1. An overlapping generations model with just private lending

1. Structure of the economy

- Time is measured in discrete periods, denoted by t, and indexed by integers: $t \in \{1, 2, 3, \dots\}$.
- The only agents are consumers. Consumers live for two consecutive periods.
- In every period t a new generation of $N(t)$ consumers is born. Members of generation t are young in period t and old in period $t+1$.
- There is only one good in each period. The good is exogenously given: it is a "gift of nature". The amount of good in period t is $Y(t)$.

Fig. 1 below represents the demographic structure of the economy.

	time period				
generation	1	2	3	4	…
0	*old*				
1	*young* →	*old*			
2		*young* →	*old*		
3			*young* →	*old*	
4				*young*	
…					
population	$N(0)+N(1)$	$N(1)+N(2)$	$N(2)+N(3)$	$N(3)+N(4)$	
amount of good	$Y(1)$	$Y(2)$	$Y(3)$	$Y(4)$	

Fig. 1. Demographic structure of the economy

- Each unit of a good cannot exist at different periods because it is assumed that the good cannot be stored and completely depreciates in one period. The amount of good $Y(t)$ available in period t can only be used for consumption in the same period t and the amount not consumed in t is lost.
- Every member i of generation t has an endowment of $w_t^i(t)$ units of the good in t and $w_t^i(t+1)$ in $t+1$. The total endowment $Y(t)$ in t is distributed among the people alive in t:

$$\sum_{i \in N(t)} w_t^i(t) + \sum_{i \in N(t-1)} w_{t-1}^i(t) = Y(t).$$

Member i of generation $t \geq 1$ consumes $c_t^i(t)$ units of the good in t and $c_t^i(t+1)$ units in $t+1$.

Definition 1.1. The consumption basket of $i \in N(t)$ is a pair $c_t^i = \left(c_t^i(t),\, c_t^i(t+1)\right)$ that establishes i's consumption $c_t^i(t)$ when young and i's consumption $c_t^i(t+1)$ when old. For $i \in N(0)$, c_0^i is just the number $c_0^i(1)$.

Definition 1.2. A consumption allocation is a sequence $\{c_t^i\}_{t\geq 0, i\in N(t)}$ of consumption baskets of members of all generations (generation 0 consists only of old people).

Definition 1.3. A consumption allocation is feasible if, for all $t \geq 1$,

$$\sum_{i\in N(t)} c_t^i(t) + \sum_{i\in N(t-1)} c_{t-1}^i(t) \leq Y(t).$$

- Consumers have preferences over their consumption baskets. When young, the preference of consumer $i \in N(t)$ is represented by a utility function u_t^i defined on the set of possible consumption baskets of consumer i.

The value $u_t^i\left(c_t^i(t), c_t^i(t+1)\right)$ is i's utility when i consumes $c_t^i(t)$ now (as a young individual) and consumes $c_t^i(t+1)$ in the future (as an old individual). When old, i's utility (in period $t+1$) only depends on $c_t^i(t+1)$, which had already been determined when i was young (in period t). For that reason, there is no need to specify a utility function for old consumers. Suffice it to assume that old consumers prefer to consume more than less of the good.

The following recapitulates the main variables of the model and the notation.

Period of time and generation	t
Number of members of generation t	$N(t)$
Amount of good available in period t	$Y(t)$
Consumption in t of individual i of generation t (i young)	$c_t^i(t)$
Consumption in $t+1$ of individual i of generation t (i old)	$c_t^i(t+1)$
Endowment in t of $i \in N(t)$	$w_t^i(t)$
Endowment in $t+1$ of $i \in N(t)$	$w_t^i(t+1)$
Utility function of member i of generation t in t (i young)	u_t^i

Remark 1.4. The above model could represent a mobile hunter-gatherer economy, with the good representing food or, more precisely, the food's calorie content.

Each young consumer i of each generation $t \geq 1$ has to choose a consumption basket c_t^i that maximizes his utility function u_t^i given the constraints that i faces. Consequently, each i makes all his decisions when young: if i chooses the basket $\left(c_t^i(t), c_t^i(t+1)\right)$ when young, his consumption $c_t^i(t+1)$ when old has been already determined. Hence, old consumers, who also aim at maximizing their utility, do not make any consumption decision: they just follow the plan defined when young. This means that the analysis can be restricted to what young individuals do.

