## 3. Interest rate and central bank

## 1. "The" interest rate of an economy

Definition 1.1. The nominal interest rate associated with a financial asset is the rate of return of the asset.

An economy has nearly as many interest rates as financial assets. The empirical evidence suggests that all of them tend to move in parallel. It is therefore reasonable to adopt the fiction that there is a unique interest rate $i$ in the economy. This rate could be taken to be the interest rate of a loan, which is itself a reference interest rate.

Definition 1.2. The interest rate $i$ of an economy is supposed to represent the average interest rate charge for a typical loan of currency.

## 2. Meaning of the interest rate

Interpretation 2.1. Reward for saving. For a moneylender, the interest rate $i$ means that he or she receives at maturity $1+i$ for every unit lent: for a moneylender, $i$ measures the profit of lending one unit of currency.

From the moneylender's perspective, thanks to the interest rate, 1 monetary unit in $t$ becomes $1+$ $i$ monetary units in $t+1$. For the moneylender, $i$ is the reward of saving: by giving up 1 today, he or she obtains $1+i$ in the future. Hence, $i$ represents the benefit of sending money to the future. This interpretation suggests a redefinition of the concept of interest rate.

Definition 2.2. The (nominal) interest rate (between periods $t$ and $t+1$ ) expresses the value in period of $t+1$ of one monetary unit of period $t$.

Interpretation 2.3. Cost of a loan. For a borrower, the interest rate $i$ means that he or she must pay $1+i$ for each unit borrowed: for a borrower, $i$ measures the cost of receiving a loan of one unit.

For the borrower's perspective, thanks to the interest rate, $1+i$ monetary units in $t+1$ can be transformed into 1 monetary unit in $t$. For the borrower, $i$ is the cost of a loan: if he or she is going to receive $1+i$ in the future, 1 unit could be obtained today. Thus, $i$ also represents the cost of bringing money from the future.

Interpretation 2.4. Measure of patience. The higher $i$, the more a borrower is willing to pay for having one unit of currency today instead of tomorrow, so the less patient the borrower is. A positive $i$ expresses a preference for the present: it is better to have money today than tomorrow.

## 3. The discount factor

Definition 3.1. The discount factor (between periods $t$ and $t+1$ ) expresses the value in period of $t$ of one monetary unit of period $t+1$.

Whereas the interest rate transforms today's money into tomorrow's money ( 1 today is $1+i$ tomorrow), the discount factor does the opposite by transforming tomorrow's money into today's money. Fig. 1 shows how the discount factor $\delta$ determines present values out of future values.


The discount factor makes 1 become $\delta$. This $\delta$ is the value in period $t$ that, when the interest rate between $t$ and $t+1$ is $i$, becomes 1 in period $t+1$. By the rule of three, $\delta=1 \cdot 1 /(1+i)=1 /(1+i)$ is the discount factor, which depends on the interest rate $i$. This leads to a more precise definition of $\delta$.

Definition 3.2. The discount factor $\delta$ between periods $t$ and $t+1$, when $i$ is the interest rate between $t$ and $t+1$, is

$$
\delta=\frac{1}{1+i}
$$

## 4. Interest rate and asset prices

Proposition 4.1. The price of a financial asset and the price of money ( $=$ the nominal interest rate, the cost of a loan) move in opposite directions.

Example 4.2. Proposition 4.1 will be illustrated for the case in which the financial asset is a T-bill. The T-bill is issued in period $t$ and matures in $t+1$. The price of the T-bill in $t$, when issued, is $P$. The face value of the T-bill is $V$, which means that, in $t+1$, the T-bill pays $V$ to the owner of the Tbill. Let $i$ be the interest rate between $t$ and $t+1$, so $i$ represents the profit of making a loan with the same maturity as the T-bill. An investor having $P$ monetary units may consider two options.

- Option 1: lend $P$. When the loan matures, in $t+1$, the investor gets $(1+i) \cdot P$.
- Option 2: buy the T-bill. When the T-bill matures, in $t+1$, the investor gets $V$.

For both options to be equally attractive, the outcomes must coincide $(1+i) \cdot P=V$. That is,

$$
\begin{equation*}
P=\frac{V}{1+i} . \tag{1}
\end{equation*}
$$

Since $V$ is a fixed given value, (1) means that the larger $i$, the smaller $P$.

## 5. Financial arbitrage

Definition 5.1. Arbitrage consists of making purchases and sales that ensure a sure profit.

Under financial arbitrage, an arbitrageur buys and sells financial assets to obtain a sure profit. It will be next argued financial arbitrage constitutes a mechanism that justifies the inverse relationship between the price of a T-bill and the interest rate established by (1).

That financial arbitrage leads to (1) will be demostrated using the proof technique known as proof by contradiction. This technique relies of the idea that a true result cannot lead to a contradiction: since everything can be demostrated from a contradiction, a true result leading to a contradiction would prove its own falsehood, a false conclusion. To this end, suppose (1) does not hold. This means that $V>(1+i) \cdot P$ or $V<(1+i) \cdot P$. Only the former possibility is handled, the latter being left as an exercise.

The economic logic of the proof relies on the idea that arbitrage opportunities (the possibility of making sure profits) cannot last. Accordingly, outcomes that create arbitrage opportunities cannot be stable and, therefore, cannot be taken as good economic predictions. Specifically, the proof will show that:
(i) $\quad V>(1+i) \cdot P$ creates arbitrage opportunities; and that
(ii) the act of profiting from arbitrage opportunities make such opportunities disappear.

So assume that $V>(1+i) \cdot P$. An arbitrageur can then obtain sure profits as follows, even having no money at all.

- Step 1: the arbitrageur borrow $P$ monetary units in $t$ and, consequently, has to repay $(1+i) \cdot P$ monetary units in $t+1$.
- Step 2: the arbitrageur purchases in $t$ a T-bill with the $P$ monetary units.
- Step 3: reached period $t+1$, the T-bill pays $V$ monetary units and, owing to $V>(1+i) \cdot P$, the arbitrageur repays the loan and pockets a profit of $V-(1+i) \cdot P>0$ monetary units.

Example 5.2. If $V=1,000, P=800$, and $i=10 \%$, each T-bill financed by a loan yields a 120 profit.

It is very likely that many arbitrageurs will be attracted by the prospect of sure benefits. Hence, significant amounts of money will be borrowed in step 1. Assuming the market for loans and the market for T-bills to be competitive (a reasonable assumption for financial markets), the borrowing of money by arbitrageurs will shift the demand for loans to the right. As depicted in Fig. 2 (left-hand side), an expanding demand for loans causes a rise in the interest rate $i$.

