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RENT-TO-OWN MARKETS ANALYSIS FROM A STRATEGIC PERSPECTIVE

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TITLE, SUMMARY, AND KEY WORDS

ENGLISH

Title:

Rent-to-own markets analysis from a strategic perspective

Summary:

In the last few years, an important shift has been produced in the luxury consumption behavior. The customer has modified its buying patterns and is becoming more open to sustainable consumption alternatives. Therefore, the new shape of the consumers priorities, is prompting the luxury fashion market to change its sales strategy.

By using a stylized model from the literature, this research aims to study the impact of the rent-to-own strategy on the luxury fashion firm's profits maximization, and consequently, its contract preferences. We applied a sequential game to three different rental contracts: wholesale contract, agent contract, and acquisition contract.

Our findings prove two main points. A luxury brand firm will not be interested in letting a rental market arise with the agency contract. The brand will choose to only serve the traditional market. We also prove that giving a second chance to buy the product has a positive effect on both the wholesale and acquisition contract firm's profit. The analysis shows that the decision choice over contracts between selecting one contract that the other, mainly depends on the consumers' valuation level. When the market highly values the item, the firm will be interested in owning a platform or choosing a wholesale contract, and consequently, offer a rental and rental and purchase option. Nevertheless, if the costumer lowly values the item, the firm will choose to not enter to the rental market.

If the company is capable of efficiently own a rental platform, the acquisition contract will show a positive outperforming opportunity for the firm. The luxury brand, instead of losing the segment of consumers that are starting to be environmentally and sustainably conscious, will attract these consumers by offering a rental service.

Key words:

Fashion luxury firm, rent-to-own strategy, profits maximization

CATALAN

Títol:

Anàlisi dels mercats de lloguer amb opció de compra des d'una perspectiva estratègica

Resum:

En els últims anys, s'ha produït un important canvi en el comportament del consum de luxe. El client ha modificat els seus patrons de compra i està més obert a alternatives de consum més sostenible. Per conseqüent, el canvi de les prioritats del consumidor està impulsant al mercat de la moda de luxe a canviar la seva estratègia de venda.

Utilitzant com a referència un model estilitzat de la literatura sobre el tema, aquest projecte pretén estudiar l'impacte d'aplicar una estratègia de lloguer amb opció a compra en el benefici de la firma de luxe i, en conseqüència, en les seves preferències a l'hora de triar el contracte de lloguer. Així doncs, apliquem un joc seqüencial a tres contractes de lloguer: contracte de venda a l'engròs, contracte d'agència i contracte d'adquisició.

Els nostres resultats demostren dos punts principals. Una empresa de marca de luxe no estarà interessada a deixar que sorgeixi un mercat de lloguer amb el contracte d’agència. La marca optarà per atendre únicament el mercat tradicional. També mostrem que donar una segona oportunitat de comprar el producte té un efecte positiu en els beneficis de la companyia de luxe, tant amb el contracte majorista com amb el d'adquisició. La nostra anàlisi reflecteix como l’elecció d’escollir un dels tres contractes depèn principalment del nivell de valoració del consumidor. Quan el consumidor considera el producte com a valuós, l’empresa estarà interessada a tenir la seva pròpia plataforma de lloguer per a oferir serveis de lloguer i de lloguer més compra. No obstant això, si el client valora poc el producte, l'empresa optarà per no entrar al mercat de lloguer escollint-ne el contracte d’agència.

Si l'empresa és capaç de posseir de manera eficient una plataforma de lloguer, el contracte d'adquisició mostrarà una oportunitat per a l'empresa. La marca de luxe, en lloc de perdre el segment de consumidors que estan començant a tenir consciència mediambiental i sostenible, atraurà a aquests clients oferint un servei de lloguer y de lloguer amb opció de compra.

Paraules clau:

Companyia de luxe, estratègia de lloguer amb opció de compra, maximització de beneficis

SPANISH

Título:

Análisis de los mercados de alquiler con opción de compra desde una perspectiva estratégica

Resumen:

En los últimos años se ha producido un importante cambio en el comportamiento del consumo de lujo. El cliente ha modificado sus patrones de compra y está más abierto a alternativas de consumo más sostenible. Consecuentemente, el cambio de las prioridades del consumidor está impulsando al mercado de la moda de lujo a cambiar su estrategia de ventas.

Utilizando como referencia un modelo estilizado, este proyecto pretende estudiar el impacto de aplicar una estrategia de alquiler con opción a compra, en el beneficio de la firma de lujo y, en sus preferencias a la hora de escoger el contrato. Para ello, aplicamos un juego secuencial a tres contratos de alquiler: contrato de venta al por mayor, contrato de agencia y contrato de adquisición.

Nuestros resultados demuestran dos puntos principales. Una firma de lujo no estará interesada en dejar que surja un mercado de alquiler con el contrato de agencia. La firma optará por atender únicamente el mercado tradicional. También mostramos que dar una segunda oportunidad de comprar el producto tiene un efecto positivo en los beneficios de la firma, tanto con el contrato mayorista como con el de adquisición. Nuestro análisis refleja cómo la elección de escoger un contrato de alquiler u otro depende principalmente del nivel de valoración del consumidor. Cuando el consumidor valora altamente un producto, la empresa estará interesada en tener su propia plataforma. No obstante, si el cliente no valora suficientemente el producto, la empresa optará por no entrar en el mercado de alquiler, y escogerá el contrato de agencia, ofreciendo únicamente un servicio de venta.

Si la compañía de lujo es capaz de mantener de manera eficiente una plataforma de alquiler, el contrato de adquisición puede ser una decisión estratégica para la firma. La marca de lujo, en vez de perder el segmento de consumidores concienciados por el medio ambiente y por la sostenibilidad, atraerá a este segmento de clientes ofreciendo un servicio de alquiler con opción de compra.

Palabras clave:

Empresa de lujo, estrategia de alquiler con opción de compra, maximización de beneficios

TABLE OF CONTENTS

1. PRESENTATION

"Continually challenge and be willing to amend your best loved ideas."

Warren Buffett

Change and challenge, two words that describe my personality. I grew up in an environment that forced me to adapt to different scenarios. These experiences have taught me that challenges are good; they help us to grow, learn and get stronger by overcoming our barriers.

This research has been another challenge that has motivated me to overcome my mental barriers. The growing importance of the sharing economy in today's global world prompted me to wonder: How is the luxury fashion industry responding to the new consumption shape? Will they suffer economic losses in the future?

In order to meet with the luxury rental market, I decided to apply the macroeconomics knowledge acquired during my bachelor's experience. The Stackelberg's model, and concepts such as the oligopoly market and the sequential game with perfect information, enabled me to analyze the market.

To carry out this research, I also needed to stylize a model of the fashion luxury market, which additionally required the concepts learned in the mathematical courses.

From the outset, the research analysis proposed was not going to be easy. It would put my foresight and constancy to the test. But I was sure that the effort would be worth it.

Last but not least, writing this project in a foreign language, such as English, has a special meaning for me. By pushing myself to venture beyond my comfort zone, I was able to improve on both a personal and a professional level.

2. INTRODUCTION

In the last few years, an important shift has been produced in the luxury consumption behavior, especially among millennials, who are increasingly inclined toward sustainability (Mishra et al., 2020). The consumer has become more environmentally conscious and is starting to give certain importance to this aspect when purchasing an item (Deloitte et al., 2021).

Nowadays, companies are taking advantage of this change, by modifying their approach and mindset. They are starting to incorporate digitalization and sustainability into their long-term strategies. Between these approaches, we find the implementation of the sharing economy model. This concept has been introduced into plenty of markets, including the vehicle industry, food, personal items, etc. Specially, the luxury fashion-rental has been one of the fastest growing second-hand markets (Campos Franco et al. (2020)).

According to a report by D’Arpizio et al. (2021), the secondhand luxury market soared to $33 billion in 2021, because of a surging demand and of an important increase in supply.

Initially, luxury brands felt threatened by the rental and secondhand platforms, mainly because of their lower prices. Nevertheless, the fashion luxury sector is well suited to the resale market. It is embedded in the philosophy of the premium quality, that lasts for generations, and that attracts the sustainable conscious consumer. Comparatively, rental can be used as a customer acquisition tool, especially among younger generations.

This research is related to the literature on sharing economy, with a special focus on the sell-or-lease strategy, the agency pricing, and the rent-to-own agreement.

During the last years the peer-to-peer item sharing stream has engaged many researchers to investigate these fields. Belk (2014) conducted a study that showed how we are entering into a new era of the post-ownership economy. In addition, Mohlmann (2015) developed a framework on the determinants of selecting a sharing option. By applying a quantitative analysis for users of B2C and C2C sharing options, he could prove the predominance of self-benefit as the main reason for using collaborative consumption services.

Weber (2014) studied the moral hazard problems in a sharing economy. The author found that in the collaborative housing sector, moral hazard competition arises between the collaborative housing service provider and the hotel.

By using an analytical model, Tian and Jiang (2018) showed that the presence of sharing platforms has a more beneficial impact on retailers than on manufacturers. They also showed that, in the sharing economy, firms should optimally increase their products quality to maximize their profit.

By considering the previous contributions, our research is based on an article by Feng et al. (2020) that analyses the luxury industry. Their paper investigates the jolt of the rental online marketplace on luxury fashion brands. The authors studied the economic consequences of two kind of contracts between firm and platform, in a big market of price-accepting consumers.

In order to pursue the analysis, they stylized a model by considering as starting point a Scenario 1, which is the conventional market for a product. In this case, a monopolistic firm sells a product. Secondly, they introduced another market to the conventional model: the rental market, from which two possible scenarios could be raised. Scenario 2a, which considers the existence of a current rental platform, and additionally, Scenario 2b which does not consider the platform's existence. The authors decided to study the first subcase, which means that they have not analyzed whether the firm might be interested in creating a rental platform ex novo.

The first contract used in their model, is the wholesale contract, in which the firm is the decider of both, the retail price charged to consumers that buy the firm's item, and the wholesale price charged to the rental platform. Afterwards, the platform chooses the rental price only for consumers interested in renting the luxury item.

The second contract option is the agency contract, where the firm decides both the retail price and the platform's rental price, in exchange for a commission fee. As a result of this commitment the platform acts as an agent for the firm.

The entrance of the luxury fashion brand into the rental platform market reflects a positive effect known as expansion, and a negative effect known as cannibalization for the firm. It positively comes from the consumers who rent the product and, in the absence of the platform, would not have bought the product. However, the consumers that rent the product but, in the in the absence of the platform, would have bought the product, have a negative effect for the brand's firm.

By applying a strategic game model, the authors show that under both rental contracts the market expansion effect induced by the rental platform overcomes the cannibalization effect. This gives an answer to Scenario 2a, motivating firms to work with platforms. Additionally, they identify the cases under which one contract is preferable to the other for the firm or for consumers.

