Competition and the Hold-Up Problem: a Setting with Non-exclusive Contracts*

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Abstract

In this work we study how the introduction of competition to the side of the market offering trading contracts affects the equilibrium investment profile in a bilateral investment game. By using a common agency framework, where contracts are not exclusive, we find that the equilibrium investment profile depends on the degree of competition in the trading game. We show that full efficiency can be implemented when competition is intense. Whenever inefficiencies emerge, the hold-up problem only affects the receiving party while the investment decision of the offering party is constrained efficient. In situations where the offering parties "tacitly" coordinate to reduce competition, full efficiency is never guaranteed. The hold-up problem affects the receiving party while the offering party tends to over invest. Nevertheless, regardless on the degree of competition, full efficiency is restored when the offering side of the market is sufficiently large.

We show that coordinating on bringing competition down is not Pareto dominant for the parties offering the contracts whenever the sensitivity of the equilibrium allocation to investment is large. Moreover, for some range of the parameters, the investing seller prefers a situation where competition is intense. This happens whenever the receiving party switches his investment decision to not invest. Finally, we rank equilibria in terms of net social welfare and we obtain that lower competitive equilibria might generate larger surpluses.

Keywords: Bilateral investment, competition, tacit coordination, Pareto dominance, hold-up problem.

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1 Introduction

In many economic situations, parties undertake relation-specific investment to increase potential gains from the relationship. Consider for instance an insurer that researches on possible contingencies to better suit the special needs for his contractor; or a seller that reduces the production cost of an intermediate good used by a single downstream producer. Even if the gains from trade increase, the ex-ante decision to undertake relation specific investment depends on the extent that the investing party can appropriate the potential gains arising from this investment. Economists have extensively studied this subject and have shown that efficient investment decisions fail to materialize whenever the investing party is not able to appropriate all the benefits generated from his specific investment. In other words, transaction-specific investments results into a fundamental transformation in market transactions, by reducing the field of available alternatives from a large number (in the ex-ante bargaining situation) to a small number (in the ex-post bidding situation) Williamson (1983). Then, economic agents get wrong investment incentives due to the problem of being held-up, and decide to undertake lower levels of investment. This has detrimental effects on resource allocation and economic welfare.

The existence of the hold-up problem is generally traced to incomplete contracts, that is, the inability of parties to write contracts depending on all relevant and publicly available information.[1] The economic literature has manly focused on two different approaches to solve this problem. The first approach, organization design, is closely related to the theory of the firm and it imposes conditions on when transactions should be undertaken trough a price mechanism - market - or by fiat - firm. It also establishes provisions for asset ownership and dictates that the residual right of control should be given to the party that undertakes specific investment, Hart (1995). The second is the long-term contract approach, and it's main focus has been to establish contractual provisions such as default or option contracts, which can be enforced in case of disagreement, and this induces parties to relax potential conflicts of interests. A relatively new approach has been to introduce competition in the market. The main mechanism is that, as long as any economic agent is able to choose the partner with whom to undertake trade, investment has also the effect of increasing the outside option, and this generates an extra incentive for investment.

In the present work, we build on this new approach but we consider the case of purely specific investment, hence, the possibility of an outside option is not feasible. At this regard, we consider an economy with a single large producer of a marketed good that needs to use a specific input provided by different sellers. Consistent with a bilateral investing game, only one of the sellers is aware of the technology that enables him to reduce the cost of input production. The buyer invests to improve her valuation for the input by adapting, for instance, her production process to this specific input. Our objective is to see how the incentives to invest, and ultimately, the equilibrium investment profile is affected by the way sellers engage in competition. We are interested in a situation where there is no competition for the market but competition in the market. Therefore, we consider a set-up where the buyer can sign trading contracts with many sellers. We find evidence of this type of contracts in the cycling industry, where large brands of elaborated cycling components such as Shimano, Specialized and Trek buy raw materials and other simple components from different suppliers. Another example is provided by the financial sector, where a large firm normally

[1] If specific investment was verifiable or enforceable ex-post, it will be in the interest of the contractual parties to write compensation schemes linked to investment. Grossman and Hart (1986), Grout (1984), Hart and Moore (1988) and Williamson (1985).

establishes multiple banking relationships, and customers also hold multiple credit cards from different networks.[2] Hence, the model that we have in mind is one with non-exclusive trading contracts.

We study how the introduction of competition to the side of the market offering trading contracts affects the equilibrium investment profile in a bilateral investment game. In a common agency framework, where contracts are not exclusive, we find that the equilibrium investment depends on the degree of competition in the trading game. Using recent results on the markets and contracts literature, we see that the upper bound on the transfer that each seller can ask depends on the threat that he is excluded from trade. Here, we proxy the level of competition by the number of active sellers that impose a threat on excluding any other seller. Hence, competition in our trading game is more aggressive the higher the number of active sellers that impose a threat of exclusion.

We show that the equilibrium investment profile depends on this "intensive" degree of competition ex-post, which arises in the trading game once specific investment is made. We show that full efficiency can be implemented when competition is intense. Whenever inefficiencies emerge, the hold-up problem only affects the receiving party while the investment decision of the offering party is constrained efficient. In situations where the offering parties "tacitly" coordinate to reduce competition, full efficiency is never guaranteed. The hold-up problem affects the receiving party while the offering party tends to over invest. With low competition, each seller obtains more that his marginal contribution to the surplus and the gains appropriated by the investing seller are larger than the increase on the trading surplus. We also find a relationship between the equilibrium investment profile and the "extensive" degree of competition, which corresponds to the number of active sellers in the industry. We obtain that the higher the number of active sellers, the equilibrium investment profile tends to efficiency regardless of the degree of competition ex-post. In picture (1), we illustrate the relation of the ex-ante investment profile with respect to the level of ex-post competition and the number of active sellers. The picture shows that higher ex-ante efficiency is achieved with a higher degree of competition ex-post or with a larger number of active sellers.

Therefore, by using the multiplicity of equilibria in our trading game, we study which equilibrium leads to larger ex-ante efficiency. Moreover, we also analyze which equilibrium is preferred by the side of the market offering the trading contracts. In our model, the most preferred equilibrium is not always the one that makes competition less severe, since the investment profile depends on how sellers compete. At this regard, the purpose of an investing party might not longer be to appropriate as much gains from the relationship as possible, because this will have an effect on the investment decision of the other party, and so in the overall potential gains. Consequently, in some situations, it is beneficial to distribute potential gains from trade evenly among different participants and this is obtained whenever competition is intense. Moreover, we show that in situations where the sellers "tacitly" coordinate to lower competition, in order to appropriate a larger proportion of the gains from trade, the results are also influenced by the sensitivity of the equilibrium allocation on investment. When a seller is more efficient that the rest, due to his specific investment, he is indirectly putting a constraint on the transfers of the other sellers. With an unchanged investment decision of the buyer, an increase of investment of the seller entails a reduction of the amount traded by the non-investing sellers. If this effect turns out to be small, the incentives of the sellers are aligned and they prefer a more favorable partition of the surplus. Conversely, whenever the effect is big, sellers' incentives are not aligned and they tend to prefer different degrees of competition. While the investing seller prefers an equilibrium

[2] Open listing agreements is another example of non-exclusivity in the real estate market.

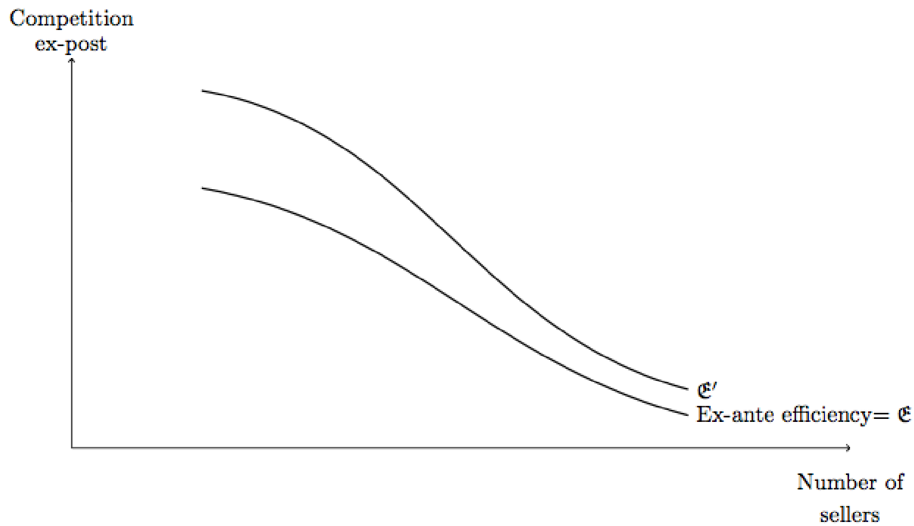


Figure 1: Iso-efficiency curves of the ex-ante investment profile. In the horizontal axis we represent the number of active sellers and it the vertical axis is the intensity of ex-post competition. The same level of ex-ante efficiency is achieved with lower levels of competition but a higher number of active sellers. Moves to the north-east of the curves represents equilibrium investment profiles closer to efficiency. Hence, we have that $\mathfrak{E}' > \mathfrak{E}$.

where the investment is maximal, this is not the case for the non-investing sellers. This comes from the fact that investments are strategic complements, then a higher investment of the buyer entails larger investment of the seller which in turn creates lower trade for the non-investing sellers.

This strategic complementarity, also explains why the maximization of social welfare is not always obtained under situations when the investment equilibrium profile is closer to efficiency. The inefficiency created to one side of the market can re-store efficiency to the other side, leading to larger potential gains from trade. Surprisingly, we show that lower competition ex-post can lead to higher levels of social welfare.

After presenting a simple numerical example, the remaining of the paper is organized as follows. In section (2), we briefly discuss the related literature. In section (3), we set up the model and introduce how competition among the offering parties takes place. Later, we proceed by solving the game backwards. Therefore, in section (4.1) we study the properties of our equilibrium allocation and in section (4.3), we characterize the equilibrium investment profile. We see that investment depends directly on competition, and we consider both, the "intensive" and "extensive" degree of competition. We proceed in section (5) with equilibria comparison. We start by analyzing which is the Pareto dominant equilibrium by the offering parties and we continue by ranking equilibria in terms of social welfare. Finally, we discuss and conclude in section (6). All proofs are relegated to the appendix.

1.1 Numerical example

Before presenting the formal model, we expose a simple numerical example that illustrates some of the results of the paper. We consider an economy with a common buyer and three competing sellers (i=1,2,3) who produce an homogeneous good. The utility and cost functions are:

$$U(X \mid \beta) = 10 + (\beta + 1)\log(X) - K \times \mathbb{1}\{\beta > 0\};$$

$$C_1(x_1 \mid \sigma) = \frac{2x_1^2}{\sigma + 1} - \psi \times \mathbb{1}\{\sigma = 1\}; \quad C_i(x_i) = 2x_i^2 \text{ for } i = 2, 3,$$

where the buyer and seller 1 are able to undertake a discreet investment decision $\beta = \{0, 9\}$ and $\sigma = \{0, 1\}$ that increases the utility of consumption and reduces the cost of production respectively. The total amount traded is the sum of the individual amounts $X = x_1 + x_2 + x_3$, and the fixed cost of investment for the buyer and the seller are $K > 0$ and $\psi > 0$ respectively. In the section where we describe the model, we elaborate on how competition between sellers takes place, here we only present the results for the two extreme equilibria. The most competitive equilibrium is the one where each sellers appropriates this marginal contribution to the trading surplus, and in the least competitive, the rent of the buyer is minimized. While the first is characterized by an even distribution of the potential gains from investment, the latter is the most favorable to the sellers. The following table presents the bounds for the fixed costs below which each party decides to invest. The under-script on the bounds represents the investment decision of the other party. The first column stands for the investment bound under efficiency, while the second and the third are for the highest and the least level of competition respectively.

Bounds	Efficient	H. competition	L. competition
K_0	5.71	0.244	0
K_σ	7.012	1.34	0.979
ψ_0	0.143	0.143	0.1614
ψ_β	1.43	1.43	1.614

Figure 2: Bounds below which the parties decide to invest. The under-script stands for the investment decision of the other side of the market.

Compared to efficiency, the common buyer decides to set a positive level of investment less often in equilibrium, but more often whenever competition between sellers is more severe. The contrary applies for the seller, whose incentive to invest is higher in a less competitive equilibrium. Observe also that investment decisions are strategic complements because the investment bounds increase with the investment of the other party. This complementarity leads to situations where, in a less competitive equilibrium, the buyer is more prompt to invest. This is the case when the seller only decides to invest in a less competitive equilibrium. In the table above, this happens whenever $\psi \in (1.43, 1.614)$ and the investment threshold for the buyer is larger in the less competitive equilibrium, because $K_\sigma^{LC} = 0.979 > 0.244 = K_0^{HC}$.

In figure (3) below, we expose the equilibrium payoffs of the sellers. In the columns, we state the possible investment equilibrium profiles and the rows stand for the two equilibria and type of sellers. This table illustrates that, because investment decisions depend on the level of competition ex-post, it might be beneficial for the sellers to coordinate on an equilibrium that is more competitive, since this gives more incentives for the buyer to invest. The red numbers represent the payoffs for the investing seller, and

$(\beta,\sigma) =$	$(0,0)$	$(\beta,0)$	$(0,\sigma)$	(β,σ)
$\pi_1^{HC}(\cdot,\cdot)$	0.202	2.02	0.346	3.465
$\pi_i^{HC}(\cdot,\cdot)$	0.202	2.02	0.143	1.43
$\pi_1^{LC}(\cdot,\cdot)$	0.214	2.14	0.375	3.755
$\pi_i^{LC}(\cdot,\cdot)$	0.214	2.14	0.147	1.47

Figure 3: Sellers' payoffs depending on the level of competition and ex-ante investment profile.

we see that he is better with higher levels of competition if the buyer changes his investment decision $\pi_1^{HC}(\beta,\sigma) = 3.465 > 0.375 = \pi_1^{LC}(0,\sigma)$. We see that this is also the case for the non investing sellers even for unchanged investment of the buyer $\pi_i^{HC}(\beta,0) = 2.02 > 1.47 = \pi_1^{LC}(\beta,\sigma)$, represented in blue in the table. Here, a more competitive equilibrium gives higher payoffs to them since it reduces the investment of the competing seller.

Consequently, since the investment ex-ante depends on the degree of competition ex-post, it is not clear that the offering parties would always prefer to "tacitly" coordinate on a lower intensity of competition or that social welfare is always maximized with more intense competition. To illustrate this last point, consider that the fixed costs of investment of the seller and the buyer are $\psi \in (1.43, 1.614)$ and $K \in (0.244, 0.979)$ respectively. Notice that, for this range of cost parameters, both equilibrium are inefficient since efficiency requires only the buyer to invest. Nevertheless, in the most competitive equilibrium nobody invests, and in the less competitive both parties invest. To see which equilibrium performs better, we compare the social surplus obtained in both equilibria $SW^{HC} = SW_{0,0}$ and $SW^{LC} = SW_{\beta,\sigma} - K - \psi$. The difference is equal to

$$SW^{HC} - SW^{LC} = SW_{0,0} - (SW_{\beta,\sigma} - K - \psi) = 9.356 - (16.51 - K - \psi) = -7.154 + K + \psi,$$

and for the extreme values of the costs $\bar{\psi} = 1.614$ and $\bar{K} = 0.979$ we get that the difference of net surplus is

$$NSW^{HC} - NSW^{LC} = -7.154 + 1.651 + 0.970 = -4.533 < 0.$$

Therefore, for this fixed costs of investment, we conclude that the lowest degree of competition ex-post does better than the highest levels of competition.[3] Hence, despite the over-investment of the seller, in the lowest competitive equilibrium, there is also investment of the buyer and this is welfare enhancing as the investment of the buyer has a big effect on social surplus. In this example, inefficiency of one side of the market restores efficiency to the other side.