On the other hand, the purchases of T-bills executed in step 2, shifts to the right that demand for T-bills (see the right-hand side of Fig. 2). This shift leads to an increase in the price $P$ of T-bills. With both $i$ and $P$ going up, $(1+i) \cdot P$ also goes up. The result is that $V-(1+i) \cdot P$ diminishes. Arbitrageurs will borrow money and buy T-bills until the gap between $V$ and $(1+i) \cdot P$ is bridged, namely, until $V=(1+i) \cdot P$ and arbitrage opportunities vanish.

## Loan market



## T-bill market



Fig. 2. The effects of arbitrage when $V>(1+i) \cdot P$

## 6. Prices of assets as present values

The concept of present value provides a second justification for equation (1). In fact, the value in $t$ +1 (the future value) of the T-bill is $V$. With interest rate $i$ between $t$ and $t+1$, the value of $V$ in $t$ (its present discounted value) is

$$
V \cdot \frac{1}{1+i}
$$

where $\frac{1}{1+i}$ is the discount factor between $t$ and $t+1$. In view of this, equation (1) states that the price of a T-bill coincides with the present discounted value of its face (future) value.

## 7. Equalization of rates of return

A third justification of (1) comes from the presumption of equalization of the interest rates of all financial assets. A justification for this presumption is that, if the equalization does not occur, financial assets with a smaller rate of return would have no demand and, consequently, they would not exist. Given that many financial assets exist, their rates of return should be the same.

The interest rate $i_{T \text {-bill }}$ implicit in (associated with or corresponding to) a T-bill is

$$
i_{T-\text { bill }}=\frac{V-P}{P} .
$$

Letting $i$ represent the interest rate of a loan, the equalization condition $i=i_{T \text {-bill }}$ leads to

$$
i=i_{T-\text { bill }}=\frac{V-P}{P}=\frac{V}{P}-1
$$

or, equivalently,

$$
1+i=\frac{V}{P}
$$

Solving for $P$ yields equation (1).
8. The central bank (CB)

Definition 8.1. The central bank is the monetary authority in an economy. It is the public institution that, typically,

- provides and regulates the money supply (M1, M2, M3);
- issues the currency (see the letters "ECB" in euro banknotes);
- controls (or pretends to control) the interest rates and/or the inflation rate;
- oversees the banking and the payment systems (the CB is the systems' supervisor);
- acts as a lender of last resort to the banking system (the CB is a banker to banks);
- establishes mininum reserve requirements and conducts the monetary policy;
- is independent of the government (though the CB may a banker to the government).

For the purposes of this course, the CB is who determines and executes the monetary policy.

## 9. Monetary policy instruments

There are three standard tools by means of which a central bank can influence the money stock.

- The quantity tool: changes in the supply of reserves to the banking system through open market operations or direct lending through standing facilities.
- The price tool: changes in the interest rate at which the CB lends (the CB's policy interest rate).
- The formal regulatory tool: changes in the reserve requirement.

10. Open market operations (OMOs)

Definition 10.1. Open market operations by the central bank are sales or purchases of financial assets (normally, government securities, like T-bills, and central bank bills) to, typically, certain counterparties (typically, the main banks of the economy).

The immediate aim of OMOs is the control of the money stock: an OMOs modifies M0 and, through the money multiplier, the change in M0 alters M1 in the desired direction.

Definition 10.2. An expansionary OMO expands the monetary base (and, therefore, the money stock) by buying financial assets: the CB gets financial assets in exchange for currency, so there is more funds in the economy (alternatively, the CB may pay the financial assets by enlarging the amount of reserves that the banks that sold the assets hold in the CB).

Definition 10.3. A contractionary OMO contracts the monetary base (and, subsequently, the money stock) by selling financial assets: the CB injects financial assets in the economy and drains currency out of it (or reduces reserves that the banks that bought the assets have in the CB).

Figs. 3 and 4 sketch an expansionary and a contractionary OMO, respectively.


Fig. 3. Expansionary OMO


Fig. 4. Contractionary OMO

According to the nature of the transaction, OMOs can be divided in two types: repo (and reverserepo) transactions and outright transactions.

Definition 10.4. An outright transaction is an OMO in which the rights embodied in the financial asset that is bought or sold are permanently transferred to the buyer (the asset is then said to be bought or sold outright).

Definition 10.5. A repurchase agreement (or repo, for short) is an OMO in which the rights embodied in the financial asset that is bought or sold are temporarily transferred to the buyer: in a repo, the seller of the financial asset must buy it back in a future date and at a preestablished price.

Definition 10.6. A reverse repurchase agreement (or reverse-repo, for short) is an OMO in which the buyer of the financial asset must sell it back in a future date and at a preestablished price.

In a repo transaction liquidity is drained (absorbed) by the central bank temporarily: the CB sells financial assets with the compromise of repurchasing them in the future. By means of a reverserepo transaction liquidity is injected by the central bank temporarily: the CB buys financial assets with an agreement of selling them back in the future; see Fig. 5.

## time $t$



Fig. 5. A reverse repurchasement agreement by the central bank (reverse-repo transaction)

## 11. Standing facilities

Definition 11.1. A standing facility is a procedure by means of which banks can borrow or lend funds directly with the central bank.

With OMOs the CB intervenes directly in financial markets. With standing facilities, the CB deals directly with some counterparties (the main banks) and afterwards lets financial markets determine how founds are allocated among financial institutions. Such facilities are passive tools to neutralize or smooth out the excessive volatility to which financial markets are prone, so that the market interest rates are in line with (or not pushed too far away from) the interest rate target of the CB (as signalled by the CB's interest rate policy).

Definition 11.2. A deposit facility is a standing facility that allows selected banks having an excess of liquidity (that is, excess funds) that cannot be used in the markets to deposit that excess in the central bank and in return be paid an interest rate normally below the market rate.

Definition 11.3. A lending facility is a standing facility that allows selected banks unable to obtain short-term liquidity in the markets to borrow directly from the CB, normally at an interest rate higher than the market rate.

## 12. The policy interest rate and the interest rate corridor

Definition 12.1. "The policy interest rate refers to a short term interest rate that the central bank uses to indicate its monetary policy stance." (Thammarak Moenjak (2014), Central banking, p. 128)

By making public the policy interest rate, the CB tries to induce market rates to be close to the policy interest rate. Whereas the policy rate signals the central bank's monetary policy stance, OMOs and standing facilities constitute tools to induce market rates to be near the policy interest rate.

In normal circumstances, competition among banks tends to ensure that borrowing and lending rates do not differ much. Temporal liquidity shortages or surpluses may induce market rates to differ significantly from the policy interest rate. In this case, appropriate expansionary or contractionary open market operations could be used to move market rates toward the goal. Knowing that the CB is willing to intervene in order to put market rates in line with the policy interest rate, banks would normally accept borrowing and lending at the CB's desired rate.

If the CB finds more convenient to regulate market conditions without stepping in (because direct intervention would be too frequent or involve too much funds), then banks can deal with temporal liquidity shortages or surpluses by resorting to the standing facilities.