On the basis of Feng et al. (2020), this research extends the authors' framework by establishing a third scenario that gives consumers the option to purchase the rented item. In analogy with the authors model, Scenario 3a, depicts a market with an existing rental platform, while Scenario 3b does not consider the existence of a platform. Specifically, the two contracts are redefined to allow consumers who rented the product to finally purchase it by making an extra payment. It is additionally added a third acquisition contract, which enables the firm to absorb an existing rental platform.

By complementing the authors paper, the main contribution of this research is to identify another way out to the lack of incentives for the firm to serve a rental market by entering to a new market: rental and purchase.

The remainder of this paper is structured as follows. Section 3 describes the model setting. Section 4 then presents the results obtained. Subsequently, Section 5 interprets the results obtained. Finally, Section 6 sums up the main findings and proposes possible directions for future research. Proofs of the formal results are collected in an Appendix.

3. THE MODEL

The model we study, which is based on the model by Feng et al. (2020), includes three agents: a designer brand firm producing a certain product, a rental platform that rents the items, and the consumers.

The model assumes that the luxury brand firm and the rental platform maximize profits and consumers utility. In addition, consumers will face four options among which to choose one: not buying nor renting the product, just renting it, renting it and afterwards buying it, and just buying the product.

Table 1 summarizes the used notation in this research.

Table 1 Notation used in the model

Symbol	Meaning
F	Designer brand firm
P	The rental platform
W	Wholesale model contract
A	Agency model contract
AC	Acquisition model contract
δ	Amount of the revenue from the renting contract charged by the platform as a commission fee
v_o	Valuation point making a consumer indifferent between renting the product or not
v_1	Valuation point making a consumer indifferent between renting the product or buying
v_2	Valuation point making a consumer indifferent between only renting or renting and buying the product
p_b^G	The retail price per unit under contract $G = W, A, AC$
p_r^G	The rental price per unit under contract $G = W, A, AC$
p	Purchase price per unit of the rented product collected by the platform
D_b^G	Total demand of the retail market under contract $G = W, A, AC$
D_r^G	Total demand of the rental market under contract $G = W, A, AC$
D_{rb}^G	Total demand of the rental and purchase market under contract $G = W, A, AC$
π_k^G	Total profit for each market agent ($k = F, P$) under contract $G = W, A, AC$

For the sake of simplicity, it is assumed that the firm produces its product at a zero cost. The firm will choose the retail price p_b, amount to be paid by consumers willing to purchase the item at the very beginning. On the other hand, p_r will be the price paid for renting the luxury item, and p will be the fixed payment to be made by consumers that decide to own the rented product.

3.1. CONSUMERS

To determine the consumer's valuation of the luxury item, we consider that the product's value is heterogeneous and that the market size is normalized to 1. Hence, as shown in Figure 1, the set of customer's valuation is given by the closed interval [0,1], where each point represents the product valuation v of a consumer.

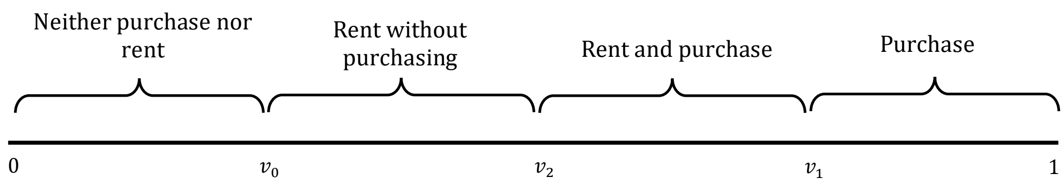

Figure 1 Consumers purchase choice for each valuation point $\boldsymbol{v} \in [0,1]$

By considering the consumer's utility from buying the new item, we obtain $u_b = v - p_b$, where p_b is the price chosen by the luxury fashion brand and the amount of money that the consumer needs to pay to own the item.

When renting the product, a consumer with valuation v will obtain a utility of $u_r = \lambda v - p_r$, where p_r is the rental price of the product and $\lambda < 1$ represents the smaller valuation that consumers attribute to renting the product in comparison with purchasing it.

By considering the consumer's option to purchase or rent the luxury item at level valuation v_1, a consumer will be indifferent between buying the item or renting when

$$v_1 - p_b = \lambda v_1 - p_r,$$

which brings as to define the threshold

$$v_1 = \frac{p_b - p_r}{1 - \lambda}.$$

Correspondingly, the set of consumers with a valuation of at least v_1 will be represented by the demand function

$$D_b = 1 - v_1 = 1 - \frac{p_b - p_r}{1 - \lambda}.$$

Similarly, we define v_0 as the threshold where a consumer is indifferent between renting or not the product

$$v_0 = \frac{p_r}{\lambda}.$$

In view of this, we can define the rental demand D_r for consumers that have a product valuation between the v_0 and v_1 as

$$D_r = v_1 - v_0 = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda} = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)}.$$

The second chance to buy the item to the rental consumers enables as to define their utility function as $u_{rb} = v - p_r - p$, where p is the fixed payment required in order to own the rented item. Consequently, the function in which a consumer with a valuation level v_2 is indifferent between only renting or renting and then buying the item is

$$\lambda v_2 - p_r = v_2 - p_r - p,$$

which brings us to define a threshold of

$$v_2 = \frac{p}{1-\lambda}.$$

Thus, the demand by consumers that will rent and next buy the item D_{rb} with a valuation of at least v_2 and at most v_1 will be as follows:

$$D_{rb} = v_1 - v_2 = \frac{p_b - p_r}{1-\lambda} - \frac{p}{1-\lambda} = \frac{p_b - p_r - p}{1-\lambda}.$$

By considering Figure 1, we impose the restriction on the valuation level λ and the bargain price p by assuming $v_0 < v_2 < v_1 < 1$. Given the fact that $v_2 < v_1$, it is reasonable to assume that $p_r + p < p_b$. That is, renting and next buying the product cannot be more expensive that just buying it. For $p_r + p > p_b$, would give no incentive to first rent and next buy.

3.2. THE FIRM AND THE PLATFORM

The firm has a menu of three contracts. The first is the wholesale contract, in which the firm charges a wholesale price p_W to the rental platform and sets a retail price p_b to their customers. The wholesale price p_W can be interpreted as a discount to the retail price with

$$p_w = \delta p_b.$$

For simplicity, instead of assuming that $\delta < 1$, we will consider an exogenous $p_w = \delta > 0$. As the unit production cost is assumed fixed, a constant δ could be interpreted as the previously agreed policy of selling the firm's luxury products to the platform at a slightly price above the production cost.

The last step in the wholesale contract is taken by the platform, which decides the rental price p_r, and collects the profits obtained from the products sold to the consumers who firstly rented the item.

When it comes to the agency contract, the renting platform acts as a go-between for the company and the customer. The firm sets both the retail price p_b and the rental price p_r. Thus, the firm's profits will be compounded by the earning obtained from selling and renting the product. On the other hand, the platform's profits will be collected from costumers that buy their rented item at a price p.

Finally, in the last contract to be considered, the firm decides to absorb the platform and collects all the profits. If the firm decides to choose this contract, it will pay an amount C to acquire the already existing platform.

The luxury firm will play a vital role in our model's analysis because renting ultimately depends on the firm's arrangement: if the firm does not provide the item at an affordable price for renting, the platform will not make profits. As a result, the company must determine its relationship strategy with the platform. The wholesale contract provides independence or no control, the agency contract provides partial control, and the acquisition contract provides complete control of the rental market.

4. INSTRUMENTAL RESULTS

The model considers the luxury brand firm, as the market leader, and the renting platform, as the follower. Both agents have all the information about their customers demand, which means that the situation is represented by a sequential game with perfect information.

The proofs of the results (Lemmas 1, 2 and 3) can be found in the Appendix.

4.1. WHOLESALE CONTRACT

Figure 2 shows the decision-making process and the timing of the wholesale contract.

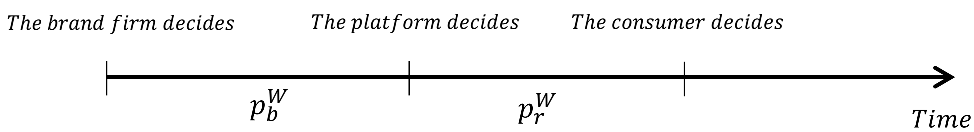

Figure 2 Timing of events of the wholesale contract

Firstly, the firm determines the retail price p_b^W. Secondly, the platform decides the price p_r^W at which it is going to rent the item. Lastly, the consumer compares p_b^W, p_r^W and p (which is exogenously given) and decides according to his/her preferences. In this contract, the platform implicitly shares with the firm the revenues from the rental market and collects the whole revenue from those who buy. So the firm is the only one selling the product without renting it, the platform is the only one renting it, and the platform is the only one selling it to those who rented the item.

We solve the game by backward induction, by using firstly the platform's optimization problem

$$maximize\ \pi_P^W = (p_r^W - \delta)D_r^W + pD_{rb}^W,$$

which enables the firm to take into account how the firm's choice of p_b influences the platform's choice of p_r:

$$p_r^W = \frac{\lambda(p_r^W - p) + \delta}{2}.$$

Secondly, the firm inserts p_r^W as a reaction function into its own maximization problem:

$$maximize\ \pi_F^W = p_b^W D_b^W + \delta D_r^W.$$

Lemma 1 summarizes the solutions for the wholesale contract. It is clearly seen that both p_b^W and p_r^W grow with the price charged by the firm to the platform δ, and fall with p, the extra payment made by consumers that purchase the rented item.

Lemma 1. Under the wholesale contract, prices, demand, and profits are:

Purchase price	$p_b^W = \frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)}$
Rental price	$p_r^W = \frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)}$
Retail demand	$D_b^W = \frac{2(1-\lambda)-\lambda p}{4(1-\lambda)}$
Rental demand	$D_r^W = \frac{p(4-3\lambda)}{4(2-\lambda)(1-\lambda)} + \frac{\lambda-2\delta}{2\lambda(2-\lambda)}$
Rent and buy demand	$D_{rb}^W = \frac{1}{2} + \frac{\lambda p}{4(1-\lambda)} - \frac{p}{1-\lambda}$
Firm's profits	$\pi_F^W = \frac{1}{4} + \frac{2\lambda(1-\lambda)+4(1-\lambda)(1+p)(2\delta-\lambda)+\lambda^2 p^2-\frac{8\delta^2}{\lambda}(1-\lambda)}{8(2-\lambda)(1-\lambda)}$
Platform's profits	$\pi_P^W = p\left(\frac{(2\delta-1)(8\lambda p+8\delta+3\lambda^2+2\lambda)}{16\lambda(2-\lambda)^3} + \frac{2-\lambda-p(2-\lambda)}{4(1-\lambda)}\right) + \frac{(2\delta-\lambda)3\lambda^3 p^2}{32(2-\lambda)^3(1-\lambda)}$

Both D_b^W and D_{rb}^W decrease with an increase of the consumers' valuation λ. Instead, rental demand will react in the opposite way, it increases when the consumers product valuation drives up.

