6. Real interest rate, Fisher equation, Fisher effect

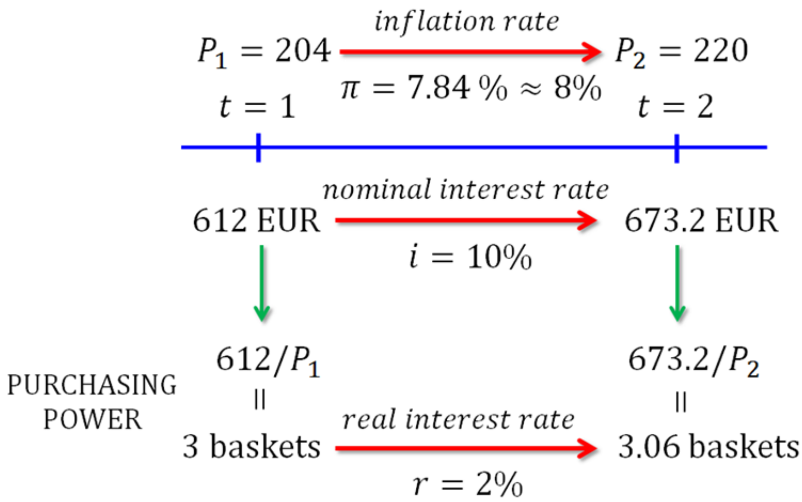
1. Real interest rate

The real interest rate r of an economy represents the purchasing power of the economy's nominal interest rate i: it is the nominal rate i expressed in terms of goods. That the nominal interest rate between period t and period $t+1$ is i means that, by lending 1 currency unit in t, one gets $1+i$ currency units in $t+1$. That the real interest rate between period t and period $t+1$ is r means that, by lending 1 unit of goods int t, one gets $1+r$ units of goods in $t+1$. Thus, r expresses purchasing power: the amount of goods obtained from each unit of good lent.

2. An example on the real interest rate

Let "goods" be represented by the CPI basket. There are two periods, $t=1$ and $t=2$. The nominal interest rate between $t=1$ and $t=2$ is $i=10\%$. The cost of the CPI basket in $t=1$ is $P_1=204$ EUR. The cost of the CPI basket in $t=2$ is $P_2=220$ EUR; see the sketch on the right. With this information, the CPI inflation rate is

$$\pi = \frac{P_2 - P_1}{P_1} = \frac{220-204}{204} = 7.84\%.$$



If 612 EUR are lent in $t=1$. Then $612\cdot(1+i) = 612\cdot(1+0.10) = 673.2$ EUR are obtained in $t=2$. In $t=1$, the purchasing power of 612 EUR is $612/P_1 = 612/204 = 3$ baskets. In $t=2$, the purchasing power of 673.2 EUR is $673.2/P_2 = 673.2/220 = 3.06$ baskets. The real interest rate r measures the change in purchasing power of the money lent. Specifically, r transforms 3 baskets into 3.06 baskets. Thus, r satisfies $3\cdot(1+r)=3.06$, so $r=0.02$ (2%).

3. The Fisher equation

The Fisher equation (1), which is an approximation of the relationship between i and r, holds that the real interest rate is the nominal interest rate minus the inflation rate. Equation (1) is usually taken as the definition of the real interest rate r of an economy.

$$\boxed{r = i - \pi} \tag{1}$$

In example §2, $i=10\%$ and $\pi = 7.84\%$ (as P jumps from 204 to 220). According to the Fisher equation, $r = i - \pi \approx 10 - 7.84 = 2.16\%$, which is close to the correct value of 2%.

4. Components of the nominal interest rate

Irving Fisher argued in 1907 that the nominal interest rate eventually, in the long run, captures the inflation rate. In this view, $i = r + \pi$: a lender expecting to earn a real interest rate r and expecting an inflation rate π will at least charge a nominal interest rate $i = r + \pi$. In some circumstances, the lender may also add a risk premium ρ to compensate the lender for taking an excessive default risk by lending a not fully creditworthy borrower. This suggests that nominal interest rates could be decomposed into at least three components: $i = r + \pi + \rho$.