Definition 12.2. The expression interest rate corridor makes reference to the combined use of standing facilities and the policy interest rate with the aim of keeping market rates within a specified corridor (fluctuation band) around the policy interest rate; see Fig. 6.


Fig. 6.
The interest rate corridor
Thammarak Moenjak (2014):
Central banking, p. 128
"... many central banks in advanced and emerging-market economies have decided to use an interest rate corridor system, with standing facilities that help put a ceiling and a floor on how far the rates that market participants charge among themselves can diverge from the policy interest rate. Central banks that have adopted an interest rate corridor system include the Federal Reserve, the Bank of England, the ECB, the Reserve Bank of Australia, the Reserve Bank of New Zealand, the Bank of Korea, the Bank Negara Malaysia, the Bangko Sentral ng Pilipinas, and the Bank of Thailand, among others".
Thammarak Moenjak (2014): Central banking, p. 128

The interest rate the CB charges for using the lending capacity is higher than the policy interest rate to induce banks to look first in markets a solution to their liquidity problem (shortage in this case). This makes the interest rate of the lending facility a ceiling for short-term market rates. A symmetric role is played by the interest rate of the deposit facility: it is a floor for market rates to encourage banks with liquidity surpluses to find borrowers in the markets, so that such banks turning to the central bank's deposit facility only as a last resort (when there is no better option).

## 13. Reserve requirements

Definition 13.1. Reserve requirements constitute the minimum amount of reserves that banks must deposit in the central bank.

Reserve requirements are usually computed as a fraction (the reserve ratio) of (sight) deposits. Reserves contribute to control the money stock by altering the portion of any deposit that has to be retained: under a zero reserve requirement, banks would have no constraint to create deposits.


Fig. 7. The central bank and the economy's payment system (for more on the payment system, see Sergio Rossi (2007): Money and payments in theory and practice)

By increasing the reserve ratio, the CB detracts lending funds from banks: less loans, less expenditure, less revenue, less deposits, smaller M1. This reduces the money multiplier: $\uparrow r \Rightarrow \downarrow m m$. $\underline{A}$ reduction of the reserve ratio has an expansive effect on M1: more fuel can be added to the flames of the money creation process.

The banks' reserves held on account at the CB are simply numbers, like deposits. An aditional purpose of the reserves system is to settle interbank payments. That is why banks must retain each day enough reserves to facilitate the interbank clearing process plus enough cash reserves to meet the withdrawal requests from depositors; see Fig 7.

## 14. An informal monetary policy tool

CBs may also resort to an unofficial ("extra-legal") regulatory tool: direct control of the quantity of bank credit. The CB informs the banks by how much they are allowed to increase lending in a certain period. Violation of the limit is punished.

This practice has been widespread in developing countries but also adopted by most developed countries (from 1945 to the 1980s). Names given are "credit controls", "corset", "lending ceilings", "credit planning scheme", "window guidance", "Kredit-Plafondierung", "encadrement du crédit"...

## 15. The CB dilemma: $i$ and M1 cannot be simultaneously controlled

Suppose the CB wants to reduce the money supply by selling T-bills. To encourage banks to buy Tbills, the CB must lower appropriately the current price of T-bills: otherwise, banks may not be willing to buy T-bills. But a reduction in the price of the T-bill raises its rate of return, so the average interest rate of the economy rises. In sum, $\downarrow$ M1 implies $\uparrow i$.

Conversely, if the CB wants to increase the money supply by buying T-bills, the demand for T-bills shifts to the right, causing a price rise. This leads to a fall in the rate of return of T-bills: 个M1 implies $\downarrow_{i}$.

Fig. 8 illustrates the impossibility of using a policy tool (a contractionary OMO) to achieve two goals (reduce both $i$ and M1). The attempt to control M1 entails a loss of control over $i$. If the sale of T-bills aims at raising $i$, the loss of control is over M1 (in the sense that it cannot be reduced).


Fig. 8. Effects of a contractionary OMOs on $i$ and M1

## 16. Endogenous and exogenous money

Definition 16.1. M1 is exogenous (set and controlled by the CB ) when the CB exercises a rigid control over M0, leaving the interest rate to settle at whatever value is necessary for loans to generate the desired level of M1.

Definition 16.2. M1 is endogenous (demand-led) when the CB supplies whatever reserves banks require to achieve a certain desired interest rate: the CB chooses the policy interest rate (which determines the price of bank loans) and M1 is next determined by "the economy" (factors behind the demand for loans by people and firms)

## 17. Regulate $i$ or M1?

In theory, if money is exogenous, by setting the level of CB reserves, the money stock could be largely controlled (given mm , controlling M0 implies controlling M1). Time ago monetary targets were chosen and the aim of CBs was to control some monetary aggregate.

To free regulated funds (mandatory reserves), banks created new financial assets that performed the same functions as regulated assets but were not subject to regulation. For the target not to be missed, the CB had to redefine the monetary aggregate to include the new assets. The banks replied with more creativity, increasing the difficulties of monetary control. The list of monetary aggregates in UK includes M0, NIBM1, M1, M2, M3, M3c, PSL1, PSL2, M4, M4c, M3H, M5...

CBs eventually gave up trying to control monetary aggregates and nowadays evidence suggests that M1 is endogenous (see Fig. 9): the CB supplies reserves on demand but sets its price (the interest rate). At that price, demand determines loans and M1. That price also guides banks to set their own interest rates. In sum, CBs currently tend to control $i$.


Fig. 8. Evidence for money endogeneity: credit creation and economic activity
Richard A. Werner (2005): New Paradigm in Macroeconomics, p. 315

## 18. Central banking in a nutshell

"Virtually every monetary economist believes that the central bank can control the monetary base... Almost all those who have worked in a central bank believe that this view is totally mistaken." Charles Goodhart
"What is it that monetary policy-makers do and how do they do it? The simple answer is that a central banker moves interest rates in order to maintain steady real growth and stable prices." Stephen Cecchetti
19. Overview of money schools

- Endogenous money school. Holds that causation runs from economic activity to money.
- Monetarist school. Money is viewed as exogenous, so causation works in reverse order: from money to economic activity.
- Credit school. There are three versions.
- Lending view. Money also affects the economy through liabilities (borrowing and credit). The reduction of bank reserves also reduces the banks' access to loanable funds, which affects negatively the supply of bank loans
- Credit rationing argument. The interest rate alone does not fully capture the links between the real and financial sectors. The availability of credit is also important, which depends on the banks' willingness to grant credit. Borrowers may be rationed when loans have collateral requirements.
- Balance sheet channel. Changes in interest rates affect the value of financial assets. This influences the access to, and the demand for, funding.
- Neutrality of money school. Contends that money is neutral, that is, has no influence (at least in the medium run) on real variables (economic activity)


## 20. Eurosystem

Definition 20.1. The Eurosystem is the monetary authority of the eurozone, the 19 EU (European Union) members that have adopted the euro as official currency. The Eurosystem consists of the ECB (European Central Bank) and the central banks of the eurozone members (the national central banks applies the monetary policy the ECB decides).