Hence, having the firm's profit's function, we obtain that

$$\frac{\partial \pi_F}{\partial \delta} = \frac{1+p}{(2-\lambda)} + \frac{2\delta}{\lambda(2-\lambda)} > 0$$

and

$$\frac{\partial \pi_F}{\partial p} = \frac{2\delta-\lambda}{2(2-\lambda)} + \frac{\lambda^2 p}{4(1-\lambda)(2-\lambda)} > 0,$$

which means that firm's profits π_F^W will increase with positive variations of p when the condition of $p > \frac{\lambda-2\delta}{8(2-\lambda)^2(1-\lambda)\lambda^2}$ is met.

4.2. AGENCY CONTRACT

Figure 3 next depicts the timing of the agency contract.

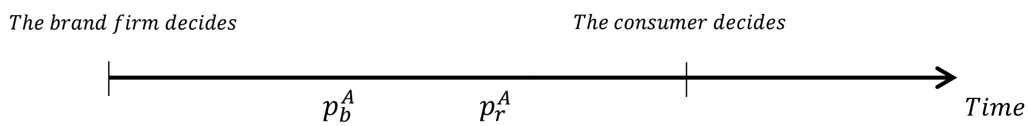

Figure 3 Timing of events of the agency contract

The firm takes its first step by deciding simultaneously the purchase price $p_b^A$and the rental price p_r^A. The consumer compares p_b^A, p_r^A and, p and makes a decision according to its preferences. As well as in the wholesale contract, the platform will passively collect the revenue from renting and buying, whereas the firm chooses p_b^A and p_r^A to

$$maximize\ \pi_{\mathrm{F}} = p_b^A D_b^A + p_r^A D_r^A.$$

Lemma 2 summarizes the results of the agency contract.

Lemma 2. Under the agency contract, prices, demand, and profits are:

Purchase price	$p_b^A = \frac{1}{2}$
Rental price	$p_r^A = \frac{\lambda}{2}$
Retail demand	$D_b^A = \frac{1}{2}$
Rental demand	$D_r^A = 0$
Rent and buy demand	$D_{rb}^A = 0$
Firm's profits	$\pi_F^A = \frac{1}{4}$
Platform's profits	$\pi_P^A = 0$

When considering the explicit condition of $0 < v_0 < v_2 < v_1 < 1$ imposed on p and λ, the prices obtained in Lemma 2 imply $v_1 = v_0 = \frac{1}{2}$. As a result, we will obtain $Dr = 0$, and consequently, $Drb = 0$.

Curiously, the results obtained show a corner solution. The firm reacts unwilling to work with the rental market by showing a $D_r^A = 0$.

4.3. ACQUISITION CONTRACT

When considering the wholesale contract, its decision-making process and its timing is shown in Figure 4.

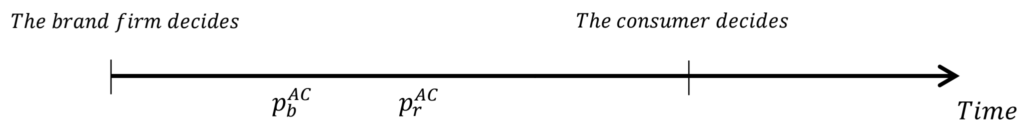

Figure 4 Timing of events of the acquisition contract

In this contract, the firm chooses both p_b^{AC} and p_r^{AC} and additionally collects the profits from renting and selling with an incurred cost of C to

$$maximize\ \pi_{\mathrm{F}} = p_b^{AC} D_b^{AC} + p_r^{AC} D_r^{AC} + p D_{rb}^{AC}.$$

Lemma 3. Under the acquisition contract, prices, demand, and profits are:

Purchase price	$p_b^{AC} = \frac{1+p}{2}$
Rental price	$p_r^{AC} = \frac{\lambda}{2}$
Retail demand	$D_b^{AC} = \frac{1-\lambda-p}{2(1-\lambda)}$
Rental demand	$D_r^{AC} = \frac{p}{2(1-\lambda)}$
Rent and buy demand	$D_{rb}^{AC} = \frac{1-\lambda-p}{2(1-\lambda)}$
Firm's profits	$\pi_F^{AC} = \frac{1}{4} + \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)$

Lemma 3 shows that the higher the price paid p to own the rented product, the greater the retail price p_b^{AC} . Instead, the rental price p_r^{AC} will only depend on the consumer's valuation λ, with which it is positively related.

Both D_b^{AC} and D_{rb}^{AC} will share the same demand: the amount of consumers initially buying the product is the same as the amount of consumers buying it after having rented it. Nevertheless, the rental demand D_r^{AC} is in general different (larger for sufficiently high p: $p > \frac{1-\lambda}{2}$).

5. MAIN RESULTS

In order to further explore the functioning of the three contracts in the luxury fashion market, we aim to compare the contracts' outcomes. Specifically, we focus on the firm's profits in each of the three contract.

Since the model's main functions are strongly correlated with the discounted value of the rental product λ, we evaluate under which conditions (parameter values), the firm chooses one contract rather than the other. The aim of the analysis is not to be exhaustive, but just to identify possibilities to prove our results.

5.1. PROFITS COMPARISON BETWEEN AGENCY AND ACQUISITION CONTRACT

In order to compare both wholesale and acquisition profits, for convenience, we assume $C = 0$.

Proposition 1 reveals that the firm's choice between the agency and the acquisition contract relies on the p value level, which is exogenously given.

If λ measures the loss of value caused by renting instead of purchasing, $1 - \lambda$ would be the extra value obtained when passing from renting to purchasing (the utility gain from renting to buying). On the other hand, p is the cost of moving from renting to buying. As a result, having a p low enough, induces consumers to buy after renting and that gives the firm incentives to create the rental platform or acquire an existing one.

If condition a) is met, then the firm will obtain a greater profit with the acquisition contract. By contrast, if $p > \frac{2}{3}(1 - \lambda)$(condition b), then the firm will choose the agency contract.

Proposition 1. The following results hold for the agency and the acquisition contracts:

a) If p is sufficiently low $\left(p < \frac{2}{3}(1 - \lambda)\right)$, then $\pi_F^{AC} > \pi_F^{A}$.

b) If p is sufficiently high $\left(p > \frac{2}{3}(1 - \lambda)\right)$, then $\pi_F^{AC} < \pi_F^{A}$.

Agency contract results obtained in Lemma 2 showed that $v_0 = v_1 = \frac{1}{2}$. In the aftermath, they reflected a firm's reaction for liquidating the platform by fixing prices that make $D_r^A = 0$, which explicitly makes $D_{rb}^A = 0$. The results reveal that if the firm considers serving both the traditional and the rental market, with an already existing rental platform in the market, it chooses not to serve the rental market. Rather than the firm sets prices to make the rental platform disappear. Or if the firm were considering setting a rental platform (to which the profits of selling the rented product are given), the final decision would be not to create the platform.

Under the acquisition contract, the firm will face a cost of C when absorbing the platform. Additionally, another possible interpretation of the formalization of the acquisition problem could be that the firm considers creating a new platform at a cost C for renting and renting and buying. In this case, it would have to solve the same optimization problem.

As Proposition 1 shows, valuation level λ decreases the p value threshold. An increase in consumers' valuation level of renting the product rather than purchasing it makes the firm more disposable to choose the acquisition contract. In this scenario, the brand will be interested in having its own rental platform. The company will take it as an opportunity to gain a new consumer segment.

By focusing on the sustainability of their items, increasing their contract's length, and improving the products quality will help the firm to gain a new highly valuable consumer. The client that previously would not have buy the luxury product, will now rent it or rent-and-purchase.

By deciding to own a rental platform, can also help the luxury firm to change the firm's philosophy and adapt it to the new costumer's mindset. In addition, it may benefit them to stand-out from their competence, as few companies are following this strategy.

We can affirm that if the condition a) is met, then firm will obtain profits from markets: the retail, the rental and the rental-and-purchase market. Nevertheless, if the condition is not satisfied then, the firm will prefer to choose the agency contract and obtain only profits from a single market, without giving the rental market to emerge.

5.2. PROFITS COMPARISON BETWEEN WHOLESALE AND AGENCY CONTRACT

Proposition 2 identifies the conditions under which the firm will opt for a wholesale contract over an agency contract, and vice versa.

Proposition 2. There is a $\lambda^* < 1$ such that:

a) for $\lambda > \lambda^*$, $\pi_F^W > \pi_F^A$

and

b) for $\lambda < \lambda^*$, $\pi_F^W < \pi_F^A$

Similarly, to the previous contracts' comparison, an increase in the valuation level λ will make the firm to be more attracted to choosing the agency contract. Specifically, when $\lambda < \lambda^*$, the brand will prefer selling only to the retail market by choosing the agency contract. On the other hand, if the valuation level is high enough to meet $\lambda > \lambda^*$, then the firm will choose the wholesale contract, and its products will be offered in retail, rental, and rental-and-purchase markets.

By Lemmas 1 and 2, $\pi_F^W > \pi_F^A$ when $\lambda > \lambda^*$; in that case $2\lambda + \frac{\lambda^2 p^2}{1-\lambda} > 4(1+p)(\lambda - 2\delta) + \frac{8\delta^2}{\lambda}$ and $\pi_F^W < \pi_F^A$ when $\lambda < \lambda^*$; in that case $2\lambda + \frac{\lambda^2 p^2}{1-\lambda} < 4(1+p)(\lambda - 2\delta) + \frac{8\delta^2}{\lambda}$. The previous conditions lead to comparing terms m and n to ascertain which profits are higher.

The λ^* restriction comes from an inequality plotted in Figure 5. The lineal representation represents the two sides of the inequality as functions of λ and that the comparison between the curves in Figure 5, establishes which of the two wholesale and agency contracts is better. It is worth to remark that we have resorted to this geometrical technique because the algebra to calculate $\lambda^*$was complex. Additionally, the used technique used enables us to show that any of the two contracts could be the best option.