Remark 1.5. Since the endowment i has as a young individual cannot be stored to be used when he becomes old, in the absence of a market for lending and borrowing the good, all individuals will consume their endowments. This would the autarkic solution: for all generation t and $i \in N(t)$, $\left(c_t^i(t), c_t^i(t+1)\right) = \left(w_t^i(t), w_t^i(t+1)\right)$.

2. The market for lending and borrowing

Assume now that a competitive market for lending and borrowing the good is created.

Definition 2.1. Let $r(t) \geq 0$ designate the (real) interest rate in t: lending 1 unit of the good in t implies receiving $1 + r(t)$ units of the good in $t + 1$. The gross interest rate is $R(t) = 1 + r(t)$.

Given that the market is competitive, every consumer takes $R(t)$ as given.

Remark 2.2. In the economy so far described, intergenerational lending is not possible: old persons in t cannot pay/collect debts in $t + 1$, since such persons are not alive in $t + 1$. Thus, lending and borrowing can only take place among members of the same generation: the young consumers.

Definition 2.3. The lending (or borrowing) of consumer i of generation t is designated by $l^i(t)$.

Having $l^i(t) > 0$ means that member i of generation t lends in t, whereas $l^i(t) < 0$ means that i borrows. Lending is written as $l^i(t)$ instead of $l_t^i(t)$ because i does not lend when old, so $l_t^i(t + 1) = 0$. The presumptions that old consumers try to maximize their utility and that their utility coincides with consumption imply that old consumers consume all the units of the good available to them. It therefore does not make sense for an old consumer to lend units of good because he will not be alive to consume the repayment of the loan in the next period. Obviously, old consumers would like to borrow, given that they will not be alive when the loan has to be paid back. The problem is that lenders will know that and, consequently, no lender would be willing to lend to an old consumer. The final conclusion of this analysis is that only young consumers participate in the market for lending and borrowing.

3. Budget constraints

Definition 3.1. Consumer i's budget constraint when young is

$$c_t^i(t) + l^i(t) \leq w_t^i(t).$$

Since i is supposed to maximize his utility from consumption, the budget constraint can be assumed to hold with equality: $c_t^i(t) + l^i(t) = w_t^i(t)$. This says that the amount $c_t^i(t)$ of good that young member i of generation t consumes plus the amount of good $l^i(t)$ that i lends (if $l^i(t) > 0$) or borrows (if $l^i(t) < 0$) must equal his own endowment $w_t^i(t)$.

Definition 3.2. Consumer i's budget constraint when old is

$$c_t^i(t + 1) \leq w_t^i(t + 1) + R(t) l^i(t).$$

It follows from the assumption that an old consumer maximizes his utility (his consumption) that the constraint holds as an equality: $c_t^i(t+1) = w_t^i(t+1) + R(t)l^i(t)$.

If $l^i(t) > 0$, then i lends when young and receives $R(t)l^i(t)$ when old. If $l^i(t) < 0$, then i borrows when young and pays back $R(t)l^i(t)$ when old.

Definition 3.3. The lifetime budget constraint (1) of consumer i is obtained by combining the constraints $c_t^i(t) + l^i(t) = w_t^i(t)$ and $c_t^i(t+1) = w_t^i(t+1) + R(t)l^i(t)$.

$$\underbrace{c_t^i(t) + \frac{c_t^i(t+1)}{R(t)}}_{\textbf{present value of lifetime consumption}} = \underbrace{w_t^i(t) + \frac{w_t^i(t+1)}{R(t)}}_{\textbf{present value of lifetime endowments}} \tag{1}$$

Condition (1) determines the set of consumption baskets $\left(c_t^i(t), c_t^i(t+1)\right)$ that are feasible to consumer i of generation t given endowments $w_t^i = \left(w_t^i(t), w_t^i(t+1)\right)$ and gross interest rate $R(t)$. The lifetime budget constraint says that the (discounted) value of a consumer's lifetime endowment coincides with the (discounted) value of the consumer's lifetime consumption.

