,0)$.

(ii) Política 1: la mateixa que en (i). Política 2: la mateixa que en (i) només que els joves de $t = 2$ que paguen els imposts tenen dotació $(2,0)$.

(iii) Política 1: la mateixa que en (i), amb l'única diferència que els joves que paguen tenen dotació $(2,0)$. Política 2: la mateixa que en (i).

(iv) Política 1: en $t = 1$, el govern fixa un impost de 0,1 que paguen els jóvenes. Política 2: en $t = 1$, el govern emet bons, pagats en $t = 2$ a parts iguals pels consumidors grans de $t = 2$ mitjançant un impost.

Exercici 3. Funcions d'estalvi. Comprova que les funcions d'estalvi donades en l'anàlisi de l'Example 2 són correctes.

Exercici 4. Equivalència de bons i imposts. Considera l'economia tal que, per a tot t i i, $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$, $N(t) = 100$, els consumidors de cada generació estan numerats de l'1 al 100, $w_t^i = (2,0)$ si i és senar i $w_t^i = (1,1)$ si i és parell. El govern vol manllevar 25 unitats del bé en el període 1 mitjançant la venda de bons i refinança el deute generat pels bons cada període emetent més bons. Troba l'esquema d'imposts i transferències que genera la mateixa assignació de consum d'equilibri que la política de refinançament del deute amb més bons.

Exercici 5. Finançament d'un bé públic. Cada generació té 100 membres: 50 d'ells ("els pobres") amb dotació $(1,0)$ i els altres 50 ("els rics") amb dotació $(4,1)$. Els consumidors, rics o pobres, empren la dotació en consum c, préstecs (privats) l i contribucions (voluntàries) e a un bé públic.

El bé públic només beneficia als consumidors joves (pots suggerir algun exemple real d'aquesta situació?). Per consegüent, la gent gran no contribueix al bé públic. La funció d'utilitat del consumidor (jove) i és $u_t^i = c_t^i(t) \cdot c_t^i(t+1) \cdot [1 + g(\sum_{j \epsilon N(t)} e^j)]$, on e^j és la contribucií del consumidor jove j. S'entén que e^j no pot ser negativa ni superior a la dotació que j té de jove.

Per a simplificar, sigui $g\left(\sum_{j \epsilon N(t)} e^j\right) = \sum_{j \epsilon N(t)} e^j$. Això pot interpretar-se com la funció de producció del bé públic: el total de contribucions $\sum_{j \epsilon N(t)} e^j$ genera el volum $g\left(\sum_{j \epsilon N(t)} e^j\right)$ de bé públic. La intuïció és que cada unitat de bé públic reforça la utilitat del consum privat: el bé públic fa més útil el consum privat del bé.

Determina quina és la contribució e^P al bé públic que, en equilibri, fa un consumidor pobre i quina és la contribució e^R que fa un consumidor ric.

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