The term of $m = 2\lambda + \frac{\lambda^2 p^2}{1-\lambda}$ (red curve in Figure 5) grows with λ. The term of $n = 4(1+p)(\lambda - 2\delta) + \frac{8\delta^2}{\lambda}$ (yellow curve in Figure 5) shows a large value with λ close to zero and decreases with λ for $\lambda < \sqrt{\frac{2\delta^2}{1+p}}$. The plot is correct as long as $\lambda^* < \sqrt{\frac{2\delta^2}{1+p}}$. This would be the case with $\frac{2\delta^2}{1+p} > 1$ with the condition that p is sufficiently small ($p < 1 - 2\delta^2$). When this occurs, the minimum of the yellow curve is higher than 1. On the other hand, if considering $\lambda^* > \sqrt{\frac{2\delta^2}{1+p}}$, then $\frac{2\delta^2}{1+p} < 1$ with the condition that δ is sufficiently small $\delta = \sqrt{\frac{1+p}{2}}$ and the yellow curve will have its minimum value in $\frac{2\delta^2}{1+p}$. In any case, a crossing seems certain for $\lambda < 1$.

Once again, this contract's comparison shows us that when the consumers' valuation level of the item is low, the firm will not be interested on entering to the rental market. Therefore, the consumer will not be disposable to pay a rental amount in order to temporally own the item, because it does not worth for him/her.

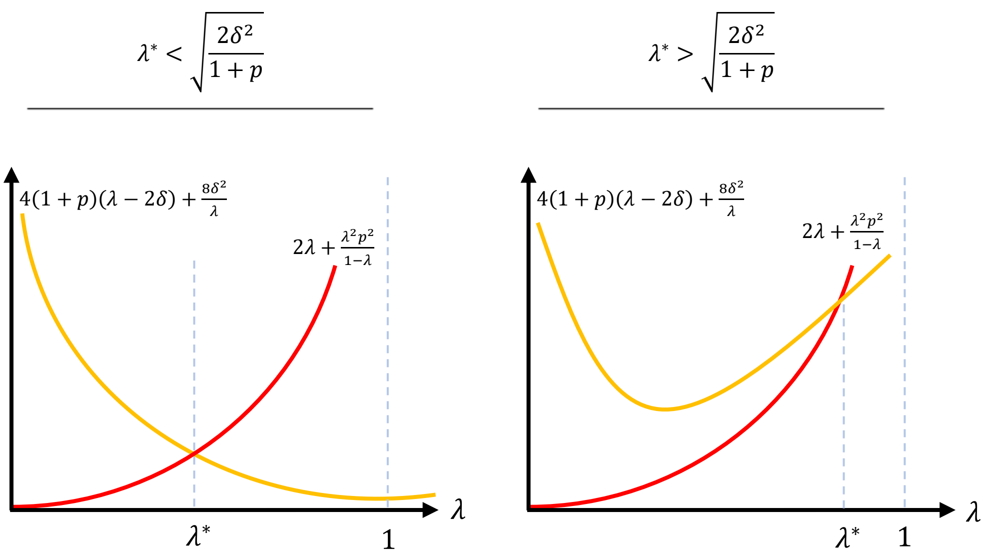

Figure 5 Representation of the conditions determining which contract (Wholesale or Agency) is better for the firm

5.3. PROFITS COMPARISON BETWEEN WHOLESALE AND ACQUISITION CONTRACT

In order to compare both wholesale and acquisition profits, for convenience, we assume $C = 0$.

The conditions that make the firm prefer one contract rather than the other can be identified in Proposition 3.

Proposition 3. If $\delta + \frac{3}{2}p > 1$, then there $\lambda^* < 1$ such that:

$a)$ for $\lambda < \lambda^*$, $\pi_F^W < \pi_F^{AC}$

and

$b)$ for some sufficiently small $\lambda > \lambda^*$, $\pi_F^W > \pi_F^{AC}$

By Lemmas 1 and 3, $\pi_F^W > \pi_F^{AC}$ when $\delta + \frac{3}{2}p > 1$; in that case $4(\delta + p) < 4\delta(1+p) - 2p(2 - 3p)$ and $\pi_F^W < \pi_F^{AC}$ when $\delta + \frac{3}{2}p < 1$; in that case $4(\delta + p) > 4\delta(1+p) - 2p(2 - 3p)$. The previous conditions lead to comparing terms m and n to ascertain which profits are higher.

The restrictions come from an inequality plotted in Figure 6. The representation shows the two sides of the inequality as functions of λ. The comparison between the curves establishes which of the two wholesale and acquisition contracts is better. In order to do so, we define a as a critical point, that enables to determine which contract is preferable to the other. The condition $\delta + \frac{3}{2}p > 1$ comes from requiring $4(\delta + p) > 4\delta(1 + p) - 2p(2 - 3p)$, to ensure that the red curve crosses the yellow one (at some point like a) before reaching its minimum. To the left of point a, due to that $\pi_F^W < \pi_F^{AC}$ firm will prefer choosing the acquisition contract (red curve). Nevertheless, to the right of point a, the wholesale contract will be better than the acquisition contract.

Figure 6 plots the two sides of the inequality as a function of λ. Both $l = 4\delta(1 + p) + \frac{\lambda^2 p^2 + 12p^2 - 6\lambda p^2}{2(1-\lambda)}$ and $g = 4p + \lambda + \frac{4\delta^2}{\lambda}$ terms will grow with λ.

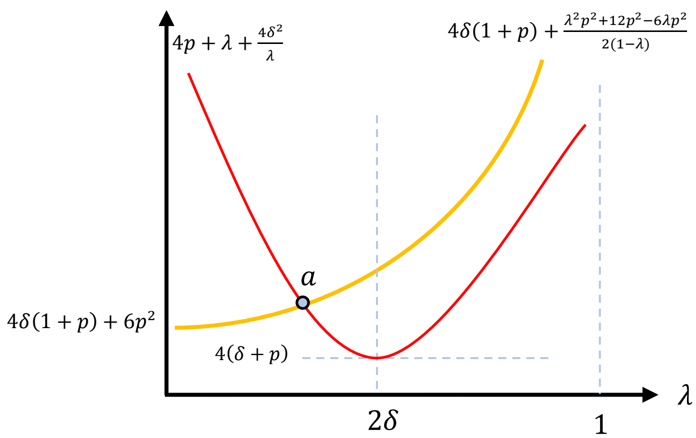

Figure 6 Representation of the conditions determining which contract (Wholesale or Acquisition) is better for the firm

In the scenario that $4(\delta + p) > 4\delta(1 + p) - 2p(2 - 3p)$, the acquisition contract will be better that the wholesale contract for small values of valuation level λ. Consequently, we can affirm that for small values of valuation, the firm will obtain a greater profit with the agency contract. If the scenario is on the other way around, firm will choose the wholesale contract.

It is also necessary to discard a double crossing shown in Figure 7. If this occurs, Proposition 3 is not entirely correct, it will mean that acquisition contract will be better for small and for large values of λ, with wholesale contract better for intermediate values. In order to discard this scenario, we show as for $\lambda > 2\delta$, the slope of the yellow curve is larger than the slope of the red curve. This is likely, because when positive the slope of the red curve is smaller than one and that of the yellow one $\frac{p^2\left(3+\lambda-\frac{\lambda^2}{2}\right)}{(1-\lambda)^2}$, which seems that this slope is larger than 1 for sufficiently large p.

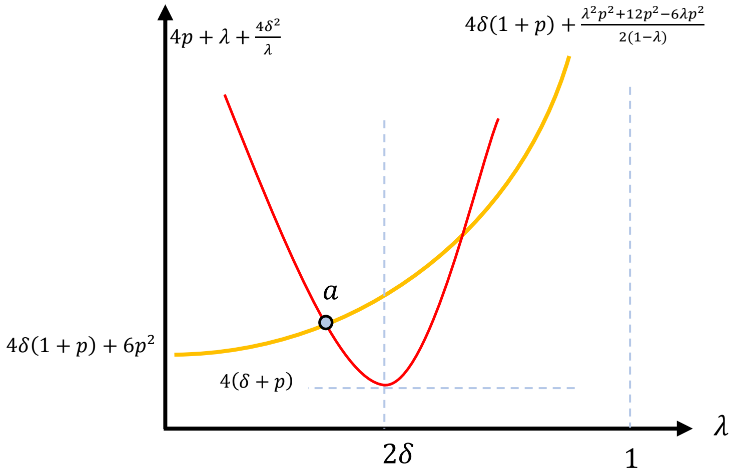

FIGURE 7 Representation of the conditions determining which contract (wholesale or acquisition) is better for the firm when observing a possible double crossing

Finally, Proposition 3 enables us to affirm as a true statement that the acquisition contract is better than the wholesale contract for $\lambda < \lambda^*$, and that the wholesale contract is better than the acquisition contract for some sufficiently small $\lambda > \lambda^*$.

6. CONCLUSIONS

The fast luxury rental market growth has gained an important role in the sharing-economy. As a result, the relevance of analyzing the performance of the firm in this relatively new market, is being significantly rewarding and useful.

When the help of Feng et al. (2020) model, we have analyzed the impact of giving a second chance to purchase a luxury product to the consumer.

To this end, we applied a strategic model game for three agents: the firm, the platform and the consumer.

Additionally, we added a new contract option to the market, making it necessary for the company to choose between using a wholesale, agency or acquisition contract when deciding whether to serve a potential rental market.

Analyzing the firm's profits changes with respect different market variables, and comparing them with the authors findings, enabled us to prove two main points.

Firstly, we show that when the firm could cooperate with a rental platform, the firm will not be interested in letting a rental market arise. If the firm considers serving both the traditional and the rental market, it chooses not to serve the rental one. In fact, the firm sets prices for buying and renting making the consumer not willing to rent, which explains why the firm obtains smaller profits with the agency contract.

Secondly, we prove that giving a second chance for buying the product, reflects a positive effect to both wholesale and acquisition firm's profits. When considering the existence of a rental platform, the firm can be interested in allowing the presence of a rental market if, simultaneously, a third market of rental and purchase arises. If the company is capable to efficiently own a rental platform, the acquisition contract will provide a positive outperforming opportunity for the firm. The luxury brand, instead of losing the segment of consumers that are starting to be environmentally and sustainable conscious, will attract this segment by offering a rental service.

By giving a second life to their products, they will not only impulse the circular economy but also, enhance the society to act and promote the sustainable consumption model. And consequently, the firm alone, will multiply markets, from one to three.

Since the consumer knows that he can own the rental item by doing an extra payment, he will be more disposable to rent the luxury item, and after deciding if owning the product or not.

To sum up, our findings show that the firm will have the capacity of increasing its profits with both wholesale and acquisition contract profits, when improving its valuation level. If the firm is capable to achieve a high valuation, he will offer the rent-to-own option.

Finally, our study only considers that in both rental and wholesale contracts, the firm also collects the rental profits. Nevertheless, future research could analyze how a proportional collection of profits when offering the rent-to-own option, would affect to firm's profits, like the Feng et al. (2020) model. Additionally, studying in detail the platform's market, to know the real cost C that would face the firm when choosing the acquisition contact, could help to understand the firm's real outperforming capacity when using this arrangement.

