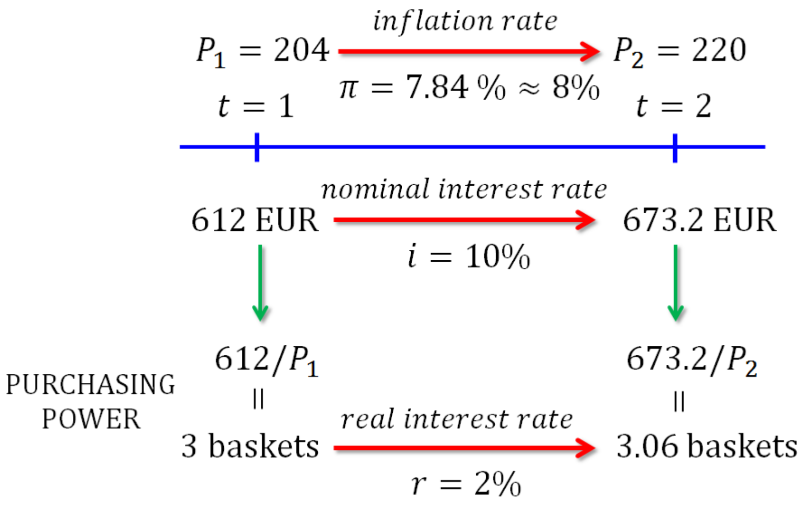
6. Real interest rate, Fisher equation, arbitrage and speculation

1. Real interest rate

The real interest rate r of an economy represents the purchasing power of the economy's nominal interest rate i: it is the nominal rate i expressed in terms of goods. That the nominal interest rate between period t and period $t+1$ is i means that, by lending 1 currency unit in t, one gets $1+i$ currency units in $t+1$. That the real interest rate between period t and period $t+1$ is r means that, by lending 1 unit of goods int t, one gets $1+r$ units of goods in $t+1$. Thus, r expresses purchasing power: the amount of goods obtained from each unit of good lent.

2. An example on the real interest rate

Let "goods" be represented by the CPI basket. There are two periods, $t = 1$ and $t = 2$. The nominal interest rate between $t = 1$ and $t = 2$ is $i = 10\%$. The cost of the CPI basket in $t = 1$ is $P_1 = 204$ EUR. The cost of the CPI basket in $t = 2$ is $P_2 = 220$ EUR; see the sketch on the right. With this information, the CPI inflation rate is



$$\pi = \frac{P_2 - P_1}{P_1} = \frac{220 - 204}{204} = 7.84\%.$$

If 612 EUR are lent in $t = 1$. Then $612 \cdot (1+i) = 612 \cdot (1 + 0.10) = 673.2$ EUR are obtained in $t = 2$. In $t = 1$, the purchasing power of 612 EUR is $612/P_1 = 612/204 = 3$ baskets. In $t = 2$, the purchasing power of 673.2 EUR is $673.2/P_2 = 673.2/220 = 3.06$ baskets. The real interest rate r measures the change in purchasing power of the money lent. Specifically, r transforms 3 baskets into 3.06 baskets. Thus, r satisfies $3 \cdot (1+r) = 3.06$, so $r = 0.02$ (2%).

3. The Fisher equation

The Fisher equation (1), which is an approximation of the relationship between i and r, holds that the real interest rate is the nominal interest rate minus the inflation rate. Equation (1) is usually taken as the definition of the real interest rate r of an economy.

$$\boxed{r = i - \pi} \qquad (1)$$

In example §2, $i = 10\%$ and $\pi = 7.84\%$ (as P jumps from 204 to 220). According to the Fisher equation, $r = i - \pi \approx 10 - 7.84 = 2.16\%$, which is close to the correct value of 2%.

4. Components of the nominal interest rate

Irving Fisher argued in 1907 that the nominal interest rate eventually, in the long run, captures the inflation rate. In this view, $i = r + \pi$: a lender expecting to earn a real interest rate r and expecting an inflation rate π will at least charge a nominal interest rate $i = r + \pi$. In some circumstances, the lender may also add a risk premium ρ to compensate the lender for taking an excessive default risk by lending a not fully creditworthy borrower. This suggests that nominal interest rates could be decomposed into at least three components: $i = r + \pi + \rho$.

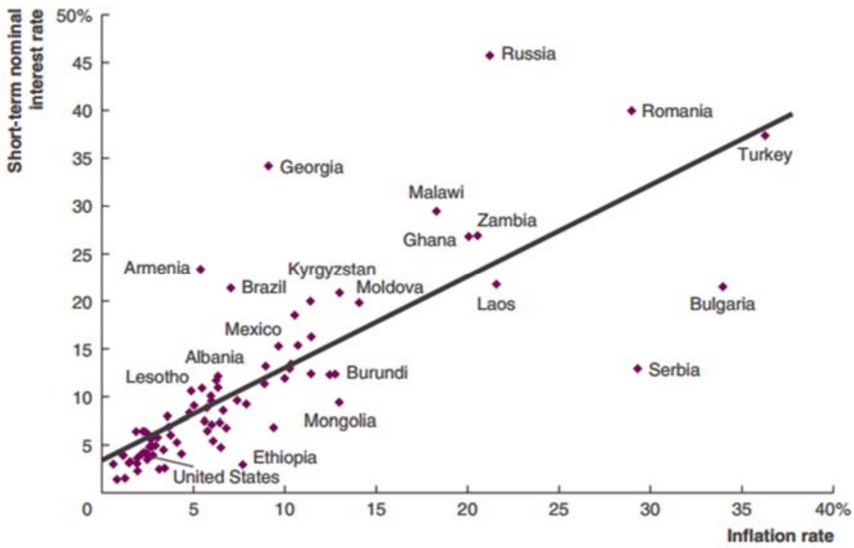
5. Negative interest rates

Negative real interest rates may arise in practice: it suffices to have $\pi > i$. In example §2, if the price level went up to, say, 269.28 instead of 220, then 673.2 EUR could only buy 2.5 baskets. Hence, after the loan is repaid one can purchase fewer baskets than the initial 3 baskets. In this case, $r = i - \pi = 10\% - 32\% = -22\%$ (from 3 to 2.5 baskets the actual loss is 16.6%). Although negative nominal interest rate might at first appear impossible, investors may be willing to accept a negative i to shelter their money. In March 2017, Spanish 12 month T-bills had a negative rate of return: $-0{,}302\%$. A negative nominal interest rate might also be observed if deflation is expected.