2 Related literature

The present work builds on the literature on markets and contracts. In this literature instead of considering the impossibility of contracting on some states of nature or actions, there are limits on the number of parties that can be part of the same contract. In our paper, we use the more recent set-up on the subject where

[3]The calculation of the social surplus and equilibrium allocation are available upon request to the author.

trading contracts are non-exclusive, hence a common agent can freely sign multiple bilateral trading contracts with different parties.[4] The first theoretical work to consider a general model of contracting between one agent and multiple principals is due to Segal (1999), where, in a general setting, he shows that with the absence of direct externalities, the contracting outcome is efficient.[5] He considers both an offer game - the common agent makes contracting - and a bidding game - where the multiple principals propose contracts to the common agent. However, in a bidding game inefficiencies can arise from the coexistence of offers made by different parties. At this regard, Bernheim and Whinston (1986) consider a bidding game and they show that an equilibrium always exists and it is efficient in the absence of direct externalities.[6]

Therefore, while it has been shown that under some mild conditions a unique efficient outcome always exists, it has been recently proven that there is multiplicity in the equilibrium payoffs, Chiesa and Denicoló (2009).[7] Restricting to non-linear schedules, the payoffs of the principals - the ones that offer the trading contract - depend on the transfers or fixed payment that they can ask for the equilibrium amount traded. In a common agency framework, this is determined by the threat of any principal to be excluded from trade. This threat pins down to which type of latent contracts are submitted.[8] If the principals submit latent contracts such that there exists optimal collective replacement of a given principal, then the equilibrium transfers are truthful in the sense that each principal appropriates his marginal contribution to the surplus. Conversely, if the latent contracts submitted are such that any principal is unilaterally replaced by the most efficient seller, the equilibrium transfers are the ones where the rent of the common agent is minimized.

In a more recent paper, Chiesa and Denicoló (2012) undertake comparative statics of this two prominent equilibria. They show that the minimum-rent equilibrium is Pareto dominant from the point of view of the parties that offer the contract and state that truthful strategies are not necessarily very attractive. This comes from the fact that the potential gains from trade are irrelevant of the distribution of rents and those who submit contracts always prefers an equilibrium where the distribution is more favorable to them. We challenge their finding by introducing a previous stage where parties can undertake specific self-investment before the contracting stage takes place. By introducing an investment stage in our game, we are able to compare equilibria with regards to the social welfare obtained. This analysis has not been carried out in the non-exclusive contract literature, where the different type of equilibria are only a different way to distribute the rents form trade between the agent to the principals, and the social surplus remains unchanged. Therefore, in our model, the redistribution of rents has implications on the investment decisions of the parties and on the final dimension of the potential gains from trade.

At this regard, the present work is closely related to the hold-up literature where an early formulation is due to Klein, Crawford and Alchian (1978) and Williamson (1979, 1983). In those papers, the hold-up problem arises due to the fact that parties are unable to bargain over specific investment once they have been made because they are unverifiable. In our model the hold-up problem does not arise from non verifiability but from the fact that investments are not contractable. One of the main conclusions of the literature is that in the absence of any contract, investment is likely to be inefficiently low under most plausible bargaining

[4]Earlier studies have centered the analysis on exclusive contracts, this is the spirit of Akerlof (1970), Rothschild and Stiglitz (1976) and Aghion and Bolton (1987).

[5]There are no externalities when the principals' payoffs depend only on their own trade with the agent.

[6]The authors consider an equilibrium where the principals submit global truthful schedules.

[7]Indeed, the authors show that the set of equilibrium payoffs is a semi-open hyper-rectangle. Additionally, Martimort and Stole (2009) show multiplicity of equilibria in a public common agency game and offer strategies for equilibrium refinement.

[8]Latent contracts are those submitted by principals but are never accepted in equilibrium.

games.[9] The literature has centered in ways of designing a mechanism to re-store the efficient levels of investment as in Aghion, Dewatripont and Rey (1994) and Chung (1991). However, our model is one à la Williamson where ex-ante contracts are not considered. This relates to the recent literature on competition and the hold-up problem as in Cole, Mailath and Postlewaite (2001a, 2001b); Mailath, Postlewaite and Samuelson (2013); Felli and Roberts (2012); Makowski (2004) and Samuelson (2013). However, all those models consider a matching mechanism where, once investment has been done, agents decide on the trading partner. Hence, investment works as a mechanism to increase the outside option which gives higher incentives to invest. Departing from this literature, we work with specific investment where the offering part of the market competes by offering trading contracts to the monopolistic side.

Finally, the present work is closely related to Roig (2013), where the author considers specific investment in a common agency game with non-exclusive contracts. However, in that model, the common buyer undertakes cooperative investment and the paper focuses on how the buyer decides to allocate the investment and the emerging market structure in equilibrium.

3 Model

We consider a bilateral investment game where a monopolistic buyer trades with many ex-ante identical sellers. In our model, there are N exogenous sellers indexed by $i \in \{1, ..., N\}$, who produce an homogeneous input that is specific for the buyer. We have a game à la Williamson consisting of two stages that are played sequentially. In stage one, specific investment takes place. Here, seller 1 can invest in a cost-reducing technology. In this way, he differentiates himself from the rest of sellers by producing more efficiently. The amount of investment is a continuous variable $\sigma \geq 0$, with a convex technology $\psi(\sigma)$. The buyer also undertakes specific investment to enhance her valuation of the total amount traded. She takes a binary decision whether or not to invest $b = \{0, 1\}$, and incurs a fixed costs of K. The way we model investment,

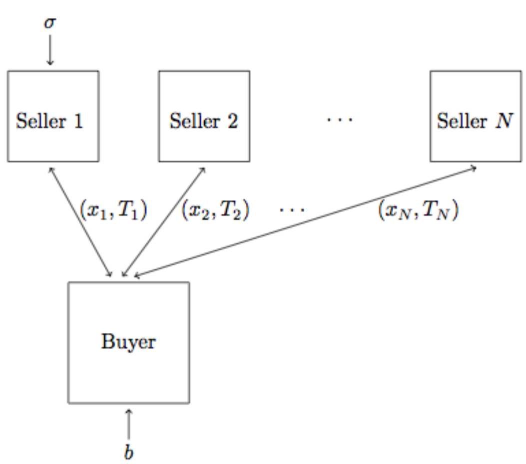


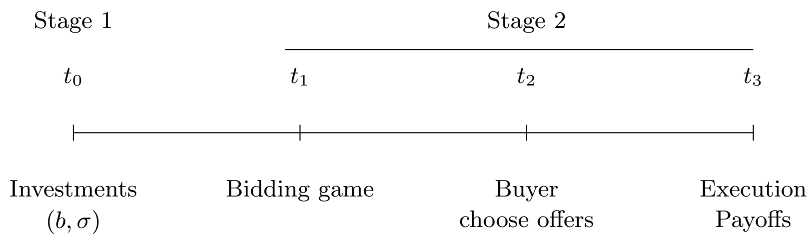
Figure 4: Bilateral investment game with N competing sellers.

is consistent with a situation where the producer of the input decides on how much to invest to reduce the

[9]See also Grossman and Hart (1986), Hart and Moore (1990).

cost of production, and the buyer decides on whether to adapt his production process to the input provided by the sellers.[10]

In stage two, trading occurs and each seller trades an amount of good with the common buyer. Here, as in Chiesa and Denicoló (2009), we consider a bidding game where each seller submits a menu of trading contracts to the buyer and the latter choses which contract to select from each of the sellers. Hence, a typical trading contract is a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ is the quantity seller i is willing to supply and $T_i \geq 0$ is the corresponding total payment or transfer from the buyer to seller i. A strategy for a seller consists of an arbitrary number of trading contracts, and we require that the null contract is always offered in equilibrium. This way, trade is voluntary and the buyer can exclude any active seller from trade. We summarize the moves of the game in the following timeline.



Hence, once the parties have undertaken specific investment, each seller submits a menu of trading contracts to the buyer, who then picks a single contract from each menu and payoffs are executed. The model is one of complete information, and the level of investment becomes public knowledge once this is undertaken. Hence, our equilibrium concept is sub-game perfect Nash (SPNE).

3.1 Payoffs and Surplus

The payoffs of the buyer and the sellers are quasi-linear in transfers.[11] The buyer obtains

$$\Pi(b) = U(X \mid b) - \sum_{i=1}^{N} T_i - K \times \mathbb{1}\{b = 1\}, \tag{3.1}$$

where $X = \sum_{i=1}^{N} x_i$ is the total quantity traded. Seller's 1 payoff is

$$\pi_1(\sigma) = T_1 - C(x_1 \mid \sigma) - \psi(\sigma). \tag{3.2}$$

For the rest of the sellers, the payoff does not directly depend on the investment profile and it is equal to

$$\pi_i = T_i - C(x_i), \quad \forall i \neq 1. \tag{3.3}$$

Finally, given the vectors of investment (b, σ) the total gains from trade are

[10] By allowing the investment of the buyer to be discreet, we are able to recover the efficient investment profile in equilibrium. If the investment were continuous we will always have underinvestment in equilibrium due to the hold-up problem.

[11] This assumption means that all parties have a constant utility for money. Furthermore, this allows us both to technically reduce the complexity of the problem and focus our analysis on welfare comparison.

$$TS^*(b,\sigma) = \max_{x_1,\ldots,x_n} \left[U(x_1 + \ldots + x_n \mid b) - C(x_1 \mid \sigma) - \sum_{i \neq 1} C(x_i) \right], \tag{3.4}$$

and $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ is the vector of quantities that solves the problem. For later use, we denote by $X^*_{\{-H\}} = \sum_{i \notin H} x_i^*$, the sum of the efficient quantities without taking the quantities of the subset of sellers H. We finish by stating the assumptions regarding the previous utility and costs functions:

1. $U'_x(\cdot) > 0, U''_{xx}(\cdot) < 0, U(X \mid \beta) > U(X)$ and $U'_x(X \mid \beta) > U'_x(X)$.
2. $C'_x(\cdot) > 0,\ C''_{xx}(\cdot) > 0,\ C'_\sigma(\cdot) < 0,\ C''_{x\sigma}(\cdot) < 0,\ \psi'_\sigma(\sigma) > 0$ and $\psi''_{\sigma\sigma}(\sigma) > 0$
3. (Inada) $\lim_{X \to 0} U'_x(\cdot) = +\infty$, $\lim_{X \to \infty} U'_x(\cdot) = 0$, $\lim_{x_i \to 0} C'_x(\cdot) = 0$ and $\lim_{x_i \to \infty} C'_x(\cdot) = +\infty$.

Assumption (1) and (2) imply that the utility and the marginal cost of production is respectively increasing and decreasing with investment. The utility function is concave and the cost function is convex. Also the cost of investment is increasing and convex. Finally, assumption (3) ensures that we have an interior solution and that it is optimal to trade with all the sellers.

3.2 Trading Game

An equilibrium in our trading game is characterized by an equilibrium allocation and an equilibrium transfer denoted by the pair (x^e, T^e). The literature has shown that under some mild conditions, the equilibrium allocation is unique and this is the one that maximizes the potential gains from trade i.e. $x_i^e = x_i^*$, $\forall i \in N$. Later in section (4.1), we see that such conditions are satisfied in our model and we state some properties of the allocation that will be later used in the paper. With respect to the equilibrium transfers, and consequently payoffs, Chiesa and Deicoló (2009), have shown that there exist multiple equilibria.[12]

Intuitively, the multiplicity of equilibrium payoffs is due to the fact that any seller can offer many other contracts than the one that will be accepted in equilibrium. Those out of equilibrium contracts, constraint the transfer that any seller i can request for his prescribed equilibrium quantity, and hence his payoffs. The upper-bound of the transfer that each seller can ask for supplying the efficient amount x^* depends on the threat of being excluded from trade, and this is related on how aggressively any other seller bids for quantities that are larger than the efficient ones. Later in section (4.2) we characterize the equilibrium transfers.

4 Analysis

Our equilibrium concept is sub-game perfect Nash equilibrium (SPNE), hence we solve the model backwards. First, we describe the properties of the equilibrium allocation in the trading game. Later, we characterize the equilibrium investment profile. Finally, we undertake equilibrium comparison where we rank equilibria with regards to Pareto dominance and social welfare.

[12] Indeed the authors provide a complete characterization of the set of the equilibrium payoffs in a common agency and they prove that the set of equilibrium payoffs form an hyper-rectangle.

4.1 Allocation in the trading game

The equilibrium allocation in the trading game depends on the investment decisions undertaken in stage one. Here, we characterize the equilibrium allocation for a given vector of investment (b, σ). Because the cost function depends only on the individual amount traded, we do not have direct externalities in our model. Hence, using the results on previous literature we know that in equilibrium each seller submits a trading contract containing the efficient allocation. The argument goes as follows, absent direct externalities, if all sellers but i have offered the efficient allocation, it is optimal for seller i to also offer the efficient allocation Bernheim and Whinston (1996) and Segal (1999). Given the schedules of trading contracts submitted by all sellers, each seller's payoff is not affected by all other trading contracts "individual efficiency". This leads to "bilateral efficiency", which means that, when submitting a contract, the seller maximizes the potential gains from trade that can be generated between him and the common buyer.

$$U\left(\bar{X}^*_{-i} + x^*_i \mid b\right) - \sum_{j\neq i} T_j - C(x^*_i \mid \sigma) > U\left(\bar{X}^*_{-i} + \hat{x}_i \mid b\right) - \sum_{j\neq i} T_j - C(\hat{x}_i \mid \sigma);\ \text{for any } \hat{x}_i \geq 0.$$

As a result, the efficient allocation is offered in equilibrium and this is implicitly given by the following system of equations:

$$\begin{aligned} U'_x(X^* \mid b) &= C'_x(x^*_1 \mid \sigma) \qquad \text{for} \quad 1. \\ U'_x(X^* \mid b) &= C'_x(x^*_i) \qquad\quad \forall i \neq 1, \end{aligned} \tag{4.1}$$

where the marginal utility equals the marginal costs of production.

We now study how the equilibrium allocation changes with the investment undertaken in stage one. The result is shown in the following lemma, whose proof is relegated to the appendix which directly comes from the previous system of equations.

Lemma 1. *i) An increase on the investment by seller* 1 *increases the amount of trade between the buyer and seller* 1*, but decreases the amount of trade with all other sellers. The total amount traded increases.*

$$\frac{\partial}{\partial \sigma} X^* > 0, \quad \frac{dx^*_1}{d\sigma} > 0 \quad \text{and} \quad \frac{dx^*_j}{d\sigma} < 0, \quad \forall j \neq 1,$$

ii) when the buyer invests the equilibrium allocation of each seller increases.

$$x^*_i(\beta, \sigma) > x^*_i(0, \sigma) \qquad \forall i \in N.$$

For a given investment of the buyer, the higher the investment undertaken by seller 1 the more efficient he becomes with respect to the other sellers. In the trading stage, this entails that the buyer substitutes trading from the other sellers to seller 1. The magnitude on the decrease of trade to the other sellers we denote by "allocative externality", and this depends on the primitives of the economy. Nevertheless, the economy in aggregate is more efficient and the total amount traded is higher.[13] With regard to the second part of the

[13] In our model we have assumed that sellers produce products that are homogeneous. However, the degree of substitutability will have a strong effect on the externality that investment by one seller creates to the equilibrium allocation of others. With perfect homogenous products, the buyer can perfectly substitute products from sellers and we expect that the indirect externality coming from investment of seller 1 is big. In our model the degree of substitutability depends on the primitives of the model,

lemma, observe that as long as the investment of the seller does not change, the relative efficiency of the sellers stays the same, therefore, if the buyer decides to invest, she trades a higher amount with every seller. Therefore, the investment of the buyer works as a public good as all sellers benefit from this investment.

Once we have established the characteristics of the equilibrium allocation, we proceed to characterize the equilibrium transfers. As recently shown by the literature of markets and contracts, there is multiplicity of equilibria.