7. APPENDIX: PROOFS OF THE RESULTS

PROOF OF LEMMA 1

Given each demand function $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)}$ and $D_{rb} = \frac{p_b - p_r - p}{1-\lambda}$, we apply the backward induction method. Our model considers firstly the platform's optimal problem,

$$\boldsymbol{maximize\ \pi_p = (p_r - \delta)D_r + pD_{rb}.}$$

Calculating the first derivative for p_r, and making it equal to zero, we obtain:

$$\frac{\partial \pi_p}{\partial p_r} = D_r + (p_r - \delta)\frac{\partial D_r}{\partial p_r} + p\frac{\partial D_{rb}}{\partial p_r} = 0$$

$$\frac{\partial \pi_p}{\partial p_r} = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} + (p_r - \delta)\left(\frac{-1}{\lambda(1-\lambda)}\right) - p\left(\frac{1}{1-\lambda}\right) = 0$$

$$\frac{\partial \pi_p}{\partial p_r} = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} + \frac{\delta - p_r}{\lambda(1-\lambda)} - \frac{\lambda p}{\lambda(1-\lambda)} = 0$$

$$\boldsymbol{p_r = \frac{\lambda(p_b - p) + \delta}{2}}$$

In order to prove the function's concavity in p_b and δ, we calculate the Hessian matrix for

$$\pi_F = p_b D_b + \delta D_r,$$

Then,

$$\nabla\, \pi_F(p_b, \delta) = \left(2 - \frac{p_b}{1-\lambda} + \frac{\delta\lambda}{\lambda(1-\lambda)}, \frac{\lambda p_b - p_r}{\lambda(1-\lambda)}\right)$$

$$H = \begin{bmatrix} \frac{\partial^2 \pi_F}{\partial {p_b}^2} & \frac{\partial^2 \pi_F}{\partial p_b \partial \delta} \\ \frac{\partial^2 \pi_F}{\partial \delta \partial p_b} & \frac{\partial^2 \pi_F}{\partial \delta^2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{1-\lambda} & \frac{1}{1-\lambda} \\ \frac{1}{1-\lambda} & 0 \end{bmatrix}$$

All the eigenvalues are negative or smaller than zero, which means being a negative semi-definite matrix.

By backward induction technique, to obtain the firm's optimum p_b function, we substitute the platforms' p_r into its maximization problem,

$$\boldsymbol{maximize\ \pi_F = p_b D_b + \delta D_r}.$$

$$D_b = 1 - \frac{p_b - \left(\frac{\lambda(p_b - p) + \delta}{2}\right)}{1 - \lambda}$$

$$D_b = 1 - \frac{2p_b - \lambda p_b + \lambda p - \delta}{2(1 - \lambda)}$$

$$\frac{\partial D_b}{\partial p_b} = \frac{\lambda - 2}{2(1 - \lambda)}$$

$$D_r = \frac{\lambda p_b - \left(\frac{\lambda(p_b - p) + \delta}{2}\right)}{\lambda(1 - \lambda)}$$

$$D_r = \frac{\lambda p_b + \lambda p - \delta}{2\lambda(1 - \lambda)}$$

$$\frac{\partial D_r}{\partial p_b} = \frac{\lambda}{2\lambda(1 - \lambda)}$$

$$\frac{\partial \pi_f}{\partial p_b} = D_b + p_b \frac{\partial D_b}{\partial p_b} + \delta \frac{\partial D_r}{\partial p_b} = 0$$

$$\frac{\partial \pi_f}{\partial p_b} = 1 - \frac{2p_b - \lambda p_b + \lambda p - \delta}{2(1 - \lambda)} + p_b\left(\frac{\lambda - 2}{2(1 - \lambda)}\right) + \delta\left(\frac{\lambda}{2\lambda(1 - \lambda)}\right) = 0$$

$$\frac{\partial \pi_f}{\partial p_b} = \frac{2(1 - \lambda)}{2(1 - \lambda)} + \frac{-2p_b + \lambda p_b - \lambda p + \delta}{2(1 - \lambda)} + \frac{\lambda p_b - 2p_b}{2(1 - \lambda)} + \frac{\delta}{2(1 - \lambda)} = 0$$

$$\frac{-2p_b + \lambda p_b - \lambda p + \delta}{2(1 - \lambda)} + \frac{\lambda p_b - 2p_b}{2(1 - \lambda)} = -\frac{2(1 - \lambda)}{2(1 - \lambda)} - \frac{\delta}{2(1 - \lambda)}$$

$$-4p_b + 2\lambda p_b - \lambda p + \delta = -2(1 - \lambda) - \delta$$

$$p_b(-2(2 - \lambda)) = -2(1 - \lambda) - 2\delta + \lambda p$$

The purchase price p_b chosen by the firm will be as follows:

$$\boldsymbol{p_b = \frac{2(1 - \lambda) + 2\delta - \lambda p}{2(2 - \lambda)}}$$

After obtaining the firms optimum price, we apply the substitution method in order to know the platform's p_r:

$$p_r = \frac{\lambda\left(\left(\frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)}\right)-p\right)+\delta}{2}$$

$$p_r = \frac{\frac{2\lambda(1-\lambda)+2\lambda\delta-\lambda^2 p-2\lambda p(2-\lambda)+2\delta(2-\lambda)}{2(2-\lambda)}}{2}$$

$$p_r = \frac{2\lambda(1-\lambda)+2\lambda\delta-\lambda^2 p-2\lambda p(2-\lambda)+2\delta(2-\lambda)}{4(2-\lambda)}$$

$$\boldsymbol{p_r = \frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)}}$$

After defining the optimum prices, we substitute the obtained functions into each demand problem:

$\therefore\ Purchase\ demand$

$$D_b = 1-\frac{p_b-p_r}{1-\lambda} = 1-\frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)(1-\lambda)}+\frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)(1-\lambda)}$$

$$D_b = 1+\frac{4\lambda-4\delta+2\lambda p-4}{4(2-\lambda)(1-\lambda)}+\frac{2\lambda-2\lambda^2+4\delta-4\lambda p+p\lambda^2}{4(2-\lambda)(1-\lambda)}$$

$$D_b = 1+\frac{6\lambda-2\lambda^2-2\lambda p+p\lambda^2-4}{4(2-\lambda)(1-\lambda)}$$

$$D_b = \frac{4(2-\lambda)(1-\lambda)}{4(2-\lambda)(1-\lambda)}+\frac{6\lambda-2\lambda^2-2\lambda p+p\lambda^2-4}{4(2-\lambda)(1-\lambda)}$$

$$D_b = \frac{8-12\lambda+4\lambda^2}{4(2-\lambda)(1-\lambda)}+\frac{6\lambda-2\lambda^2-2\lambda p+p\lambda^2-4}{4(2-\lambda)(1-\lambda)}$$

$$D_b = \frac{2(2-\lambda)(1-\lambda)}{4(2-\lambda)(1-\lambda)}-\frac{\lambda p(2-\lambda)}{4(2-\lambda)(1-\lambda)}$$

$$\boldsymbol{D_b = \frac{2(1-\lambda)-\lambda p}{4(1-\lambda)}}$$

$$\frac{\partial D_b}{\partial p} = -\frac{\lambda}{4(1-\lambda)} < 0$$

$$\frac{\partial D_b}{\partial \lambda} = -\frac{2+p}{4(1-\lambda)} < 0$$

Due to that $\frac{\partial D_b}{\partial p} < 0$ and $\frac{\partial D_b}{\partial \lambda} < 0$, the firm's demand will decrease with positive changes in these variables.

$\therefore$ *Rental demand*

$$D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} = \frac{\left(\frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)}\right)\lambda - \frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)}}{\lambda(1-\lambda)}$$

$$D_r = \frac{4\lambda(1-\lambda)+4\delta\lambda-3\lambda^2 p-4\delta+4\lambda p}{4\lambda(2-\lambda)(1-\lambda)} - \frac{2\lambda(1-\lambda)}{4\lambda(2-\lambda)(1-\lambda)}$$

$$\boldsymbol{D_r = \frac{p(4-3\lambda)}{4(2-\lambda)(1-\lambda)} + \frac{\lambda-2\delta}{2\lambda(2-\lambda)}}$$

We as well calculate the rental demand reaction when p changes:

$$\frac{\partial D_r}{\partial p} = \frac{4-3\lambda}{4(2-\lambda)(1-\lambda)} > 0$$

A change in the bargain price p, has a positive effect to the rental demand.

$\therefore$ *Rental and purchase demand*

$$D_{rb} = \frac{p_b - p_r - p}{1-\lambda} = \frac{\frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)} - \frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)} - p}{1-\lambda}$$

$$D_{rb} = \frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)(1-\lambda)} - \frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)(1-\lambda)} - \frac{p}{1-\lambda}$$

$$D_{rb} = \frac{2-2\lambda+2\delta-\lambda p}{2(2-\lambda)(1-\lambda)} - \frac{2\lambda-2\lambda^2+4\delta-4\lambda p+\lambda^2 p}{4(2-\lambda)(1-\lambda)} - \frac{p}{1-\lambda}$$

$$D_{rb} = \frac{2(2-\lambda)(1-\lambda)}{4(2-\lambda)(1-\lambda)} + \frac{\lambda p(2-\lambda)}{4(2-\lambda)(1-\lambda)} - \frac{p}{1-\lambda}$$

$$\boldsymbol{D_{rb} = \frac{1}{2} + \frac{\lambda p}{4(1-\lambda)} - \frac{p}{1-\lambda}}$$

When calculating the rental and purchase demand reaction when p changes we obtain:

$$\frac{\partial D_{rb}}{\partial p} = \frac{\lambda+4p\lambda-8p}{4(2-\lambda)(1-\lambda)} > 0$$

If $p < \frac{\lambda}{4(2-\lambda)}$, then changes in p will increase the rental and purchase demand.

