Complementary Appendix of

Domestic and International Research Joint Ventures: The Effect of Collusion

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A Complementary Appendix: Second-order and stability conditions

In this appendix, we elucidate the conditions that ensure positive quantities and compliance with second-order and stability conditions in all the scenarios considered, i.e., we prove the following claim.

Claim 1 *Imposing* $\gamma \geqslant \underline{\gamma} = 9.6$ *is sufficient to ensure compliance with second-order and stability conditions.*

A.1 Second-order conditions

♦ *Base case (no RJVs)*

It can be verified that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2 \pi_{ij} / \partial x_{ij}^2 < 0$ (see Eq. (6)) we obtain

$$\gamma > \gamma_1 \equiv \frac{4}{25} \left[4 - \beta \left(1 + 2\lambda\right)\right]^2 . \tag{1}$$

A sufficient condition for Eq. (1) to be true, is that $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}} \gamma_1 \equiv \gamma_1^{\lambda} = \frac{4}{25}(4-\beta)^2$.

♦ *Domestic RJVs without collusion at the production stage*

It can be confirmed that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}^2 < 0$ and $\partial^2(\pi_{1j}+\pi_{2j})/\partial x_{2j}^2 < 0$ (see Eq. (8)) we obtain

$$\gamma > \gamma_2 \equiv \frac{4}{25}\left[17+\beta\left(17\beta-16-12\lambda(1+\beta)+8\lambda^2\beta\right)\right], \tag{2}$$

and positivity of the determinant requires $(\gamma_2-\gamma)^2 - \left\{\frac{8}{25}[1+2\beta(\lambda-2)][\beta(1+2\lambda)-4]\right\}^2 > 0$, which is observed when

$$\gamma > \gamma_3 \equiv \max\left\{4(\beta-1)^2, \frac{4}{25}[\beta(4\lambda-3)-3]^2\right\}. \tag{3}$$

A sufficient condition for Eqs. (2) and (3) to be true, is that $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}} \gamma_2 \equiv \gamma_2^{\lambda} = \frac{4}{25}\left(17\beta^2-16\beta+17\right)$ and $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}} \gamma_3 \equiv \gamma_3^{\lambda} = \max\{4(\beta-1)^2, \frac{36}{25}(\beta+1)^2\}$, respectively.

♦ *International RJVs without collusion at the production stage*

It can be verified that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2(\pi_{iA}+\pi_{iB})/\partial x_{iA}^2 < 0$ and $\partial^2(\pi_{iA}+\pi_{iB})/\partial x_{iB}^2 < 0$ (see Eq. (9)) we obtain

$$\gamma > \gamma_4 \equiv \frac{4}{25}\{17+\beta[\beta(2+\lambda(13\lambda-2))-22\lambda-6]\}, \tag{4}$$

and positivity of the determinant requires $(\gamma_4-\gamma)^2 - \left\{\frac{8}{25}[1-\beta(3\lambda-1)][\beta(1+2\lambda)-4]\right\}^2 > 0$, which is observed when

$$\gamma > \gamma_5 \equiv \max\left\{4(\lambda\beta-1)^2, \frac{4}{25}[\beta(\lambda-2)+3]^2\right\}. \tag{5}$$

A sufficient condition for Eqs. (4) and (5) to be true, is that $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}} \gamma_4 \equiv \gamma_4^{\lambda} = \frac{4}{25}\left(2\beta^2-6\beta+17\right)$ and $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}} \gamma_5 \equiv \gamma_5^{\lambda} = \max\left\{4, \frac{1}{25}(7-5\beta)^2\right\} = 4$, respectively.

♦ *Domestic RJVs with collusion at the production stage*

It can be confirmed that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}^2 < 0$ and $\partial^2(\pi_{1j}+\pi_{2j})/\partial x_{2j}^2 < 0$ we obtain

$$\gamma > \gamma_6 \equiv \frac{4}{9}[\beta(\lambda-1)-1]^2, \tag{6}$$

and positivity of the determinant requires $(\gamma_6 - \gamma)^2 - \gamma_6^2 > 0$, which is observed when

$$\gamma > \gamma_7 \equiv 2\gamma_6. \tag{7}$$

A sufficient condition for Eqs. (6) and (7) to be true, is that $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_6 \equiv \gamma_6^{\lambda} \equiv \frac{4}{9}(\beta+1)^2$ and $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_7 \equiv \gamma_7^{\lambda} = 2\gamma_6^{\lambda}$, respectively.

♦ *International RJVs with collusion at the production stage*

It can be verified that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2 (\pi_{iA} + \pi_{iB}) / \partial x_{iA}^2 < 0$ and $\partial^2 (\pi_{iA} + \pi_{iB}) / \partial x_{iB}^2 < 0$ we obtain

$$\gamma > \gamma_8 \equiv \frac{1}{9}\left\{8 + 2\beta\left[\beta\left(1+\lambda^2\right) - 4\right]\right\}, \tag{8}$$

and positivity of the determinant requires $(\gamma_8 - \gamma)^2 - \left[\frac{4}{9}\beta\lambda(2-\beta)\right]^2 > 0$, which is observed when

$$\gamma > \gamma_9 \equiv \frac{2}{9}\left[\beta(\lambda - 1) + 2\right]^2. \tag{9}$$

A sufficient condition for Eqs. (8) and (9) to be true, is that $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_8 \equiv \gamma_8^{\lambda} \equiv \frac{1}{18}\left(5\beta^2 - 18\beta + 17\right)$ and $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_9 \equiv \gamma_9^{\lambda} \equiv \frac{1}{18}(5 - 3\beta)^2$, respectively.

As a result of comparing the previous second-order conditions and using the bounds γ_h^{λ} for $h = 1, ..., 9$, we compute the lower bound for γ as:[1]

$$\gamma \geqslant \max_{0 \leq \beta \leq 1} \{\gamma_1^{\lambda}, ..., \gamma_9^{\lambda}\} = \max_{0 \leq \beta \leq 1} \{4, \frac{36}{25}(\beta+1)^2\} = 5.76. \blacksquare$$

A.2 Stability conditions

Stability of equilibria is ensured when the Jacobian of first derivatives of profits with respect to R&D investments is negative definite (for further details, see chapter 2 in Vives (2001): 'Oligopoly pricing: old ideas and new tools,' MIT Press, Massachussets). This matrix is symmetric with the following structure

$$\begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix}.$$

[1] It can be verified that $\gamma_1^{\lambda} < \gamma_5^{\lambda}$, $\gamma_2^{\lambda} < \gamma_5^{\lambda}$, $\gamma_4^{\lambda} < \gamma_5^{\lambda}$, $\gamma_6^{\lambda} < \gamma_7^{\lambda} < \gamma_5^{\lambda}$, and $\gamma_8^{\lambda} < \gamma_9^{\lambda} < \gamma_5^{\lambda}$. In addition, the first bound in γ_3^{λ} is also lower than γ_5^{λ}, i.e., $4(\beta-1)^2 < 4$.