4. The consumer's decision problem

Assumption 4.1. Each consumer $i \in N(t)$, for $t \geq 1$, is assumed to choose a consumption basket c_t^i that maximizes u_t^i given w_t^i and $R(t)$. Formally, consumer i's aim is to

$$\begin{aligned} &maximize_{\{c_t^i(t),\, c_t^i(t+1)\}} \;\; u_t^i\left(c_t^i(t), c_t^i(t+1)\right) \\ &subject\ to\;\; c_t^i(t) + \frac{c_t^i(t+1)}{R(t)} = w_t^i(t) + \frac{w_t^i(t+1)}{R(t)}. \end{aligned} \tag{2}$$

Remark 4.2. Every utility function u_t^i is typically assumed to satisfy the properties ensuring that indifference curves are differentiable, decreasing, and (strictly) convex (u_t^i is increasing, strictly quasi-concave, and continuously differentiable).

- Method 1 for solving the consumer's problem: Lagrange multiplier approach.

The solution to problem (2) can be obtained by solving problem (3).

$$\begin{aligned} maximize_{\{c_t^i(t),\, c_t^i(t+1)\}} \; L\left(c_t^i(t), c_t^i(t+1), \lambda\right) &= u_t^i\left(c_t^i(t), c_t^i(t+1)\right) + \\ \lambda\left(w_t^i(t) + \frac{w_t^i(t+1)}{R(t)} - c_t^i(t) - \frac{c_t^i(t+1)}{R(t)}\right) \end{aligned} \tag{3}$$

The first-order conditions to solve (3) are given by (4), (5), and (6) below.

$$0 = \frac{\partial L}{\partial c_t^i(t)} = \frac{\partial u_t^i}{\partial c_t^i(t)} - \lambda \tag{4}$$

$$0 = \frac{\partial L}{\partial c_t^i(t+1)} = \frac{\partial u_t^i}{\partial c_t^i(t+1)} - \frac{\lambda}{R(t)} \tag{5}$$

$$0 = \frac{\partial L}{\partial \lambda} = w_t^i(t) + \frac{w_t^i(t+1)}{R(t)} - c_t^i(t) - \frac{c_t^i(t+1)}{R(t)} \tag{6}$$

Condition (6) is the budget constraint (1). Solving for λ in (4), $\lambda = \frac{\partial u_t^i}{\partial c_t^i(t)}$. Solving for λ in (5), $\lambda = \frac{\partial u_t^i}{\partial c_t^i(t+1)} R(t)$. Therefore, $\frac{\partial u_t^i}{\partial c_t^i(t)} = \frac{\partial u_t^i}{\partial c_t^i(t+1)} R(t)$. Equivalently,

$$R(t) = \frac{\partial u_t^i / \partial c_t^i(t)}{\partial u_t^i / \partial c_t^i(t+1)} \,. \tag{7}$$

Definition 4.3. The term $\frac{\partial u_t^i}{\partial c_t^i(t)}$ is the marginal utility that the (young) consumer i obtains from the current consumption $c_t^i(t)$. The term $\frac{\partial u_t^i}{\partial c_t^i(t+1)}$ is the marginal utility that the (young) consumer i obtains from the future consumption $c_t^i(t+1)$. The ratio between the two is i's marginal rate of substitution MRS_t^i.

$$MRS_t^i = \frac{\partial u_t^i / \partial c_t^i(t)}{\partial u_t^i / \partial c_t^i(t+1)}$$

Geometrically, the MRS_t^i evaluated at $c_t^i = \left(c_t^i(t), c_t^i(t+1)\right)$ is the slope (in absolute value) of the indifference curve containing the basket c_t^i. The economic interpretation is that MRS_t^i represents the increase in $c_t^i(t+1)$ necessary to keep utility constant given a decrease of $c_t^i(t)$.

Remark 4.4. The properties of the utility function listed in Remark 4.2 ensure that, when the first-order maximization conditions hold, the second-order conditions also hold. They also ensure that the solution is unique

Proposition 4.5. The solution $\left(c_t^i(t), c_t^i(t+1)\right)$ of consumer i's problem solves (1) and (7).

- Method 2 for solving the consumer's problem: substitution of (1) into the utility function.