To calculate each agent's profits, we substitute the previously variables obtained: p_b, D_b, p_r, D_r, and D_{rb} in each profits function:

$\therefore$ *Firm's profits*

Firm's profits are determined by the following function,

$$\boldsymbol{\pi_F = p_b D_b + \delta D_r}.$$

$$\pi_F = \left(\frac{2(1-\lambda)+2\delta-\lambda p}{2(2-\lambda)}\right)\left(\frac{2(1-\lambda)-\lambda p}{4(1-\lambda)}\right)+\delta\left(\frac{(4-3\lambda)p}{4(2-\lambda)(1-\lambda)}+\frac{\lambda-2\delta}{2\lambda(2-\lambda)}\right)$$

$$\pi_F = \frac{(1-\lambda)}{2(2-\lambda)}+\left(\frac{\lambda^2p^2-2\delta\lambda p}{8(2-\lambda)(1-\lambda)}\right)+\delta\left(\frac{(4-3\lambda)p}{4(2-\lambda)(1-\lambda)}+\frac{\lambda-2\delta}{2\lambda(2-\lambda)}\right)$$

$$\pi_F = \frac{(1-\lambda)}{2(2-\lambda)}+\frac{\lambda^2p^2-2\delta\lambda p}{8(2-\lambda)(1-\lambda)}+\frac{4p\delta-3\lambda p\delta}{4(2-\lambda)(1-\lambda)}+\frac{\lambda\delta-2\delta^2}{2\lambda(2-\lambda)}$$

$$\boldsymbol{\pi_F = \frac{1}{4}+\frac{2\lambda(1-\lambda)+4(1-\lambda)(1+p)(2\delta-\lambda)+\lambda^2p^2-\frac{8\delta^2}{\lambda}(1-\lambda)}{8(2-\lambda)(1-\lambda)}}$$

$$\frac{\partial\pi_F}{\partial\delta} = \frac{1+p}{(2-\lambda)}+\frac{2\delta}{\lambda(2-\lambda)} > 0$$

$$\frac{\partial\pi_F}{\partial p} = \frac{2\delta-\lambda}{2(2-\lambda)}+\frac{\lambda^2 p}{4(1-\lambda)(2-\lambda)} > 0$$

As $\frac{\partial\pi_F}{\partial\delta} > 0$, the firm's profits increase with δ. Additionally, firm will obtain a positive profit when

$$p > \frac{\lambda-2\delta}{8(2-\lambda)^2(1-\lambda)\lambda^2}.$$

$\therefore$ *Platform's profits*

Platform's profits are determined by the following function,

$$\boldsymbol{\pi_p = (p_r-\delta)D_r + pD_{rb}}.$$

$$\pi_p = \left(\frac{2\lambda(1-\lambda)+4\delta-\lambda p(4-\lambda)}{4(2-\lambda)}-\delta\right)\left(\frac{p(4-3\lambda)}{4(2-\lambda)(1-\lambda)}+\frac{\lambda-2\delta}{2\lambda(2-\lambda)}\right)+p\left(\frac{1}{2}+\frac{\lambda p}{4(1-\lambda)}-\frac{p}{1-\lambda}\right)$$

$$\pi_p = \left(\frac{2\lambda(1-\lambda)-\lambda p(4-\lambda)-4\delta(1-\lambda)}{4(2-\lambda)}\right)\left(\frac{p(4-3\lambda)}{4(2-\lambda)(1-\lambda)} + \frac{\lambda-2\delta}{2\lambda(2-\lambda)}\right) + p\left(\frac{1}{2} + \frac{\lambda p}{4(1-\lambda)} - \frac{p}{1-\lambda}\right)$$

$$\pi_p = \left(\frac{8\lambda p^2+8\delta p+3\lambda^2 p+2\lambda p}{8(2-\lambda)^2} + \frac{3\lambda^3 p^2}{16(2-\lambda)^2(1-\lambda)}\right)\left(\frac{2\delta-\lambda}{2\lambda(2-\lambda)}\right) + p\left(\frac{1}{2} + \frac{\lambda p}{4(1-\lambda)} - \frac{p}{1-\lambda}\right)$$

$$\pi_p = \left(\frac{(2\delta-1)(8\lambda p+8\delta+3\lambda^2+2\lambda)p}{16\lambda(2-\lambda)^3} + \frac{(2\delta-\lambda)3\lambda^3 p^2}{32(2-\lambda)^3(1-\lambda)}\right) + p\left(\frac{1}{2} + \frac{\lambda p}{4(1-\lambda)} - \frac{p}{1-\lambda}\right)$$

$$\boldsymbol{\pi_p = p\left(\frac{(2\delta-1)(8\lambda p+8\delta+3\lambda^2+2\lambda)}{16\lambda(2-\lambda)^3} + \frac{2-\lambda-p(2-\lambda)}{4(1-\lambda)}\right) + \frac{(2\delta-\lambda)3\lambda^3 p^2}{32(2-\lambda)^3(1-\lambda)}}$$

PROOF OF LEMMA 2

Given each demand function $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)}$ and $D_{rb} = \frac{p_b - p_r - p}{1-\lambda}$. The firm chooses simultaneously the retail price p_b and the rental price p_r by considering its profits function,

$$\boldsymbol{maximize\ \pi_{\mathrm{F}} = p_b D_b + p_r D_r}.$$

To prove the function's concavity in p_b and p_r, we calculate the Hessian matrix for π_{F},

$$\nabla\,\pi_F(p_b, p_r) = \left(1 - \frac{2p_b - p_r}{1-\lambda} + \frac{\lambda p_r}{\lambda(1-\lambda)}, \frac{p_b}{1-\lambda} + \frac{\lambda p_b - 2p_r}{\lambda(1-\lambda)}\right)$$

$$H = \begin{bmatrix} \frac{\partial^2 \pi_F}{\partial {p_b}^2} & \frac{\partial^2 \pi_F}{\partial p_b \partial p_r} \\ \frac{\partial^2 \pi_F}{\partial p_r \partial p_b} & \frac{\partial^2 \pi_F}{\partial {p_r}^2} \end{bmatrix} = \begin{bmatrix} \frac{-2}{1-\lambda} & \frac{2}{1-\lambda} \\ \frac{2}{1-\lambda} & \frac{-2}{\lambda(1-\lambda)} \end{bmatrix}$$

Given that all the eigenvalues are negative, it is a negative-definite matrix (concave).

To obtain prices, we calculate the partial derivative for p_b and p_r, and make them equal to zero:

$$\frac{\partial \pi_F}{\partial p_b} = D_b + p_b \frac{\partial D_b}{\partial p_b} + p_r \frac{\partial D_r}{\partial p_b} = 0$$

$$\frac{\partial \pi_F}{\partial p_b} = \left(1 - \frac{p_b - p_r}{1-\lambda}\right) + p_b\left(\frac{-1}{1-\lambda}\right) + p_r\left(\frac{\lambda}{\lambda(1-\lambda)}\right) = 0$$

$$\frac{\partial \pi_F}{\partial p_b} = \frac{1-\lambda}{1-\lambda} + \frac{2p_r - 2p_b}{1-\lambda} = 0$$

$$\frac{\partial \pi_F}{\partial p_b} = \frac{1-\lambda+2p_r-2p_b}{1-\lambda} = 0$$

$$\frac{2p_r}{1-\lambda} = \frac{\lambda - 1 + 2p_b}{1-\lambda}$$

$$p_r = \frac{2p_b + \lambda - 1}{2}$$

$$\boldsymbol{p_r = p_b - \frac{1-\lambda}{2}}$$

$$\frac{\partial \pi_F}{\partial p_r} = p_b \frac{\partial D_b}{\partial p_r} + D_r + p_r \frac{\partial D_r}{\partial p_r} = 0$$

$$\frac{\partial \pi_F}{\partial p_r} = p_b \left(\frac{1}{1-\lambda}\right) + \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} + p_r \left(\frac{-1}{\lambda(1-\lambda)}\right) = 0$$

$$\frac{\partial \pi_F}{\partial p_r} = \frac{\lambda p_b}{\lambda(1-\lambda)} + \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} - \frac{p_r}{\lambda(1-\lambda)} = 0$$

$$\frac{\partial \pi_F}{\partial p_r} = \frac{2\lambda p_b - 2p_r}{\lambda(1-\lambda)} = 0$$

$$2\lambda p_b - 2p_r = 0$$

$$2\lambda p_b = 2p_r$$

By considering the previous price functions, we apply the substitution method in order to obtain p_b,

$$\frac{2\lambda p_b - 2\left(\frac{2p_b + \lambda - 1}{2}\right)}{\lambda(1-\lambda)} = 0$$

$$\frac{2\lambda p_b - 2p_b - \lambda + 1}{\lambda(1-\lambda)} = 0$$

$$2\lambda p_b - 2p_b = -(1-\lambda)$$

$$p_b(-2(1-\lambda)) = -(1-\lambda)$$

$$p_b = \frac{1-\lambda}{2(1-\lambda)}$$

$$\boldsymbol{p_b = \frac{1}{2}}.$$

Once obtained $\boldsymbol{p_b = \frac{1}{2}}$, the rental price p_r chosen by the firm will be as follows,

$$p_r = p_b - \frac{1-\lambda}{2} = \frac{1}{2} - \frac{1-\lambda}{2}$$

$$\boldsymbol{p_r = \frac{\lambda}{2}}.$$

After defining the optimum prices, we substitute the values into each demand function:

$\therefore$ *Purchase demand*

$$D_b = 1 - \frac{p_b - p_r}{1-\lambda} = 1 - \frac{\frac{1}{2} - \frac{\lambda}{2}}{1-\lambda}$$

$$D_b = 1 - \frac{1-\lambda}{2(1-\lambda)}$$

$$D_b = \frac{1-\lambda}{2(1-\lambda)}$$

$$\boldsymbol{D_b = \frac{1}{2}}$$

The platform will obtain a fixed retail demand.

$\therefore$ *Rental demand*

$$D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} = \frac{\lambda\left(\frac{1}{2}\right) - \left(\frac{\lambda}{2}\right)}{\lambda(1-\lambda)}$$

$$\boldsymbol{D_r = \frac{\frac{\lambda}{2} - \frac{\lambda}{2}}{\lambda(1-\lambda)} = 0}$$

$\therefore$ *Rental and purchase demand*

By the assumption that $0 < v_0 < v_2 < v_1 < 1$, and the previously obtained values of $p_r = \frac{1}{2}$ and $p_r = \frac{\lambda}{2}$ values, then

$$v_1 = \frac{p_b - p_r}{1-\lambda} = \frac{\frac{1}{2} - \frac{\lambda}{2}}{1-\lambda}$$

$$\boldsymbol{v_1 = \frac{1}{2}}$$

and

$$v_0 = \frac{p_r}{\lambda} = \frac{\boldsymbol{\frac{\lambda}{2}}}{\boldsymbol{\lambda}}$$

$$\boldsymbol{v_0 = \frac{1}{2}},$$

which implies that $v_1 = v_0 = \frac{1}{2}$. The firm chooses prices to make the rental demand $D_r = 0$, and consequently, $D_{rb} = 0$.

$\therefore Firm's\ profits$

Firm's profits are determined by the following function,

$$\boldsymbol{\pi_F = p_b D_b + p_r D_r.}$$

$$\pi_F = \frac{1}{2} \cdot \frac{1}{2}$$

$$\boldsymbol{\pi_F = \frac{1}{4}}$$

$\therefore Platform's\ profits$

Since the firm chooses prices to make the rental $D_r = 0$, the platform's profits will be null.