The Jacobian of first derivatives is negative definite if

$$A < 0, \tag{10}$$

$$(A - B)(A + B) > 0, \tag{11}$$

$$2BCD + A\left(A^2 - B^2 - C^2 - D^2\right) < 0, \tag{12}$$

$$\left[(A + B)^2 - (C + D)^2\right]\left[(A - B)^2 - (C - D)^2\right] > 0. \tag{13}$$

The condition in Eq. (10) is already guaranteed by second-order conditions.

Claim 2 *Conditions in Eqs. (11)-(13) are satisfied iff*

$$A - B < 0, \tag{14}$$

$$A + B < 0, \tag{15}$$

$$(A + B)^2 - (C + D)^2 > 0, \tag{16}$$

$$(A - B)^2 - (C - D)^2 > 0. \tag{17}$$

Proof. First, note that Eqs. (14) and (15) guarantee that Eq. (11) holds and Eqs. (16) and (17) guarantee that Eq. (13) holds. Finally, Eq. (12) can be rewritten as:

$$(A - B)^2\left(2A(A + B) - (C + D)^2\right) > (C - D)^2(A - B)(A + B). \tag{18}$$

Under Eq. (17), Eq. (18) holds iff

$$2A(A + B) - (C + D)^2 > (A - B)(A + B), \text{ or} \tag{19}$$

$$(A + B)^2 - (C + D)^2 > 0, \tag{20}$$

which is Eq. (16). ■

♦ *Base case (no RJVs)*
In this scenario

$$\begin{aligned} A &\equiv \partial^2\pi_{ij}/\partial x_{ij}^2 = \frac{1}{25}\left\{64 - 25\gamma + 4\beta\left[1 + 2\lambda\right]\left[-8 + \beta\left(1 + 2\lambda\right)\right]\right\}, \\ B &\equiv \partial^2\pi_{ij}/\partial x_{ij}\partial x_{kj} = \frac{4}{25}\left[1 - 2\beta\left(2 - \lambda\right)\right]\left[-4 + \beta\left(1 + 2\lambda\right)\right], \text{ and} \\ C &= D \equiv \partial^2\pi_{ij}/\partial x_{ij}\partial x_{il} = \frac{4}{25}\left[-4 + \beta\left(1 + 2\lambda\right)\right]\left[1 + \beta\left(1 - 3\lambda\right)\right]. \end{aligned}$$

Thus, Eq. (17) holds directly and Eqs. (14)-(16) become

$$\begin{aligned}
\gamma &> \gamma_{10} \equiv \frac{4}{5}(1-\beta)\left[4-\beta(1+2\lambda)\right], && (21)\\
\gamma &> \gamma_{11} \equiv \frac{4}{25}\left[4-\beta(1+2\lambda)\right]\left[3+\beta(3-4\lambda)\right], && (22)\\
\gamma &> \gamma_{12} \equiv \max\left\{\frac{4}{5}\left[4-\beta(1+2\lambda)\right](1+\beta-2\beta\lambda), \frac{4}{25}\left[4-\beta(1+2\lambda)\right](1+\beta+2\beta\lambda)\right\}. && (23)
\end{aligned}$$

A sufficient condition for Eqs. (21)-(23) to be true is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{10} \equiv \gamma_{10}^{\lambda} \equiv \frac{4}{5}(4-\beta)(1-\beta)$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{11} \equiv \gamma_{11}^{\lambda} = \frac{12}{25}(4-\beta)(1+\beta)$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{12} \equiv \gamma_{12}^{\lambda} = \max\left\{\frac{4}{5}(4-\beta)(1+\beta), \frac{24}{25}\right\} = \frac{4}{5}(4-\beta)(1+\beta)$, respectively.

♦ *Domestic RJVs without collusion at the production stage*

In this scenario

$$\begin{aligned}
A &\equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{ij}^2 = \frac{1}{25}\left\{68-25\gamma+4\beta\left[-16+17\beta-12\lambda(1+\beta)+8\beta\lambda^2\right]\right\},\\
B &\equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{2j} = \frac{8}{25}\left[1-2\beta(2-\lambda)\right]\left[-4+\beta(1+2\lambda)\right], \text{ and}\\
C &= D \equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{il} = \frac{4}{25}\left[-3+\beta(-3+4\lambda)\right]\left[1+\beta(1-3\lambda)\right],
\end{aligned}$$

for $i=1,2$ and $j,l=A,B$. Thus, Eq. (17) holds directly and Eqs. (14)-(16) become

$$\begin{aligned}
\gamma &> \gamma_{13} \equiv 4(1-\beta)^2, && (24)\\
\gamma &> \gamma_{14} \equiv \frac{4}{25}\left[3+\beta(3-4\lambda)\right]^2, && (25)\\
\gamma &> \gamma_{15} \equiv \max\left\{\frac{4}{5}\left[3+\beta(3-4\lambda)\right](1+\beta-2\beta\lambda), \frac{4}{25}\left[3+\beta(3-4\lambda)\right](1+\beta+2\beta\lambda)\right\}. && (26)
\end{aligned}$$

A sufficient condition for Eqs. (24)-(26) to be true is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{13} \equiv \gamma_{13}^{\lambda} \equiv 4(1-\beta)^2$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{14} \equiv \gamma_{14}^{\lambda} = \frac{36}{25}(1+\beta)^2$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{15} \equiv \gamma_{15}^{\lambda} = \max\left\{\frac{12}{5}(1+\beta)^2, \frac{1}{2}(1+\beta)^2\right\} = \frac{12}{5}(1+\beta)^2$, respectively.