The primary objective of the Eurosystem is price stability. Secondary objectives are financial stability and financial integration. The Eurosystem is different from the European System of Central Banks (ESCB), whose members are the ECB and the central banks of all the EU members. http://en.wikipedia.org/wiki/Eurosystem | http://en.wikipedia.org/wiki/Eurozone
21. European Central Bank


The ECB was established by the Treaty of Amsterdam in 1998. It constitutes the core of the Eurosystem and the ESCB. The ECB is responsible for the conduct of the monetary policy since the 1st of January, 1999. The Eurosystem can be identified with the ECB, as both have the same decision-making bodies, listed next.

- Governing Council. It is the main decision-making body of the ECB. Formulates the monetary policy for the eurozone (liquidity and key ECB interest rates decisions). Ensures the performance of the tasks assigned to the Eurosystem.
- Executive Board (6 members). Implements the monetary policy following the guidelines and decisions by the Governing Council. It manages the day-to-day business of the ECB.
- General Council. It is a transitional body: will be dissolved once all EU members have adopted the euro. It helps to coordinate eurozone members with the rest of EU members.
http://www.ecb.int/ecb/html/index.en.html


## 22. Main refinancing operations (MROs) and standard tenders

Definition 22.1. The main refinancing operations constitute the ECB's basic tool of monetary policy and enjoy the following characteristics: (i) they are liquidity-providing reverse transactions; (ii) executed regularly each week in a decentralized manner by the national central banks of the Eurosystem; (iii) normally have a maturity of one week; and (iv) conducted in the form of standard tenders: a fixed rate (volume) tender or a variable rate (interest) tender. http://www.ecb.int/pub/pdf/other/gendoc2006en.pdf

Definition 22.2. In a fixed rate tender MRO the ECB specifies the interest rate in advance. Participants next bid the amount of money they would like to transact at the given interest rate.

Since October 2008, MROs are conducted at fixed rates. Since 9 September 2014, the fixed rate has been set at $0.05 \%$, the lowest in the history of the ECB.
http://www.ecb.europa.eu/stats/monetary/rates/html/index.en.html
The late-2000s financial crisis was triggered by a liquidity shortfall that caused some large US financial institutions to collapse. To enhance the provision of liquidity, the Governing Council decided on 15 October 2008 to conduct all longer-term refinancing operations through a fixed rate tender procedure with full allotment (measure of enhanced credit support). On 7 November 2013 the Governing Council decided to continue conducting its main, special-term and longer-term refinancing operations as fixed rate tender procedures with full allotment for as long as necessary, and at least until July 2015. http://www.ecb.europa.eu/mopo/implement/omo/html/index.en.html

Example 22.3. Fixed rate tender MRO. The ECB wants to provide liquidity and decides to allot $€ 300$ million at a given interest rate $i$. Only four counterparties (banks) submit a bid: B1 $=160$, B2 $=80, \mathrm{~B} 3=100$, and $\mathrm{B} 4=60$ EUR millions. Thus, total demand is 400 . The percentage of allotment is $300 / 400=0.75=75 \%$. Each bank is then allotted the $75 \%$ of its bid: B1 receives $120=160$. $75 \%$; B2 gets $60=80 \cdot 75 \%$; B3 is assigned $75=100 \cdot 75 \%$; and B4 obtains $45=60 \cdot 75 \%$.

Definition 22.4. In a variable rate tender MRO participants "bid the amounts of money and the interest rates at which they want to enter into transactions with the national central banks".

## 23. Example of a variable tender MRO

Example 23.1. The ECB decides to provide liquidity by an amount of $€ 70$ million. Only two banks bid, B1 and B2. The bid consists of a list of interest rates and quantities demanded at each rate; see Table 10. For instance, at $5 \%$, B1 asks for $€ 7$ million and B 2 asks for $€ 3$ million. Column 4 indicates total bids: 10 at $5 \%$; 30 at $4 \%$; 50 at $3 \%$; and 110 at $2 \%$. Column 5 displays cumulative bids: up to $5 \%, 10$; up to $4 \%, 40$; up to $3 \%, 90$; and up to $2 \%, 200$.

| $i$ | bids by banks |  | total <br> bids | сити lative bids | allotment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | B1 | B2 |  |  | B1 | B2 |  |
| 5\% | 7 | 3 | 10 | 10 | 7 | 3 |  |
| 4\% | 10 | 20 | 30 | 40 | 10 | 20 |  |
| 3\% | 20 | 30 | 50 | 90 | $\begin{gathered} 20 \cdot 60 \% \\ =12 \end{gathered}$ | $\begin{gathered} 30 \cdot 60 \% \\ =18 \end{gathered}$ |  |
| 2\% | 40 | 70 | 110 | 200 | - | - |  |
| Total <br> The ECB wants to supply 70 |  |  |  |  | 29 | 41 | 70 |

Banks ask for 200, whereas the ECB only supplies 70. To deermine the allotment, the ECB starts with the highest rate (5\%) and fully allots the bids (10). Consequently, only 60 million remain to be assigned.

At the next rate (4\%), banks ask for 30 . As there are 60 to be allocated, banks receive 30 , so 30 are left to be assigned.

Table 10. Bids and allotment in a variable tender MRO

At $3 \%$ banks ask for 50 . Since there are only 30 available, the percentage of allotment is $30 / 50=$ $60 \%$. As in the fixed rate tender, this percentage is applied to the bids at $3 \%$. To sum up, B1 is allotted 29 and B 2 is assigned 41.

Definition 23.2. The marginal interest rate of a variable tender is the smallest interest rate at which some bid is (maybe partially) satisfied.

In Example 23.1, the marginal interest rate is the rate associated with the first cumulative bid larger than the amount the ECB wants to inject: the cumulative bid is 90 and the rate is $3 \%$.

Above the marginal interest rate of the tender full allotment holds: all banks are given what they ask for. Below that rate, no bid is considered. There are two basic procedures to determine the interest rate applied to allotments.

- If the allotment procedure is conducted according to a multiple rate (American) auction, B1 gets $7($ at $5 \%)+10($ at $4 \%)+12($ at $3 \%)=29$ and B2 receives $3($ at $5 \%)+20($ at $4 \%)+18($ at $3 \%)=41$.
- If the allotment procedure follows a fixed rate (Dutch) auction, all bids are paid at the marginal rate: B1 receives 29 at the marginal rate (3\%) and B2 gets 41 also at the marginal rate (3\%)


## 24. Reserve requirements

http://www.ecb.europa.eu/mopo/implement/mr/html/calc.en.html|http://www.ecb.int/mopo/implement/mr/html/calc.en.html

The minimum reserve system pursues the stabilization of money market interest rates and the regulation of liquidity.