6. An example with negative interest rate

Let the nominal interest rate be 1% and the inflation rate 0.25%. The real interest rate (using the Fisher equation) is 0.75%. Imagine that the inflation rate is expected to be –1%. In this case, a negative nominal interest rate equal to –0.25% is still capable of ensuring a real interest rate of 0.75%. The lesson of this example is that the relevant variable for lenders is the real not the nominal interest rate: the nominal interest rate is instrumental, not an end it itself.

7. The Fisher effect



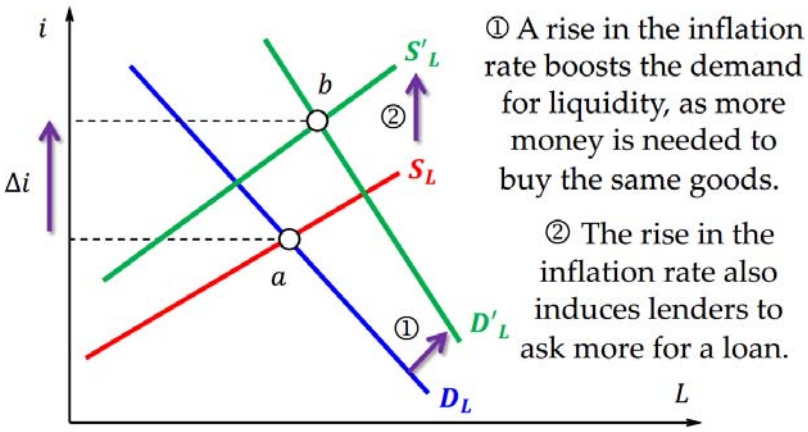
The Fisher hypothesis states that the real interest rate is approximately constant. The Fisher effect (an implication of the Fisher hypothesis) asserts that there is a one-to-one relationship between i and π: every additional point of the inflation rate becomes an additional point of the nominal interest rate. The chart on the right (RG Hubbard et al., 2012, *Macroeconomics*, p. 204) shows empirical evidence in support of the Fisher effect: economies with high inflation rates tend to have high nominal interest rates.

8. Justification of the Fisher effect

Equation $i = r + \pi$ explains why lenders would ask for a higher nominal interest rate to get back the purchasing power lost due to a price increase. For example, if $P_0 = 100$, $P_1 = 110$ and $P_2 = 132$, it follows that $\pi_1 = 10\%$ and $\pi_2 = 20\%$. Let $r_1 = 5\%$: from period $t = 0$ to $t = 1$ lenders get a 5% increase in purchasing power. This means that lending in $t = 0$ money equivalent to 1 basket, the equivalent to 1,05 baskets is received in $t = 1$. That is, if 100 EUR are lent in $t = 0$, 115.5 EUR will be received in $t = 1$. By (1), the i_1 ensuring that $r_1 = 5\%$ when $\pi_1 = 10\%$ is $i_1 = r_1 + \pi_1 = 15\%$. Assuming the Fisher hypothesis, $r_2 = r_1 = 5\%$. If i_2 remained at 15%, by lending 110 EUR (the basket value in $t = 1$), $110 \cdot (1 + i_2) = 110 \cdot 1 + 0{,}15 = 126.5$ EUR would be received in $t = 2$. As $P_2 = 132$, the purchasing power of 126.5 EUR is 0.958 baskets: purchasing power is lost. By (1), the i_2 needed to preserve the purchasing power of a money loan is $i_2 = r_2 + \pi_2 = 5\% + 20\% = 25\%$: from $t = 1$ to $t = 2$, π goes up 10 points and i also goes up 10 points.

9. The Fisher effect in the liquidity market

The figure on the right shows how to obtain the Fisher effect in the liquidity market model. First, with higher prices, borrowers will likely demand more liquidity to maintain their consumption. More liquidity will be demanded if prices are expected to rise in the future (so purchases are made now before prices go up). And second, a rise in the inflation rate encourages lenders to charge a higher nominal interest rate for supplying liquidity.



10. Arbitrage and speculation

Arbitrage refers to market transactions that, taking advantage of price differences, generate a sure profit. Speculation is the same as arbitrage with the only difference that transactions do not guarantee a sure profit. Whereas a speculator is taking a risk, an arbitrageur obtains a risk-free profit. Almost nothing lies outside the scope of arbitration and speculation: commodities, bonds, currencies, shares, options, real estate, derivatives, futures contracts…

11. Interest rate and price of financial assets (T-bills in particular)

The price of a financial asset and the nominal interest rate move in opposite directions. This result is shown next for the case in which the financial asset is a T-bill. The T-bill is issued in period t and matures in $t+1$. The price of the T-bill in t, when issued, is P. The face value of the T-bill is V, which means that, in $t+1$, the T-bill pays V to the owner of the T-bill. Let i be the interest rate between t and $t+1$, so i represents the profit of making a loan with the same maturity as the T-bill. An investor having P monetary units may consider two options.