♦ *International RJVs without collusion at the production stage*

In this scenario

$$\begin{aligned}
A &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}^2 = \frac{1}{25}\left\{68-25\gamma+4\beta\left[-6-22\lambda+\beta\left(2+\lambda\left[13\lambda-2\right]\right)\right]\right\},\\
B &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{iA}\partial x_{iB} = \frac{8}{25}\left[1+\beta(1-3\lambda)\right]\left[-4+\beta(1+2\lambda)\right],\\
C &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kj} = \frac{4}{25}\left[-3+\beta\left(19-3\beta-12\lambda(1+\beta)+13\beta\lambda^2\right)\right], \text{ and}\\
D &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kl} = \frac{4}{25}\left[1+\beta(1-3\lambda)\right]\left[-3-\beta(3-4\lambda)\right],
\end{aligned}$$

for $i,k=1,2,\ k\neq i$ and $j,l=A,B,\ l\neq j$. Thus, Eqs. (14)-(17) become

$$\begin{aligned}
\gamma &> \gamma_{16}\equiv 4\left(1-\beta\lambda\right)^2, &(27)\\
\gamma &> \gamma_{17}\equiv \frac{4}{25}\left[3-\beta\left(2-\lambda\right)\right]^2, &(28)\\
\gamma &> \gamma_{18}\equiv \max\left\{\frac{4}{5}\left(1-\beta\right)\left[3-\beta\left(2-\lambda\right)\right],\frac{4}{25}\left[3-\beta\left(2-\lambda\right)\right]\left[1+\beta\left(1+2\lambda\right)\right]\right\}, &(29)\\
\gamma &> \gamma_{19}\equiv \max\left\{4\left(1-\beta\right)\left(1-\beta\lambda\right),4\left(1-\beta\lambda\right)\left[1+\beta\left(1-2\lambda\right)\right]\right\}. &(30)
\end{aligned}$$

A sufficient condition for Eqs. (27)-(30) to be true is that $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{16}\equiv\gamma_{16}^{\lambda}\equiv 4$, $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{17}\equiv\gamma_{17}^{\lambda}=\frac{1}{25}\left(7-5\beta\right)^2$, $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{18}\equiv\gamma_{18}^{\lambda}=\max\left\{\frac{2}{5}\left(7-5\beta\right)\left(1-\beta\right),\frac{4}{25}\left(7-5\beta\right)\right\}$, and $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{19}\equiv\gamma_{19}^{\lambda}=\max\left\{4\left(1-\beta\right),4\left(1+\beta\right)\right\}=4\left(1+\beta\right)$, respectively.

♦ *Domestic RJVs with collusion at the production stage*

In this scenario

$$\begin{aligned}
A &\equiv \partial^2\left(\pi_{1j}+\pi_{2j}\right)/\partial x_{ij}^2=\frac{1}{9}\left\{4-9\gamma+4\beta\left[2+\beta\left(1-\lambda\right)\right]\left[1-\lambda\right]\right\},\\
B &\equiv \partial^2\left(\pi_{1j}+\pi_{2j}\right)/\partial x_{1j}\partial x_{2j}=\frac{4}{9}\left[1+\beta\left(1-\lambda\right)\right]^2, \text{ and}\\
C &= D\equiv\partial^2\left(\pi_{1j}+\pi_{2j}\right)/\partial x_{1j}\partial x_{il}=\frac{2}{9}\left[1+\beta\left(1-\lambda\right)\right]\left[-1+\beta\left(-1+4\lambda\right)\right],
\end{aligned}$$

for $i=1,2$ and $j,l=A,B$. Thus, Eq. (17) holds directly and Eqs. (14)-(16) become

$$\begin{aligned}
\gamma &> 0, &(31)\\
\gamma &> \gamma_{20}\equiv\frac{8}{9}\left[1+\beta\left(1-\lambda\right)\right]^2, &(32)\\
\gamma &> \gamma_{21}\equiv\max\left\{\frac{4}{9}\left[1+\beta\left(1-\lambda\right)\right]\left[1+\beta\left(1+2\lambda\right)\right],\frac{4}{3}\left[1+\beta\left(1-\lambda\right)\right]\left[1+\beta\left(1-2\lambda\right)\right]\right\}. &(33)
\end{aligned}$$

Eq. (31) holds by construction. A sufficient condition for Eqs. (32) and (33) to be true is that $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{20}\equiv\gamma_{20}^{\lambda}\equiv\frac{8}{9}\left(1+\beta\right)^2$ and $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{21}\equiv\gamma_{21}^{\lambda}=\max\left\{\frac{1}{2}\left(1+\beta\right)^2,\frac{4}{3}\left(1+\beta\right)^2\right\}=\frac{4}{3}\left(1+\beta\right)^2$, respectively.

♦ *International RJVs with collusion at the production stage*

In this scenario

$$
\begin{aligned}
A &\equiv \partial^2 \left(\pi_{iA}+\pi_{iB}\right)/\partial x_{ij}^2 = \frac{1}{9}\left\{8-9\gamma-2\beta\left[4-\beta\left(1+\lambda^2\right)\right]\right\}, \\
B &\equiv \partial^2 \left(\pi_{iA}+\pi_{iB}\right)/\partial x_{iA}\partial x_{iB} = \frac{4}{9}\beta\lambda\left(2-\beta\right), \\
C &\equiv \partial^2 \left(\pi_{iA}+\pi_{iB}\right)/\partial x_{ij}\partial x_{kj} = \frac{2}{9}\left\{-2+\beta\left[5+\beta\left(-2+\lambda^2\right)\right]\right\}, \text{ and} \\
D &\equiv \partial^2 \left(\pi_{iA}+\pi_{iB}\right)/\partial x_{ij}\partial x_{kl} = \frac{2}{9}\beta\lambda\left(1+\beta\right),
\end{aligned}
$$

for $i,k=1,2$, $k\neq i$ and $j,l=A,B$, $l\neq j$. Thus, Eqs. (14)-(17) become

$$\gamma > \gamma_{22} \equiv \frac{2}{9}\left[2-\beta\left(1+\lambda\right)\right]^2, \tag{34}$$

$$\gamma > \gamma_{23} \equiv \frac{2}{9}\left[2-\beta\left(1-\lambda\right)\right]^2, \tag{35}$$

$$\gamma > \gamma_{24} \equiv \max\left\{\frac{2}{3}\left(1-\beta\right)\left[2-\beta\left(1-\lambda\right)\right], \frac{2}{9}\left[2-\beta\left(1-\lambda\right)\right]\left[1+\beta\left(1+2\lambda\right)\right]\right\}, \tag{36}$$

$$\gamma > \gamma_{25} \equiv \max\left\{\frac{2}{3}\left(1-\beta\right)\left[2-\beta\left(1+\lambda\right)\right], \frac{2}{9}\left[2-\beta\left(1+\lambda\right)\right]\left[1+\beta\left(1-2\lambda\right)\right]\right\}. \tag{37}$$

A sufficient condition for Eqs. (34)-(37) to be true is that $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}}\gamma_{22} \equiv \gamma_{22}^{\lambda} \equiv \frac{2}{9}\left(2-\beta\right)^2$, $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}}\gamma_{23} \equiv \gamma_{23}^{\lambda} = \frac{1}{18}\left(5-3\beta\right)^2$, $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}}\gamma_{24} \equiv \gamma_{24}^{\lambda} = \max\left\{\frac{1}{3}\left(5-3\beta\right)\left(1-\beta\right), \frac{2}{9}\left(5-3\beta\right)\right\}$, and $\gamma > \max\limits_{0\leqslant\lambda\leqslant\bar{\lambda}}\gamma_{25} \equiv \gamma_{25}^{\lambda} = \max\left\{\frac{2}{3}\left(1-\beta\right)\left(2-\beta\right), \frac{2}{9}\left(1+\beta\right)\left(2-\beta\right)\right\}$, respectively.