Solving for $c_t^i(t+1)$ in (1) yields $c_t^i(t+1) = R(t)[w_t^i(t) - c_t^i(t)] + w_t^i(t+1)$. After inserting this into the utility function u_t^i, (2) becomes

$$max_{\{c_t^i(t)\}} \; u_t^i\left(c_t^i(t), R(t)[w_t^i(t) - c_t^i(t)] + w_t^i(t+1)\right).$$

Since $c_t^i(t+1) = R(t)[w_t^i(t) - c_t^i(t)] + w_t^i(t+1)$, take the total derivative of the utility function u_t^i

$$du_t^i = \frac{\partial u_t^i}{\partial c_t^i(t)} dc_t^i + \frac{\partial u_t^i}{\partial c_t^i(t+1)} \frac{\partial c_t^i(t+1)}{\partial c_t^i(t)} dc_t^i \,,$$

that is,

$$\frac{du_t^i}{dc_t^i(t)} = \frac{\partial u_t^i}{\partial c_t^i(t)} - \frac{\partial u_t^i}{\partial c_t^i(t+1)} R(t).$$

To maximize u_t^i, it must be that $\frac{du_t^i}{dc_t^i(t)} = 0$. As a result, (7) is obtained. This and $c_t^i(t+1) = R(t)[w_t^i(t) - c_t^i(t)] + w_t^i(t+1)$, which is the budget constraint (1), solve the problem. Hence, both methods involve the same two conditions to solve the consumer's problem: (1) and (7).

5. The savings function

Definition 5.1. The savings $s^i(t)$ of consumer i of generation t is defined as $s^i(t) = w_t^i(t) - c_t^i(t)$: the difference between i's endowment and i's consumption.

Remark 5.2. Since old individuals consume all they can, the savings function refers to young individuals.

Definition 5.3. Using (1) and (7), the demand function for consumption C_t^i of consumer i of generation t when i is young is obtained: $c_t^i(t) = C_t^i\big(w_t^i(t), w_t^i(t+1), R(t)\big)$.

The demand function C_t^i expresses i's consumption (when he is young) as a function of both his liftime endowments $w_t^i(t)$ and $w_t^i(t+1)$, and the gross interest rate $R(t)$. Given C_t^i and $s^i(t) = w_t^i(t) - c_t^i(t)$, it is easy to determine the corresponding savings function $S_t^i\big(w_t^i(t), w_t^i(t+1), R(t)\big)$.

Definition 5.4. The savings $s^i(t)$ of consumer i of generation t is when i is $S_t^i = w_t^i(t) - C_t^i$.

6. A Cobb-Douglas example

Example 6.1. Each consumer i (young in period t) has utility function $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$. Consumer i's endowments are left unspecified as $w_t^i(t)$ and $w_t^i(t+1)$.

For Example 6.1, (7) becomes $R(t) = c_t^i(t+1)/c_t^i(t)$. By (1), $c_t^i(t+1) = R(t)[w_t^i(t) - c_t^i(t)] + w_t^i(t+1)$. Accordingly, the demand function for consumption (when young) is $c_t^i(t) = \frac{1}{2}\left(w_t^i(t) + \frac{w_t^i(t+1)}{R(t)}\right)$. Consumption depends positively on wealth and negatively on the interest rate.

The corresponding savings function is $s^i(t) = w_t^i(t) - \frac{1}{2}\left(w_t^i(t) + \frac{w_t^i(t+1)}{R(t)}\right) = \frac{1}{2}\left(w_t^i(t) - \frac{w_t^i(t+1)}{R(t)}\right)$. In view of this, a rise in the interest rate stimulates savings (verify that $\frac{\partial s^i(t)}{\partial R(t)} > 0$).

7. General competitive equilibrium

Definition 7.1. A general competitive equilibrium (GCE) is a sequence $\{\hat{R}(t)\}_{t\geq 1}$ of (gross real) interest rates and a consumption allocation $\{\hat{c}_t^i\}_{t\geq 0, i\in N(t)}$ such that E1 and E2 next hold.

E1. For every period $t \geq 1$ and $i \in N(t)$, $\hat{c}_t^i$ maximizes u_t^i given $\hat{R}(t)$ and i's endowments w_t^i (for $t = 0$, $\hat{c}_t^i$ is just the available wealth of the old).