- Option 1: lend P. When the loan matures, in $t+1$, the investor gets $(1+i)\cdot P$.
- Option 2: buy the T-bill. When the T-bill matures, in $t+1$, the investor gets V.

For both options to be equally attractive, the two outcomes must coincide; that is, $(1+i)\cdot P = V$. Solving for P,

$$\boxed{P = \frac{V}{1+i}}\,. \qquad (2)$$

Since V is a fixed given value, (2) means that the larger i, the smaller P.

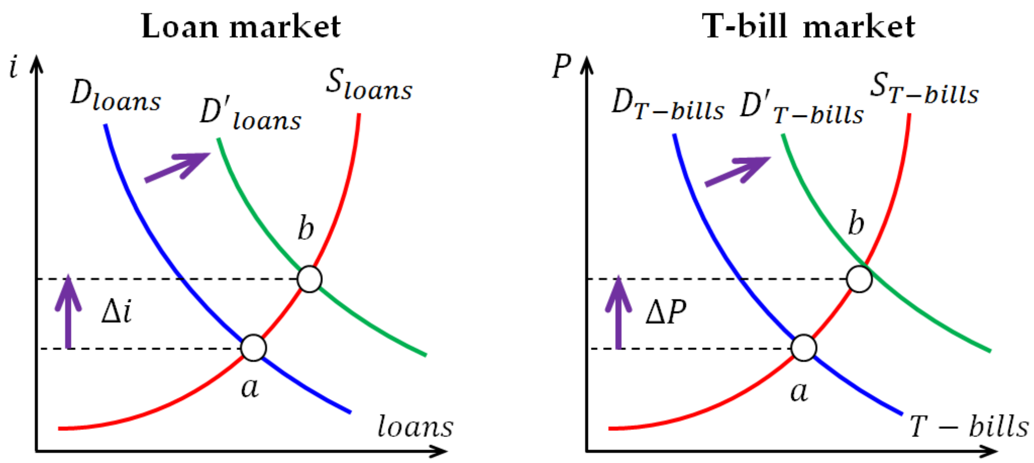
12. Financial arbitrage and inverse relationship between interest rate and price of T-bills

Arbitrage consists of making purchases and sales that ensure a sure profit. Under financial arbitrage, an arbitrageur buys and sells financial assets to obtain a sure profit. It will be next argued that financial arbitrage justifies the inverse relationship between the price of a T-bill and the interest rate established by (2). To this end, suppose there are arbitrageurs looking for opportunities to make a sure profit and that (2) does not hold; that is, $V > (1+i)\cdot P$ or $V < (1+i)\cdot P$. Only the former possibility is handled, the latter being left as an exercise. If $V > (1+i)\cdot P$, then an arbitrageur could obtain sure profits as follows, even having no money.

• **Step 1.** The arbitrageur borrows P monetary units in t and, consequently, has to repay in $t + 1$ $(1 + i) \cdot P$ monetary units.

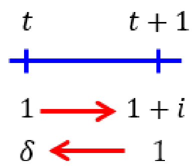
• **Step 2.** The arbitrageur purchases in t a T-bill with the P monetary units.

• **Step 3.** Reached period $t + 1$, the T-bill pays V monetary units and, as $V > (1 + i) \cdot P$, the arbi-trageur repays the loan and pockets a profit of $V - (1 + i) \cdot P > 0$ monetary units (example: if V= 1.000, P= 800 and i = 10%, each T-bill bought with borrowed money yields a profit of 120).



Assuming competitive markets (see graphs on the left), step 1 shifts the demand for loans (for liquidity) function to the right and the interest rate i goes up. The purchase of T-bills executed in step 2 shifts the demand for T-bills function to the right. This increases the price P of T-bills. With both i and P going up, $(1 + i) \cdot P$ also goes up. The result is that $V - (1 + i) \cdot P$ diminishes. Arbitrageurs will borrow money and buy T-bills until the gap between values V and $(1 + i) \cdot P$ is closed; that is, until $V = (1 + i) \cdot P$. In this case, arbitrage opportunities vanish. This reasoning proves that $V > (1 + i) \cdot P$ cannot hold for long in the presence of a financial arbitrage.

13. Discount factor



The discount factor δ between periods t and $t + 1$, when i is the interest rate between t and $t + 1$, is $\delta = \frac{1}{1+i}$. The discount factor (between periods t and $t + 1$) expresses the value in period of t of one monetary unit of period $t + 1$. Whereas the interest rate transforms today's money into tomorrow's money (1 today is $1 + i$ tomorrow), the discount factor does the opposite by transforming tomorrow's money into today's money. The sketch below shows how the discount factor δ determines present values out of future values. The discount factor makes 1 become δ. This δ is the value in period t that, when the interest rate between t and $t + 1$ is i, becomes 1 in period $t + 1$. By the rule of three, $\delta = 1 \cdot 1/(1 + i) = 1/(1 + i)$ is the discount factor, which depends on the interest rate i. This leads to a more precise definition of δ.

14. The price of T-bills as a present value

The concept of present value provides a second justification of equation (2). The value in $t + 1$ (the future value) of the T-bill is V. With interest rate i between t and $t + 1$, the value of V in t (its present discounted value) is $V \cdot \frac{1}{1+i}$, where $\frac{1}{1+i}$ is the discount factor between t and $t + 1$. In view of this, equation (2) states that the price of a T-bill coincides with the present discounted value of its face (future) value.