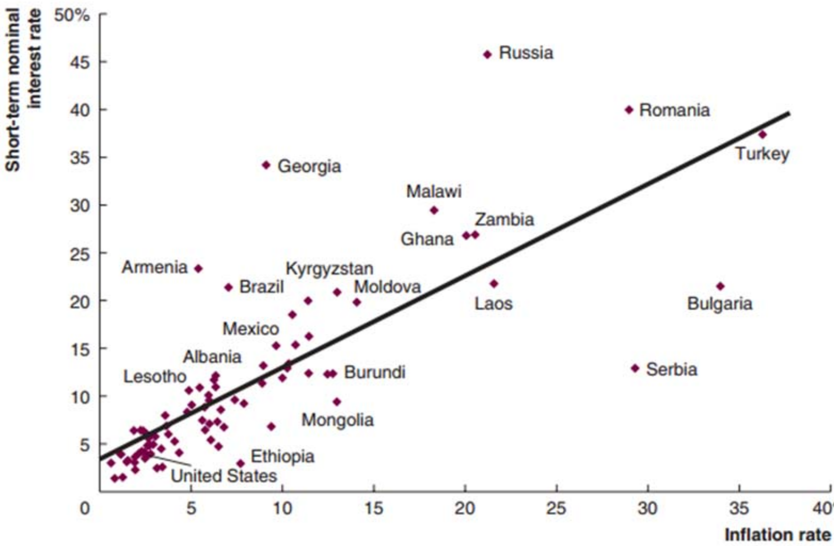
5. **Negative interest rates**

Negative real interest rates may arise in practice: it suffices to have $\pi > i$. In example §2, if the price level went up to, say, 269.28 instead of 220, then 673.2 EUR could only buy 2.5 baskets. Hence, after the loan is repaid one can purchase fewer baskets than the initial 3 baskets. In this case, $r = i - \pi$ = 10% − 32% = −22% (from 3 to 2.5 baskets the actual loss is 16.6%). Although negative nominal interest rate might at first appear impossible, investors may be willing to accept a negative i to shelter their money. In March 2017, Spanish 12 month T-bills had a negative rate of return: −0,302%. A negative nominal interest rate might also be observed if deflation is expected.

6. **An example with negative interest rate**

Let the nominal interest rate be 1% and the inflation rate 0.25%. The real interest rate (using the Fisher equation) is 0.75%. Imagine that the inflation rate is expected to be –1%. In this case, a negative nominal interest rate equal to –0.25% is still capable of ensuring a real interest rate of 0.75%. The lesson of this example is that the relevant variable for lenders is the real not the nominal interest rate: the nominal interest rate is instrumental, not an end it itself.

7. **The Fisher effect**



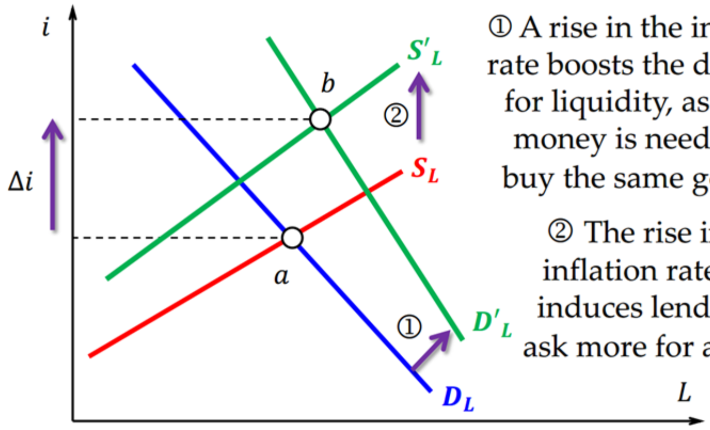
The Fisher hypothesis states that the real interest rate is approximately constant. The Fisher effect (an implication of the Fisher hypothesis) asserts that there is a one-to-one relationship between i and π: every additional point of the inflation rate becomes an additional point of the nominal interest rate. The chart on the right (RG Hubbard et al., 2012, *Macroeconomics*, p. 204) shows empirical evidence in support of the Fisher effect: economies with high inflation rates tend to have high nominal interest rates.