PROOF OF LEMMA 3

Considering the demand functions of $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)}$ and $D_{rb} = \frac{p_b - p_r - p}{1-\lambda}$, the firm determines the optimal renting p_b, and retail p_r price simultaneously, by solving the following problem:

$$\boldsymbol{maximize\ \pi_f = p_b D_b + p_r D_r + p D_{rb} - C}$$

To prove the function's concavity in p_b and p_r, we calculate the Hessian matrix for π_{F},

$$\nabla\, \pi_F(p_b, p_r, p) = 1 - \frac{2p_b - p_r - p}{1-\lambda} + \frac{\lambda p_r}{\lambda(1-\lambda)}, \frac{p_b - p}{1-\lambda} + \frac{\lambda p_b - 2p_r}{\lambda(1-\lambda)}, \frac{p_b - p_r - 2p}{1-\lambda}.$$

$$H = \begin{bmatrix} \frac{\partial^2 \pi_F}{\partial {p_b}^2} & \frac{\partial^2 \pi_F}{\partial p_b \partial p_r} & \frac{\partial^2 \pi_F}{\partial p_b \partial p} \\ \frac{\partial^2 \pi_F}{\partial p_r \partial p_b} & \frac{\partial^2 \pi_F}{\partial {p_r}^2} & \frac{\partial^2 \pi_F}{\partial p_r \partial p} \\ \frac{\partial^2 \pi_F}{\partial p \partial p_b} & \frac{\partial^2 \pi_F}{\partial p \partial p_r} & \frac{\partial^2 \pi_F}{\partial p^2} \end{bmatrix} = \begin{bmatrix} \frac{-2}{1-\lambda} & \frac{2}{1-\lambda} & \frac{1}{1-\lambda} \\ \frac{2}{1-\lambda} & \frac{-2}{1-\lambda} & \frac{-1}{1-\lambda} \\ \frac{1}{1-\lambda} & \frac{-1}{1-\lambda} & \frac{-2}{1-\lambda} \end{bmatrix}$$

All the eigenvalues are negative, which means being a negative-definite matrix (concave).

Calculating the first derivative for p_b and p_r, and making them equal to zero, we obtain:

$$\frac{\partial\pi_F}{\partial p_b} = D_b + p_b\frac{\partial D_b}{\partial p_b} + p_r\frac{\partial D_r}{\partial p_b} + p\frac{\partial D_{rb}}{\partial p_b} = 0$$

$$\frac{\partial\pi_F}{\partial p_b} = \left(1 - \frac{p_b - p_r}{1-\lambda}\right) + p_b\left(\frac{-1}{1-\lambda}\right) + p_r\left(\frac{\lambda}{\lambda(1-\lambda)}\right) + p\left(\frac{1}{1-\lambda}\right) = 0$$

$$\frac{\partial\pi_F}{\partial p_b} = \left(\frac{1-\lambda-p_b+p_r}{1-\lambda}\right) - \left(\frac{p_b}{1-\lambda}\right) + \left(\frac{p_r}{1-\lambda}\right) + \left(\frac{p}{1-\lambda}\right) = 0$$

$$\frac{1-\lambda-2p_b+2p_r+p}{1-\lambda} = 0$$

$$p_r = \frac{\lambda+2p_b-p-1}{2}$$

$$\frac{\partial\pi_F}{\partial p_r} = p_b\frac{\partial D_b}{\partial p_r} + D_r + p_r\frac{\partial D_r}{\partial p_r} + p\frac{\partial D_{rb}}{\partial p_r}$$

$$\frac{\partial\pi_F}{\partial p_r} = p_b\left(\frac{1}{1-\lambda}\right) + \left(\frac{\lambda p_b - p_r}{\lambda(1-\lambda)}\right) + p_r\left(\frac{-1}{\lambda(1-\lambda)}\right) + p\left(\frac{-1}{1-\lambda}\right) = 0$$

$$\frac{\partial\pi_F}{\partial p_r} = \left(\frac{\lambda p_b}{\lambda(1-\lambda)}\right) + \left(\frac{\lambda p_b - p_r}{\lambda(1-\lambda)}\right) - \left(\frac{p_r}{\lambda(1-\lambda)}\right) - \left(\frac{\lambda p}{\lambda(1-\lambda)}\right) = 0$$

$$\frac{2\lambda p_b - 2p_r - \lambda p}{\lambda(1-\lambda)} = 0$$

We apply the substitution method to obtain the values of p_b and p_r,

$$\frac{2\lambda p_b - 2\left(\frac{\lambda+2p_b-p-1}{2}\right) - \lambda p}{\lambda(1-\lambda)} = 0$$

$$\frac{2\lambda p_b}{\lambda(1-\lambda)} - \left(\frac{\lambda+2p_b-p-1}{\lambda(1-\lambda)}\right) - \frac{\lambda p}{\lambda(1-\lambda)} = 0$$

$$2\lambda p_b - \lambda - 2p_b + p + 1 - \lambda p = 0$$

$$2\lambda p_b - 2p_b = \lambda - p - 1 + \lambda p$$

$$p_b(-2(1-\lambda)) = -(1-\lambda) - p(1-\lambda)$$

$$p_b = \frac{(1-\lambda) + p(1-\lambda)}{2(1-\lambda)}$$

$$\boldsymbol{p_b = \frac{1+p}{2}}$$

Given $p_b = \frac{1+p}{2}$, we get p_r:

$$p_r = \frac{\lambda + 2\left(\frac{1+p}{2}\right) - p - 1}{2}$$

$$p_r = \frac{\lambda + (1+p) - p - 1}{2}$$

$$\boldsymbol{p_r = \frac{\lambda}{2}}$$

After defining the optimum prices, we substitute the obtained functions into each demand problem.

$\therefore$ *Purchase demand*

$$D_b = 1 - \frac{p_b - p_r}{1-\lambda} = 1 - \frac{\left(\frac{1+p}{2}\right) - \left(\frac{\lambda}{2}\right)}{1-\lambda}$$

$$D_b = 1 + \frac{\lambda - 1 - p}{2(1-\lambda)}$$

$$\boldsymbol{D_b = \frac{1 - \lambda - p}{2(1-\lambda)}}$$

We calculate the partial derivative of D_b with respect to p and λ in order to know its reaction to parameter changes:

$$\frac{\partial D_b}{\partial p} = -\frac{1}{2(1-\lambda)} < 0$$

$$\frac{\partial D_b}{\partial \lambda} = -\frac{1}{2(1-\lambda)} < 0$$

As both derivatives are negative, we can say that an increase in p and λ will have a negative impact on the purchase demand.

To calculate each agent's profits, we substitute the previously variables obtained (p_b, D_b, p_r, D_r, and D_{rb}) in each profits function:

$\therefore$ *Rental demand*

$$D_r = \frac{\lambda p_b - p_r}{\lambda(1-\lambda)} = \frac{\lambda\left(\frac{1+p}{2}\right) - \left(\frac{\lambda}{2}\right)}{\lambda(1-\lambda)}$$

$$D_r = \frac{\left(\frac{\lambda p}{2}\right)}{\lambda(1-\lambda)}$$

$$\boldsymbol{D_r} = \frac{\boldsymbol{p}}{\mathbf{2(1-\boldsymbol{\lambda})}}$$

We calculate the partial derivative of D_r for p and λ in order to know its reaction for the variables changes:

$$\frac{\partial D_r}{\partial p} = \frac{1}{2(1-\lambda)} > 0$$

$$\frac{\partial D_r}{\partial \lambda} = -\frac{p}{2(1-\lambda)^2} > 0$$

Since, both derivatives are positive, an increase in p and λ will have a positive impact on the purchase demand.

$\therefore$ *Rental and purchase demand*

$$D_{rb} = \frac{p_b - p_r - p}{1-\lambda} = \frac{\left(\frac{1+p}{2}\right) - \left(\frac{\lambda}{2}\right) - p}{1-\lambda}$$

$$D_{rb} = \frac{\left(\frac{1+p-\lambda}{2}\right) - p}{1-\lambda}$$

$$D_{rb} = \frac{1+p-\lambda}{2(1-\lambda)} - \frac{p}{1-\lambda}$$

$$\boldsymbol{D_{rb}} = \frac{\mathbf{1} - \boldsymbol{\lambda} - \boldsymbol{p}}{\mathbf{2(1-\boldsymbol{\lambda})}}$$

When calculating the partial derivatives of D_{rb} for p and λ in order to know its reaction for the variables changes:

$$\frac{\partial D_{rb}}{\partial p} = -\frac{1}{2(1-\lambda)} < 0$$

$$\frac{\partial D_{rb}}{\partial \lambda} = -\frac{1}{2(1-\lambda)} < 0$$

To calculate the firm's profits, we substitute p_b, D_b, p_r, D_r, and D_r in each profits function.

$\therefore Firm's\ profits$

The firm's profits are determined by the following function:

$$\boldsymbol{\pi_F = p_b D_b + p_r D_r + p D_{rb}}$$

$$\pi_F = \left(\frac{1+p}{2}\right)\left(1 + \frac{\lambda - 1 - p}{2(1-\lambda)}\right) + \left(\frac{\lambda}{2}\right)\left(\frac{p}{2(1-\lambda)}\right) + p\left(\frac{1-\lambda-p}{2(1-\lambda)}\right)$$

$$\pi_F = \left(\frac{1-\lambda(1+p)-p^2}{4(1-\lambda)}\right) + \left(\frac{\lambda p}{4(1-\lambda)}\right) + \left(\frac{p-\lambda p-p^2}{2(1-\lambda)}\right)$$

$$\pi_F = \left(\frac{1-\lambda-\lambda p-p^2}{4(1-\lambda)}\right) + \left(\frac{\lambda p}{4(1-\lambda)}\right) + \left(\frac{2p-2\lambda p-2p^2}{4(1-\lambda)}\right)$$

$$\pi_F = \frac{2p(1-\lambda) - 3p^2 + (1-\lambda)}{4(1-\lambda)}$$

$$\boldsymbol{\pi_F = \frac{1}{4} + \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)}$$

$$\frac{\partial \pi_F}{\partial p} = \frac{(2-\lambda) - 3p}{2 - 2\lambda}$$

$$\frac{\partial \pi_F}{\partial \lambda} = \frac{-3p^2 - 4}{(4-4\lambda)^2} = \frac{-3p^2 - 4}{(4-4\lambda)^2}$$

$$\frac{\partial \pi_F}{\partial \lambda} = \frac{-3p^2 - 4}{16 - 32\lambda + 16\lambda^2}$$

As $\frac{\partial \pi_F}{\partial p} > 0$, the firm's profits increase with p.

As $\frac{\partial \pi_F}{\partial \lambda} > 0$, the firm's profits decrease with λ.