As a result of comparing the previous stability conditions and using the bounds γ_h^{λ} for $h = 10, ..., 25$, we compute the lower bound for γ as:[2]

$$\gamma \geqslant \underline{\gamma} \equiv \max_{0\leq\beta\leq1}\left\{\gamma_{10}^{\lambda}, ..., \gamma_{25}^{\lambda}\right\} = 9.6. \blacksquare$$

[2] It can be confirmed that $\gamma_{10}^{\lambda} < \gamma_{12}^{\lambda}$, $\gamma_{11}^{\lambda} < \gamma_{12}^{\lambda} < 4.8$, $\gamma_{13}^{\lambda} < 4$, $\gamma_{14}^{\lambda} < \gamma_{15}^{\lambda} < 9.6$, $\gamma_{16}^{\lambda} < \gamma_{19}^{\lambda}$, $\gamma_{17}^{\lambda} < 1.96$, $\gamma_{18}^{\lambda} < 5.6$, $\gamma_{19}^{\lambda} < 8$, $\gamma_{20}^{\lambda} < \gamma_{21}^{\lambda} < 16/3$, $\gamma_{22}^{\lambda} < 8/9$, $\gamma_{23}^{\lambda} < 25/15$, $\gamma_{24}^{\lambda} < 2$, and $\gamma_{25}^{\lambda} < 4/3$.

B Complementary Appendix: Extended Proof of Proposition 1

In the base case, maximization of the stage-1 profit function (i.e., Eq. (3)) yields the following SPNE values

$$x_{ij}^{0} = \frac{2(4-\beta-2\beta\lambda)(2(a-c)-t)}{25\gamma+2(\beta+2\beta\lambda-4)(2\beta+4\beta\lambda+2)}, \tag{38}$$

$$h_{ij}^{0} = \frac{5}{2}\gamma\frac{2(a-c)-t}{25\gamma+4(\beta+2\beta\lambda-4)(\beta+2\beta\lambda+1)}+\frac{t}{2}, \tag{39}$$

$$e_{ij}^{0} = \frac{5}{2}\gamma\frac{2(a-c)-t}{25\gamma+4(\beta+2\beta\lambda-4)(\beta+2\beta\lambda+1)}-\frac{t}{2}, \tag{40}$$

$$q_{j}^{0} = 10\gamma\frac{2(a-c)-t}{25(\gamma-1)+(2\beta+4\beta\lambda-3)^2}, \tag{41}$$

$$\pi_{ij}^{0} = \left(\frac{25\gamma^2}{2}-\frac{\gamma(2(4-\beta-2\beta\lambda))^2}{2}\right)\left(\frac{2(a-c)-t}{25\gamma+4(\beta+2\beta\lambda-4)(\beta+2\beta\lambda+1)}\right)^2+\frac{t^2}{2}. \tag{42}$$

In the case of a domestic RJV, maximization of the stage-1 profit function (i.e., Eq. (8)) yields the following SPNE values

$$x_{ij}^{D} = \frac{2(3\beta-4\beta\lambda+3)(2(a-c)-t)}{25\gamma-2(2\beta+4\beta\lambda+2)(3\beta-4\beta\lambda+3)}, \tag{43}$$

$$h_{ij}^{D} = \frac{5}{2}\gamma\frac{2a-2c-t}{25\gamma-2(2\beta+4\beta\lambda+2)(3\beta-4\beta\lambda+3)}+\frac{t}{2}, \tag{44}$$

$$e_{ij}^{D} = \frac{5}{2}\gamma\frac{2a-2c-t}{25\gamma-2(2\beta+4\beta\lambda+2)(3\beta-4\beta\lambda+3)}-\frac{t}{2}, \tag{45}$$

$$q_{j}^{D} = 10\gamma\frac{2(a-c)-t}{25\gamma-12-4\beta(2(3+\lambda)+\beta(1+2\lambda)(3-4\lambda))}, \tag{46}$$

$$\pi_{ij}^{D} = \left(\frac{25\gamma^2}{2}-\frac{\gamma(2(3\beta-4\beta\lambda+3))^2}{2}\right)\left(\frac{2(a-c)-t}{25\gamma-2(2\beta+4\beta\lambda+2)(3\beta-4\beta\lambda+3)}\right)^2+\frac{t^2}{2} \tag{47}$$

In the case of an international RJV, maximization of the stage-1 profit function (i.e., Eq. (9)) yields the following SPNE values

$$x_{ij}^{I} = \frac{2\left(3-2\beta+\beta\lambda\right)\left(2\left(a-c\right)-t\right)}{25\gamma-2\left(3-2\beta+\beta\lambda\right)\left(2\beta+4\beta\lambda+2\right)}, \tag{48}$$

$$h_{ij}^{I} = \frac{5}{2}\gamma\frac{2a-2c-t}{25\gamma-2\left(3-2\beta+\beta\lambda\right)\left(2\beta+4\beta\lambda+2\right)}+\frac{t}{2}, \tag{49}$$

$$e_{ij}^{I} = \frac{5}{2}\gamma\frac{2a-2c-t}{25\gamma-2\left(3-2\beta+\beta\lambda\right)\left(2\beta+4\beta\lambda+2\right)}-\frac{t}{2}, \tag{50}$$

$$q_{j}^{I} = 10\gamma\frac{2\left(a-c\right)-t}{25\gamma-2\left(3-2\beta+\beta\lambda\right)\left(2\beta+4\beta\lambda+2\right)}, \tag{51}$$

$$\pi_{ij}^{I} = \left(\frac{25\gamma^{2}}{2}-\frac{\gamma\left(2\left(3-2\beta+\beta\lambda\right)\right)^{2}}{2}\right)\left(\frac{2\left(a-c\right)-t}{25\gamma-2\left(3-2\beta+\beta\lambda\right)\left(2\beta+4\beta\lambda+2\right)}\right)^{2}+\frac{t^{2}}{2} \tag{52}$$

A comparison between Eqs. (38)-(42) and Eqs. (43)-(47) yields directly $q_j^0 > q_j^D$, $h_{ij}^0 > h_{ij}^D$, $e_{ij}^0 > e_{ij}^D$, $x_{ij}^0 > x_{ij}^D$, $\pi_{ij}^0 < \pi_{ij}^D$ for $\lambda > \frac{4\beta-1}{2\beta} \equiv \lambda_2^*$; and $q_j^0 \leq q_j^D$, $h_{ij}^0 \leq h_{ij}^D$, $e_{ij}^0 \leq e_{ij}^D$, $x_{ij}^0 \leq x_{ij}^D$, $\pi_{ij}^0 \geq \pi_{ij}^D$ for $\lambda \leq \lambda_2^*$.