15. Equalization of rates of return

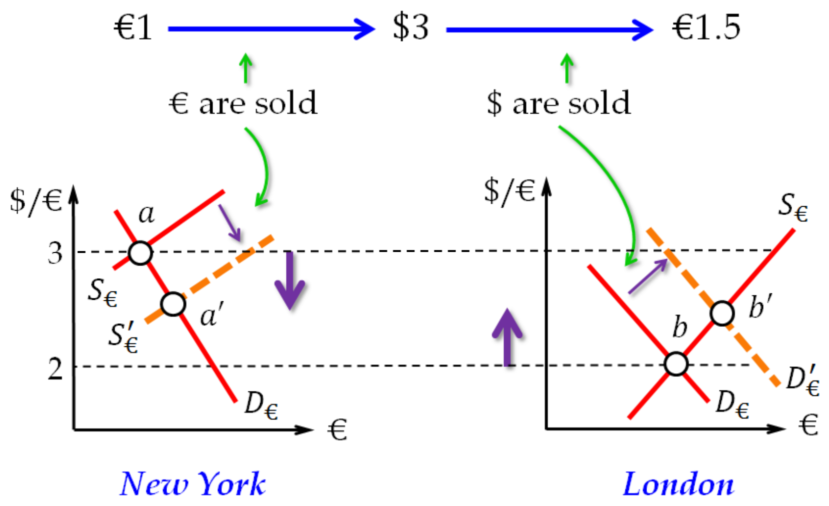
A third justification of (2) comes from presuming the equalization of the interest rates of all financial assets; otherwise, financial assets with a smaller rate of return would have no demand and, hence, would not exist. The interest rate $i_{T\text{-}bill}$ of a T-bill is $i_{T\text{-}bill} = \frac{V-P}{P}$. Letting i represent the interest rate of a loan, the equalization condition $i = i_{T\text{-}bill}$ leads to $i = i_{T\text{-}bill} = \frac{V-P}{P} = \frac{V}{P} - 1$, or, equivalently, $1 + i = \frac{V}{P}$. Solving for P yields equation (2).

16. Spatial arbitrage

Spatial arbitrage <u>exploits price differences in different locations</u>. As an example, suppose $e_L = 2$ \$/€ in London and $e_N = 3$ \$/€ in New York; see the figure next. An arbitrageur would buy euros where they are 'cheap' (in London, where buying €1 just takes \$2) to sell them were they are 'expensive' (in NY, where you need \$3 to get €1). The sequence

$$€1 \rightarrow^{\text{sold in NY}} \$3 \rightarrow^{\text{sold in L}} €1.5$$

generates a sure profit of €0.5 per euro (a 50% profit rate).

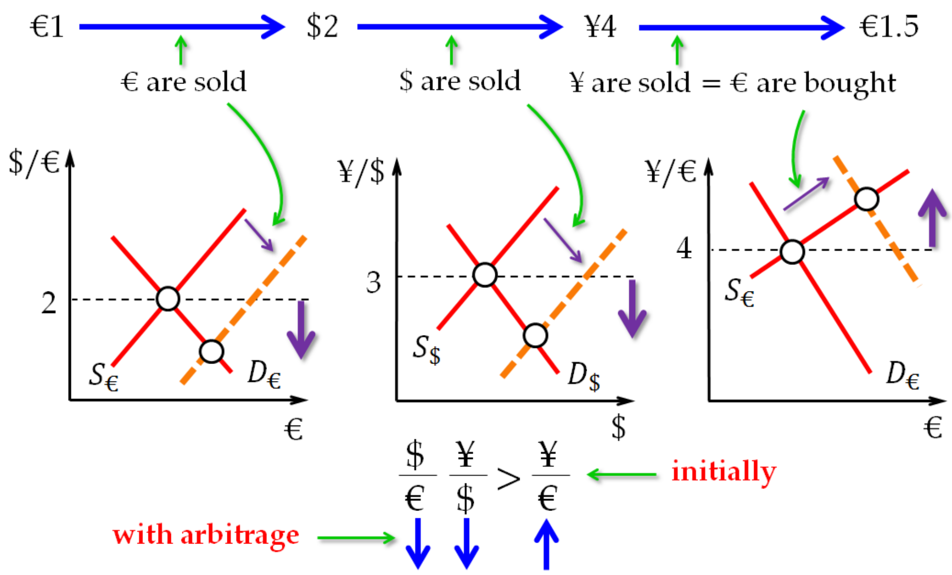


The cycle may be continued: €1 → \$3 → €1.5 → \$4.5 → €2.25 → \$6.75 → €3.375 → … These transactions eventually alter prices.

• By buying euros in London, $D_€$ shifts to the right and ↑e in London: the euro <u>appreciates where it is "cheap"</u> (right-hand of the figure on the left).

• By selling euros in New York, arbitrageurs shift $S_€$ to the right in New York, so ↓e in New York : the euro <u>depreciates where it is 'expensive'</u> (left-hand side of the figure on the left).

Summing up, $e_L = 2$ \$/€ rises and $e_N = 3$ \$/€. Eventually (even in minutes), both prices will converge to some value between $e = 2$ and $e = 3$. Reached that value, spatial arbitrage is no longer be possible and, as a result, the exchange rate in both markets, in New York and in London, will coincide.



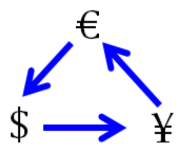
17. Triangular arbitrage

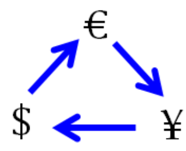
Triangular arbitrage takes advantage of <u>price imbalances</u> involving at least three currencies to obtain a sure profit by buying/selling the three currencies. To illustrate triangular arbitrage, let exchange rates be 2 \$/€, 3 ¥/\$, and 4 ¥/€; see the figure on the left. <u>Triangular arbitrage can only occur if the product of two rates is not equal to the third one</u> (the product of the three rates is meaningful if one currency cancels out). The product 3 ¥/\$ · 4 ¥/€ is not meaningful, as no currency cancels out. By taking the inverse $\frac{1}{3}$ \$/¥ of 3 ¥/\$ a meaningful product obtains: $\frac{1}{3}$ \$/¥ · 4 ¥/€ = $\frac{4}{3}$ \$/€ ≠ 2 \$/€.