The reserve ratio is around $2 \%$ ( $1 \%$ since 18 Jan 2012). Compliance is determined on the basis of averages of reserve holdings during the maintenance period. Reserves are remunerated at the average of MROs rates. Reserve holdings above the minimum are not remunerated. Noncompliance is penalized. Some reserve maintenance statistics:

- from $13 / 02 / 08$ to $11 / 03 / 08$ average required reserves amounted to $€ 200$ billion, the remuneration rate was $4.1 \%$, and the penalty rate for deficiencies $7.5 \%$;
- from $21 / 01 / 09$ to $10 / 02 / 09$ : $€ 221$ billion, $2 \%, 5.5 \%$;
- from $08 / 12 / 09$ to $19 / 01 / 10$, € $£ 10$ billion, $1 \%, 4.25 \%$;
- from $15 / 02 / 12$ to $13 / 03 / 12$, €104.2 billion, $1 \%, 4.25 \%$;
- from $08 / 10 / 14$ to $11 / 11 / 14$ : €105.7 billion, $0.05 \%, 2.80 \%$;
- from $28 / 10 / 15$ to $08 / 12 / 15$ : €113.061 billion, $0.05 \%, 2.80 \%$.
http://www.ecb.europa.eu/mopo/implement/mr/html/index.en.html


## 25. Standing facilities

http://en.wikipedia.org/wiki/Overnight_market | http://www.ecb.europa.eu/mopo/implement/sf/html/index.en.html

Definition 25.1. The Eurosystem allows counterparties (banks) to use two standing facilities:

- the marginal lending facility, to obtain overnight liquidity from the national central banks against eligible assets; and
- the deposit facility, to make overnight deposits with the national central banks.

The aim of the two standing facilities offered by the Eurosystem, the marginal lending facility and the deposit facility, is three-fold:

- to provide and absorbe overnight liquidity;
- signal the general stance of monetary policy; and
- bound overnight market interest rates.

Definition 25.2. The overnight market is the money market for loans with the shortest maturity. In the overnight market the borrowed funds (plus interest) must be repaid at the start of the next day (liquidity is provided "overnight").

Overnight rates provide information concerning the liquidity conditions in the economy. Lack of confidence among banks (they are afraid of lending each other) and the existence of tight liquidity conditions may easily cause overnight rates to shoot up. Fig. 11 displays the evolution of the Euribor (the Euro Interbank Offered Rate). The one-week rates approximate overnight rates.

The interest rates corresponding to the two standing facilities defines the ECB's interest rate corridor for overnight rates, because the interest rate on the marginal lending facility is a ceiling for overnight market rates and the interest rate on the deposit facility is normally a floor rate.

## 26. Interest rates set by the ECB <br> http://www.ecb.int/home/html/index.en.html | http://www.ecb.europa.eu/stats/monetary/rates/html/index.en.html

- The interest rate of MROs. Of the three interest rates set by the ECB, this is the key one: it is the rate at which banks can regularly borrow from the ECB. Since 10 Sep 2014, the fixed rate MROs, the procedure currently adopted, has been set at $0.05 \%$. When the procedure changed to fixed rate in October 2008, 4.25\% was the current minimum bid rate accepted in a variable rate MRO.
- The interest rate on the marginal lending facility. This what banks must pay for overnight lending. Currently ( 25 Feb 2016) and since 10 Sep 2014, it is $0.3 \%$.
- The interest rate on the deposit facility. This is what banks obtain from parking funds, as an overnight deposit, at national central banks. Since 9 Dec 2015 it has been set at $-0.3 \%$. The previous value, was $-0.2 \%$. The previous one to that, $-0.1 \%$, represented the first instance of a negative interest rate set by the ECB (11 Jun 2014).

Why establish negative interest rates? From the real sector perspective, the lower the interest rate, the more is people encouraged to spend. Hence, a negative interest rate looks like a desperate message from the central bank asking people to spend money without dilation.

From the financial sector perspective, the interest rate on the deposit facility has been lowered to negative values to maintain the interest rate corridor: as the MRO rate was, on 11 Jun 2014, reduced from $0.25 \%$ to $0.15 \%$ and the rate on the deposit facility was already $0 \%$, it was deemed necessary to also diminish the deposit rate.

The claim justifying the need to preserve the corridor is that, if the above rates were kept too close to each other, it would be more difficult to maintain a liquidity market in which banks lend to each other. Fig. 12 shows the interest rate corridor defined by the ECB decisions.
https://www.ecb.europa.eu/home/html/faqinterestrates.en.html


Fig. 11.Euribor, 2 Jan 2007 - 23 Feb 2016


Fig. 12. ECB key interest rates (25 Feb 2016)

## 27. The standard liquidity market model

Definition 27.1. The (standard) liquidity (or loan or loanable funds) market model is a model to determine the nominal interest rate.

It is similar to a competitive market model in which market equilibrium specifies the nominal interest rate. A market demand function represents the demand for liquidity (for loans, for credit) in the economy. A sort of market supply function represents the supply of liquidity (loans, credit) in the economy.

It would more reasonable to model the liquidity market at least as a two-tier market, the top tier involving the big players (central bank and main banks) and the bottom tier consisting of everyone except the central bank. But for simplicity, and as a first approach, all agents (the big and the small fish) will be put together in a single market.

## 28. The supply of liquidity

Definition 28.1. The direct supply of liquidity is provided by banks (who supply consumers, firms, and other banks) and the central bank (who supplies banks).

Definition 28.2. The indirect supply of liquidity corresponds to purchases of (interest-bearing) financial assets.

Purchasing a financial asset supplies liquidity since the buyer of an asset gives money in exchange. Consequently, the seller of a financial asset is in practice obtaining a loan of money. The difference is that a bank's loan is not generally marketable, whereas interest-bearing assets can be resold (a lender can easily become a borrower).

## 29. The market supply of liquidity function

Definition 29.1. The market supply of liquidity function relates the total volume of liquidity supplied to the nominal interest rate at which this volume is supplied.

The supply of liquidity function represents the decisions by lenders (savers). The agents creating the supply of liquidity are banks, financial intermediaries, buyers of interest-bearing financial assets, and the central bank.


Fig. 13. Supply of liquidity function

The supply of liquidity function is not a competitive supply function because the agents generating the supply are persumed to set the interest rate at which they are willing to provide liquidity. Fig. 13 illustrates this idea. Pick any volume of liquidity supplied, like $L_{a}$. Choose a sufficiently low interest rate, so that a point like $a^{\prime}$ can be identified. In this case, the presumption is that $a^{\prime}$ cannot represent a point of the supply of liquidity function because lenders would ask for a higher interest rate. Hence, at a point like $a^{\prime}$, the tendency for the interest rate is to increase.