4.2 Equilibrium Transfers

In this section, we are going to follow closely the work of Chiesa and Deicoló (2009). They state that the equilibrium transfer from the part offering the trading contracts equals the threat of being excluded from trade, and this threat depends on how aggressive other sellers compete for out of equilibrium trading quantities.[14] Departing from their work, we allowing any subset of active sellers J to offer out of equilibrium trading contracts to collectively replace any seller i.

Definition 1. *J_i is the set of sellers who offer an out of equilibrium contract to collectively replace any seller i, and $|J_i|$ is the cardinality of the set.*

Hence, by adapting the equilibrium transfers from Chiesa and Deicoló (2009) by allowing specific investment and by the set of sellers J, the equilibrium transfer for seller 1 is

$$T_1^{J_1} = U\left(X^* \mid b\right) - \left(V\left(\cdot \mid X^*_{-\{J,i\}}, b\right) + \sum_{j \in J} C(x_j^*)\right), \tag{4.2}$$

where

$$V\left(\cdot \mid X^*_{-\{J,i\}}, b\right) = \max_{\{x_j\}_{j \in J}} \left[U\left(X^*_{-\{J,i\}} + \sum_{j \in J} x_j \mid b\right) - \sum_{j \in J} C(x_j)\right] \tag{4.3}$$

is the maximal trading surplus that can be generated given a fixed amount of $X^*_{-\{J,i\}}$. We denote by the vector $\tilde{\mathbf{x}}^J = \left\{\tilde{x}_j^J, ..., \tilde{x}_J^J\right\}$ for $j \in J$ the trading quantities that maximize the problem above. The equilibrium transfers for the rest of the sellers is easily obtained.[15]

The equilibrium transfer represented in (4.2) tells us that the maximum that a seller can ask, for selling the efficient amount, equals the maximum trading surplus, minus the optimal one that can be obtained by the rest of the sellers of the economy holding constant the allocation of the sellers who do not submit out of equilibrium trading contracts. Hence, if the seller was to ask a larger transfer, the buyer will decide to exclude him from trade. Due to the convexity of the production function, in our model, we proxy the degree

that is, on the convexity of the cost function. Conversely, if products have some degree of heterogeneity, the buyer will not reduce much the amount that she trades with other sellers after an increase of investment of seller 1. Therefore, the degree of the indirect externality will depend on the product substitutability. I am thankful to professor Sánchez-Pagués for this observation.

[14]The authors provide a characterization of the strategies for the sellers that will support the equilibrium where there is optimal unilateral replacement. Here, we do not characterize such strategy profiles, but we use its methodology to obtain the equilibrium transfer for any set of sellers J_i who effectuate optimal collective replacement.

[15]Chiesa and Denicoló (2009) provide the characterization of the equilibrium strategies that sustain an equilibrium where there is unilateral replacement. This is shown in proposition (1) of their paper. By following a similar procedure, we can show that the buyer will never get a larger payoffs by selecting a combination of the out of equilibrium trading contracts offered by the sellers. The proof for the case where competition is the most severe, i.e. $J_i = N \setminus \{i\}$ is available upon request to the author and this be can easily extended by any $J \in N$.

of competition by the number of active sellers who submit out of equilibrium contracts to collectively replace any seller i. Therefore, for a given vector of investment (b, σ), the highest degree of competition is obtained when all active sellers submit out of equilibrium trading contracts to replace any seller i, that is, $J = N \setminus \{i\}$. Conversely, the lowest degree of competition is achieved when the set J is a singleton, and there is only one seller that unilaterally replaces any other seller by submitting an out of equilibrium trading contract. A level of intermediate competition happens when the set J is in between the two previous cases.

With the equilibrium transfers that we have obtained we characterize the equilibrium payoffs. This is stated in the following proposition.

Proposition 1. *For a subset of sellers J_i, and a investment profile (b, σ) the equilibrium payoffs are given by:*

$$\pi_1^J(b, \sigma) = TS^*(b, \sigma) - \tilde{TS}_{-1}^J(b) - \psi(\sigma); \text{ for } i = 1, \tag{4.4}$$

$$\pi_i^J(b, \sigma) = TS^*(b, \sigma) - \tilde{TS}_{-i}^J(b, \sigma); \quad \forall i \in N, \text{ and } i \neq 1, \tag{4.5}$$

and the one of the common buyer is

$$\Pi^J(b, \sigma) = TS^*(b, \sigma) - \sum_i \left(TS^*(b, \sigma) - \tilde{TS}_{-i}^J(b, \sigma) \right) - K \times \mathbb{1}\{b = 1\}, \tag{4.6}$$

where $\tilde{TS}_{-1}^J(b) = \max_{\{x_j\}_{j \in J}} \left[U \left(X^*_{-\{J,i\}} + \sum_{j \in J} x_j \mid b \right) - \sum_{j \in J} C(x_j) \right] - \sum_{j \neq J,i} C(x_j^*)$; $\tilde{TS}_{-i}^J(b, \sigma) = \max_{\{x_j\}_{j \in J}} \left[U \left(X^*_{-\{J,i\}} + \sum_{j \in J} x_j \mid b \right) - C(x_1 \mid \sigma) - \sum_{j \in J \setminus \{1\}} C(x_j) \right] - \sum_{j \neq J,i} C(x_j^*)$.
ii) When $|J| < N - 1$ each seller obtains more than his marginal contribution to the surplus.

The proof is relegated to the appendix. Whenever collective replacement is undertaken by all active sellers i.e. $J = N \setminus \{i\}$, we obtain that each seller only appropriates his marginal contribution to the surplus and the trading gains are evenly distributed to all players. In this equilibrium, and for a given investment profile, the partition of the trading surplus is the one that maximizes the rent of the buyer.[16] Conversely, when sellers "tacility" coordinate to allow only a sub set of them to offer out of equilibrium contracts, the distribution of the gains from trade is biased in favor of the sellers and the rent of the common buyer is reduced. Due to the convexity of costs of production, the lower the number of sellers in J the smaller is the rent of the buyer and this is minimized whenever the set J is a singleton.

We finish this section by introducing a lemma that will be very useful for the rest of the paper. This refers to the total amount traded when any seller i is excluded from trade.

Lemma 2. *For any investment profile (b, σ) and for any J, the total amount traded is higher whenever all sellers are active.*

$$X^*(b, \sigma) > X^*_{-\{J,i\}}(b, \sigma) + \sum_{j \in J} \tilde{x}_j^J(b, \sigma).$$

and

$$\tilde{x}_j^{J'}(b, \sigma) > x_j^J(b, \sigma); \quad \forall j \in J, J' \text{ and } J' \subset J.$$

[16]The literature have established that in this case sellers submit truthful schedules. A strategy is said to be truthful relative to a given action if it truly reflects the sellers' marginal preference for another action relative to the given action. However, in a framework with a private common agency with no direct externalities, truthful means that each principal can ask for payments that differ from his true valuations of the proposed trade only by a constant.

The formal proof is in the appendix. The intuition is that, due to the convexity of the cost function, the increase of the total amount traded due to an extra seller always dominates the increase on the amount traded from the subset of sellers J. It is immediate to see that the individual amount that any seller $j \in J$ submits in his out of equilibrium contract, needed to replace seller i, is larger than his efficient amount. Because they aim at excluding one seller, they have to offer a larger amount to the buyer.

We now proceed to study how the equilibrium investment profile depends on the degree of competition among sellers, or equivalently, on the number of sellers who offer an out of equilibrium trading contract to collectively replace any seller i.

4.3 Investment profile

We start by characterizing the efficient investment and we proceed with the equilibrium investment profile. We see that the decisions to invest of both sides of the market depend on how competition between the parties offering the trading contracts take place, since this have a direct effect on the part of the trading surplus that each player is able to appropriate.

4.3.1 Efficient investment

The efficient vector of investment is the one that arises when the investing parties appropriate all the gains coming from investment. The efficient investment is then characterized by the solution of the following system of equations:

$$\psi'_\sigma(\sigma_{\mathbf{E}}) = -C'_\sigma(x_1^*(b, \sigma_{\mathbf{E}}^b) \mid \sigma_{\mathbf{E}}^b), \qquad \forall\, b \tag{4.7}$$

$$K \begin{cases} \leq TS^*(\beta, \sigma_{\mathbf{E}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0) - \Big(\psi(\sigma_{\mathbf{E}}^\beta) - \psi(\sigma_{\mathbf{E}}^0)\Big) \equiv K_{\mathbf{E}} & \text{then } b = \beta \\ > K_{\mathbf{E}} & \text{then } b = 0. \end{cases} \tag{4.8}$$

The seller sets the level of investment such as the marginal benefit equals the marginal cost of investment. Similarly, the buyer invests if the fixed cost of investment K is lower than the net social benefit, which is represented by the threshold $K_{\mathbf{E}}$. A characteristic of the efficient investment profile, that also carries over in equilibrium, is that investments are strategic complements. Hence the more one of the parties invests, the higher are the incentives of the other party to increase own's investment.[17] Here, this result comes from a variant of super-modularity. From lemma (1), we know that the investment of one party always increases the total amount of trade, and through this trade decisions, the value of investment by one party increases the marginal return to the other party's investment.

[17]We have proven in lemma (1) that the amount traded with each seller increases if the buyer is investing, this implies that for a given level of seller's investment we have $x_1^*(\beta, \sigma) > x_1^*(0, \sigma)$ and together with assumption $C''_{x\sigma}(\cdot) < 0$ we obtain that the right hand side of (4.7) increases with the level of investment of the buyer:

$$\partial(rhs) = -C'_\sigma(x_1^*(\beta, \sigma) \mid \sigma) + C'_\sigma(x_1^*(0, \sigma) \mid \sigma) = -\int_{x_1^*(0,\sigma)}^{x_1^*(\beta,\sigma)} C''_{x\sigma}(\tau) d\tau > 0$$

The last equality comes from the fundamental theorem of calculus and by application of lemma (1). A similar argument can be used to see that the investment threshold of the buyer increases with the investment of the seller. In the case that the investments are continuous, we have investment complementarity if the function $TS^*(\beta, \sigma)$ is super-modular in (β, σ) (i.e $TS^*_{\beta\sigma}(\beta, \sigma) > 0$; see *Donald Topkins 1978).*

4.3.2 Equilibrium investment profile

In the efficient investment profile, the marginal cost of investment equals the marginal benefit. In equilibrium, we see that as long as the investing party does not appropriate all the benefits coming from investment, the implementation of the efficient investment profile is not possible. Interestingly, we see that efficiency can only be implemented whenever competition between sellers is severe. In the analysis that follows, we consider both the "intensive" and "extensive" degree of competition. The former takes into account how the sellers establish competition in the trading game, and this depends on the number of sellers who summit out of equilibrium trading contracts to collectively replace any other seller, i.e. the cardinality of the set J. The latter considers how the equilibrium investment profile depends on the number of active sellers in the industry.

4.3.3 "Intensive" degree of competition

The degree of "intensive" ex-post competition is characterized by the threat that any seller i is excluded from trade and this threat depends on the number of sellers J that undertake optimal replacement. Because we consider a noncooperative game, the equilibrium investment decisions are best-response actions. Therefore, an equilibrium in the investing game is:

Definition 2. *A vector* (b^J, σ^J) *constitutes an equilibrium, if and only if*

$$b^J \in arg\max_b \ \Pi^J(b, \sigma^J)$$
$$\sigma^J \in arg\max_\sigma \ \pi_1^J(b^J, \sigma).$$

Because the equilibrium payoff depends on how sellers compete, we obtain a direct link between the equilibrium investment profile and competition. We are first interested to know if the efficient investment profile can be implemented in equilibrium. At this regard, we introduce the following proposition.

Proposition 2. *The efficient investment profile is implementable if and only if competition in the trading game is the most aggressive i.e.* $|J| = N - 1$.

Investment decisions depend on how each party appropriates the gains coming from investment. Whenever competition among sellers is the most severe, each seller obtains his marginal contribution to the gains from trade, hence the investing seller appropriates the increase of the trading surplus coming from investment. We find that this is never the case when sellers "tacitly" coordinate to reduce ex-post competition. In this case, each seller obtains more than his marginal contribution to the surplus, and this distorts the incentives to invest efficiently. Hence, given the investment decision of the buyer, the seller always invests efficiently in the most competitive equilibrium. Because the buyer takes the efficient level of investment, under some values for the fixed cost of investment, we obtain the result. We refer to the appendix for a formal proof.

From the previous result, we can easily characterize the investment profile when competition in the trading game is the most severe. This is introduced in the following corollary whose proof is relegated to the appendix.

Corollary 1. *When, in the most competitive equilibrium, the buyer fails to take the optimal level of investment, the equilibrium investment profile is characterized by underinvestment.*

The previous two results state that the investment decision of the seller is constrained efficient, that is, for a given investment of the common buyer, the seller always takes the efficient investment decision. However, whenever the buyer takes sub-optimal investment decisions, because she does not appropriate all the benefits coming from her investment, the equilibrium investment profile is characterized by the hold-up problem, and both parties underinvest. Downward distortions of investment arise because of strategic complementarity. In the figure below, we compare this equilibrium investment profile from the efficient one. The line in red represents the region where inefficiency occurs and this is characterized by underinvestment.

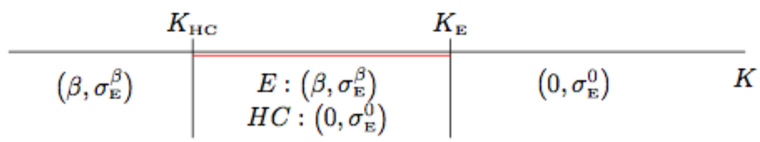


Figure 5: Equilibrium investment profile when the competition ex-post is the most aggressive. In the horizontal line there is the fixed cost of investment of the buyer and K_{HC}, K_E stand for the investing thresholds for the most competitive equilibrium and efficiency respectively. Full efficiency is implemented whenever $K \notin (K_{HC}, K_E)$.

After stating that with lower levels of ex-post competition the efficient investment profile cannot be implemented, we proceed by analyzing what are the characteristics of the investment when sellers "tacitly" coordinate to reduce competition. The result is stated in the following proposition.

Proposition 3. *Whenever sellers "tacitly" coordinate to reduce the level of ex-post competition, i.e.* $J < N - 1$*, we obtain that:*
i) for a given investment of the buyer, the magnitude of seller's over-investment depends on the level of ex-post competition and the degree of the "allocative externality", and this is equal to

$$\gamma(J) = - \sum_{m \notin J,1} \left(\int_{X^*}^{X^*_{-\{J,1\}} + \sum_{j \in J} \tilde{x}_j^J} U''_{xx}(\tau) d\tau \right) \frac{dx_m^*}{d\sigma},$$

and this decreases with the level of ex-post competition i.e. $\frac{\partial \gamma(J)}{\partial J} < 0$.
ii) When the buyer's investment decision is not efficient, this is characterized by underinvestment, and for a given investment of the seller, the region of costs below which the buyer invests increases with the level of ex-post competition i.e. $\frac{\partial K^J}{\partial J} > 0$.

Again the formal proof is in the appendix. Here, we observe that contrary to the case where competition ex-post is the most severe, the investment of the seller is distorted upwards. This is due to the fact that he does not only appropriate all the direct gains coming from his investment, but also part of the payoffs from the other sellers. For a fixed investment of the buyer, we see that $\gamma(J)$ decreases with J. In other words, the lower is the level of competition - which implies a smaller J - the distortion of investment will be larger. Therefore, with the same investment decision of the buyer, the investment of the seller is monotonically decreasing with the level of competition. Moreover, the amount of over-investment depends on how the equilibrium allocation of the non investing sellers changes with respect to the investment of the former.

The larger this "allocative externality" the more the seller over-invests. This is because the threat of being replaced depends on his investment profile through the equilibrium allocation that remains unchanged, the larger the investment the more costly will be to replace him. Hence, the investing seller can ask a larger transfer at the expense of the other sellers. He then enjoys an extra surplus that is "stolen" from the non-investing sellers. This is never the case when competition is the most sever, where the previous effect would vanish due to the envelop condition.