PROOF OF PROPOSITION 1

Comparing the profit of the firm under the agency contract π_F^A with that under the acquisition contract π_F^{AC}, we obtain:

$$\pi_F^A > \pi_F^{AC}$$

$$\frac{1}{4} > \frac{1}{4} + \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)$$

$$\frac{1}{4} > \frac{1}{4} + \frac{2p - 2\lambda p - 3p^2}{4(1-\lambda)}$$

$$4 - 4\lambda > 2p - 2\lambda p - 3p^2 + 4 - 4\lambda$$

$$-3p^2 + (2 - 2\lambda)p = 0$$

$$p_{1,2} > \frac{-(2-2\lambda) \pm (-2\lambda + 2)}{2(-3)}$$

$$p_1 > \frac{-(2-2\lambda) - (-2\lambda + 2)}{2(-3)} = \mathbf{0}$$

$$\boldsymbol{p_1} > \frac{-(\mathbf{2} - \mathbf{2}\boldsymbol{\lambda}) + (-\mathbf{2}\boldsymbol{\lambda} + \mathbf{2})}{\mathbf{2}(-\mathbf{3})} = \frac{\mathbf{2}}{\mathbf{3}}(\mathbf{1} - \boldsymbol{\lambda}).$$

The result reveals that if $p > \frac{2}{3}(1-\lambda)$ then $\pi_F^A > \pi_F^{AC}$ but if $p < \frac{2}{3}(1-\lambda)$ then $\pi_F^A < \pi_F^{AC}$.

Proof of Proposition 2

Comparing the profit of the firm under the agency contract π_F^A with that under the wholesale contract π_F^W, we obtain:

$$\pi_F^A < \pi_F^W$$

$$\frac{1}{4} < \frac{1}{4} + \frac{2\lambda(1-\lambda) + 4(1-\lambda)(1+p)(2\delta - \lambda) + \lambda^2 p^2 - \frac{8\delta^2}{\lambda}(1-\lambda)}{8(2-\lambda)(1-\lambda)}$$

$$2\lambda(1-\lambda) + 4(1-\lambda)(1+p)(2\delta - \lambda) + \lambda^2 p^2 - \frac{8\delta^2}{\lambda}(1-\lambda) > 0$$

$$\lambda^2 p^2 > \frac{8\delta^2}{\lambda}(1-\lambda) - 2\lambda(1-\lambda) - 4(1-\lambda)(1+p)(2\delta - \lambda)$$

$$\lambda^2 p^2 > \frac{8\delta^2}{\lambda} - 2\lambda - 4(1+p)(2\delta - \lambda)$$

$$\mathbf{2}\boldsymbol{\lambda} + \frac{\boldsymbol{\lambda}^{\mathbf{2}}\boldsymbol{p}^{\mathbf{2}}}{\mathbf{1} - \boldsymbol{\lambda}} < \mathbf{4}(\mathbf{1} + \boldsymbol{p})(\boldsymbol{\lambda} - \mathbf{2}\boldsymbol{\delta}) + \frac{\mathbf{8}\boldsymbol{\delta}^{\mathbf{2}}}{\boldsymbol{\lambda}}$$

In order to know the effect of λ in $2\lambda + \frac{\lambda^2 p^2}{1-\lambda} < 4(1+p)(\lambda - 2\delta) + \frac{8\delta^2}{\lambda}$, for convenience we define as $m = 2\lambda + \frac{\lambda^2 p^2}{1-\lambda}$ and $n = 4(1+p)(\lambda - 2\delta) + \frac{8\delta^2}{\lambda}$. By doing the partial derivative for λ we get

$$\boldsymbol{\frac{\partial m}{\partial \lambda} = 2 + 2\lambda p^2 - 4\lambda}$$

and

$$n = 4\lambda^2 + 4\lambda^2 p - 8\delta\lambda - 8\lambda\delta p + 8\delta^2$$

$$n = 4\lambda + 4\lambda p - 8\delta - 8\delta p + \frac{8\delta^2}{\lambda}$$

$$\boldsymbol{\frac{\partial n}{\partial \lambda} = 4 + 4p - \frac{8\delta^2}{\lambda^2}}$$

$$4 + 4p - \frac{8\delta^2}{\lambda^2} = 0$$

$$\lambda^2 + \lambda^2 p = 2\delta^2$$

$$\lambda^2 = \frac{2\delta^2}{1+p}$$

$$\boldsymbol{\lambda = \sqrt{\frac{2\delta^2}{1+p}}}.$$

PROOF OF PROPOSITION 3

Comparing the profit of the firm under the wholesale contract π_F^W with that under the acquisition contract π_F^{AC}, we obtain:

$$\pi_F^W > \pi_F^{AC}$$

$$\frac{1}{4} + \frac{2\lambda(1-\lambda) + 4(1-\lambda)(1+p)(2\delta-\lambda) + \lambda^2 p^2 - \frac{8\delta^2}{\lambda}(1-\lambda)}{8(2-\lambda)(1-\lambda)} > \frac{1}{4} + \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)$$

$$\frac{2\lambda(1-\lambda) + 4(1-\lambda)(1+p)(2\delta-\lambda) + \lambda^2 p^2 - \frac{8\delta^2}{\lambda}(1-\lambda)}{8(2-\lambda)(1-\lambda)} > \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)$$

$$\frac{4(1-\lambda)\left(2\lambda(1-\lambda) + 4(1-\lambda)(1+p)(2\delta-\lambda) + \lambda^2 p^2 - \frac{8\delta^2}{\lambda}(1-\lambda)\right)}{8(2-\lambda)(1-\lambda)} > \frac{p}{2}\left(1 - \frac{3p}{2(1-\lambda)}\right)$$

$$\frac{2\lambda(1-\lambda)+4(1-\lambda)(1+p)(2\delta-\lambda)+\lambda^2p^2-\frac{8\delta^2}{\lambda}(1-\lambda)}{2(2-\lambda)} > 2(1-\lambda)p-3p^2$$

$$2\lambda(1-\lambda)+4(1-\lambda)(1+p)(2\delta-\lambda)+\lambda^2p^2-\frac{8\delta^2}{\lambda}(1-\lambda) > 2p(2-\lambda)(2(1-\lambda)-3p)$$

$$2\lambda+4(1+p)(2\delta-\lambda)+\frac{\lambda^2p^2}{1-\lambda}-\frac{8\delta^2}{\lambda} > 2p(2-\lambda)(2-\frac{3p}{1-\lambda})$$

$$8\delta+8\delta p-2\lambda-4\lambda p+\frac{\lambda^2p^2}{1-\lambda}-\frac{8\delta^2}{\lambda} > (4p-2\lambda p)\left(\frac{2-2\lambda-3p}{1-\lambda}\right)$$

$$8\delta(1+p)-2\lambda-4\lambda p+\frac{\lambda^2p^2}{1-\lambda}-\frac{8\delta^2}{\lambda} > \frac{8p-12\lambda p-12p^2+4\lambda^2p+6\lambda p^2}{1-\lambda}$$

$$8\delta(1+p)-2\lambda-4\lambda p+\frac{\lambda^2p^2}{1-\lambda}-\frac{8\delta^2}{\lambda} > 4p(2-\lambda)-\frac{6p^2(2-\lambda)}{1-\lambda}$$

$$8\delta(1+p)+\frac{\lambda^2p^2}{1-\lambda}+\frac{6p^2(2-\lambda)}{1-\lambda} > 2\left(4p+\lambda+\frac{4\delta^2}{\lambda}\right)$$

$$8\delta(1+p)+\frac{p^2(\lambda^2+12-6\lambda)}{1-\lambda} > 2\left(4p+\lambda+\frac{4\delta^2}{\lambda}\right)$$

$$\boldsymbol{4\delta(1+p)+\frac{p^2(\frac{\lambda^2}{2}+6-3\lambda)}{1-\lambda} > 4p+\lambda+\frac{4\delta^2}{\lambda}}$$

In order to know the effect of λ in $4\delta(1+p)+\frac{p^2(\frac{\lambda^2}{2}+6-3\lambda)}{1-\lambda} > 4p+\lambda+\frac{4\delta^2}{\lambda}$, for convenience we define as $l = 4\delta(1+p)+\frac{p^2(\frac{\lambda^2}{2}+6-3\lambda)}{1-\lambda}$ and $g = 4p+\lambda+\frac{4\delta^2}{\lambda}$. By doing the partial derivative with respect λ we get

$$l = 4\delta(1+p)+\frac{\frac{\lambda^2p^2}{2}+6p^2-3\lambda p^2}{1-\lambda}$$

$$l = 4\delta(1+p)+\frac{\lambda^2p^2+12p^2-6\lambda p^2}{2(1-\lambda)}$$

$$\frac{\partial l}{\partial \lambda} = \frac{(2\lambda p^2-6p^2)\cdot(2-2\lambda)-(\lambda^2p^2+12p^2-6\lambda p^2)\cdot(-2)}{(2(1-\lambda))^2} = 0$$

$$\frac{(2\lambda p^2 - 6p^2)\cdot 2(1-\lambda) + (2\lambda^2 p^2 + 24p^2 - 12\lambda p^2)}{4(1-\lambda)^2} = 0$$

$$\frac{(2\lambda p^2 - 6p^2)\cdot 2(1-\lambda) - (\lambda^2 p^2 + 12p^2 - 6\lambda p^2)\cdot(-2)}{\left(2(1-\lambda)\right)^2} = 0$$

$$\frac{(2\lambda p^2 - 6p^2)\cdot (1-\lambda) + (\lambda^2 p^2 + 12p^2 - 6\lambda p^2)}{2(1-\lambda)^2} = 0$$

$$\frac{p^2(\lambda-3)}{1-\lambda} + \frac{\lambda^2 p^2 + 12p^2 - 6\lambda p^2}{2(1-\lambda)^2} = 0$$

$$\frac{p^2(\lambda-3)}{1-\lambda} + \frac{\lambda^2 p^2}{2(1-\lambda)^2} + \frac{6p^2 - 3\lambda p^2}{(1-\lambda)^2} = 0$$

$$\frac{p^2}{1-\lambda}\left(\lambda - 3 + \frac{\lambda^2}{2(1-\lambda)} + \frac{6-3\lambda}{1-\lambda}\right) = 0$$

$$\frac{p^2}{1-\lambda}\left(\frac{3+\lambda}{1-\lambda} - \frac{\lambda^2}{2(1-\lambda)}\right) = 0$$

$$\boldsymbol{\left(\frac{p}{1-\lambda}\right)^2\left(3+\lambda-\frac{\lambda^2}{2}\right) > 0}$$

and

$$g = 4p + \lambda + \left(\frac{4\delta^2}{\lambda}\right)$$

$$\frac{\partial g}{\partial \lambda} = 1 - \frac{4\delta^2}{\lambda^2} = 0$$

$$1 - \frac{4\delta^2}{\lambda^2} = 0$$

$$\lambda^2 = 4\delta^2$$

$$\boldsymbol{\lambda > 2\delta}.$$

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