A comparison between Eqs. (38)-(42) and Eqs. (48)-(52) yields directly $q_j^0 > q_j^I$, $h_{ij}^0 > h_{ij}^I$, $e_{ij}^0 > e_{ij}^I$, $x_{ij}^0 > x_{ij}^I$, $\pi_{ij}^0 < \pi_{ij}^I$ for $\lambda < \frac{1+\beta}{3\beta} \equiv \lambda_1^*$; and $q_j^0 \leq q_j^I$, $h_{ij}^0 \leq h_{ij}^I$, $e_{ij}^0 \leq e_{ij}^I$, $x_{ij}^0 \leq x_{ij}^I$, $\pi_{ij}^0 \geq \pi_{ij}^I$ for $\lambda \geq \lambda_1^*$.

Finally, a comparison between Eqs. (43)-(47) and Eqs. (48)-(52) yields directly $q_j^D > q_j^I$, $h_{ij}^D > h_{ij}^I$, $e_{ij}^D > e_{ij}^I$, $x_{ij}^D > x_{ij}^I$, $\pi_{ij}^D < \pi_{ij}^I$ for $\lambda < 1$; and $q_j^D \leq q_j^I$, $h_{ij}^D \leq h_{ij}^I$, $e_{ij}^D \leq e_{ij}^I$, $x_{ij}^D \leq x_{ij}^I$, $\pi_{ij}^D \geq \pi_{ij}^I$ for $\lambda \geq 1$.

As a consequence, regarding the comparison of quantities in Fig. 1, we have $q_j^0 > q_j^I$ and $q_j^0 > q_j^D$ in region I; $q_j^I > q_j^0 > q_j^D$ (since $\lambda > 1$) in region II; and $q_j^D > q_j^0 > q_j^I$ (since $\lambda < 1$) in region III. Regarding the comparison of profits, we have: $\pi_{ij}^D > \pi_{ij}^I > \pi_{ij}^0$ in region I (in Fig. 1) for $\lambda > 1$; $\pi_{ij}^I > \pi_{ij}^D > \pi_{ij}^0$ in region I for $\lambda < 1$; $\pi_{ij}^D > \pi_{ij}^0 > \pi_{ij}^I$ in region II; and $\pi_{ij}^I > \pi_{ij}^0 > \pi_{ij}^D$ in region III. These results will be used in Complementary Appendix C. ■

C Complementary Appendix: Equilibrium analysis

In this appendix, we perform an equilibrium analysis. The purpose of this analysis is twofold. On the one hand, it justifies the symmetric cases considered in the consumer welfare analysis (where either two domestic or two international RJVs are formed) since no asymmetric outcomes occur in equilibrium (i.e., where only one RJV is formed). On the other hand, it allows us to compare private and social interests and to derive some policy implications out of this comparison. The complexity of the analysis requires to include some simplifying assumptions to get conclusive results. More precisely, the exercise is performed for the parameter values considered in Fig. 2, i.e., $\gamma = 10$ and $\frac{t}{(a-c)} = \frac{4}{11}$, to make easier the comparison between private and social incentives. Although the analysis is not exhaustive, it is appropriate for a moderate range of these parameter values and reveals some interesting insights. In addition, changes in γ and t affect simultaneously both the private and the social profitability of RJVs.[3]

C.1 RJVs without collusion at the production stage

First, we consider two games in which two-partner RJVs can be formed. In the *domestic game*, the (two) domestic firms in each of the (two) countries decide whether or not to form a RJV. In the *international game*, there are two couples of international partners that decide whether or not to form a RJV. These games can be represented in normal form (where players, strategies, and payoffs are displayed) in the following way,

Domestic game

	Form	No form
Form	(π^D, π^D)	(π^{DN}, π^{ND})
No form	(π^{ND}, π^{DN})	(π^0, π^0)

International game

	Form	No form
Form	(π^I, π^I)	(π^{IN}, π^{NI})
No form	(π^{NI}, π^{IN})	(π^0, π^0)

where firm-market subscripts are omitted such that π^D refers to π_{ij}^D, etc. After computing the equilibrium of these two games,[4] we need to consider them jointly to obtain the final equilibrium outcome, given that domestic and international RJVs cannot occur simultaneously in our setting.

⧫ *Domestic game*

The unilateral incentive to form a domestic RJV is derived from studying the best-reply of domestic firms given that the firms located in the other country do not form a RJV, i.e., computing $\pi^{DN} - \pi^0$. This exercise yields three areas depending on the sign of the difference, as depicted below.

[3] An increase in γ makes R&D more costly and discourages RJVs. An increase in t makes international RJVs with collusion more profitable (both privately and socially).

[4] The precise values these profit expressions are complex and are available from the authors on request.

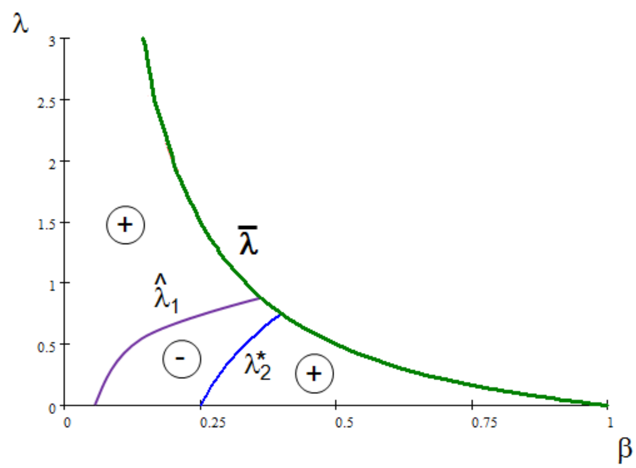

Fig. $A1$: $\pi^{DN} - \pi^{0}$

The function λ_2^* is the same as in Fig. 1 and $\widehat{\lambda}_1$ is another threshold value.[5]

The best-reply of domestic firms given that the firms located in the other country form a RJV is obtained from the difference $\pi^{D} - \pi^{ND}$ and also yields three areas depending on the sign of the difference in the way displayed below.