Now, maintaining the amount of liquidity $L_{a}$, consider a sufficiently high interest rate, so point $a^{\prime \prime}$ would result. In that case, lenders will probably believe that the interest rate they charge is excessive (which may increase the risk of default) and, accordingly, they may be willing to reduce it. Thus, a point like $a^{\prime \prime}$ can neither represent a point of the supply of liquidity function since at that point the tendency for the interest rate is to fall.

One may proceed in a similar way by considering points between $a^{\prime}$ and $a^{\prime \prime}$ : for points above but close to $a^{\prime}$ the tendency will still be for the interest rate to go up; and for points below but close to $a^{\prime \prime}$ the tendency will still be for the interest rate to go down. It is then reasonable to expect that at some intermediate point, like $a$, the two tendencies will neutralize each other, in the sense that lenders feel that the "right" interest rate to charge for supplying the volume of liquidity $L_{a}$ is $i_{a}$. The pair ( $L_{a}, i_{a}$ ) would be a point on the market supply of liquidity function.

The same exercise could be replicated for any other volume of liquidity, like $L_{b}$. A sufficiently low interest rate will be considered insufficient to supply $L_{b}$ (so the interest rate will be risen) and a sufficiently high interest rate will be deemed excessive (hence, it will be lowered). At some point, such as $b$, the two forces will be balanced and another point on the supply funcion will be found.

Hypothesis 29.2. The market supply of liquidity function is assumed upward sloping: in order to supply more liquidity, lenders ask for a higher interest rate.

## 30. The demand for liquidity

Definition 30.1. The direct demand for liquidity corresponds to loan applications typically addressed to banks; for instance, the demand for loans for house purchase.

Definition 30.2. The indirect demand for liquidity is given by the sale or issuing of interestbearing financial assets, like T-bills, government bonds, corporate bonds...

There is no substantial difference between the two components of liquidity demand: when a bank accepts a loan application, it is as if the applicant sold a financial asset to the bank (the loan).

## 31. The market demand for liquidity function

Definition 31.1. The market demand for liquidity function establishes, for each value of the nominal interest rate, the total volume of liquidity demanded at that rate.

The demand for liquidity function represents the decisions by borrowers (investors). The agents generating (net) demand for liquidity are consumers (consumer credits, loans for house purchase), firms (trade credit, issuance of corporate bonds), and the government (T-bills, bonds).


Fig. 14. Demand for liquidity function

Though the demand for liquidity function is interpreted as a competitive demand function, it can be motivated in a way similar to the supply function; see Fig. 14. Pick any interest rate, like $i_{a}$. For a sufficiently low volume of liquidity demanded, like $L$, it is likely that borrowers would ask for more liquidity. Hence, at a point like $a^{\prime}$, the tendency for the demand for liquidity is to grow. Similarly, for a sufficiently large volume of liquidity (like the one corresponding to $a^{\prime \prime}$ ) it is likely that, given $i_{a}$, borrowers would like to cut borrowing (at $a^{\prime \prime}$ it is "excessive")

One may proceed in a similar way by considering points between $a^{\prime}$ and $a^{\prime \prime}$ : for points close to $a^{\prime}$ the tendency is for the demand for liquidity to increase and for points around $a^{\prime \prime}$ the tendency is for the demand for liquidity to fall. It is then reasonable to expect that at some intermediate point, like $a$, the two tendencies will neutralize each other, in the sense that borrowers feel that the "right" volume of liquidity to demand is $L_{a}$ when the interest rate charged is $i_{a}$. The pair ( $i_{a}, L_{a}$ ) would be a point on the market demand for liquidity function.

The same exercise could be replicated for any other interest rate, like $i_{b}$. A sufficiently low demand for liquidity will be considered insufficient under interest rate $i_{b}$ (so demand will rise)
and a sufficiently high demand for liquidity will be deemed excessive (and, accordingly, it will be lowered). At some point, such as $b$, the two forces will offset each other and another point on the demand funcion will be identified.

Hypothesis 31.2. The market demand for liquidity function is assumed downward sloping: the higher the interest rate (the price of obtaining liquidity), the smaller the demand for liquidity.

## 32. Liquidity market equilibrium

Fig. 15 depicts the supply of liquidity function $S_{L}$ and the associated tendency for the interest rate outside the function. This representation suggests that the supply of liquidity function can be defined as the set of pairs $(i, L)$ such that, when lenders supply the volume $L$ of liquidity and charge the interest rate $i$, there is no tendency for $i$ to change.


Fig. 15. Dynamics behind the supply function


Fig. 16. Dynamics behind the demand function

Fig. 16 shows the demand for liquidity function $D_{L}$ and the corresponding tendency for the demand for liquidity outside the function. The graph suggests that the demand for liquidity function can be defined as the set of pairs $(i, L)$ such that, at the given interest rate $i$, there is no tendency for the demand for liquidity $L$ to change.

Fig. 17 which puts Figs. 15 and 16 together, is the graphical representation of the liquidity market model. The arrows indicate the market dynamics corresponding to each pair ( $i, L$ ) on the four regions of the plane. For instance, in the region consisting of the points below both functions (see point $a$ in Fig. 18), the tendency of both the interest rate and the demand for liquidity is to rise. The combination of these two tendencies is represented by the arrow pointing to the north east. Fig. 18 depicts the dynamical process starting from $a$. The dynamical process is supposed to converge to the only point $e$ on the plane where there is no tendency to change. This point is given by the intersection of the two functions. At point $e$, there is no pressure on $i$ to change because $e$ is on the supply function and there is no pressure on $L$ to change as $e$ is on the demand function.


Fig. 17. Dynamics behind the supply function


Fig. 18. Convergence to market equilibrium

Definition 32.1. A (liquidity) market equilibrium is a pair $\left(i^{*}, L^{*}\right)$ such that:

- when the interest rate is $i^{*}$, the total volume of liquidity demanded is $L^{*}$; and
- the interest rate at which suppliers are willing to supply exactly the amount $L^{*}$ is $i^{*}$.

Remark 32.2. Every market equilibrium is graphically represented by a point ( $i, L$ ) where the supply and demand functions intersect.

Hypothesis 32.3. There are points $(i, L)$ on the market demand for liquidity function and $\left(i^{\prime}, L\right)$ on the market supply of liquidity function such that $i>i^{\prime}$.

Hypothesis 32.3 means that, for some volume of liquidity $L$, the vertical line that is drawn from value $L$ upwards crosses the supply function before the demand function. For instance, this occurs in Fig. 17 for values $L<L^{*}$. Intuitively, Hypothesis 32.3 holds that, for some volume of liquidity, borrowers are willing to pay for it more than lenders are asking to supply it. In other words, there is room for a market to exist: demand can afford being supplied.

Definition 32.4. Call "standard" the liquidity market model that satisfies Hypotheses 29.2, 31.2, and 32.3.