The following corollary states how the equilibrium investment looks like and ex-ante inefficiencies may arise in both sides of the market.

Corollary 2. *Whenever the buyer takes efficient investment decisions the seller over-invests.*
i) If the investment decision of the buyer is not efficient, the inefficiency created is two-sided:
A) the buyers underinvests, and
B) the seller over-invests or underinvest depending on how investment affects the equilibrium allocation of the non investing sellers. Over-investment appears in equilibrium if

$$-\frac{dx_j^*}{d\sigma} > \frac{\int_{x_1^*(\beta,\sigma_E^\beta)}^{x_1^*(0,\sigma_J^0)} C''_{x\sigma}(\tau)d\tau}{(N\setminus\{1\}-J)\times\int_{X^*(0,\sigma_J^0)}^{X^*_{-\{J,i\}}(0,\sigma_J^0)+\sum_{j\in J}\tilde{x}_j(0,\sigma_J^0)} U''_{xx}(\tau)d\tau} = \lambda(J),$$

The formal proof is in the appendix, and the figure below represents the equilibrium investment decisions and how they differ from the efficient ones. One red line represents a situation where inefficiency occurs to

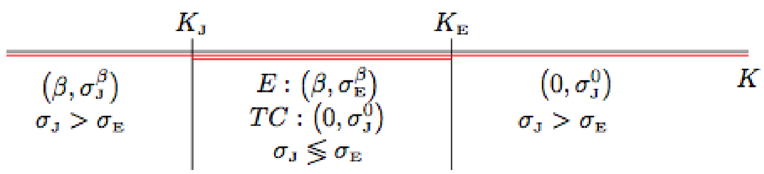


Figure 6: Equilibrium investment profile when the competition ex-post is the most aggressive. In the horizontal line there is the fixed cost of investment of the buyer and K_J, K_E stand for the investing thresholds in equilibrium and efficiency respectively. Full efficiency is never implemented and it can be double-sided when $K \in (K_J, K_E)$.

only one side of the market, and two lines represents inefficiencies arising to both sides of the market.

For the sake of clarity, we proceed to represent graphically the equilibrium investment profile in the different types of equilibria. Accordingly, the evolution of the investment of the seller regarding the level of competition is represented in figure (7). We observe that the investment decision of the seller depends crucially on the level of competition ex-post and the degree of the "allocative externality" that he creates to the other sellers. This does not only determines the slope of the curve represented in the figure, but also the investment decision of the buyer which is represented in figure (8). It is interesting to see that whether the investment threshold is monotone or not, depends on the "allocative externality" that the investing seller creates to his competitors. In general, when the effect that the investing seller have on the equilibrium amount traded of the other sellers is small, a more unfavorable partition of the surplus coming from lower competition dominates the constraining effect on the equilibrium transfers. As a result, a lower level of competition entails that the buyer decides to switch his investment decision from investment to non-investment. Conversely, whenever the "allocative externality" is large, the constraint created to the

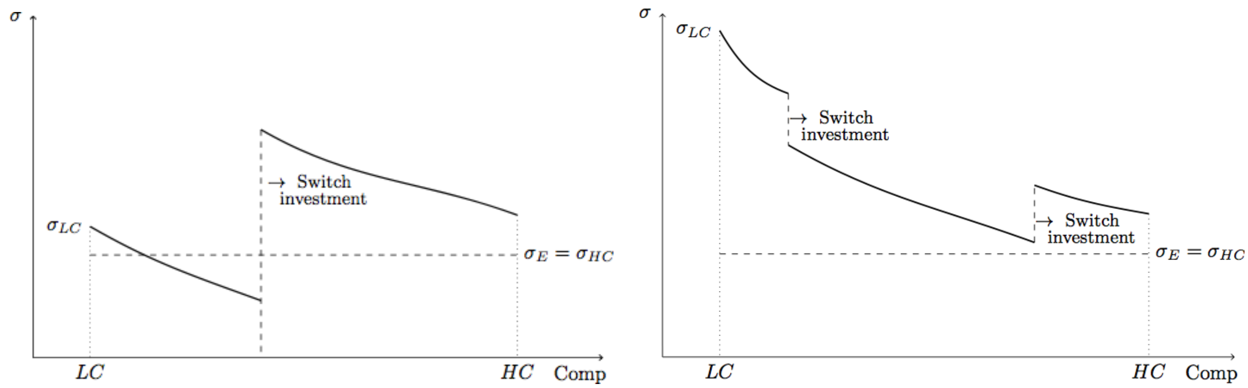


Figure 7: Equilibrium investment profile of the seller depending on the level of ex-post competition. We start from a situation where the buyer invests in the most competitive equilibrium. The picture on the left stands for a situation where the "allocative externality" is moderate and when the buyer switches from investing to non-investing there is a drop on the seller's investment - due to strategic complementarity - that stays below the efficiency level. The one on the right stands for a situation where the "allocative externality" is large and the level of investment is always above the efficiency level. We also see that the buyer's investment decision is not monotone and she may decide to invest with high and low levels of competition but not with intermediate ones.

transfers of the non-investing sellers dominates the more unfavorable partition of the surplus. Here, lower competition might make the buyer to undertake a positive level of investment, that will not come about with higher levels of competition. In this case, replacement by any seller i is cheaper to undertake with a lower number of sellers and this depends on the investment decision of the seller. For an exhaustive analysis

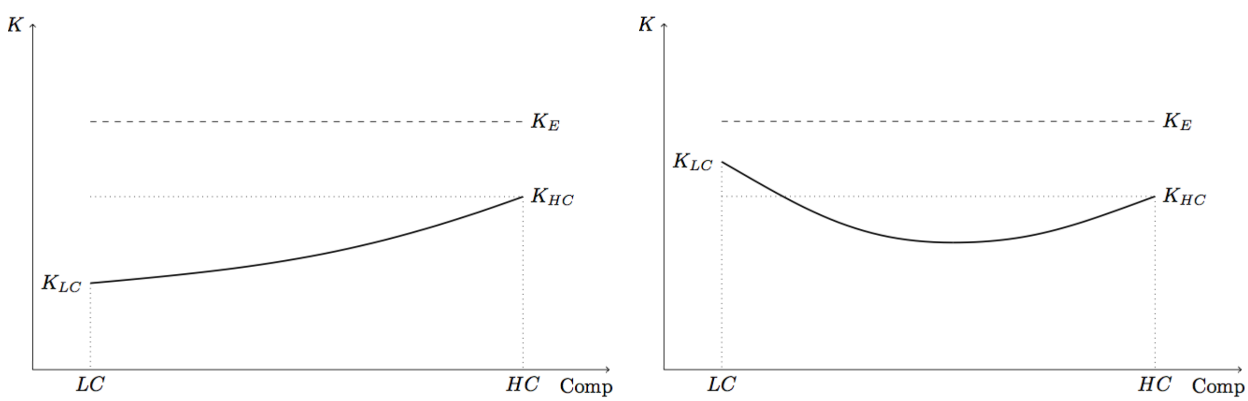


Figure 8: Equilibrium investment thresholds below which the buyer decides to invest. The figure on the left represents a situation where the lower proportion of the gains from trade that the buyer appropriates the milder is competition dominates the higher investment of the seller. The curve in monotonically decreasing form more to less level of competition. The one on the right is a situation where the investment of the seller dominates and there exist a region where for lowers levels of competition the efficient investment for the buyer is restored.

on the comparisons of the thresholds below which the buyer decides to invest we refer to the appendix (C), where we compare the investment thresholds in the most competitive equilibrium with the ones obtained when the sellers decide to "tacitly" coordinate to reduce the level of competition.

So far we have established how the ex-post level of competition affects the investment decisions of both sides of the market. We now proceed to study how the equilibrium investment profile is affected by the number of active sellers in the industry and we denote this, the "extensive" degree of competition in the market.

4.3.4 "Extensive" degree of competition

In this section, we study how the equilibrium investment profile depends on the number of active sellers that offer trading contracts to the common buyer. We obtain that the larger the number of sellers, the higher is the constraint that sellers impose to each other, and hence the lower the transfer that they obtain in equilibrium. Unsurprisingly, this has an effect on the equilibrium investment profile and the result is stated in the following proposition.

Proposition 4. *Full efficiency is implemented regardless of the level of ex-post competition, provided that the number of active sellers is sufficiently large.*

The derivation is in the appendix and this comes from the fact that, as the number of sellers increases, each seller is able to appropriate less from the trading surplus. In the limit, each seller only obtain his marginal contribution to the surplus. The buyer also invests efficiently since with a higher number of active sellers each one of them is able to appropriates less from the trading gains generated by the investment of the former. Whit a large enough number of sellers, the buyer appropriates all the benefits coming from his investment. The link between unilateral investment decisions and the number of active sellers is illustrated in the following picture, where we relate unilateral investment decisions with the number of active sellers.

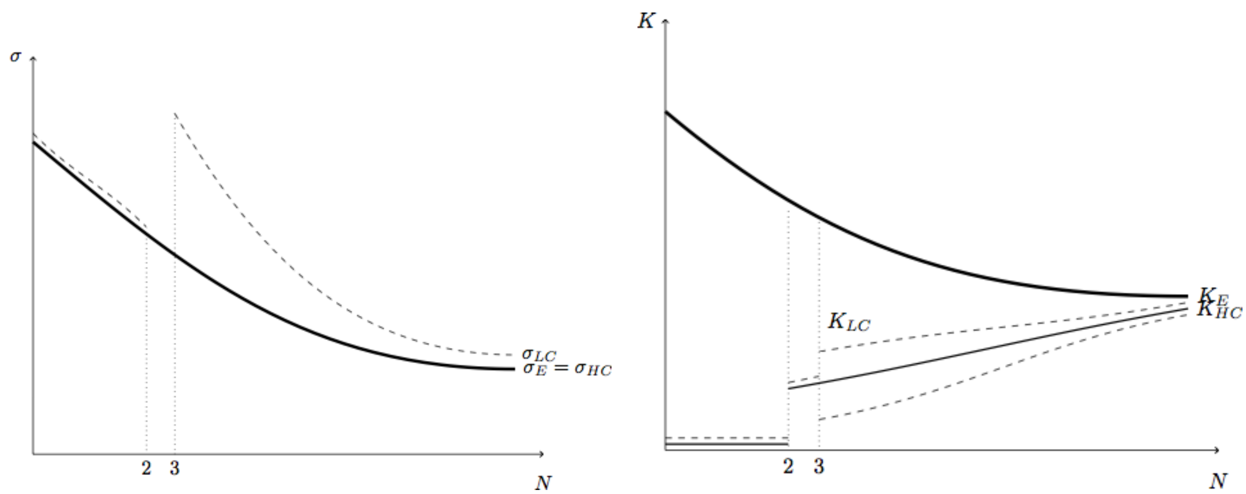


Figure 9: Unilateral investment decisions as a function of number of active sellers. On the left, the investment of the seller and on the right, the threshold below which the buyer decides to invest. The thick solid line stands for the efficient investment profile, the solid line represents a situation where competition is the most severe. The dashed line is the one corresponding to the lowest level of competition.

This illustrates that with one active seller, we have a situation of a bilateral monopoly. Because we consider a bidding game, the buyer is completely held-up and he never invests. Conversely, the investment decision of the seller is the efficient one. When we move to more than two sellers, we start to obtain a situation where the buyer decides to invests, this is because sellers start to compete for the trading contracts and the former is able to appropriate part of the benefits coming from his investment. With low levels of competition, the seller over-invests as he gets more than his marginal contribution to the surplus. However, since competition increases with a larger number of sellers, in the limit the seller is only able to obtain his marginal contribution to the surplus. Moreover, since each seller obtains less rents when the number of sellers increases, the buyer is able to get all the benefits coming for her investment. Thus, when the number of sellers is sufficiently large, the investment equilibrium profile tends to efficiency.

With the characterization of the equilibrium investment profile, we proceed to undertake equilibrium comparison. We start by introducing the concept of Pareto optimality and we indicate which equilibrium is the one preferred by the parties offering the trading contracts. Later, we depart form the analysis of surplus distribution and we establish which equilibrium performs best in terms of social welfare.

5 Comparison of equilibria

We see that both the concept of Pareto dominance and equilibrium ranking in terms of social welfare crucially depend on the "allocative externality" that the investing seller creates to the rest of the competing sellers.

5.1 Pareto optimality

In this section, we analyze which equilibrium gives higher payoffs to the side of the market offering the trading contracts. It is obvious that with a given investment profile, the offering parties will always prefer an equilibrium where the level of competition is less severe. This is always the case when the trading gains remain unchanged.[18] However, since in our model the equilibrium investment profile depends on the level of competition ex-post,"tacitly" coordinating on lowering the level of competition may not always be preferred. For the part of the investing seller, the trade-off is whether a more favorable distribution might have an effect on the investing decision of the buyer. For the non-investing sellers, in addition, there is also the investment decision of the seller and how this affects their equilibrium allocation. Hence, considering the degree of the "allocative externality" created by the investing seller will be fundamental. The result is presented in the following proposition.

Proposition 5. *Whenever the "allocative externality" is small i.e.* $\frac{\partial K_J}{\partial J} > 0$ *:*
i) the least competitive equilibrium is Pareto dominant for the sellers if the investment decision of the buyer is equilibrium invariant,
ii) otherwise, Pareto dominance is attained with an intermediate level of competition.
Whenever the "allocative externality" is big i.e. $\frac{\partial K_J}{\partial J} < 0$*, the least competitive equilibrium is never Pareto dominant,*
i) while it is always preferred for the investing seller,
ii) the non-investing sellers are always better-off with the most competitive equilibrium.

The formal proof is relegated to the appendix and we see that this result comes at no surprise. For convenience we have defined the "allocative externality" of being either big or small depending on whether

[18] This is the case in Chiesa and Denicoló (2009, 2012) that state that the minimum rent equilibrium - the least competitive - is Pareto dominant. This result comes form the fact that as long as parties do not invest, the social surplus stays the same, all the sellers are identical and an equilibrium of the trading game represents only a split of the surplus.

the investment threshold of the buyer either increases or decreases with regards to the level of ex-post competition. We have shown that a larger "allocative externality" creates a larger investment of the seller, and due to investment complementarity, this gives more incentives for the buyer to invest. A lower level of competition is associated to both an increase of the incentives of the seller and a less favorable partition of the trading gains to the buyer. However, if the "allocative externality" is sufficiently big, the increase of the equilibrium investment of the seller dominates the lower partition of the surplus and the buyer is better-off with a lower level of competition. As a result, the incentives for the buyer to invest increase with lower levels of ex-post competition.

Whenever the "allocative externality" is large the investing seller is able to appropriate a substantial part of the payoffs of the non investing sellers. With a sufficiently big "allocative externality" this makes him to prefer a less competitive equilibrium regardless of the investment decision of the buyer. For the same token, the non-investing parties will prefer a more competitive equilibrium because the investing seller has less incentives to increase investment. Conversely, whenever the "allocative externality" turns out to be small two effects explain our result. On the one hand, the level of investment of the seller is quite similar regardless of the level of competition ex-post, and the part of payoffs from the non-investing sellers appropriated by the investing seller is small. Hence, with the same investment decision of the buyer, all sellers prefer a more favorable distribution of the surplus at the expense of the buyer, and this is achieved by "tacitly" coordinating on a less competitive equilibrium. However, if the buyer decides not to invest, whenever the level of competition is low, parties may prefer a higher competition as the total surplus generated is bigger. Those results are represented in figure below, where individual payoffs of the offering side of the market are related to the level of competition ex-post.

Because in equilibrium the investment of one party is affected by the other one and those are strategic complements, we can be sure that the investing seller and the buyer are always better-off the higher the investments in equilibrium. However, this is not always the case for the non investing sellers where the investment of the buyer and the seller go in opposite directions. The seller who is investing obtains a higher payoff as the investment is superior in a less competitive equilibrium when the investment of the buyer is equilibrium invariant. For the non-investing sellers, they will prefer a division of the surplus that is more favorable as long as the investment of the seller is not very different in all possible equilibria. As we have seen, an increase of the level of investment by the seller creates an "allocative externality" to the other sellers as they trade less with the common buyer. This negative externality then dominates a more favorable partition of the surplus when the difference of investments of the seller is large.