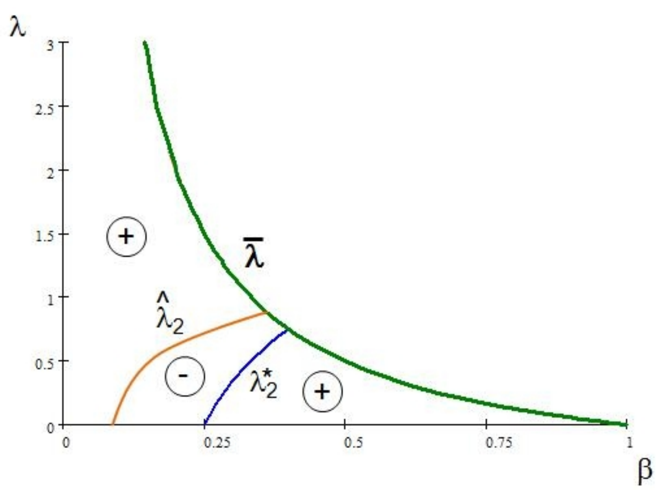

Fig. $A2$: $\pi^{D} - \pi^{ND}$

[5]The precise value of $\widehat{\lambda}_1$ is complex and is available from the authors on request.

The function λ_2^* is the same as in Fig. 1 and $\widehat{\lambda}_2$ is another threshold value.[6]

Finally, the equilibrium arises from the joint analysis of Figs. $A1$ and $A2$, which is shown in the figure below.

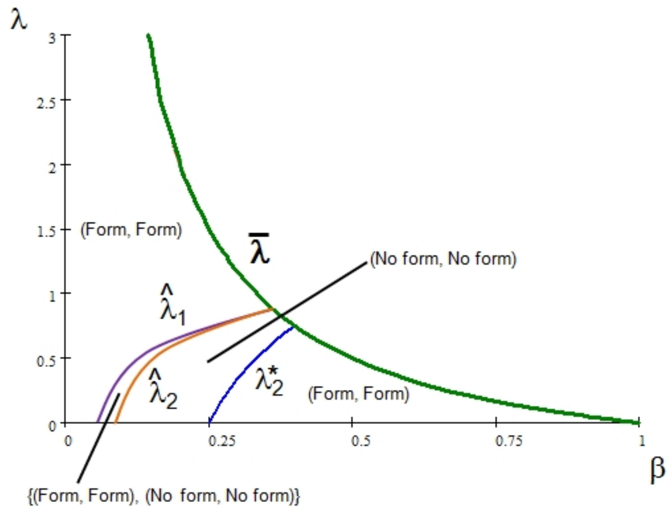

Fig. $A3$: Equilibrium – domestic game without collusion

The equilibrium always involves forming a domestic RJV, except for the central region delimited by functions λ_2^* and $\widehat{\lambda}_1$. More precisely, a multiple equilibrium arises in the region comprised between $\widehat{\lambda}_2$ and $\widehat{\lambda}_1$.

♦ *International game*

The unilateral incentive to form an international RJV is derived from studying the best-reply of two international partners given that the other firms do not form a RJV, i.e., computing $\pi^{IN} - \pi^0$. This exercise yields three areas depending on the sign of the difference, as depicted below.

[6] The precise value of $\widehat{\lambda}_2$ is complex and is available from the authors on request.

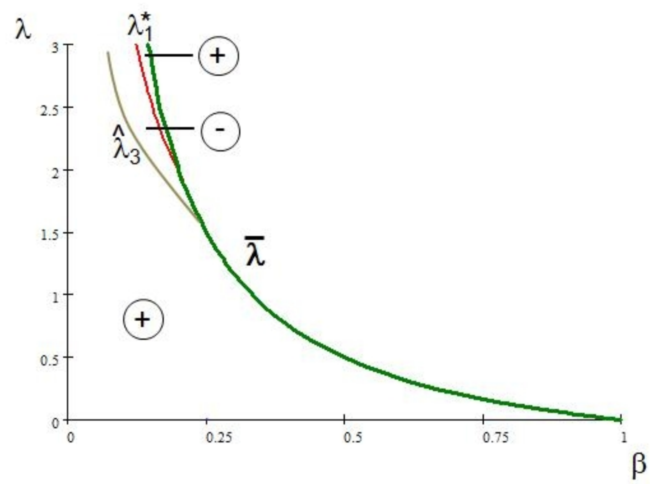

Fig. $A4$: $\pi^{IN} - \pi^0$

The function λ_1^* is the same as in Fig. 1 and $\widehat{\lambda}_3$ is another threshold value.[7]

The best-reply of two international partners given that the other firms form a RJV is obtained from the difference $\pi^I - \pi^{NI}$ and also yields three areas depending on the sign of the difference in the way displayed below.

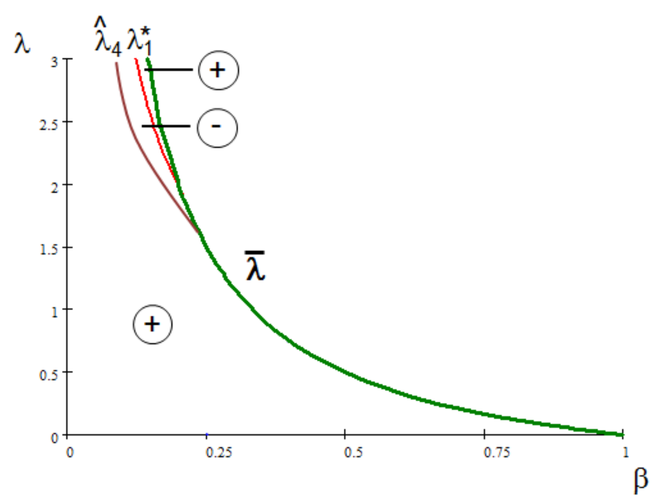

Fig. $A5$: $\pi^I - \pi^{NI}$

[7] The precise value of $\widehat{\lambda}_3$ is complex and is available from the authors on request.

The function λ_1^* is the same as in Fig. 1 and $\widehat{\lambda}_4$ is another threshold value.[8]

Finally, the equilibrium arises from the joint analysis of Figs. $A4$ and $A5$, which is shown in the figure below.