8. **Justification of the Fisher effect**

Equation $i = r + \pi$ explains why lenders would ask for a higher nominal interest rate to get back the purchasing power lost due to a price increase. For example, if $P_0 = 100$, $P_1 = 110$ and $P_2 = 132$, it follows that $\pi_1 = 10\%$ and $\pi_2 = 20\%$. Let $r_1 = 5\%$: from period $t = 0$ to $t = 1$ lenders get a 5% increase in purchasing power. This means that lending in $t = 0$ money equivalent to 1 basket, the equivalent to 1,05 baskets is received in $t = 1$. That is, if 100 EUR are lent in $t = 0$, 115.5 EUR will be received in $t = 1$. By (1), the i_1 ensuring that $r_1 = 5\%$ when $\pi_1 = 10\%$ is $i_1 = r_1 + \pi_1 = 15\%$. Assuming the Fisher hypothesis, $r_2 = r_1 = 5\%$. If i_2 remained at 15%, by lending 110 EUR (the basket value in $t = 1$), $110 \cdot (1 + i_2) = 110 \cdot 1 + 0{,}15 = 126.5$ EUR would be received in $t = 2$. As $P_2 = 132$, the purchasing power of 126.5 EUR is 0.958 baskets: purchasing power is lost. By (1), the i_2 needed to preserve the purchasing power of a money loan is $i_2 = r_2 + \pi_2 = 5\% + 20\% = 25\%$: from $t = 1$ to $t = 2$, π goes up 10 points and i also goes up 10 points.

9. The Fisher effect in the liquidity market

The figure on the right shows how to obtain the Fisher effect in the liquidity market model. First, with higher prices, borrowers will likely demand more liquidity to maintain their consumption. More liquidity will be demanded if prices are expected to rise in the future (so purchases are made now before prices go up). And second, a rise in the inflation rate encourages lenders to charge a higher nominal interest rate for supplying liquidity.



10. Interest rate and price of financial assets (T-bills in particular)

The price of a financial asset and the nominal interest rate move in opposite directions. This result is shown next for the case in which the financial asset is a T-bill. The T-bill is issued in period t and matures in $t+1$. The price of the T-bill in t, when issued, is P. The face value of the T-bill is V, which means that, in $t+1$, the T-bill pays V to the owner of the T-bill. Let i be the interest rate between t and $t+1$, so i represents the profit of making a loan with the same maturity as the T-bill. An investor having P monetary units may consider two options.

- Option 1: lend P. When the loan matures, in $t+1$, the investor gets $(1+i)\cdot P$.
- Option 2: buy the T-bill. When the T-bill matures, in $t+1$, the investor gets V.

For both options to be equally attractive, the two outcomes must coincide; that is, $(1+i)\cdot P = V$. Solving for P,

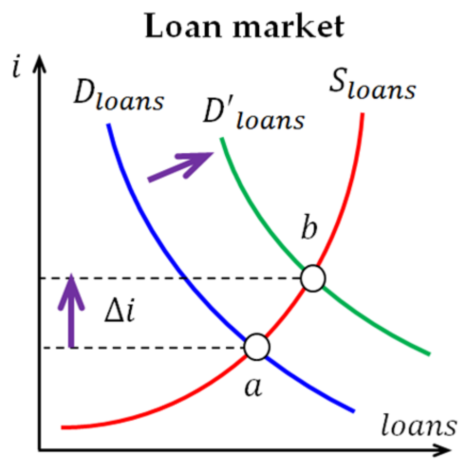
$$\boxed{P = \frac{V}{1+i}}. \tag{2}$$

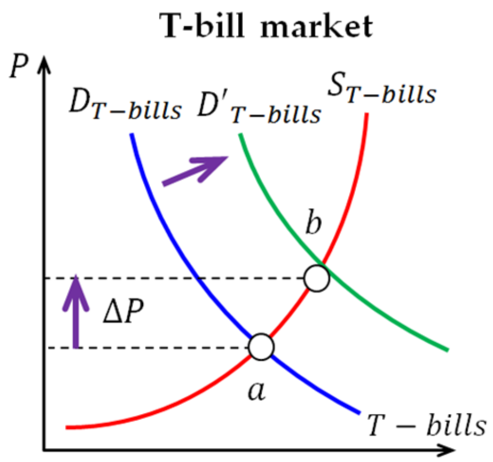
Since V is a fixed given value, (2) means that the larger i, the smaller P.