E2. For every period $t \geq 1$, the goods market clearing condition is satisfied; formally, $\sum_{i\in N(t)} \hat{c}_t^i(t) + \sum_{i\in N(t-1)} \hat{c}_{t-1}^i(t) = Y(t) = \sum_{i\in N(t)} w_t^i(t) + \sum_{i\in N(t-1)} w_{t-1}^i(t)$.

Condition E1 asserts that, in every period $t \geq 1$ and for each young consumer i in t, $\hat{c}_t^i$ is the value of i's demand function for consumption given $\hat{R}(t)$ and i's endowment vector w_t^i. In other words, E1 simply says that $\hat{c}_t^i$ is obtained using (1) and (7).

Condition E2 holds that the market for the good is in equilibrium in every period $t \geq 1$. That is, for each $t \geq 1$, the aggregate (or total) demand for the good $\sum_{i\in N(t)} \hat{c}_t^i(t) + \sum_{i\in N(t-1)} \hat{c}_{t-1}^i(t)$ equals the aggregate (or total) supply of the good $\sum_{i\in N(t)} w_t^i(t) + \sum_{i\in N(t-1)} w_{t-1}^i(t)$. In view of the fact that there is no production of the good, total supply coincides with the total endowment $Y(t)$.

Remark 7.2. There are only two markets in this economy: the market for the good and the market for loans of the good. As only those who are young in t lend or borrow in t, the loan market is in equilibrium when $\sum_{i\in N(t)} l^i(t) = 0$. By Walras' law, condition $\sum_{i\in N(t)} l^i(t) = 0$ could replace condition E2 in Definition 7.1.

It follows from the budget constraint $c_t^i(t) + l^i(t) = w_t^i(t)$ when i is young that $l^i(t) = w_t^i(t) - c_t^i(t)$. On the other hand, by definition, $s^i(t) = w_t^i(t) - c_t^i(t)$. As a consequence, $l^i(t) = s^i(t)$: what i lends or borrows is exactly what i saves. When i lends ($l^i(t) > 0$), i is saving; when i borrows ($l^i(t) < 0$), i is dissaving. The point is that $\sum_{i\in N(t)} l^i(t) = 0$ amounts to $\sum_{i\in N(t)} s^i(t) = 0$: in a GCE, aggregate savings must be zero (total lending should equal total borrowing).

As a result of the previous analysis, if $\{\hat{R}(t)\}$ and $\{\hat{c}_t^i\}$ constitute a GCE, then, for each $\hat{R}(t)$, $\sum_{i\in N(t)} S^i\left(w_t^i(t), w_t^i(t+1), \hat{R}(t)\right) = 0$.

Conversely, if $\{\hat{R}(t)\}$ is such that, for all $\hat{R}(t)$, $\sum_{i\in N(t)} S^i\left(w_t^i(t), w_t^i(t+1), \hat{R}(t)\right) = 0$, then, for some $\{\hat{c}_t^i\}$, $\{\hat{R}(t)\}$ and $\{\hat{c}_t^i\}$ constitute a GCE.

8. Computing a GCE: an example

Example 8.1. For all t: $\left(w_t^i(t), w_t^i(t+1)\right) = (4, 1)$; $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$; and $N(t) = 200$.

Example 8.1 has the same utility functions as Example 6.1. As shown in Section 6, $s^i(t) = \frac{1}{2}\left(w_t^i(t) - \frac{w_t^i(t+1)}{R(t)}\right)$. Thus, the equilibrium condition (given by E2) $0 = \sum_{i \in N(t)} s^i(t) = 100\left(4 - \frac{1}{R(t)}\right)$ leads to $R(t) = 1/4$.

With $R(t) = 1/4$, for all i, $s^i(t) = \frac{4 - 1 \cdot 4}{2} = 0$. This means that <u>no individual saves</u>: there is no lending nor borrowing and, accordingly, consumption in each period coincides with the endowment in that period.

9. Failure of Pareto efficiency

Definition 9.1. A consumption allocation $C = \{c_t^i\}_{t \geq 0, i \in N(t)}$ is <u>Pareto efficient</u> if there does not exist another consumption allocation $\tilde{C} = \{\tilde{c}_t^i\}_{t \geq 0, i \in N(t)}$ such that P1 and P2 below are met.