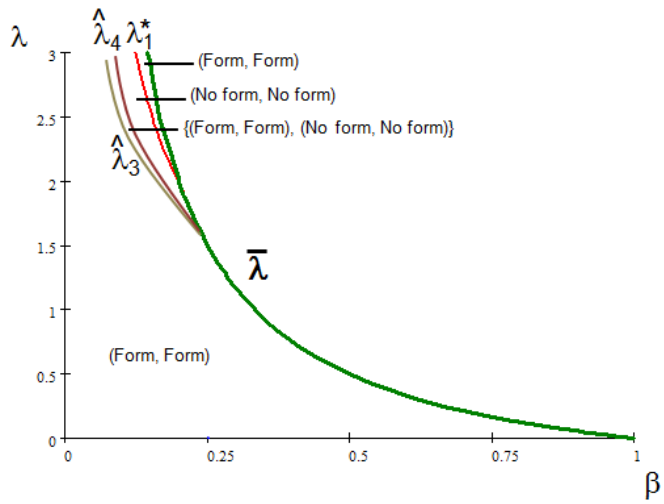

Fig. $A6$: Equilibrium – international game without collusion

The equilibrium always involves forming an international RJV, except for the central region delimited by functions $\widehat{\lambda}_3$ and λ_1^*. More precisely, a multiple equilibrium arises in the region comprised between $\widehat{\lambda}_3$ and $\widehat{\lambda}_4$.

♦ *Conclusion*

To be able to provide an accurate equilibrium prediction, we need to consider simultaneously the domestic and the international games, i.e., Figs. $A3$ and $A6$.

First, we need to compare (i) π^D and π^0 in the region $\lambda \in \left(\widehat{\lambda}_3, \lambda_1^*\right)$ because (Form, Form) is the equilibrium in the domestic game whereas (No form, No form) can be the equilibrium in the international game, and (ii) π^I and π^0 in the region $\lambda \in \left(\lambda_2^*, \widehat{\lambda}_1\right)$ because (No form, No form) can be the equilibrium in the domestic game whereas (Form, Form) is the equilibrium in the international game. From Complementary Appendix B, we know that $\pi^D > \pi^0$ for $\lambda > \lambda_2^*$ and that $\pi^I > \pi^0$ for $\lambda < \lambda_1^*$. Therefore, in both cases (Form, Form) is the final equilibrium.

Second, we will assume that firms will form the *best* RJV in cases when both the domestic and the international can arise as an equilibrium outcome. Therefore, we need to compare π^I and

[8] The precise value of $\widehat{\lambda}_4$ is complex and is available from the authors on request.

π^D. From Complementary Appendix B, we know that $\pi^I > \pi^D$ for $\lambda < 1$.

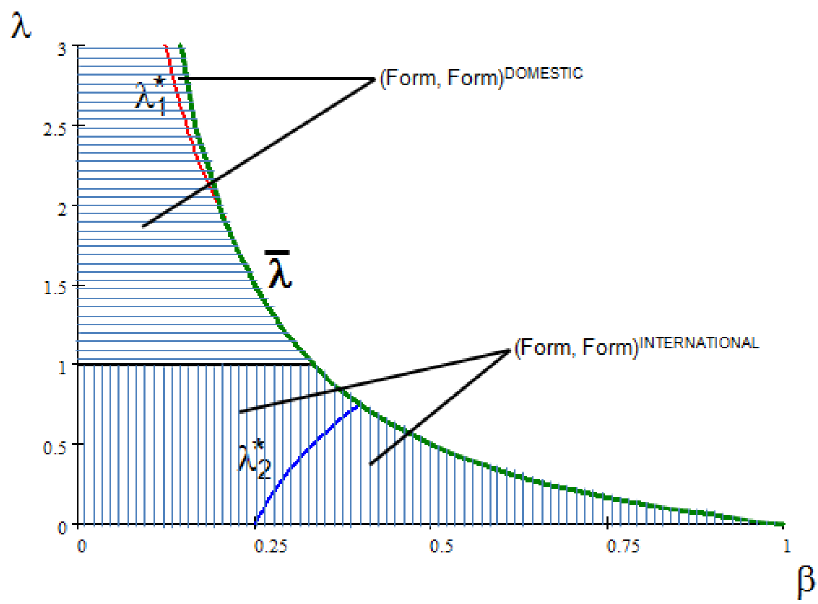

Fig. $A7$: Equilibrium $-$ domestic and international game without collusion

Thus, in the absence of collusion, firms always engage in RJVs and the bound $\lambda \lesseqgtr 1$ determines the type of agreement. On the one hand, when international spillovers are small (i.e., $\lambda < 1$) firms use international RJVs to *internalize* the externalities stemming from cross-border cooperation agreements. On the other hand, when international spillovers are large (i.e., $\lambda > 1$), firms do not need cross-border agreements to benefit from foreign R&D and therefore they prefer domestic agreements. Comparing Figs. 1 and $A7$, we observe that (i) in the northern region (i.e., above λ_1^*), international RJVs maximize consumer welfare but firms prefer domestic RJVs, (ii) in the upper central region (i.e., $\lambda \in (1, \lambda_1^*)$), no RJV maximizes consumer welfare but firms prefer domestic RJVs, (iii) in the lower central region (i.e., $\lambda \in (\max.\{0, \lambda_2^*\}, 1)$), no RJV maximizes consumer welfare but firms prefer international RJVs, and (iv) in the eastern region (i.e., on the right of λ_2^*), domestic RJVs maximize consumer welfare but firms prefer international RJVs. Therefore, there is an important conflict between private and public incentives: although domestic (international) RJVs are socially desirable when domestic (international) spillovers are large, they are not observed in equilibrium because firms already benefit from the other firms' R&D.

C.2 RJVs with collusion at the production stage

In this subsection, we replicate the previous analysis now in the presence of collusion at the production stage. Players and strategies are the same as before, but profits change due to the presence of collusion. Thus, the *domestic* and *international games* are represented in normal form in the following way, where subscript C denotes collusion.

Domestic game	Form	No form
Form	(π_C^D, π_C^D)	(π_C^{DN}, π_C^{ND})
No form	(π_C^{ND}, π_C^{DN})	(π_C^0, π_C^0)

International game	Form	No form
Form	(π_C^I, π_C^I)	(π_C^{IN}, π_C^{NI})
No form	(π_C^{NI}, π_C^{IN})	(π_C^0, π_C^0)

⧫ *Domestic game*

The unilateral incentive to form a domestic RJV is derived from studying the best-reply of domestic firms given that the firms located in the other country do not form a RJV, i.e., computing $\pi_C^{DN} - \pi_C^0$. This difference is always negative for $0 \leqslant \lambda \leqslant \overline{\lambda} \equiv (1-\beta)/2\beta$.[9]

The best-reply of domestic firms given that the firms located in the other country form a RJV is obtained from the difference $\pi_C^D - \pi_C^{ND}$, which is also negative in our relevant region for any combination of β and λ.