Therefore, Pareto dominance of a less competitive equilibrium is not robust when we introduce specific investment by the trading partners. The intuition behind this result is that under some situations and because both parts of the market are undertaking specific investment, it might be better to agree on a more even distribution of potential gains from trade between all parties than a more asymmetric one, since the latter might induce one of the parties to withdraw from investment.

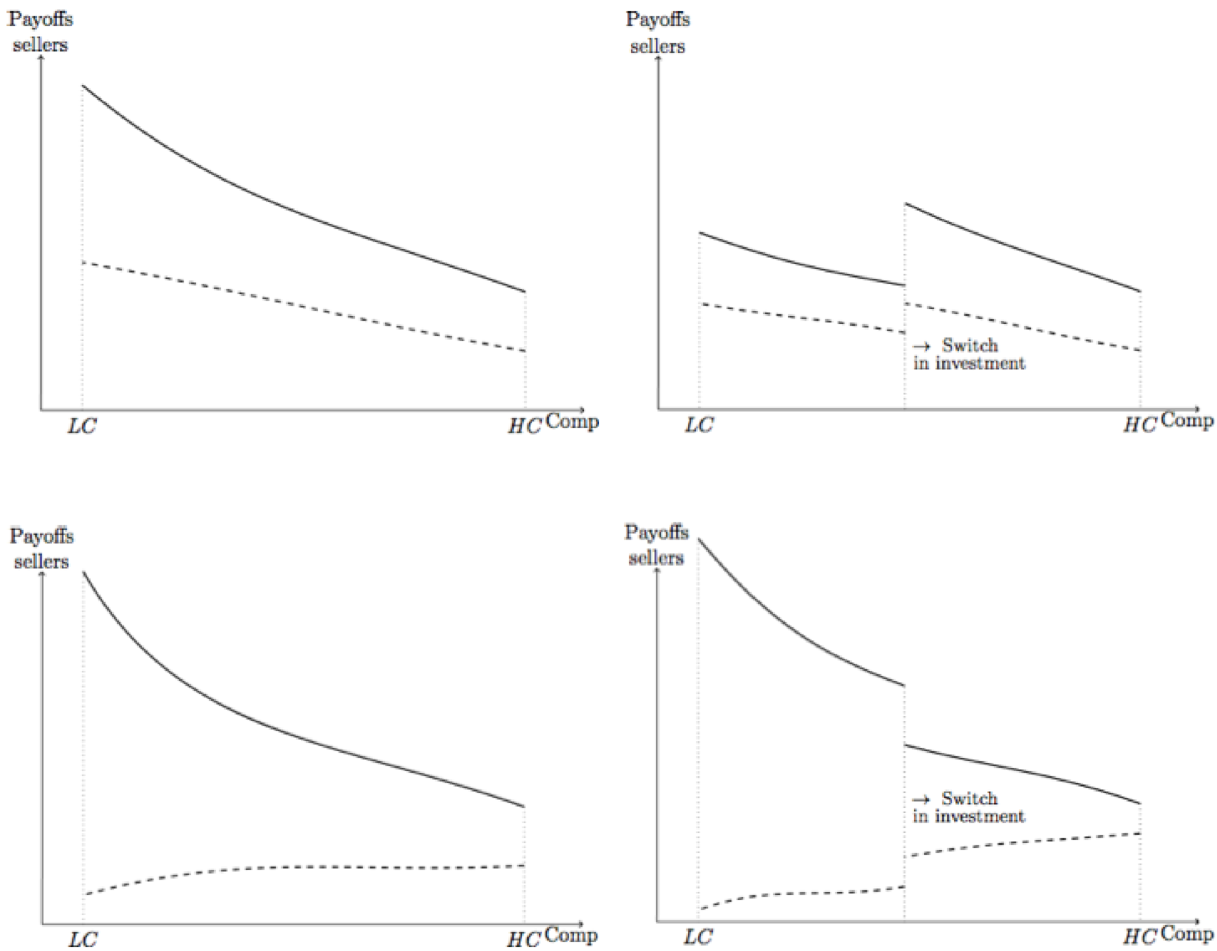


Figure 10: Payoffs of the sellers with the level of ex-post competition. The figure in the top represents the situation where the "allocative externality" is small and the one on the bottom whenever this is large. The black line represents the payoff of the investing seller and the dashed line is the one for the non-investing sellers. The picture on the left represents a situation where the equilibrium investment of the buyer is equilibrium invariant and the on the right is a situation where the investment of the buyer changes regarding the equilibrium ex-post.

5.2 Social Welfare

Here we rank equilibria according to the social surplus obtained in equilibrium. This is equal to the total gains from trade minus the costs of investment:

$$SW^*(b,\sigma) = TS^*(b,\sigma) - K \times \mathbb{1}\{b=1\} - \psi(\sigma).$$

From the previous analysis we have seen that ex-ante inefficiencies are more prompt to emerge whenever the ex-post competition is less severe. Hence, the most aggressive equilibrium in general performs better in terms of social welfare. However, we have also established that investments in our setting are strategic complements. Therefore, we surprisingly find that decreasing the level of ex-post competition may entail larger social surpluses whenever the "allocative externality" created to the non-investing sellers is sufficiency

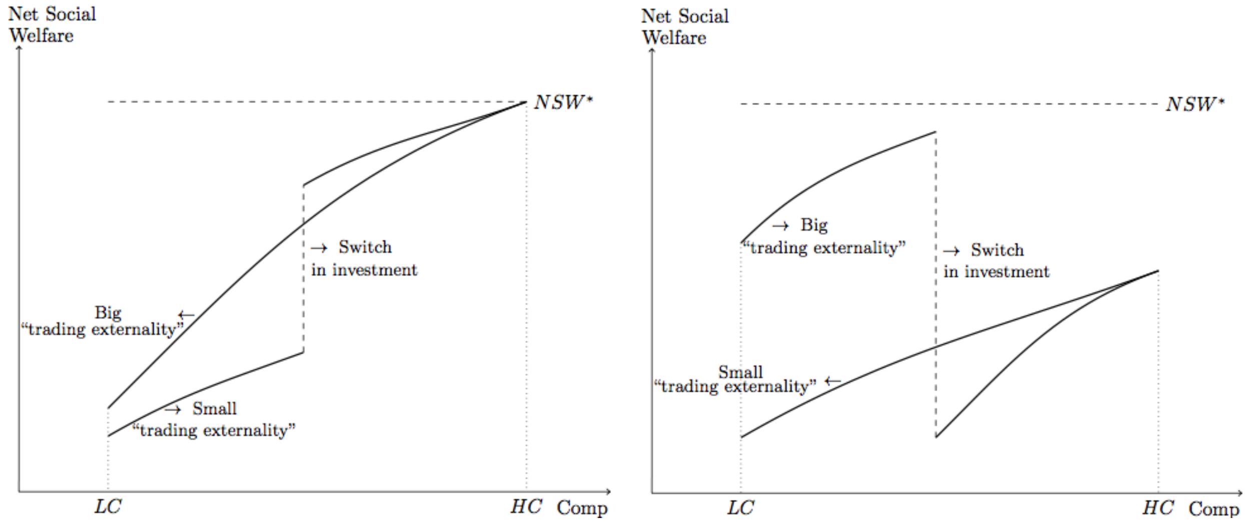


Figure 11: Net social welfare as a function of the level of competition. The figure on the left stands for the situation where the investment of the buyer in the higher level of competition coincides with the efficient one and the figure on the right is when the contrary occurs. Jumps in the curves stand for a switch in the investing decision of the buyer and higher slope of the curves represented larger levels of "allocative externality". Observe that only when the investment decision of the buyer is not efficient in the highest level of competition, there is the possibility that the buyer invests whenever the competition is less severe. This can only occur whenever the "allocative externality" is sufficiently big.

big. Hence, the inefficiencies in the investment arising to the side of the seller works as a mechanism to restore the efficient investment of the buyer. As long as this latter investment has a meaningful contribution to the social surplus, this is welfare enhancing. The following theorem states the result of this section.

Theorem 1. *When in the most competitive equilibrium, the investment decision of the buyer is not efficient and the "allocative externality" is sufficiently big, social welfare is maximized with an intermediate level of competition. Otherwise, maximization always occurs with the highest level of competition.*

Here, we have seen that a high distortion of the investment created by one side of the market works as a mechanism to restore the efficient investment of the other side and social welfare is increased. The results stated in the proposition are represented in figure (11), where we see that the net social welfare is monotonically decreasing with the lowest level of competition in situations where the investment decision of the buyer is equilibrium invariant. The figure presents jumps whenever the investment decision of the buyer depends on the equilibrium played. Social welfare is maximized with an intermediate level of competition if the decision of the buyer switches from non-investing to investing. This is represented on the right hand side of the figure.[19]

[19] We are aware that the result stated in the proposition depends on the fact that only one of the sellers is investing. As long as investment heavily distorts the allocation of the non-investing parties, any subgroup might be cheaper than collective replacement. As a result the buyer enjoys higher payoffs in the former and she undertakes the efficient investment decision. We then wonder if this result is robust by allowing all sellers take specific investment. We talk about this case later in the discussion.

6 Conclusion and discussion

We have seen that with the introduction of specific investment in both sides of the market, the equilibrium played in the trading game is not only a way to redistribute rents between the sellers and the buyer but it has also an effect on the size of the potential gains from trade. In previous analysis, it has been stated that an equilibrium where the competition is maximal is not necessarily very attractive from the part of the market offering the contracts. This is because the offering part can "tacitly" coordinate to reduce competition in order to obtain a more favorable partition of the potential gains from trade. However, in the current work we have seen that, in general, an equilibrium with higher competition displays more efficient investment profiles which implies a larger social surplus. Yet, we have found that as long as the "allocative externality" created by the investing seller is sufficiently big, "tacitly" coordinating to reduce competition might perform better than higher competition in terms of social welfare. This result might seem counterintuitive as a better outcome is achieved with a larger distortion. However, this is possible in our model due to the fact that specific investment is undertaken by both sides of the market. We have shown that with a large "allocative externality" the investing seller invests more and replacement by a smaller subgroup of sellers might be cheaper. As a result, the buyer might decide to invest in a lower competitive equilibrium.

Therefore, one question to ask is: what equilibrium is likely to arise? This issue has already been addressed in the literature but there does not exist a clear answer.[20] However, in our model this is a question of great importance due to its affects on welfare and not only because of redistributive implications. Despite the fact that we can not be sure of the equilibrium played in the trading game, we think that an external player might induce some set of equilibria to be played. Is might be the case in markets that has been recently liberalized, where an external player has to ensure that real competition exists in the market with the objective to maximize total social welfare. In our model, inducing equilibria have to do with the number of trading contracts that sellers are allowed to offer in equilibrium, and a regulator might be able to induce one equilibrium or the other by imposing restrictions on the number of these contracts.[21]

For simplicity we have considered the case that only one of the sellers knows the new technology and can undertake specific investment to reduce the cost of production. At this regard, we have considered the analysis of a bilateral investing game. Nevertheless, a natural extension of the model is to consider the case where all sellers have knowledge of the new technology allowing them to reduce the production costs. However, even if unilateral investment decisions are easy to obtain and coincide with the ones obtained in this paper, the characterization of the equilibrium investment profile looks more complicated. This is so because the investment of the buyer between the sellers are strategic complements while the ones between sellers are strategic substitutes. However, we believe that the strategic substitutability between investment of the sellers is of second order, and an increase in the investment of the buyer makes all sellers to invest more in equilibrium. Hence, the same results will be obtained but now the preference over one equilibrium coincides for all sellers. A major change might be that replacement undertaken by a smaller subgroup might not be less costly than the one undertaken by a larger group. Therefore, we may lose the result of higher social welfare whenever competition is less intense.

Another possible extension is to consider a setting without a monopolistic buyer. In this case, non-

[20]Some of the works addressing equilibrium selection are *Martimort and Stole (2009)* and *Klemperer and Meyer (1989).*

[21]For a formalization of the minimum cardinality that will support the least competitive equilibrium, we refer to the original work of Chiesa and Denicoló (2009) proposition (3). We conjecture that any equilibrium where competition is aggressive, the number of trading contracts needs to be larger.

exclusivity also comes from the fact that a seller can sign multiple non-exclusive contracts with different buyers. In such a case a buyer differentiates and creates an indirect externality to the others if she decides to invest. We believe that the equilibrium menus offered are complicated to obtain, and we conjecture that the competitive advantage that the buyer obtains with respect to the rest might induce him to over-invest. At this regard, the model might have some similarities with the ones regarding investment and matching. Finally, in the present paper we have not considered a contract mechanism that is able to restore the efficient level of investment. Therefore, it is left for a topic of future research, if there exist a pre-contractual design that would enable to achieve the efficient equilibrium investment profile is situations where sellers "tacitly" coordinate to reduce the level of ex-post competition.[22]

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Appendices

A Appendix

Here we consider some lemmas that will be useful for the analysis of equilibrium investment profile.

Lemma 3. *The total gains from trade are bigger when the buyer invests, that is,* $TS^*(\beta,\sigma^\beta) > TS^*(0,\sigma^0)$.

Proof. This is easy to see because:

$$\begin{aligned}
TS^*(\beta,\sigma^\beta) &= U(X^*(\beta,\sigma^\beta)\mid\beta) - C(x_1^*(\beta,\sigma^\beta)\mid\sigma^\beta) - \sum_{i\neq 1} C(x_i^*(\beta,\sigma^\beta)) \\
&> U(X^*(\beta,\sigma^0)\mid\beta) - C(x_1^*(\beta,\sigma^\beta)\mid\sigma^\beta) - \sum_{i\neq 1} C(x_i^*(\beta,\sigma^\beta)) \\
&= U(X^*(\beta,\sigma^0)\mid\beta) - U(X^*(\beta,\sigma^0)) + U(X^*(\beta,\sigma^0)) - C(x_1^*(\beta,\sigma^\beta)\mid\sigma^\beta) - \sum_{i\neq 1} C(x_i^*(\beta,\sigma^\beta)) \\
&\geq U(X^*(\beta,\sigma^0)\mid\beta) - U(X^*(\beta,\sigma^0)) + TS^*(0,\sigma^0) \\
\implies \quad & TS^*(\beta,\sigma^\beta) - TS^*(0,\sigma^0) \geq U(X^*(\beta,\sigma^0)\mid\beta) - U(X^*(\beta,\sigma^0)) > 0
\end{aligned}$$

The first inequality comes from efficiency and the last inequality comes by assumption $U(X^*\mid\beta) - U(X^*) > 0 \quad \forall X$. □

Lemma 4. *The total gains from trade are is bigger with a higher investment of the seller,* $TS^*(b,\sigma\prime) > TS^*(b,\sigma)$ *for* $\sigma\prime > \sigma$.