This means that there are arbitrage opportunities. There are six exchange sequences:

(1) € → $ → ¥ (2) € → ¥ → $ (3) $ → € → ¥
(4) $ → ¥ → € (5) ¥ → $ → € (6) ¥ → € → $.

But (1) is equivalent to both (3) and (5) because all generate the cycle € → $ → ¥ → €. And (2), (4), and (6) are equivalent because all generate the cycle € → ¥ → $ → €. As a a consequence, there are two ways of trying to exploit price differences, represented by the two exchange cycles shown just above. One the cycles generates profits; the other, losses. The right-hand cycle produces a loss: €1 → ¥4 → $4/3 → €2/3. The left-hand one yields a profit: €1 → $2 → ¥6 → €1.5. As noticed, $\frac{\$}{¥}\cdot\frac{¥}{€} < \frac{\$}{€}$: going directly from $ to € is more profitable than going indirectly through ¥. The step '€1 → $2' makes the dollar appreciate: $/€ falls. The step '$2 → ¥6' makes the yen appreciate: $/¥ rises. The step '¥6 → €1.5' makes the euro appreciate: ¥/€ rises. Thus the gap between going directly or indirectly between any two currencies tends to close.





18. How to become a millionaire in one day

Let e = 2 $/€ today and suppose I expect e' = 1.9 $/€ tomorrow. Imagine that the overnight (daily) interest rate is 3‰. If my expectation is correct, I can become a millionaire tomorrow. This is the recipe. I ask for a loan of, say, €100 million. Tomorrow I will have to return this amount plus €300,000. With my €100 million, and given the exchange rate e = 2 $/€, I purchase $200 million. I could lend those dollars for a day, but since the day has been hard enough I just wait for tomorrow.

Tomorrow comes and I am right. I then sell the $200 million at the rate e' = 1.9 $/€ and get €105,263,157 (the additional cents, left as a tip). I next repay my €100 million debt plus the loan interest of €300,000. And I finally search for a fiscal paradise that would welcome my remaining €4,963,157…

What if I am wrong and, for instance, e' = 2.1 $/€? Then I have a little problem, since, at the rate e', I can only obtain €95,238,095.23 from my $200 million. That means that I incur a big loss.

19. Short-selling

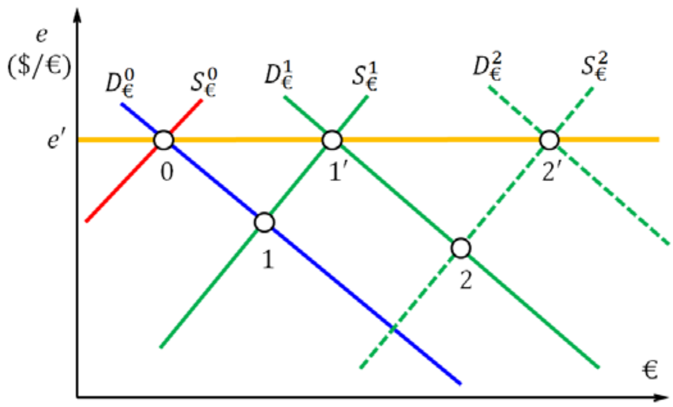
Short-selling consists of (i) borrowing some good or financial asset to (ii) sell it, expecting to make a profit by (iii) buying the good or asset later, when it is time to return it to the lender, at a smaller price (so the short-seller expects to make a profit from a price decline).

Going long is the strategy opposite to short-selling: an asset is bought expecting that its price will rise.

The example in §18 illustrates short selling: I assumed a debt in euros because I expected a depreciation of the euro. Hence, by purchasing dollars, I expected to obtain next more euros for the same dollars, so that the debt could be repaid with cheaper euros.

To limit market volatility, some restrictions to short selling were imposed in September 2008. In fact, short selling is capable of triggering currency crises.

20. Currency crises and speculative attacks



A currency crisis typically arises when a fixed exchange rate cannot be defended, that is, achieved through central bank intervention in the currency market. What if market participants believe that a given exchange rate cannot be defended? They will probably engage in short-selling: expecting the euro to lose value, they will ask for loans in euros, and buy dollars with them. This shifts $S_€$ to the right, so the euro loses value. And here it is a self-fulfilling prophecy: what agents do in response to what they expect to occur contributes to cause what they expect to occur.

The figure above represents the events that hasten a currency crises. The fixed exchange rate is e' and the market is initially at point 0. A speculative attack unfolds through a massive sale of euros, to repurchase them later cheaper, at a smaller rate. This attack shifts $S_€$ from $S^0_€$ to $S^1_€$, moving the market equilibrium from point 0 to point 1. To defend the fixed rate, the central bank reacts by selling dollars, shifting $D_€$ from $D^0_€$ to $D^1_€$. Market equilibrium then moves from 1 to 1′.