As a consequence, No form is a dominant strategy and the equilibrium of the domestic game is always (No form, No form).

⧫ *International game*

The unilateral incentive to form an international RJV is derived from studying the best-reply of two international partners given that the other firms do not form a RJV, i.e., computing $\pi_C^{IN} - \pi_C^0$. This exercise yields two areas depending on the sign of the difference, as depicted below,

[9] The precise details on the computations are available from the authors on request.

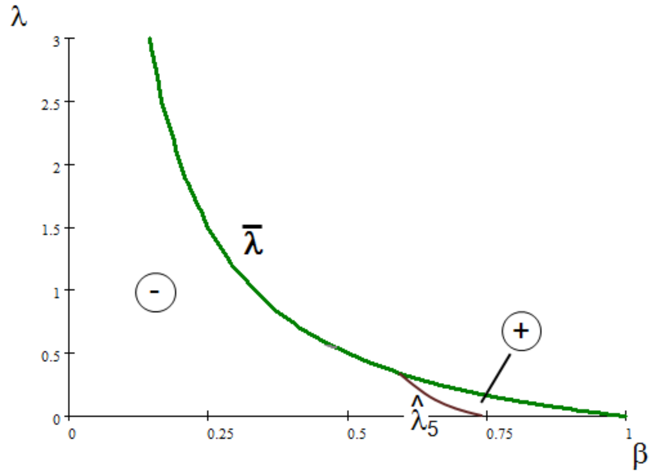

Fig. $A8$: $\pi_C^{IN} - \pi_C^0$

where the function $\widehat{\lambda}_5$ is another threshold value.[10]

The best-reply of two international partners given that the other firms form a RJV is obtained from the difference $\pi_C^I - \pi_C^{NI}$, which is always positive in our relevant region for any combination of β and λ.

As a consequence, the equilibrium is as shown in the figure below.

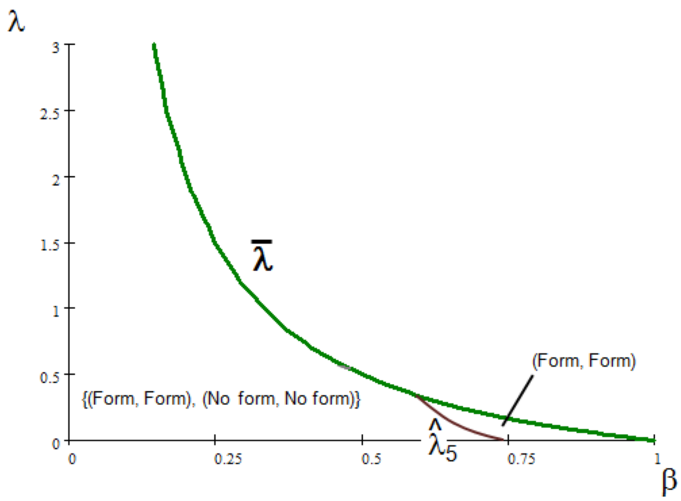

Fig. $A9$: Equilibrium $-$ international game with collusion

[10]The precise value of $\widehat{\lambda}_5$ is complex and is available from the authors on request.

The equilibrium always involves forming an international RJV, but (No form, No form) is also an equilibrium in the western region of Fig. $A9$.

♦ *Conclusion*

The simultaneous consideration of the domestic and the international games is straightforward given that No form is a dominant strategy in the domestic game. Thus, the equilibrium is plotted in Fig. $A10$ below, where λ_3^* (which appears in Fig. 2) has been included to better compare firms and consumers' interests.

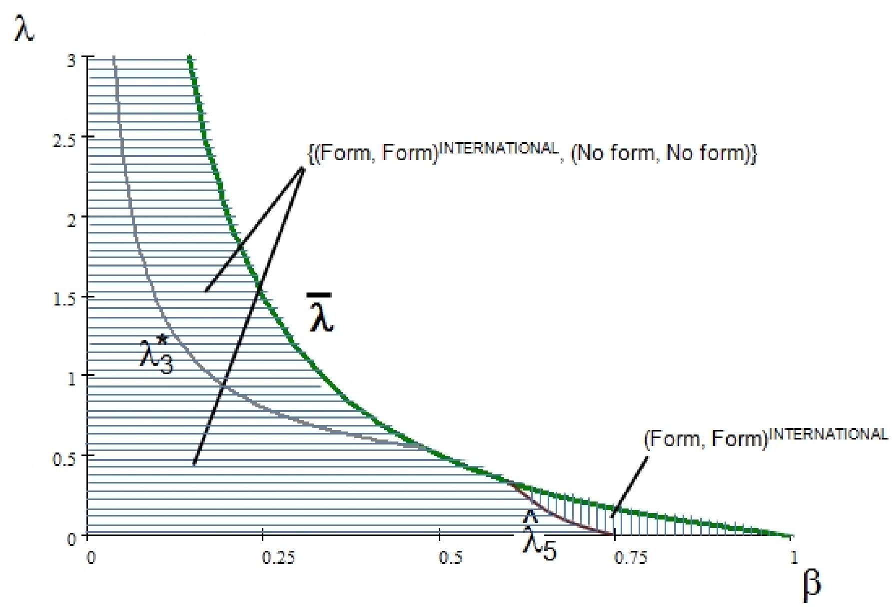

Fig. $A10$: Equilibrium − domestic and international game with collusion

Thus, in the presence of collusion, firms never engage in domestic RJVs and may always engage in international RJVs, although staying alone may also be an equilibrium for sufficiently low values of β. Comparing Figs. 2 and $A10$, we observe that (i) in the northern region (i.e., above λ_3^*), there is no private-public conflict when the equilibrium is of the type (Form, Form)$^{\text{INTERNATIONAL}}$ but there is a conflict when the equilibrium is (No form, No form) because consumer welfare is maximized under international RJVs, (ii) in the central region (below λ_3^* and on the left of $\widehat{\lambda}_5$), there is no private-public conflict when the equilibrium is of the type (No Form, No Form) but there is a conflict when the equilibrium is (Form, Form)$^{\text{INTERNATIONAL}}$ because consumer welfare is maximized in the absence of RJVs, and (iii) in the eastern region (i.e., on the right of $\widehat{\lambda}_5$), there is again a conflict because no RJVs maximize consumer welfare but firms prefer international RJVs. In conclusion, while both consumers and firms dislike domestic RJVs, international RJV formation is always an equilibrium because they are formed as a device to save internationalization costs. $\blacksquare$