Proof. We consider the case where $b = 0$ but the case where $b = \beta$ is analogous.

$$\begin{aligned}
TS^*(0,\sigma) &= U(X^*(0,\sigma)) - C(x_1^*(0,\sigma)\mid\sigma) - \sum_{i\neq 1} C(x_i^*(0,\sigma)) \\
&< U(X^*(0,\sigma)) - C(x_1^*(0,\sigma)\mid\sigma\prime) - \sum_{i\neq 1} C(x_i^*(0,\sigma)) + U(X^*(0,\sigma\prime)) - U(X^*(0,\sigma\prime)) \\
&< U(X^*(0,\sigma)) - U(X^*(0,\sigma\prime)) + TS^*(0,\sigma\prime) \\
\implies \quad & TS^*(0,\sigma\prime) - TS^*(0,\sigma) \geq U(X^*(0,\sigma\prime)) - U(X^*(0,\sigma)) = \int_{X^*(0,\sigma)}^{X^*(0,\sigma\prime)} U_x'(\tau)d\tau > 0
\end{aligned}$$

where the strict inequality comes from lemma (1) that $X^*(0,\sigma\prime) > X^*(0,\sigma)$ for any $\sigma\prime > \sigma$. □

Lemma 5. *The increase on the total gains from trade by an extra seller are higher when the buyer is investing:*

$$TS^*(\beta,\sigma^\beta) - TS^*_{-i}(\beta,\sigma^\beta) \geq TS^*(0,\sigma^0) - TS^*_{-i}(0,\sigma^0) \quad \textit{for} \quad i \neq 1$$

Proof. We will make explicit use of lemma (3). Observe that the previous expression is equivalent to $TS^*(\beta,\sigma^\beta) - TS^*(0,\sigma^0) \geq TS^*_{-i}(\beta,\sigma^\beta) - TS^*_{-i}(0,\sigma^0)$ and by lemma (3) we know that

$$TS^*(\beta,\sigma^\beta) - TS^*(0,\sigma^0) \geq U(X^*(\beta,\sigma^\beta)\mid\beta) - U(X^*(\beta,\sigma^\beta)) = \underline{D},$$

we proceed by obtaining the upper bound of the difference $TS^*_{-i}(\beta,\sigma^\beta) - TS^*_{-i}(0,\sigma^0)$:

$$
\begin{aligned}
&TS^*_{-i}(\beta,\sigma^\beta) = U\left(\sum_{j\neq 1}\tilde{x}^{N-1}(\beta,\sigma^\beta) \mid \beta\right) - C\left(\tilde{x}_1^{N-1}(\beta,\sigma^\beta) \mid \sigma^\beta\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j^{N-1}(\beta,\sigma^\beta)\right)\\
&\leq U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - C\left(\tilde{x}_1^{N-1}(\beta,\sigma^0) \mid \sigma^0\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j^{N-1}(\beta,\sigma^0)\right)\\
&= U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right) + U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right) - C\left(\tilde{x}_1^{N-1}(\beta,\sigma^0) \mid \sigma^0\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j^{N-1}(\beta,\sigma^0)\right)\\
&\leq U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right) + TS^*_{-i}(0,\sigma^0)\\
&\implies \quad TS^*_{-i}(\beta,\sigma^\beta) - TS^*_{-i}(0,\sigma^0) \leq U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right) = \overline{D}.
\end{aligned}
$$

Where the first two inequalities come from efficiency and it is easy to see that $\underline{D} - \overline{D} > 0$ as

$$
\begin{aligned}
\underline{D} - \overline{D} &= U(X^*(\beta,\sigma^0) \mid \beta) - U(X^*(\beta,\sigma^0)) - \left[U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right)\right]\\
&= U(X^*(\beta,\sigma^0) \mid \beta) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0) \mid \beta\right) - \left[U(X^*(\beta,\sigma^0)) - U\left(\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)\right)\right]\\
&= \int_{\sum_{j\neq i}\tilde{x}^{N-1}(\beta,\sigma^0)}^{X^*(\beta,\sigma^0)} \left(U'_x(\tau \mid \beta) - U'_x(\tau)\right) d\tau > 0
\end{aligned}
$$

which is positive by lemma (2) and assumption $U'_x(\tau|\beta) > U'_x(\tau)$ □

Lemma 6. *The increase on the total welfare given by an extra seller is higher when the buyer is investing,*

$$TS^*(\beta,\sigma^\beta) - TS^*_{-1}(\beta) - \psi(\sigma^\beta) \geq TS^*(0,\sigma^0) - TS^*_{-1}(0) - \psi(\sigma^0).$$

Proof. We are going to proceed by contradiction. Take that the net profit of the seller when the agent invest is strictly lower than when he does not invest.

$$TS^*(\beta,\sigma^\beta) - TS^*_{-1}(\beta) - \psi(\sigma^\beta) < TS^*(0,\sigma^0) - TS^*_{-1}(0) - \psi(\sigma^0),$$

and by the optimality of the investment decision of the seller we have shown that $\sigma^\beta > \sigma^0$ but then the seller could reduce the amount of investment when the agent invest and set $\sigma^\beta = \sigma^0$ but the we have that $\psi(\sigma^\beta) = \psi(\sigma^0)$ and the previous expression is:

$$TS^*(\beta,\sigma^0) - TS^*_{-1}(\beta) < TS^*(0,\sigma^0) - TS^*_{-1}(0),$$

but this contradicts what we have proven in lemma (5). □

Lemma 7. *The difference obtained in the gains from trade form collective replacement to any other replacement undertaken by $J < N-1$ is higher if the buyer is investing.*

$$TS^*_{-1}(\beta) - \tilde{TS}^J_{-1}(\beta,\sigma^\beta_J) > TS^*_{-1}(0) - \tilde{TS}^J_{-1}(0,\sigma^0_J).$$

Proof. By using the same procedure as in lemma (5) we obtain:

$$
\begin{aligned}
&TS^*_{-1}(\beta) - \tilde{TS}^J_{-1}(\beta, \sigma^\beta_{\mathbf{J}}) - TS^*_{-1}(0) + \tilde{TS}^J_{-1}(0, \sigma^0_{\mathbf{J}}) \\
&\geq U\left(\sum_{j\neq i} \tilde{x}^{N-1}_j \mid \beta\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J} \tilde{x}_j \mid \beta\right) - \left[U\left(\sum_{j\neq i} \tilde{x}^{N-1}_j\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J} \tilde{x}_1\right)\right] \\
&= \int_{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_1}^{\sum_{j\neq 1}\tilde{x}^{N-1}_j} \big(U'_x(\tau \mid \beta) - U'_x(\tau)\big)\, d\tau > 0
\end{aligned}
$$

and this is positive by lemma (2) and by assumption $U(X \mid \beta) > U(X)$. Also by the facts that the investment of the seller makes the difference to increase. □

Lemma 8. *When the buyer invests in J' but not in J when $J' \subset J$, the non-investing seller is always better in the most competitive equilibrium. For any $J' \subset J$ then:*

$$TS^*(0, \sigma^0_J) - \tilde{TS}^J_{-i}(0, \sigma^0_J) > TS^*(\beta, \sigma^\beta_{J'}) - \tilde{TS}^{J'}_{-i}(\beta, \sigma^\beta_{J'})$$

Proof. We proceed by contradiction, consider the case that:

$$TS^*(0, \sigma^0_{\mathbf{J}}) - \tilde{TS}^J_{-i}(0, \sigma^0_{\mathbf{J}}) < TS^*(\beta, \sigma^\beta_{\mathbf{J}'}) - \tilde{TS}^{J'}_{-i}(\beta, \sigma^\beta_{\mathbf{J}'})$$

but then as it has been shown before, it cannot be the case that $K_{J'} > K_J$, since in this case replacement by a lower number of sellers is more expensive and then it cannot be that with J' then $b = \beta$ and at the same time that for J then $b = 0$. Then, we reach a contradiction. Finally, by the monotonicity of the investment of the seller we obtain that the maximum payoff for the non investing sellers is in the highest competitive equilibrium which is given by: $TS^*(0, \sigma^0_{\mathbf{HC}}) - TS^*_{-i}(0, \sigma^0_{\mathbf{HC}})$. □

B Appendix

Proof of lemma (1): We start by proving how seller's investment affects the equilibrium allocation. We consider the case where $b = 0$ but this is analogous for $b = \beta$. Differentiating the first-order conditions given in (4.1) for x^*_j with respect to σ we obtain:

$$U''_{xx}(X^*) \times \sum_{h=1}^N \frac{dx^*_h}{d\sigma} = C''_{xx}(x^*_j) \times \frac{dx^*_j}{d\sigma}. \tag{B.1}$$

Since the left hand side is independent of j we find that all $\frac{dx^*_j}{d\sigma}$ have the same sign. Now suppose also $\frac{dx^*_1}{d\sigma}$ has that same sign. Then also the sum has that same sign and since $U''_{xx}(\cdot) < 0$ and $C''_{xx}(\cdot) > 0$ this leads to a contradiction. Now suppose $\frac{dx^*_1}{d\sigma} < 0$. The other signs therefore have to be positive. By (B.1) we find $\sum_{h=1}^N \frac{dx^*_h}{d\sigma} < 0$. But the first-order condition for x^*_1, differentiated with respect to σ is:

$$U''_{xx}(X^*) \times \sum_{h=1}^N \frac{dx^*_h}{d\sigma} = C''_{xx}(x^*_1 \mid \sigma) \times \frac{dx^*_1}{d\sigma} + C''_{x\sigma}(x^*_1 \mid \sigma), \tag{B.2}$$

which would then have a positive left hand side and a negative right hand side due to $C''_{x\sigma}(\cdot) < 0$ - a contradiction. We thus have shown the second and third point. Again by (B.1) the first claim follows from $\frac{\partial}{\partial\sigma} X^* = \sum_{h=1}^N \frac{dx^*_h}{d\sigma}$ and the level of the "allocative externality" is implicitly characterized in expression (B.1). We proceed by analyzing the effect that the investment of the buyer has on the equilibrium allocation. Again, we are going to make use of the conditions for the equilibrium allocation represented in equation (4.1), and for a fixed investment of the seller we get:

$$C'_x(x_1^* \mid \sigma) = U'_x(X^* \mid \beta) > U'_x(X^*) = C'_x(x_1^* \mid \sigma) \qquad \text{for} \quad 1$$
$$C'_x(x_i^*) = U'_x(X^* \mid \beta) > U'_x(X^*) = C'_x(x_i^*) \qquad \text{for} \quad i \neq 1$$

The strict inequality is by assumption and by the convexity of the cost function we obtain the result.

Proof of proposition (1): Following Chiesa and Denicoló (2009), we have obtained that the equilibrium transfer for seller 1 is

$$T_1^J = U\left(X^* \mid b\right) - \left(V\left(\cdot \mid X^*_{-\{J,i\}}, b\right) + \sum_{j\in J} C(x_j^*)\right),$$

Finally, by operating, we rewrite the equilibrium transfer like

$$\begin{aligned}
T_1^J &= U(X^* \mid b) - \max_{\{x_j\}_J}\left[U\left(X^*_{-\{J,i\}} + \sum_{j\in J} x_j \mid b\right) - \sum_{j\in J} C(x_j)\right] - \sum_{j\in J} C(x_j^*)\\
&= U(X^* \mid b) - \sum_{j\in J} C(x_j^*) - \left[U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j^J \mid b\right) - \sum_{j\in J} C(\tilde{x}_j^J)\right]\\
&= U(X^* \mid b) - \sum_{j\in J} C(x_j^*) - \left[U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j^J \mid b\right) - \sum_{j\in J} C(\tilde{x}_j^J)\right] + \left[\sum_{j\notin J,1}\left(C(x_j^*) - C(x_j^*)\right)\right]\\
&+ \left[C(x_1^* \mid \sigma) - C(x_1^* \mid \sigma)\right]\\
&= TS^*(b,\sigma) - \left[U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j^J \mid b\right) - \sum_{j\in J} C(\tilde{x}_j^J) - \sum_{j\neq J,1} C(x_j^*)\right] + C(x_1^* \mid \sigma)\\
&= TS^*(b,\sigma) - \tilde{TS}^J_{-i}(b,\sigma) + C(x_1^* \mid \sigma),
\end{aligned}$$

and by putting this to the payoff functions in (3.1) and (3.3), we obtain the equilibrium payoffs stated in the proposition.

We proceed to show point (ii). It states that for a given investment profile (b,σ) each sellers obtains more than his marginal contribution when they coordinate on reducing competition ex-post. This is equivalent to show that $TS^*_{-i}(b,\sigma) > \tilde{TS}^J_{-i}(b,\sigma)$. We consider the case where $b = 0$ but $b = \beta$ is similar, and we take the equilibrium transfer for the investing seller.

$$\begin{aligned}
\tilde{TS}^J_{-1}(\cdot) &= U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j \mid b\right) - \sum_{j\in J} C(\tilde{x}_j) - \sum_{j\neq J,1} C(x_j^*)\\
&= U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j\right) - \sum_{j\in J} C(\tilde{x}_j) - \sum_{j\neq J,1} C(x_j^*) + \left[U\left(\sum_{j\neq 1}\tilde{x}_j^{N-1}\right) - U\left(\sum_{j\neq 1}\tilde{x}_j^{N-1}\right)\right]\\
&\leq U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j\right) - U\left(\sum_{j\neq i}\tilde{x}_j^{N-1}\right) + TS^*_{-i}(\cdot)\\
&\Longrightarrow TS^*_{-1}(\cdot) - \tilde{TS}^J_{-1}(\cdot) \geq U\left(\sum_{j\neq 1}\tilde{x}_j^{N-1}\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j\right) = \int_{X^*_{-\{J,1\}} + \sum_{j\in J}\tilde{x}_j}^{\sum_{j\neq 1}\tilde{x}_j^{N-1}} U'_x(\tau)d\tau > 0
\end{aligned}$$

Where the last inequality comes from lemma (2). From the above, it is also true that for any $J' \subset J$ we obtain that $\tilde{TS}^J_{-i}(\cdot) > \tilde{TS}^{J'}_{-i}(\cdot)$.

Proof of lemma (2): We are going to consider the case where $b = 0$ but this is similar for $b = \beta$. Also, consider any $J \subset N$. Whenever the investment profile is the same, we know that $\sum_{h \neq J,i} x_h^* = X^*_{-\{J,i\}}$, and the expression above is equivalent to $\sum_{j\in J} x_j^* + x_i^* > \sum_{j \in J} \tilde{x}_j^J$. Therefore since $x_i^* > 0$ if $\sum_{j\in J}\left(x_j^* - \tilde{x}_j^J\right) > 0$ we are done. Observe that for a given investment profile, if the above it is true, it has to be true for any $j \in J$, hence $x_j^* > \tilde{x}_j^J$. If the contrary occurs, $x_j^* < \tilde{x}_j^J$, then from the equilibrium allocation we have:

$$U'_x\left(X^*_{-\{J,i\}} + \sum_{j\in J} \tilde{x}_j^J\right) = C'_x(\tilde{x}_j) > C'_x(x_j^*) = U'_x(X^*),$$

and by concavity of U we prove the claim. The previous also implies that for any $j \in J$ we have $\tilde{x}_j^J > x_j^*$. Using the same procedure we can easily prove that for any $J' \subseteq J$ we have:

$$X^*_{-\{J,i\}} + \sum_{j\in J} \tilde{x}_j^J \geq X^*_{-\{J',i\}} + \sum_{j\in J'} \tilde{x}_j^{J'},$$

and by using the same argument as before, we obtain that $\tilde{x}_j^{J'} \geq \tilde{x}_j^J$.

Proof of proposition (2): As it will be clear later, depending on the fixed cost parameter the buyer undertakes the efficient investment. Therefore, in order to show existence of efficiency in the equilibrium investment profile, we pay attention to seller's investment. We first show the "if" part of the proposition. The payoff of the seller in the most competitive equilibrium is:

$$\pi_1^{HC} = TS^*(b,\sigma) - TS^*_{-1}(b),$$

and the term $TS^*_{-1}(b)$ does not depend on the amount invested σ. Therefore using $TS^*(b,\sigma)$ given by expression (3.4) and by the envelope-theorem, the first-order condition for the seller 1 is given by:

$$\psi'_\sigma(\sigma_{\mathbf{HC}}) = -C'_\sigma(x_1^*(b,\sigma^b_{\mathbf{HC}})|\sigma^b_{\mathbf{HC}}), \tag{B.3}$$

and this coincides with the efficient one obtained in expression (4.7). Because the seller receives the full marginal social surplus from his own investment, he becomes the residual claimant and invests efficiently. Therefore, whenever the investment decision of the buyer coincides with the efficient one i.e. $b_{\mathbf{HC}} = b_{\mathbf{E}}$ the equilibrium vector in the most competitive equilibrium is efficient.