Proposition 32.5. The standard liquidity market model has a unique market equilibrium.
Proof. By Hypothesis 32.3, there are points $\left(i^{\prime}, L^{\prime}\right)$ on the demand function $D_{L}$ and ( $i^{\prime \prime}, L^{\prime}$ ) on the supply function $S_{L}$ such that $i^{\prime}>i^{\prime \prime}$. This is represented in Fig. 19. Since $D_{L}$ is decreasing by Hypothesis 31.2, when the demand function is drawn on the left of point $a$, the function goes up. Similarly, as $S_{L}$ is increasing by Hypothesis 29.2, when the supply function is drawn on the left of point $b$, the function goes down. Consequently, $D_{L}$ does not cross $S_{L}$ to the left of $L^{\prime}$. Now, given that $D_{L}$ slopes downward and $S_{L}$ slopes upward, both functions must intersect at some point $e$ on

the right of $L^{\prime}$. Moreover, once $e$ is reached, $S_{L}$ will remain above $D_{L}$, for which reason no further intersection can occur.-

Remark 32.6. As suggested by Fig. 18, the presumed dynamics underlying the supply and demand functions ensure convergence to the only market equilibrium of the standard model.

Fig. 19. Proof of Proposition 32.5

## 33. Shifting the liquidity demand function

Remark 33.1. Rule of thumb for shifting the demand function. Any event that, for any given interest rate, stimulates [discourages] the demand for liquidity (for loans, for credit) shifts the market demand for liquidity function to the right [left].

Example 33.2. Events that will pressumably shift the demand function to the right: having more consumers, more firms, a higher budget deficit, the expectation of a higher inflation rate, an improvement in indices of business or consumer confidence, an increase in wealth or profits (may be), an increase in the foreign demand for domestic loans... The opposite changes will tend to shift the demand function to the left.


Fig. 20. Effects of a demand shift to the right


Fig. 21. Effects of a supply shift to the left

Fig. 20 shows the effect on the market of equilibrium of an expansion of the demand for liquidity: a demand shift to the right causes a rise in both the equilibrium interest rate and the volume of liquidity (as market equilibrium goes from $a$ to $b$ ).

## 34. Shifting the liquidity supply function

Remark 34.1. Rule of thumb for shifting the supply function. Any event that, for any given interest rate, stimulates [discourages] the supply of liquidity shifts (at least the typical section of the) market supply function of liquidity to the right [left].

Example 34.2. Events that will pressumably shift the supply function to the right: having more banks, the expectation of a higher inflation rate, an increase in the consumers' or the firms' saving rate, expansionary open market operations by the central bank, lifting credit controls imposed on banks, fiscal advantages granted for purchasing financial assets... The opposite changes will tend to shift the supply function to the left.

Fig. 21 shows the effect on the market of equilibrium of a contraction of the supply of liquidity: a supply shift to the left causes a rise in the equilibrium interest rate and a decline in the volume of liquidity (as market equilibrium goes from $a$ to $b$ ).

## 35. Simultaneous shifts of supply and demand functions

Fig. 22 analyzes the effect on market equilibrium of a simultaneous shift of both functions to the left. To establish the result of such changes it is convenient to investigate each shift separately and next combine the effects. If, on Fig. 22, only the change in demand took place, in equilibrium, both liquidity and interest rate would fall. If only the supply change occurred, then, in equilibrium, liquidity would fall but the interest rate would shoot up. Therefore, when both changes happen, it follows that liquidity decreases but the effect on the interest rate cannot be determined: the demand shift pressures down, whereas the supply shift pressures up. Without knowing which effect dominates, the net impact on the interest rate remains uncertain. Fig. 22 shows two possibilities: if the supply function shifts from $S_{L}$ to $S_{L}{ }^{\prime}$, the demand impact dominates and the interest rate falls (equilibrium going from $a$ to $a^{\prime}$ ); if the supply function shifts from $S_{L}$ to $S_{L}{ }^{\prime \prime}$, the supply impact dominates and the interest rate goes up (equilibrium going from $a$ to $a^{\prime \prime}$ ).


Fig. 22. Effects of a demand and supply shifts to the left


Fig. 23. Supply to the left, demand to the right

Fig. 23 considers the case of a demand surge and a supply contraction. Each event separately causes a rise in the equilibrium interest rate; thus, when combined, they produce an interest rate increase. On the other hand, the demand surge increases equilibrium liquidity, whereas the supply crunch causes a liquidity contraction. The net effect is uncertain: equilibrium liquidity could go up (when market equilibrium moves from $a$ to $a^{\prime \prime}$ ) or down (from $a$ to $a^{\prime}$ ).

## 36. The real interest rate

Definition 36.1 (informal). The real interest rate $r$ of an economy represents the purchasing power of the economy's nominal interest rate $i$ : it is the nominal rate $i$ expressed in terms of goods.

That the nominal interest rate between period $t$ and period $t+1$ is $i$ means that, by lending 1 currency unit in $t$, one gets $1+i$ currency units in $t+1$.

That the real interest rate between period $t$ and period $t+1$ is $r$ means that, by lending 1 unit of goods int $t$, one gets $1+r$ units of goods in $t+1$. Therefore, $r$ expresses purchasing power: the amount of goods obtained from each unit of good lent.

Example 36.2. Let "goods" be represented by the CPI basket. There are two periods, $t$ $=1$ and $t=2$. The nominal interest rate between $t=1$ and $t=2$ is $i=10 \%$. The cost of the CPI basket in $t=1$ is $P_{1}=204$ EUR. The cost of the CPI basket in $t=2$ is $P_{2}=$ 220 EUR; see Fig. 24. With this information, the CPI inflation rate is

$$
\pi=\frac{P_{2}-P_{1}}{P_{1}}=\frac{220-204}{204}=7.84 \%
$$



Fig. 24. Calculating the real interest rate

Now suppose 612 EUR are lent in $t=1$. Then $612 \cdot(1+i)=612 \cdot(1+0.10)=673.2$ EUR are obtained in $t=2$. In $t=1$, the purchasing power of 612 EUR is $612 / P_{1}=612 / 204=3$ baskets. In $t$ $=2$, the purchasing power of 673.2 EUR is $673.2 / P_{2}=673.2 / 220=3.06$ baskets. The real interest rate $r$ measures the change in purchasing power of the money lent. Specifically, $r$ transforms 3 baskets into 3.06 baskets. Thus, $r$ satisfies $3 \cdot(1+r)=3.06$, so $r=0.02(2 \%)$.

## 37. The Fisher equation

Definition 37.1. The Fisher equation (2), which is an approximation of the relationship between $i$ and $r$, holds that the real interest rate is the difference between the nominal interest rate and the inflation rate.

$$
\begin{equation*}
r=\pi-i \tag{2}
\end{equation*}
$$

Definition 37.2. The real interest rate $r$ of an economy is defined by equation (2).