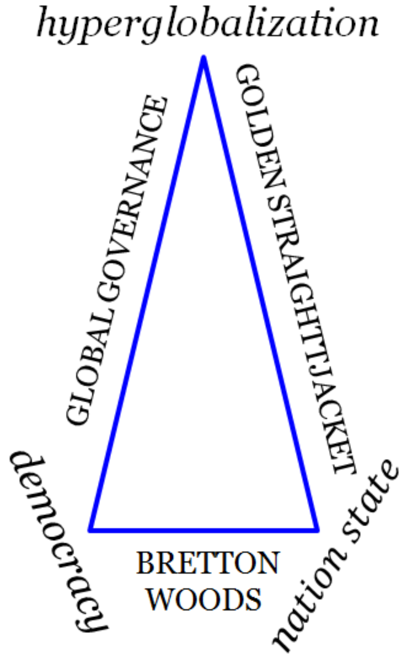
Excursus. A priori, a currency is equally likely to appreciate or depreciate. In this respect, mounting a speculative attack without further information is a 50-50 bet, which does not look promising for a speculator. Moreover, in this case, some speculators may bet that the currency is going to depreciate and others that it is going to appreciate, so the two attacks may cancel each other or, in any event, may make the defense of the fixed rate easier. Therefore, to ignite a speculative attack some reason must point to a specific unidirectional and persistent modification in the exchange rate. There must be some objective feature of the economy creating a tendency for the currency to either appreciate or depreciate. That feature will coordinate all speculators to bet in only one direction: either all believe that an appreciation will occur or all believe that it will be a depreciation. Accordingly, a speculative attack is most likely to be conducted for some objective reason whose effects on the currency the attack exacerbates. It is worth noticing as well that it is easier for the central bank to fight a speculative attack based on the bet that the currency is going to appreciate: in that case, to sustain the fixed exchange rate, the CB needs only to sell what it owns in abundance, namely, the domestic currency. Consequently, the archetypal currency crisis will arise when a speculative attack is launched against the currency because some feature in the economy automatically leads the currency to depreciate (for instance, a domestic inflation rate higher than the inflation rate in the rest of the world).

A second attack shifts $S_€$ from $S^1_€$ to $S^2_€$, reaching point 2. If the central bank still has enough dollar reserves, equilibrium may be moved to 2′. If not, the attack is successful and market equilibrium remains at 2: the attack has led to a sharp decline in the exchange rate. In this case, the government accepts the new exchange and devaluates the currency (reduces the fixed exchange rate).

A famous, successful speculative attack took place on the 16th of September, 1992 (the Black Wednesday), when George Soros became famous for forcing the British government to withdraw from the European Exchange Rate Mechanism (a fixed exchange rate agreement, predecessor of the euro). Soros made a gain of over $1 billion by short selling pound sterlings. Newspapers revealed that the British Treasury spent £27 billion trying to sustain the value of the pound.

21. Rodrik's trilemma

The term '<u>globalization</u>' refers to the process and consequences of the <u>opening up of domestic markets</u> (from both the real and the financial sector) <u>to the international markets</u>.



Rodrik's fundamental political trilemma of the world economy expresses the tension between national democracy and global markets; see the right figure. "<u>We cannot have hyperglobalization, democracy, and national self-determination all at once</u> […] If we want hyperglobalization and democracy, we need to give up on the nation state. If we must keep the nation state and want hyperglobalization too, then we must forget about democracy. And if we want to combine democracy with the nation state, then it is bye-bye deep globalization." Dani Rodrik (2011): *The globalization paradox: democracy and the future of the world economy*.

There are three options to handle the tension between national democracy and global markets that the trilemma expresses. "We can <u>restrict democracy</u> in the interest of minimizing international transaction costs, disregarding the economic and social whiplash that the global economy occasionally produces. We can <u>limit globalization</u>, in the hope of building democratic legitimacy at home. Or we can <u>globalize democracy,</u> at the cost of national sovereignty." D. Rodrik (2011): *The globalization paradox*.

• **Option 1. The Golden Straightjacket.** Hyperglobalization means that national borders do not interfere at all with the circulation of goods, services, and capital. If a nation state becomes hyperglobalized, then domestic regulations and taxes must be consistent with the requirements of hyperglobalization and, in particular, with <u>ensuring that the domestic economy remains attractive to, and earns the confidence of, international investors and traders</u>. Therefore, domestic policy must be subordinated to comply with the conditions of economic globalization by adopting such policies as:

- a strict monetary policy ('tight money');
- "flexible" labour markets;
- deregulation, privatization, and minimize public intervention ('small government');
- keep taxes (particularly, capital and corporate taxes) low;
- maintain the economy sufficiently open to the rest of the world ('open borders').

When this set of policies is adopted it is said that the nation state wears The Golden Straightjacket on. The government putting on this jacket is free from domestic (economic or social) obligations or constraints. The requirements of the global economy dictate the domestic policy. Signs of wearing the jacket:

- economic policy-making institutions (central banks, market regulators) turn 'independent' from democratic control;
- social insurance is reduced (privatized);
- corporate taxes and the top income taxes lowered; and
- policy goals are subordinated to keep market confidence.