To show the "only if" part, we take any $J \subset N$ and $J \neq N \setminus \{1\}$, and we obtain that the equilibrium payoffs of seller 1 is:

$$\pi_1^J(b,\sigma) = TS^*(b,\sigma) - \tilde{TS}^J_{-1}(b,\sigma),$$

and calculating the first order condition and applying the envelope theorem we obtain that the equilibrium investment profile is characterized by:

$$\psi'_\sigma(\sigma) = -C'_\sigma(x_1^*(b,\sigma^b)|\sigma^b) - \frac{\partial\left(\tilde{TS}^J_{-1}(b,\sigma)\right)}{\partial\sigma},$$

where the extra term depends on the investment of the seller from the allocation that remains unchanged $X^*_{-\{J,1\}}(b,\sigma)$. As a result, $\frac{\partial\left(\tilde{TS}^J_{-1}(b,\sigma)\right)}{\partial\sigma} \neq 0$ and this creates a distortion of the investment of the seller. Hence, we conclude that full efficiency is only implemented whenever the competition between sellers is the most severe.

Proof of corollary (1): The investment decision of the seller is the one in proposition (2) and the one for the buyer in the most competitive equilibrium is:

$$K \begin{cases} \leq TS^*(\beta, \sigma_{\mathbf{E}}^{\beta}) - TS^*(0, \sigma_{\mathbf{E}}^{0}) - \kappa_{\mathbf{HC}} \equiv K_{\mathbf{HC}} & \text{then } b = \beta \\ > K_{\mathbf{HC}} & \text{then } b = 0 \end{cases} \tag{B.4}$$

where the term $\kappa_{\mathbf{HC}}$ is the difference in the payoff of the sellers when the buyer decides to invest and it is equal to:

$$\begin{aligned} \kappa_{\mathbf{HC}} &\equiv \pi_1^{HC}(\beta, \sigma_{\mathbf{E}}^{\beta}) - \pi_1^{HC}(0, \sigma_{\mathbf{E}}^{0}) + \sum_{i \neq 1} \left[\pi_i^{HC}(\beta, \sigma_{\mathbf{E}}^{\beta})) - \pi_i^{HC}(0, \sigma_{\mathbf{E}}^{0})) \right] \\ &= TS^*(\beta, \sigma^{\beta}) - TS^*_{-1}(\beta) - TS^*(0, \sigma^0) + TS^*_{-1}(0) \\ &+ \sum_{i \neq 1} \left[TS^*(\beta, \sigma^{\beta}) - TS^*_{-i}(\beta, \sigma^{\beta}) - TS^*(0, \sigma^0) + TS^*_{-i}(0, \sigma^0) \right] \end{aligned}$$

The magnitude $\kappa_{\mathbf{HC}}$ then represents how much the sellers benefit from the investment of the buyer and are the gains that cannot be appropriated by the latter. By making an explicit use of the lemmas in appendix (A) we show that the appropriation of the gains by the sellers is bigger than the cost of investment $\kappa_{\mathbf{HC}} > \psi(\sigma_{\mathbf{E}}^{\beta}) - \psi(\sigma_{\mathbf{E}}^{0})$. We show that $\kappa > \psi(\sigma_{\mathbf{E}}^{\beta}) - \psi(\sigma_{\mathbf{E}}^{0})$ by splitting κ in two parts $A = \sum_{i \neq 1} \left[TS^*(\beta, \sigma^{\beta}) - TS^*_{-i}(\beta, \sigma^{\beta}) - TS^*(0, \sigma^0) + TS^*_{-i}(0, \sigma^0) \right]$ and $B = TS^*(\beta, \sigma^{\beta}) - TS^*_{-1}(\beta) - TS^*(0, \sigma^0) + TS^*_{-1}(0)$. In lemma (5), we show that $A > 0$ and in lemma (6) we show that $B > \psi(\sigma_{\mathbf{E}}^{\beta}) - \psi(\sigma_{\mathbf{E}}^{0})$.

This implies that the threshold of the cost of the buyer below which she invests is lower compared to the efficient one $K_{\mathbf{HC}} < K_{\mathbf{E}}$. Thus, as the buyer cannot appropriate all the gains coming from his investment she underinvests whenever the fix cost of investment is $K \in (K_{\mathbf{HC}}, K_{\mathbf{E}})$ as $b_{\mathbf{HC}} \neq b_{\mathbf{E}} = \beta$. Finally, since investments are strategic complements, this implies that the seller also underinvests in equilibrium, i.e. $\sigma_{\mathbf{HC}} < \sigma_{\mathbf{E}}$. Hence, as long as the investment of the seller increases the total amount traded as shown in lemma (1), the threshold for the investment of the buyer also increases with the investment of the seller. $\frac{\partial K_{HC}}{\partial \sigma} = -\frac{\partial \kappa_{\mathbf{HC}}}{\partial \sigma} > 0$. And this coincides with the change of the payoffs for the buyer.

$$\frac{\partial \Pi^{HC}(\cdot)}{\partial \sigma} = -\frac{\partial \kappa_{\mathbf{HC}}}{\partial \sigma} = -\sum_{i \neq 1} \int_{x_1^*(\beta,\sigma) + \tilde{x}_1^{N-1}(0,\sigma)}^{\tilde{x}_1^{N-1}(\beta,\sigma) + x_1^*(0,\sigma)} C''_{x\sigma}(\tau) d\tau > 0,$$

and this is positive by $\tilde{x}_1^{N-1}(\beta, \sigma) + x_1^*(0, \sigma) > x_1^*(\beta, \sigma) + \tilde{x}_1^{N-1}(0, \sigma)$ and assumption $C''_{x\sigma}(\cdot) < 0$.

Proof of proposition (3): We start by showing point (i). From proposition (2) we know that the seller's investment fails to be efficient. Here, we show that there exist over-investment and we give it's magnitude. We take the first order condition for the seller. Then, from the equilibrium payoff of the seller 1 and the envelope condition, we obtain that for any $J < N - 1$:

$$\begin{aligned} \psi'_\sigma(\sigma_{\mathbf{J}}) &= -C'_\sigma(x_1^*(b, \sigma_{\mathbf{J}}) \mid \sigma_{\mathbf{J}}) - \sum_{m \neq J,1} \left(U'_x \left(X^*_{-\{J,1\}} + \sum_{j \in J} \tilde{x}_j \right) - C'_x(x_j^*) \right) \times \frac{dx_m^*}{d\sigma} \\ &= -C'_\sigma(x_1^*(b, \sigma_{\mathbf{J}}) \mid \sigma_{\mathbf{J}}) - \sum_{m \neq J,1} \left(U'_x \left(X^*_{-\{J,1\}} + \sum_{j \in J} \tilde{x}_i \right) - U'_x(X^*) \right) \times \frac{dx_m^*}{d\sigma}, \end{aligned} \tag{B.5}$$

where the transformation in the second line is due to the fact that, at the equilibrium allocation, marginal benefit equals marginal cost, i.e. $U'_x(X^*) = C'_x(x_j^*),\ \forall j \in N$. Comparing this condition with the efficient one in (4.7), we see that the difference is the additional term $\gamma(J) \equiv -\sum_{m \neq J,1} \left(U'_x \left(X^*_{-\{J,1\}} + \sum_{j \in J} \tilde{x}_j \right) - U'_x(X^*) \right) \frac{dx_m^*}{d\sigma}$, and by applying the fundamental theorem of calculus we get:

$$\gamma(J) \equiv -\sum_{m\neq J,1}\left(U'_x\left(X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j\right)-U'_x(X^*)\right)\times\frac{dx^*_m}{d\sigma} = -\sum_{m\neq J,1}\left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j}U''_{xx}(\tau)d\tau\right)\times\frac{dx^*_m}{d\sigma}>0,$$

and the whole expression is positive. By lemma (2) and the concavity of U, we know that the part in brackets is positive. By lemma (1) we know that the amount traded with the sellers that are not investing is decreasing with the amount invested by the seller. Therefore, this term is strictly positive which means that the seller over-invests and it's magnitude depends on the "allocative externality" that the investment of the seller creates to the non-investing sellers. In order to show that the degree of over-investment decreases with the level of competition, i.e. $\frac{\partial\gamma(F)}{\partial J}<0$, we calculate how the previous expression varies with an increase in J. By applying Leibniz rule we obtain:

$$\frac{\partial\gamma(J)}{\partial J}=\left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j}U''_{xx}(\tau)d\tau\right)\times\frac{dx^*_m}{d\sigma}-U''_{xx}\left(X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j\right)\times\underbrace{\frac{\partial\left(X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j\right)}{\partial J}}_{(+)}\times\frac{dx^*_m}{d\sigma}<0.$$

And the sign of the specified is due to lemma (2).
Again point (ii) is more involved and the investment decision of the buyer is given by:

$$K\begin{cases}\leq TS^*(\beta,\sigma^\beta_{\mathbf{J}})-TS^*(0,\sigma^0_{\mathbf{J}})-\kappa_{\mathbf{J}}\equiv K_{\mathbf{J}} & \text{then } b=\beta\\ >K_{\mathbf{J}} & \text{then } b=0\end{cases}\tag{B.6}$$

where the extra term $\kappa_{\mathbf{J}}$ is the difference in the payoff of the sellers when the buyer invests. Again this represents how much the sellers benefit form the investment of the buyer and those benefits can not be appropriate by the latter.

$$\begin{aligned}\kappa_{\mathbf{J}} &\equiv \pi^J_1(\beta,\sigma^\beta_{\mathbf{J}})-\pi^J_1(0,\sigma^0_{\mathbf{J}})+\sum_{i\neq 1}\left[\pi^J_i(\beta,\sigma^\beta_{\mathbf{J}}))-\pi^J_i(0,\sigma^0_{\mathbf{J}}))\right]\\ &=TS^*(\beta,\sigma^\beta)-\tilde{TS}^i_{-1}(\beta,\sigma^\beta)-TS^*(0,\sigma^0)+\tilde{TS}^i_{-1}(0,\sigma^0)\\ &+\sum_{i\neq 1}\left[SW^*(\beta,\sigma^\beta)-\tilde{TS}^1_{-i}(\beta,\sigma^\beta)-TS^*(0,\sigma^0)+\tilde{TS}^1_{-i}(0,\sigma^0)\right].\end{aligned}$$

And for a given investment of the seller, the threshold of the buyer is below the efficient one i.e. $K_{\mathbf{LC}}<K_{\mathbf{E}}$. Again, this comes from the fact that in equilibrium, the buyer is not able to appropriate all benefits coming from investment and the proof is the same as in corollary (1) and we do not repeat it here. Moreover, with a given level of investment by the investing seller, each seller is able to appropriate a larger amount of the gains from trade as shown in proposition (1), and this reduces the rents of the buyer. Therefore, the incentives for the buyer to invest decreases the lower the level of competition ex-post.

Proof of corollary (2): The first point comes directly from proposition (3). Point (A) states that whenever the buyer does not take the efficient investment decision, this is characterized by underinvestment. We take the extreme case, and this is when the investment ex-post is the least competitive equilibrium. Hence, we have to show that $K_{LC}<K_E$ and hence:

$$\begin{aligned}K_{LC}\leq K_E \iff & TS^*(\beta,\sigma^\beta_{\mathbf{LC}})-TS^*(0,\sigma^0_{\mathbf{LC}})-\kappa_{LC}\leq TS^*(\beta,\sigma^\beta_{\mathbf{E}})-TS^*(0,\sigma^0_{\mathbf{E}})-\left(\psi\left(\sigma^\beta_{\mathbf{E}}\right)-\psi\left(\sigma^0_{\mathbf{E}}\right)\right)\\ \rightarrow\quad & \psi\left(\sigma^\beta_{\mathbf{E}}\right)-\psi\left(\sigma^0_{\mathbf{E}}\right)\leq TS^*(\beta,\sigma^\beta_{\mathbf{E}})-\tilde{TS}^i_{-1}(\beta,\sigma^\beta_{\mathbf{LC}})-\left(TS^*(0,\sigma^0_{\mathbf{E}})-\tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})\right)\\ &+\sum_{i\neq 1}\left[TS^*(\beta,\sigma^\beta_{\mathbf{E}})-\tilde{TS}^1_{-i}(\beta,\sigma^\beta_{\mathbf{LC}})-\left(TS^*(0,\sigma^0_{\mathbf{E}})-\tilde{TS}^1_{-i}(0,\sigma^0_{\mathbf{LC}})\right).\right]\end{aligned}$$

By using the same procedure as in lemma (5) we can see that in general the last part in brackets is positive. Therefore, to show the above, we need that:

$$TS^*(\beta,\sigma^\beta_{\mathbf{E}}) - \tilde{TS}^i_{-1}(\beta,\sigma^\beta_{\mathbf{LC}}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})\right) \geq \psi\left(\sigma^\beta_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right)$$

Here we apply lemma (7) that states $\tilde{TS}^i_{-1}(\beta,\sigma^\beta_{\mathbf{LC}}) < TS^*_{-1}(\beta) - TS^*_{-1}(0) + \tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})$, and by introducing this the the previous expression we have that:

$$TS^*(\beta,\sigma^\beta_{\mathbf{E}}) - \tilde{TS}^i_{-1}(\beta,\sigma^\beta_{\mathbf{LC}}) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})\right) > TS^*(\beta,\sigma^\beta_{\mathbf{E}}) - \left[TS^*_{-1}(\beta) - TS^*_{-1}(0) + \tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})\right]$$
$$- \left(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}^i_{-1}(0,\sigma^0_{\mathbf{LC}})\right) = TS^*(\beta,\sigma^\beta_{\mathbf{E}}) - TS^*_{-1}(\beta) - \left(TS^*(0,\sigma^0_{\mathbf{E}}) - TS^*_{-1}(0)\right) > \psi\left(\sigma^\beta_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right)$$

where the last inequality comes by lemma (6). Hence, in general the threshold of investment in the least competitive equilibrium is lower than the efficiency one.
To show point B) we need to compare the right hand side of the expression determining the investment in the least competitive equilibrium (B.5) evaluated at $b=0$, with the right hand side of expression determining the efficient investment (4.7) evaluated at $b=\beta$.

$$Rhs^{LC}(0) = -C'_\sigma(x^*_1(0,\sigma^0_{\mathbf{J}}) \mid \sigma) - \sum_{m\neq J,1}\left(\int_{X^*}^{X^*_{-\{J1\}}+\sum_{j\in J}\tilde{x}_j} U''_{xx}(\tau)d\tau\right)\frac{dx^*_m}{d\sigma}; \quad Rhs^E(\beta) = -C'_\sigma(x^*_1(\beta,\sigma^\beta_{\mathbf{E}}) \mid \sigma),$$

and we will have that the efficient investment is higher if:

$$Rhs^E(\beta) > Rhs^{LC}(0) \to -C'_\sigma(x^*_1(\beta,\sigma^\beta) \mid \sigma) > -C'_\sigma(x^*_1(0,\sigma^0) \mid \sigma) - \sum_{m\neq J,1}\left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_j} U''_{xx}(\tau)d\tau\right)\frac{dx^*_m}{d\sigma}$$

$$\to \int_{x^*_1(\beta,\sigma^\beta)}^{x^*_1(0,\sigma^0)} C''_{x\sigma}(\tau)d\tau > -\sum_{m\neq J,1}\left(\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_i} U''_{xx}(\tau)d\tau\right)\frac{dx^*_j}{d\sigma} \to -\frac{dx^*_j}{d\sigma} < \frac{\int_{x^*_1(\beta,\sigma^\beta)}^{x^*_1(0,\sigma^0)} C''_{x\sigma}(\tau)d\tau}{(N\setminus\{1\}-i)\times\int_{X^*}^{X^*_{-\{J,1\}}+\sum_{j\in J}\tilde{x}_i} U''_{xx}(\tau)d\tau}$$

otherwise, the contrary occurs. That is, if the trading externality is large, the investing seller invests more than the efficiency level regardless of the investment decision of the buyer.