P1. For some $t \geq 1$ and some $i \in N(t)$, $u_t^i(\tilde{c}_t^i) > u_t^i(c_t^i)$.

P2. For all $t \geq 1$ and all $i \in N(t)$, $u_t^i(\tilde{c}_t^i) \geq u_t^i(c_t^i)$.

That consumption allocation C is Pareto efficient means that for no other $\tilde{C}$ some consumer i has more utility and no consumer i has less utility.

Remark 9.2. The First Fundamental Welfare Theorem states that, for the standard atemporal Walrasian economies, the allocation of goods in a Walrasian (or general competitive) equilibrium is Pareto efficient. For economies represented by overlapping generation models this result fails: <u>the consumption allocation of a GCE need not be Pareto efficient</u>.

Example 9.3. In Example 8.1, the GCE was such that $\left(c_t^i(t), c_t^i(t+1)\right) = (4, 1)$ for all $t \geq 1$ (the old in period 1 consume $w_0^i(1) = 1$). It will be shown that this allocation is not Pareto efficient.

To this end, for each period $t \geq 1$, define a bijection $\beta_t : N(t) \to N(t-1)$. This bijection exists because the sets $N(t)$ and $N(t-1)$ have the same number of members: 200. Each mapping β_t matches every young consumer $i \in N(t)$ in period t with some old consumer $\beta_t(i) \in N(t-1)$ from the same period t. Let a new consumption allocation $\tilde{C}$ be defined from the GCE consumption allocation by letting each young consumer i transfer ε units of the good to the old consumer $\beta_t(i)$. The old are all clearly better off in $\tilde{C}$: whereas in the GCE consumption allocation each old consumer of each generation consumes 1 unit of the good, in $\tilde{C}$ each such individual consumes $1 + \varepsilon$.

As regards the young consumers, the utility of each young consumer in the GCE consumption allocation is $u_t^i(4,1) = 4$. After the transfer, in $\tilde{C}$, the utility is $u_t^i(4-\varepsilon, 1+\varepsilon) = (4-\varepsilon)(1+\varepsilon) = 4 + \varepsilon(3-\varepsilon)$. For sufficiently small ε (specifically, for $\varepsilon < 3$), $u_t^i(4-\varepsilon, 1+\varepsilon) > 4$. Consequently, all the young consumers are better off in $\tilde{C}$. This proves that the GCE allocation is not Pareto efficient.

10. Exercicis

Exercici 1. Restricció pressupostària vital. Considera el pla on es mesura $c_t^i(t)$ en l'eix d'abscisses i es mesura $c_t^i(t+1)$ en l'eix d'ordenades. Representa gràficament en el pla anterior la recta que defineix la restricció (1) i identifica el valor del pendent.

Exercici 2. Consum i estalvi. Troba les funcions de consum i estalvi si:

(i) $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t)^\alpha \cdot c_t^i(t+1)^\beta$, on α i β són constants positives;
(ii) $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t)^\alpha + c_t^i(t+1)^\beta$, on α i β són constants positives;
(iii) en l'apartat (i), α i β són constants negatives.

Exercici 3. Equilibri. Determina l'equilibri general competitiu en els dos casos (i) i (ii) de l'Exercici 2 si, de grans, els consumidors no tenen cap dotació, de joves tenen una unitat i hi ha 100 consumidors en cada generació.

Exercici 4. Equilibri i Paretoeficiència. Verifica que l'equilibri general competitiu de la següent economia no és Paretoeficient:

(i) cada generació està formada per 100 consumidors;
(ii) la dotació de cada consumidor jove és 2;
(iii) la dotació de cada consumidor gran és 1; i
(iv) per a tot i i t, $u_t^i\left(c_t^i(t), c_t^i(t+1)\right) = c_t^i(t) \cdot c_t^i(t+1)$.

Exercici 5. Equilibri amb lladregots i escurabosses. Cada generació té 100 membres: 80 ("els pobres") amb dotació $(1,0)$ i els altres 20 ("els rics") amb dotació $(4,2)$. Tots els consumidors joves de totes les generacions tenen la funció d'utilitat $u_t^i = c_t^i(t) \cdot c_t^i(t+1)$.