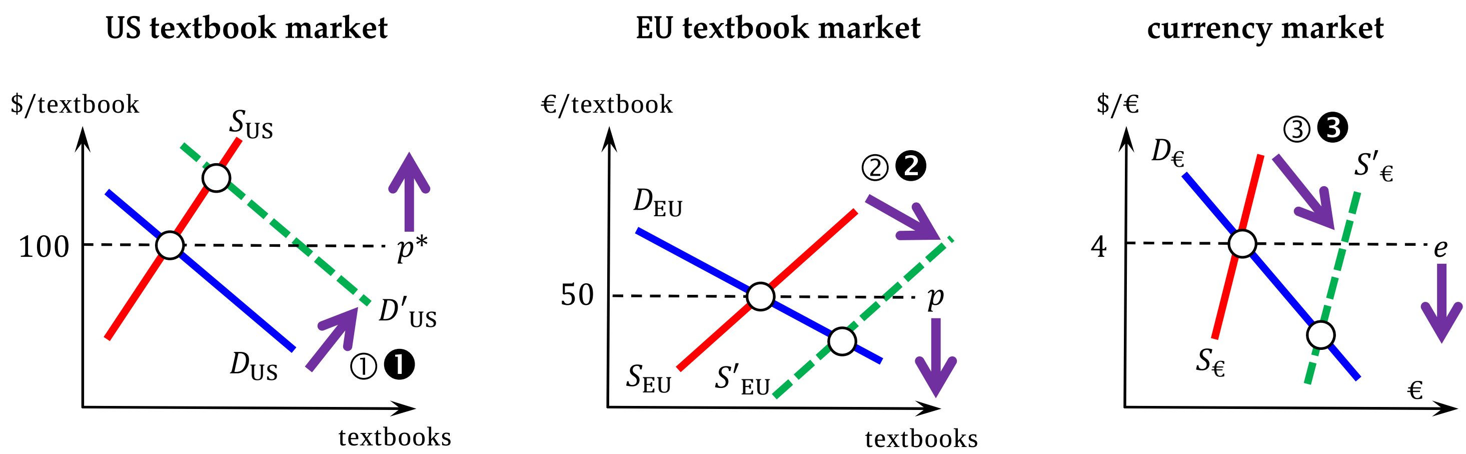
• **Option 2. Bretton Woods compromise** ('thin' version of globalization). This compromise merely implies a reduced international discipline: <u>each nation state enjoys sufficient freedom to pursue domestic goals</u>, like development levels, as long as restrictions on capital flows are implemented. Since nation states can follow their own paths of development, domestic differences can be maintained and enlarged.

• **Option 3. Global governance**. The global governance option involves removing the nation state in order to have democratic policies and hyperglobalization. This option amounts to relocating politics to the global level, in the sense that <u>rule making becomes supranational</u> (the European Union is a regional example). The difficulties with option 3 emerge from the possibility that there is too much diversity among nation states to make global federalism a practical option.

Rodrik (*One economics, many recipes: globalization, institutions, and economic growth*, 2007, p. 43) claims that "Sustaining growth is more difficult than igniting it". A generalization of this observation is that <u>it is harder for an economy to remain in a (non-spontaneous) state than to achieve it</u>. Globalization seems to illustrate this generalization: <u>more effort is necessary for an economy to remain globalized than to become globalized</u>. Argentina in the mid-1990s become hyperglobalized very quickly, but the cost of maintaining that state turned out to be unsustainable and led to the catastrophic crisis of 2001.

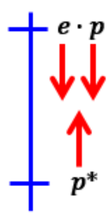
22. Purchasing power parity and commercial arbitrage

<u>Commercial arbitrage</u> consists of buying goods where they are cheap and selling them where they are expensive. In the absence of transportation costs, PPP can be justified by commercial arbitrage. To illustrate the justification, suppose that only one good can be traded between Euroland and the US: Macroeconomic textbooks. The price of a textbook in the US is $p^* = \$100$; in Euroland, $p = €50$. Letting $e = 4$ \$/€, the price in dollars of a Euroland textbook is 4 \$/€ $\cdot$ €50 = \$200. Consequently, textbooks are cheap in the US. Commercial arbitrageurs would proceed as shown below (it is assumed that textbooks can be sent from one economy to the other at no transport cost).



①=Americans buy textbooks in US ②= sell them in EU ③= and convert € in to \$

❶=Europeans convert € into \$ ❷= buy textbooks in US ❸= and sell them in EU



• If the arbitrageur is an American, then he or she will buy textbooks in the US to subsequently ship them to Euroland; once sold there, euros are converted into dollars.

- If the arbitrageur is a European, then he or she will first convert euros are converted into dollars, buy textbooks in the US to finally ship them to Euroland and sell them there.

The purchase of books in the US tends to rise p^*. The sale of those books in Euroland make p fall. More dollars demanded lower e. Initially, $4{\cdot}50 = e \cdot p > p^* = 100$. By arbitrage, $e \cdot p$ tends to fall and p^* tends to rise. Eventually, $e \cdot p = p^*$. This condition stops arbitrage and makes e reach the PPP value p^*/p .

23. Relative purchasing power parity

Relative purchasing power parity, the dynamic version of the (absolute) purchasing power parity, holds that the exchange rate moves to neutralize inflation differentials. Concretely, define the rate of appreciation of the exchange rate between two currencies as $\hat{e} = \frac{e - e_{-1}}{e_{-1}}$, where e is the current value of the exchange rate and e_{-1} is its value in the immediately preceding period. Let π denote the domestic inflation rate, and π^* the foreign inflation rate, between these two periods. The exact version of the parity is given by (3), whereas its common formulation, an approximation of (3), is given by (4).

$$1 + \hat{e} = \frac{1 + \pi^*}{1 + \pi} \qquad (3)$$

$$\hat{e} \approx \pi^* - \pi \qquad (4)$$

If the euro is the domestic currency, the dollar is the foreign currency, and the units of e are \$/€, then (3) asserts that the rate of appreciation of the euro is approximately equal to the difference between the US inflation rate and the European inflation rate.

An an example, let $\pi^* = 5\%$ and $\pi = 25\%$. Then, by (3), $\hat{e} \approx 5 - 25 = -20\%$: the euro must depreciate by 20% to compensate for the fact that European prices grow 20 points faster than American prices.