11. Financial arbitrage and inverse relationship between interest rate and price of T-bills

Arbitrage consists of making purchases and sales that ensure a sure profit. Under financial arbitrage, an arbitrageur buys and sells financial assets to obtain a sure profit. It will be next argued that financial arbitrage justifies the inverse relationship between the price of a T-bill and the interest rate established by (2). To this end, suppose there are arbitrageurs looking for opportunities to make a sure profit and that (2) does not hold; that is, $V > (1+i)\cdot P$ or $V < (1+i)\cdot P$. Only the former possibility is handled, the latter being left as an exercise. If $V > (1+i)\cdot P$, then an arbitrageur could obtain sure profits as follows, even having no money.

- **Step 1.** The arbitrageur borrows P monetary units in t and, consequently, has to repay in $t+1$ $(1+i)\cdot P$ monetary units.
- **Step 2.** The arbitrageur purchases in t a T-bill with the P monetary units.
- **Step 3.** Reached period $t+1$, the T-bill pays V monetary units and, as $V > (1+i)\cdot P$, the arbitrageur repays the loan and pockets a profit of $V - (1+i)\cdot P > 0$ monetary units (example: if V= 1.000, P= 800 and i = 10%, each T-bill bought with borrowed money yields a profit of 120).

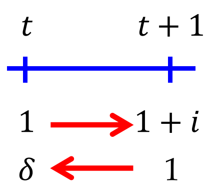




Assuming competitive markets (see graphs on the left), step 1 shifts the demand for loans (for liquidity) function to the right and <u>the interest rate i goes up</u>. The purchase of T-bills executed in step 2 shifts the demand for T-bills function to the right. This increases the price P of T-bills. With both i and P going up, $(1 + i) \cdot P$ also goes up. The result is that $V - (1 + i) \cdot P$ diminishes. <u>Arbitrageurs will borrow money and buy T-bills until</u> the gap between values V and $(1 + i)\cdot P$ is closed; that is, until $V = (1 + i) \cdot P$. In this case, <u>arbitrage opportunities vanish</u>. This reasoning proves that $V > (1 + i) \cdot P$ cannot hold for long in the presence of a financial arbitrage.

12. Discount factor

The <u>discount factor</u> δ between periods t and $t + 1$, <u>when i is the interest rate</u> between t and $t + 1$, is $\delta = \frac{1}{1+i}$. The <u>discount factor</u> (between periods t and $t + 1$) expresses the value in period of t of one monetary unit of period $t + 1$. Whereas the interest rate transforms today's money into tomorrow's money (1 today is $1 + i$ tomorrow), the discount factor does the opposite by transforming tomorrow's money into today's money. The sketch below shows how the discount factor δ determines present values out of future values.



The discount factor makes 1 become δ. This δ is the value in period t that, when the interest rate between t and $t + 1$ is i, becomes 1 in period $t + 1$. By the rule of three, $\delta = 1 \cdot 1/(1 + i) = 1/(1 + i)$ is the discount factor, which depends on the interest rate i. This leads to a more precise definition of δ.

13. The price of T-bills as a present value

The concept of present value provides a second justification of equation (2). The value in $t + 1$ (the future value) of the T-bill is V. With interest rate i between t and $t + 1$, the value of V in t (its present discounted value) is $V \cdot \frac{1}{1+i}$, where $\frac{1}{1+i}$ is the discount factor between t and $t + 1$. In view of this, equation (2) states that <u>the price of a T-bill coincides with the present discounted value of its face (future) value</u>.

14. Equalization of rates of return

A third justification of (2) comes from presuming the <u>equalization of the interest rates of all financial assets</u>; otherwise, financial assets with a smaller rate of return would have no demand and, hence, would not exist. The interest rate $i_{T\text{-}bill}$ of a T-bill is $i_{T\text{-}bill} = \frac{V-P}{P}$. Letting i represent the interest rate of a loan, the equalization condition $i = i_{T\text{-}bill}$ leads to $i = i_{T\text{-}bill} = \frac{V-P}{P} = \frac{V}{P} - 1$, or, equivalently, $1 + i = \frac{V}{P}$. Solving for P yields equation (2).