Example 37.3. In Example 36.2, $i=10 \%$ and $\pi=7.84 \%$ (as $P$ jumps from 204 to 220). According to the Fisher equation, $r=i-\pi \approx 10-7.84=2.16 \%$, which is close to the correct value of $2 \%$.

Irving Fisher argued in 1907 that the nominal interest rate eventually, in the long run, captures the inflation rate. In this view, $i=r+\pi$ : a lender expecting to earn a real interest rate $r$ and expecting an inflation rate $\pi$ will at least charge a nominal interest rate $i=r+\pi$. In some circumstances, the lender may also add a risk premium $\rho$ to compensate the lender for taking an excessive default risk by lending a not fully creditworthy borrower. This suggests that nominal interest rates could be decomposed into at least three components, as shown in (3).

$$
\begin{equation*}
i=r+\pi+\rho \tag{3}
\end{equation*}
$$

## 38. Negative real interest rates and negative nominal interest rates

Negative real interest rates may arise in practice: it suffices to have $\pi>i$. In Example 37.2, if the price level raised to, say, 269.28 instead of 220 , then 673.2 EUR could only buy 2.5 baskets. Hence, after the loan is repaid one can purchase fewer baskets than the initial 3 baskets. In this case, $r=i$ $-\pi=10 \%-32 \%=-22 \%$ (from 3 to 2.5 baskets the actual loss is $16.6 \%$ ).

Although negative nominal interest rate might at first appear impossible, see by way of illustration http://uk.reuters.com/article/2012/07/05/denmark-ratesidUKL6E8I5A8520120705: investors were willing to accept a negative $i$ to shelter their money. A negative nominal interest rate might also be observed if deflation is expected.

Example 38.1. Let the nominal interest rate be $1 \%$ and the inflation rate $0.25 \%$. The real interest rate (computed using the Fisher equation) is $0.75 \%$. Imagine that the inflation rate is expected to be $-1 \%$. In this case, a negative nominal interest rate equal to $-0.25 \%$ is still capable of ensuring a real interest rate of $0.75 \%$. The lesson of this example is that the relevant variable for lenders is the real not the nominal interest rate: the nominal interest rate is instrumental, not an end it itself.

## 39. The Fisher effect

Definition 39.1. The Fisher hypothesis states that the real interest rate is approximately constant.

Definition 39.2. The Fisher effect (an implication of the Fisher hypothesis) asserts that there is a one-to-one relationship between $i$ and $\pi$ : every additional point of the inflation rate becomes an additional point of the nominal interest rate.

The Fisher effect is consistent with the empirical evidence: economies with high inflation rates tend to be economies with high nominal interest rates; see Fig. 25.


Fig. 25. Evidence for the Fisher effect, RG Hubbard, AP O'Brien, M Rafferty (2012), Macroeconomics, p. 204

According to (3), it is natural to expect that, when the inflation rate rises, lenders will demand a higher nominal interest rate to get back the purchasing power lost due to the price increase. Example 39.3 illustrates numerically this presumption.

Example 39.3. Let $P_{0}=100, P_{1}=110$, and $P_{2}=132$, so $\pi_{1}=10 \%$ and $\pi_{2}=20 \%$. Suppose $r_{1}=5 \%$ : from period $t=0$ to $t=1$ lenders get a $5 \%$ increase in purchasing power. This means that for each equivalent to 1 basket lent in $t=0$, the equivalent of 1.05 baskets are received in $t=1$. That is, if 100 EUR are lent in $t=0,115.5$ EUR will be received in $t=1$. Using the Fisher equation, the $i_{1}$ ensuring that $r_{1}=5 \%$ when $\pi_{1}=10 \%$ is $i_{1}=r_{1}+\pi_{1}=15 \%$.

Assume the Fisher hypothesis, so $r_{2}=r_{1}=5 \%$. If $i_{2}$ remained at $15 \%$, by lending $€ 110$ (the basket value in $t=1$ ), the amount received in $t=2$ would be $110 \cdot\left(1+i_{2}\right)=110 \cdot 1+0.15=126.5$. As $P_{2}$ $=132$, the purchasing power of $€ 126.5$ is 0.958 baskets: there is a loss of purchasing power. By the Fisher equation, the $i_{2}$ needed to preserve the purchasing power of a money loan is $i_{2}=r_{2}+\pi_{2}=$ $5 \%+20 \%=25 \%$ : from $t=1$ to $t=2, \pi$ goes up 10 points and $i$ also goes up 10 points.


Fig. 26 shows how to obtain the Fisher effect in the liquidity market model. On the one hand, to be able to afford their consumption plans, borrowers will likely demand more liquidity (this reaction is uncertain). On the other hand, a rise in the inflation rate encourages lenders to charge a higher nominal interest rate for supplying liquidity.

Fig. 26. The Fisher effect in the liquidity market model

## 40. The Lucas paradox

Definition 40.1. Orthodox macroeconomic theory predicts that capital (lending) should flow from the richer to the poorer economies. The Lucas paradox is the observation that such flows are not occurring.

In a 1990 paper, Nobel in Economics laureate Robert Lucas, Jr. estimated that, if orthodox macroeconomic theory were true, the return to investment in India in 1988 should be around 58 times higher than in the United States. Such monumental return differential should make capital to flow from the United States to India. Yet this flow has not been observed.

In fact, it is likely that the real interest rate will substantially differ between richer and poorer economies. In a poor economy, by definition, GDP per capita is low and, accordingly, savings are low. In addition, lack of productive capital (which lies behind a low GDP per capita level) implies that the return to capital will also tend to be high. Scarce supply of savings combined with high demand for capital lead to high real interest rates. The reverse is expected to occur in a rich economy. As a consequence, given that capital is mobile internationally, it is natural to predict a flow of funds from richer to poorer economies. One reason why such a flow has not been observed is that investment (lending) in poorer economies is riskier.

- Investors may lack relevant information: poorer economies are typically less transparent than richer ones.
- There is also exchange rate risk, that is, that the currency of the poor economy receiving investment will fall with respect to the currency of the domestic economy of the investor. If this fall occurs, the investor incurs a loss when converting the invested funds back into the investor's currency.
- Investors may believe that the default risk is higher in a poor (less well known) than in a rich (better known) economy. This belief can be justified by the fact that poorer economies are weak agents in international capital markets (it is harder for them to obtain foreign funds) and historically have been politically and/or socially more unstable than rich countries.
- In general, the environment surrounding a poor economy tends to be more unstable or unpredictable. For example, governments may lack credibility insofar as they are prone to make frequent changes in regulations and taxes.

In these circumstances, it would not be surprising to observe funds flowing from poorer to richer economies, where investment, despite being probably less profitable, is safer. This will cause real interest rate differences between rich and poor economies to widen rather than to contract.