Absolute PPP implies relative PPP but not vice versa. In fact, if absolute PPP holds, then (3) can be obtained as follows.

$$1 + \hat{e} = 1 + \frac{e - e_{-1}}{e_{-1}} = 1 + \frac{e}{e_{-1}} - 1 = \frac{e}{e_{-1}} \overset{\text{PPP}}{=} \frac{\frac{P^*}{P^*_{-1}}}{\frac{P}{P_{-1}}} = \frac{\frac{P^*}{P}}{\frac{P^*_{-1}}{P_{-1}}} = \frac{1 + \frac{P^*}{P} - 1}{1 + \frac{P^*_{-1}}{P_{-1}} - 1} = \frac{1 + \pi^*}{1 + \pi}$$

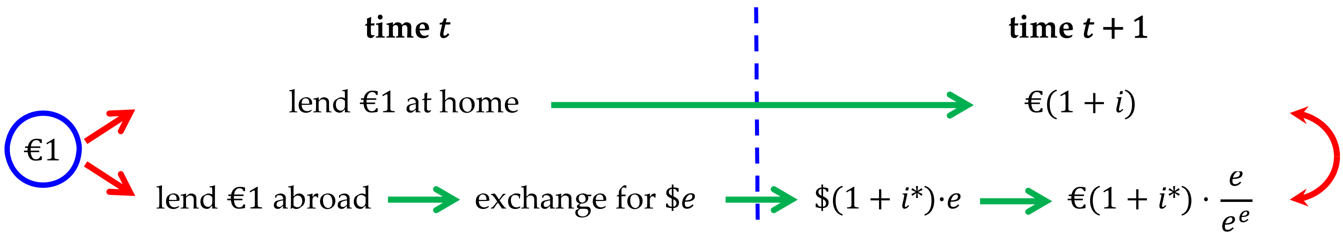
24. Uncovered interest rate parity

The uncovered interest rate parity establishes a relationship between the domestic interest rate i, the foreign interest rate i^*, and the expected rate of appreciation $\hat{e}^e = \frac{e^e - e}{e}$ of the domestic currency with respect to the foreign currency. The exact version of the uncovered interest rate parity is given by (5), whereas its usual formulation is given by (6) (taxes are expressed in per one terms).

$$\hat{e}^e = \frac{i^* - i}{1 + i} \qquad (5)$$

$$\hat{e}^e \approx i^* - i \qquad (6)$$

Equation (5) can be justified by the equality of returns from investing domestically and investing abroad. Specifically, suppose an investor has €1 to lend in period t and the loan is repaid in $t + 1$. The domestic interest rate between t and $t + 1$ is i. The foreign interest rate between t and $t + 1$ is i^*. The exchange rate in t is e \$/€. The investor faces at least the following two options; see the figure below.



• Option 1. <u>To lend the euro at home in t</u>. In this case, in $t + 1$, the investor receives €$(1 + i)$.

• Option 2. <u>To lend the euro abroad in t</u>. This option involves exchanging the euro for e dollars (since the exchange rate is e \$/€) in order to next lend the e dollars abroad. As a result, in $t + 1$, the investor receives \$$(1 + i^*)\cdot e$. If, in t, the investor expects the exchange rate in $t + 1$ to be e^e, then the investor expects to obtain €$(1 + i^*)\cdot e/e^e$.

The presumption that both options produce the same result implies that $1 + i = (1 + i^*)\cdot\frac{e}{e^e}$ or, equivalently, $\frac{e}{e^e}\cdot(1 + i) = 1 + i^*$, which is equivalent to $\left(1 + \frac{e}{e^e} - 1\right)\cdot(1 + i) = 1 + i^*$. That is, $(1 + \hat{e}^e)\cdot(1 + i) = 1 + i^*$. Therefore, $1 + \hat{e}^e + i + \hat{e}^e\cdot i = 1 + i^*$. Solving for $\hat{e}^e$ leads to (5). The approximation (6) of (5) follows if i is small enough. For sufficiently small i, $1 + i \approx i$ and consequently (6) approximates (5).

An interpretation of (6) is as follows. Suppose the foreign interest rate i^* abroad is larger than the domestic interest rate i. This means that $i^* - i > 0$. The interest parity condition (6) contends that, in this case, an appreciation of the domestic currency should be expected: $\hat{e}^e > 0$ (which means that $e^e > e$). This appreciation is required to compensate for the fact that investing abroad is, in terms of interest rates, more profitable. In other words, <u>the higher return obtained by investing abroad is reduced by a loss when converting the foreign currency back into the domestic currency</u>, so that the net result is the same as if investment had been at home.

The term 'uncovered' refers to the fact that e^e is an expectation, not an actual value: the option of investing abroad is not covered against the risk of predicting the exchange rate wrongly.

If the domestic interest is higher than the foreign interest, $i^* - i < 0$, then (6) implies expecting a depreciation of the domestic currency: $\hat{e}^e < 0$ (that is, $e^e < e$).

To illustrate this parity, suppose $i = 5\%$ and $i^* = 25\%$. Then, by (6) , it must be that $\hat{e}^e \approx i^* - i = 0.25 - 0.05 = 0.2 = 20\%$: the expectation should be that domestic currency will appreciate by 20%. Using the exact version (5) of the interest parity, $\hat{e}^e = \frac{i^* - i}{1+i} = \frac{0.25 - 0.05}{1.05} = \frac{0.20}{1.05} = \frac{20}{105} = \frac{4}{51} \approx 0.238 = 23.8\%$. Since 5% is not a small value for the domestic interest, (5) and (6) differ significantly.

If parities (4) and (6) hold, and if expectations are correct, $\hat{e} \approx \pi^* - \pi$, $\hat{e}^e \approx i^* - i$, and $\hat{e}^e = \hat{e}$. Hence, $\pi^* - \pi \approx i^* - i$: <u>the inflation differential between countries reflects the interest differential</u>.