(i) Troba el vector de consum d'equilibri de cada consumidor jove.
(ii) Per a cada període, determina el vector de consum agregat d'equilibri del total de rics (i també del total de pobres) del període.

Per a tot t, cada jove pobre en t roba b unitats del grup de rics en t. El robatori agregat puja a $80 \cdot b$: $3 \cdot b$ unitats es prenen de cada jove ric i b unitats es prenen de cada ric gran. En conseqüència, el total apropiat pels joves pobres $(80 \cdot b)$ coincideix amb el que perden els rics $(20 \cdot 3 \cdot b + 20 \cdot b)$.

(iii) Torna a respondre a (i) i a (ii) si $b = 1$. Causa el robatori un augment o una reducció de la desigualtat?

Exercici 6. Imposts. Cada generació està formada per tres grups: 1, 2 i 3. Cada grup està format per n individus idèntics. La funció d'utilitat de cada jove és $u_t = c_t \cdot c_{t+1}$, on c_t és el consum que l'individu fa de jove i c_{t+1} el consum que el mateix individu farà de gran. La funció d'utilitat de cada individu gran coincideix amb el seu consum. Cada individu del grup 1 té, com a dotació, zero unitats del bé de jove i una unitat del bé de gran. Cada individu del grup 2 té, com a dotació, una unitat del bé de jove i zero unitats del bé de gran. Cada individu del grup 3 no té dotació del bé, ni de jove ni de gran. Una llei sagrada establerta en temps immemorial dicta que, cada període, els membres joves dels grups 1 i 2 han de pagar τ unitats del bé (aquest import és el mateix cada període i suficientment petit per a què tothom el pugui pagar). La llei mana que la recaptació total de l'impost en el període sigui distribuïda, en el mateix període, de manera igualitària entre els membres del grup 3, però no especifica si els destinataris de la transferència han de ser el joves del grup 3 o els grans del grup 3.

(i) Determina l'equilibri general, i la utilitat corresponent de cada individu, en els dos casos: cas 1, la transferència es fa als joves del grup 3; cas 2, la transferència es fa als grans del grup 3. (ii) Jutja quina opció consideres més recomanable.

11. Bibliografia

- McCandless, George T. i Neil Wallace (1991): *Introduction to Dynamic Macroeconomic Theory: An Overlapping Generations Approach*, Harvard University Press, Harvard, Massachusetts.

Probablement, el millor manual per a començar a estudiar el model de generacions encavalcades. Introdueix poc a poc cada element del model i analitza amb deteniment la resolució del model bàsic i de diverses extensions. El text d'aquests apunts en fonamenta en els capítols 1 i 2.

- Kehoe, Timothy J. (1987): "El modelo de generaciones sucesivas", Cuadernos Económicos de I.C.E. [Información Comercial Española] 1987/1, 9-30.

És una referència per als interessats en una descripció del model en castellà. Es tracta d'un article de revista. Per aquest motiu, la presentació és menys didàctica, atès que no detalla tot el desenvolupament. És més útil per a qui coneix mínimament el model i vol conèixer en quines altres direccions es pot estendre.

- Ihori, Toshihiro (1996): *Public finance in an overlapping generations economy*, Macmillan Press, Londres.

Un text una mica més avançat (més directe) que el text de McCandless. No s'atura a considerar economies sense producció. Més útil per a considerar el paper del govern en l'economia (imposts, deute, pensions. Té capítols sobre la introducció del diner en el model i la consideració d'economies obertes.

- Wickens, Michael (2008): *Macroeconomic Theory: A Dynamic General Equilibrium Approach*, Princeton University Press, Princeton i Oxford.

És un gran manual per a estudiar la teoria macroeconòmica moderna amb microfonaments. És una mica dens per a començar. A canvi, és molt complet: és un vademècum dels models dinàmics d'equilibri general. L'inconvenient és que no dedica gaire exclusivament al model de generacions encavalcades (seccions 6.3 i 9.4.1)Theory: A Dynamic General Equilibrium Approach

- Acemoglu, Daron (2009): *Introduction to modern economic growth*, Princeton University Press, Princeton, New Jersey.

Un manual total de la teoria del creixement econòmic. El capítol 9 està reservat a analitzar el creixement econòmic en el model de generacions encavalcades.