Proof of proposition (4): We proceed by construction and we consider the case when the number of active sellers tends to infinity. We consider first the investment decision of the seller and take the case where the distortion is maximal. Hence, if in this situation, investment tends to efficiency, so will be for any $J\in N$. Second, we obtain that for any equilibrium, the investment threshold of the buyer tends to the efficient one.
Regarding the investment of the seller, we take the highest level of distortion and this is the one when competition ex-post is the least severe, or $|J|=1$:

$$\gamma^1(N) \equiv -\sum_{m\neq 1,i}\left(\int_{X^*(N)}^{X^*_{-\{1,i\}}(N)+\tilde{x}_i(N)} U''_{xx}(\tau)d\tau\right)\times\frac{dx^*_m}{d\sigma}. \tag{B.7}$$

Observe that the magnitude of this object depends on the difference between the efficient amount traded and the one obtained with unilateral replacement which equals to $x^*_i(N)+x^*_1(N)-\tilde{x}_i(N)>0$. We now show that this difference tends to zero when the number of active sellers is arbitrarily large and so the expression within the brackets in (B.7) tends to zero. At this purpose we make use of the following two lemmas. The following lemma shows how the individual and aggregate amount of trade evolves with an increase of sellers.

Lemma 9. *For a given investment profile (b,σ) the amount that each seller trades with the buyer decreases with the* *number of active sellers, but the aggregate level of trade is higher.*

$$x_i^*(\Delta N) < x_i^*(N) \quad \forall i \in N \quad \textit{and} \quad X^*(\Delta N) > X^*(N).$$

Proof. The results comes directly from the concavity of the utility function and the convexity of the cost function. In order to ease notation, we do not consider investment. We define $\Delta N = N + 1$ and with a number of ΔN active sellers, the amount traded in equilibrium needs to satisfy:

$$U_x'\left(\sum_{i=1}^{\Delta N} x_i^*(\Delta N)\right) = C_x'\left(x_i^*(\Delta N)\right).$$

We proof the claim by contradiction, assume that $x_i^*(\Delta N) \geq x_i^*(N)\ \forall\ i \in N$, and since $\Delta N > N$ we have that $\sum_i^{\Delta N} x_i^*(\Delta N) > \sum_i^N x_i^*(N)$ and by the concavity of $U(\cdot)$ and optimality it has to be the case that:

$$C_x'\left(x_i^*(\Delta N)\right) = U_x'\left(\sum_{i=1}^{\Delta N} x_i^*(\Delta N)\right) < U_x'\left(\sum_{i=1}^{N} x_i^*(N)\right) = C_x'\left(x_i^*(N)\right) \quad \forall i \in N,$$

but the convexity of $C_x'(\cdot)$ implies that $x^*(\Delta N) < x^*(N)$, which leads to a contradiction. From the previous, we see that that $X^*(\Delta N) > X^*(N)$ comes directly. □

Therefore as the number of seller increase, the amount $x_1^*(N)$ decreases and $\lim_{N\to\infty} x_1^*(N) \approx 0$. Regarding how the amount $\tilde{x}_i(N)$ evolves with the number of sellers, we know that this object is the solution of the function $V(X^*_{-\{1,i\}})$ introduced in expression (4.3) in the appendix. The properties of this function are introduced in the following lemma.

Lemma 10. *The function $V\left(X^*_{-\{1,i\}}\right)$ is well defined, strictly increasing and strictly concave in $X^*_{-\{1,i\}}$. The maximizer $\tilde{x}_i\left(X^*_{-\{1,i\}}\right)$ is decreasing in $X^*_{-\{1,i\}}$.*

Proof. That the function $V\left(X^*_{-\{1,i\}}\right)$ is well defined follows form the Inada conditions. By the envelop theorem we have $V_x'\left(X^*_{-\{1,i\}}\right) > 0$ and $V_{xx}''\left(X^*_{-\{1,i\}}\right) < 0$, which implies that the function is strictly increasing and strictly concave. By the implicit function theorem, we find that:

$$\frac{\partial \tilde{x}_i\left(X^*_{-\{1,i\}}\right)}{\partial X^*_{-\{1,i\}}} = \frac{U_{xx}''(\cdot)}{C_{xx}''(\cdot) - U_{xx}''(\cdot)} < 0.$$

Thus, it is decreasing with an increase of the unchanged equilibrium allocation due to the concavity of the utility function and the convexity of the cost function. □

Hence, an increase of the number of sellers make $X^*_{-\{1,i\}}(N)$ increase as shown in lemma (9), and by the previous lemma we know that the amount $\tilde{x}_1(N)$ decreases. In the limit, we have that $\lim_{N\to\infty}[\tilde{x}_i(N)] \approx x_i^*(N)$. Thus, we have shown that with an arbitrarily number of active sellers, the difference between the upper and the lower integrand of (B.7) tends to zero. We conclude that the extra term causing the inefficiencies in the seller's investment disappear and this tends to efficiency $\lim_{N\to\infty}[\zeta(N)] \approx 0 \Longrightarrow \sigma_{\mathbf{LC}} \approx \sigma_{\mathbf{E}}$.
We now show that the investment thresholds of the buyer also converge whenever the number of active sellers is sufficiently large. The investment threshold for any $J \subset N$ is:

$$K_J \equiv TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(0, \sigma_{\mathbf{J}}^0) - \kappa_{\mathbf{J}}.$$

From above, we know that the investment of the seller tends to efficiency $\sigma_{\mathbf{J}} \approx \sigma_{\mathbf{E}}$, which implies that the first part of the threshold also tends to efficiency $\lim_{N\to\infty}\left[TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(0, \sigma_{\mathbf{J}}^0)\right] \approx TS^*(\beta, \sigma_{\mathbf{E}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0)$. By the same argument as before we can also show that the appropriation of gains from trade by the sellers coming from an investment of the buyer tends to zero when the number of sellers is arbitrarily large. Finally, the investing seller appropriation of investment tends to the private cost of investment, that is, $\lim_{N\to\infty}[\kappa_J] \approx \psi(\sigma^{\beta}_{\mathbf{E}}) - \psi(\sigma^{0}_{\mathbf{E}})$. Hence, with all things considered we have that $\lim_{N\to\infty}[K_J] \approx K_E$ and the equilibrium investment profile tends to efficiency.

Proof of proposition (5): We begin by considering the case where the "allocative externality" is small. In this case, we have established that the investment threshold of the buyer is monotonically decreasing the lower the level of competition ex-post. The lower partition of the surplus appropriated by the buyer with lower competition dominates the higher investment of the seller. We start with the case that the investment of the buyer is the same regardless of the level of competition ex-post. To see that the investing seller is better-off with lower levels of competition we only need to verify that his investment increases the lower is the level of competition and this is the case since we know that $\frac{\partial \gamma(J)}{\partial J} < 0$. For the non-investing seller, it is easy to see that for any $J \subset N$ and $J \neq N \setminus \{i\}$ we obtain:

$$\begin{aligned} TS^*(b,\sigma^b_{\mathbf{J}}) - \tilde{TS}^J_{-i}(b,\sigma^b_{\mathbf{J}}) &> TS^*(b,\sigma^b_{\mathbf{E}}) - TS^*_{-i}(b,\sigma^b_{\mathbf{E}}) \\ &\implies TS^*_{-i}(b,\sigma^b_{\mathbf{E}}) - \tilde{TS}^J_{-i}(b,\sigma^b_{\mathbf{J}}) > TS^*(b,\sigma^b_{\mathbf{E}}) - TS^*(b,\sigma^b_{\mathbf{J}}) \approx 0 \\ &\implies TS^*_{-i}(b,\sigma^b_{\mathbf{E}}) - \tilde{TS}^J_{-i}(b,\sigma^b_{\mathbf{J}}) > 0. \end{aligned}$$

The right hand side of the second line is close to zero due to the fact that when the "trading externality" is small, the investment of the seller is similar regardless to the equilibrium ex-post $\sigma^b_{\mathbf{J}} \approx \sigma^b_{\mathbf{E}}$. The third line is positive by point ii) in proposition (1). In a situation where the investment of the buyer depends on the equilibrium played ex-post and because the "allocative externality" is small, we know that for any $J \subset N$ we have that the investment threshold in the most competitive equilibrium is the largest $K_{HC} > K_J$. Because the externality is small then there exist $J \subset N$ and $\sigma^{\beta}_{\mathbf{J}} > \sigma^{0}_{\mathbf{LC}}$. This implies that the largest payoff of the investing seller is achieved with an intermediate level of competition. With regards to the non-investing sellers, we have that, the largest payoffs is achieved with an intermediate level of competition. By the same procedure as before, we know that $TS^*(\beta,\sigma^{\beta}_{\mathbf{J}}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}_{\mathbf{J}}) > TS^*(\beta,\sigma^{\beta}_{\mathbf{E}}) - TS^*_{-i}(\beta,\sigma^{\beta}_{\mathbf{E}})$. Therefore, we have that the largest payoff is attained with a level of intermediate competition if:

$$TS^*(\beta,\sigma^{\beta}_{\mathbf{J}}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}_{\mathbf{J}}) > TS^*(0,\sigma^{0}_{\mathbf{LC}}) - \tilde{TS}^1_{-i}(0,\sigma^{0}_{\mathbf{LC}}). \tag{B.8}$$

Because the "allocative externality" is small, we know that with a given investment of the buyer, the investment of the seller will be similar. Hence, by lemma (5) and proposition (1) we know that:

$$\begin{aligned} TS^*(\beta,\sigma^{\beta}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}) &> TS^*(0,\sigma^{0}) - \tilde{TS}^J_{-i}(0,\sigma^{0}) \\ TS^*(0,\sigma^{0}) - \tilde{TS}^1_{-i}(0,\sigma^{0}) &> TS^*(0,\sigma^{0}) - \tilde{TS}^J_{-i}(0,\sigma^{0}). \end{aligned}$$

By summing up both expressions we have

$$\tilde{TS}^1_{-i}(0,\sigma^{0}) < TS^*(\beta,\sigma^{\beta}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}) + TS^*(0,\sigma^{0}) - 2\left[TS^*(0,\sigma^{0}) - \tilde{TS}^J_{-i}(0,\sigma^{0})\right]$$

and by putting this in equation (B.8) we obtain:

$$\begin{aligned} TS^*(\beta,\sigma^{\beta}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}) &> TS^*(0,\sigma^{0}) - TS^*(\beta,\sigma^{\beta}) + \tilde{TS}^J_{-i}(\beta,\sigma^{\beta}) - TS^*(0,\sigma^{0}) \\ &\quad + 2\left[TS^*(0,\sigma^{0}) - \tilde{TS}^J_{-i}(0,\sigma^{0})\right] \\ &\implies 2\left[TS^*(\beta,\sigma^{\beta}) - \tilde{TS}^J_{-i}(\beta,\sigma^{\beta})\right] > 2\left[TS^*(0,\sigma^{0}) - \tilde{TS}^J_{-i}(0,\sigma^{0})\right], \end{aligned}$$

where the last inequality holds again by lemma (5).

Whenever the "allocative externality" is big, the proof is more involved. We have seen that if the "allocative externally" is big there exists a subset of agents that undertake collective replacement $J \subset N$ such that $K_J > K_{HC}$. For the investing seller it is easy to see that, as long as the investment is higher with a less competitive equilibrium his payoffs are also higher. This is always the case due to the complementarity of investment and that the buyer may decide to invest with a lower the degree of competition. For the non investing seller the proof is simple. In the case where the fixed costs of investment is such that the buyer invests in a lower competitive equilibrium we have that the non-investing sellers are better with the highest competitive equilibrium. Therefore, we have that:

$$\begin{aligned} &TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - \tilde{TS}_{-i}^J(\beta, \sigma_{\mathbf{J}}^\beta) < TS^*(0, \sigma_{\mathbf{E}}^0) - TS_{-i}^*(0, \sigma_{\mathbf{E}}^0) \\ &\Longrightarrow TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0) < \tilde{TS}_{-i}^J(\beta, \sigma_{\mathbf{J}}^\beta) - TS_{-i}^*(0, \sigma_{\mathbf{E}}^0); \quad \forall J \in N \end{aligned}$$

where the left hand side represents the whole gain coming from the investment of the buyer and this is positive. However, the right hand side is bigger and represents the gain in surplus when the buyer is investing whenever any seller $i \neq 1$ is excluded from trade and this is proved in lemma (8) in the appendix. Observe that whenever the investment decision of the buyer is equilibrium invariant, we also have that the maximal surplus is attained with the highest level of competition since

$$TS^*(\beta, \sigma_{\mathbf{E}}^\beta) - TS_{-i}^*(\beta, \sigma_{\mathbf{E}}^\beta) > TS^*(0, \sigma_{\mathbf{E}}^0) - TS_{-i}^*(0, \sigma_{\mathbf{E}}^0) > TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - \tilde{TS}_{-i}^J(\beta, \sigma_{\mathbf{J}}^\beta); \quad \forall J \in N.$$

Proof of theorem (1): We start with the case when the trading externality is small. In the previous section we stated that with a small "allocative externality" the investment threshold of the buyer is the biggest with the highest degree of competition, i.e $K_{HC} \geq K_J \; \forall J \subset N$. This entails that the investment decision of the buyer is only efficient in any "tacitly" coordinating equilibrium whenever it is also in the most competitive equilibrium. Then, it is immediate to see that, because the investment decision of the seller in any "tacitly" coordinating equilibrium is inefficient as shown in proposition (2), we obtain that the highest level of social welfare is obtained when competition is the most severe.

We proceed by considering by analyzing when the "allocative externality" is big. In this case, we have established that the investment threshold of the buyer in an equilibrium where sellers tacitly coordinate to bring competition down, might be above the one corresponding to the highest competitive equilibrium. Here, we show that there exists a situation where the social welfare is bigger with less competition. Therefore, in what follows, we consider the case where there exist a $J \subset N$ such that $K_{HC} < K_J$. We define the difference in net social surplus as:

$$D(\cdot) = SW^J(\beta, \sigma^\beta) - SW^{HC}(0, \sigma^0) = TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - K - \psi(\sigma_{\mathbf{J}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0) + \psi(\sigma_{\mathbf{E}}^0).$$

Since we want to know if there exists a situation where a less competitive equilibrium does better, we take the lowest possible value of the cost of investment of the buyer, which is $K = K_{HC} = TS^*(\beta, \sigma_{\mathbf{E}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0) - \kappa_{HC}$. By introducing this in the previous expression we obtain that the lower bound of the difference is given by:

$$\begin{aligned} \bar{D}(\cdot) &= TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(\beta, \sigma_{\mathbf{E}}^\beta) + TS^*(0, \sigma_{\mathbf{E}}^0) + \kappa_{HC} - \psi(\sigma_{\mathbf{J}}^\beta) - TS^*(0, \sigma_{\mathbf{E}}^0) + \psi(\sigma_{\mathbf{E}}^0) \\ &= TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(\beta, \sigma_{\mathbf{E}}^\beta) + \kappa_{HC} - \left(\psi(\sigma_{\mathbf{J}}^\beta) - \psi(\sigma_{\mathbf{E}}^0)\right) \\ &> TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(\beta, \sigma_{\mathbf{E}}^\beta) + \psi(\sigma_{\mathbf{E}}^\beta) - \psi(\sigma_{\mathbf{E}}^0) - \left(\psi(\sigma_{\mathbf{J}}^\beta) - \psi(\sigma_{\mathbf{E}}^0)\right) \\ &= TS^*(\beta, \sigma_{\mathbf{J}}^\beta) - TS^*(\beta, \sigma_{\mathbf{E}}^\beta) - \left(\psi(\sigma_{\mathbf{J}}^\beta) - \psi(\sigma_{\mathbf{E}}^\beta)\right), \end{aligned}$$

where the first inequality comes from the proof of proposition (3). Therefore, we will obtain that the difference is positive, whenever the increase in the social surplus due to a higher investment of the seller is bigger than the cost, i.e. $TS^*(\beta, \sigma^\beta_{\mathbf{J}}) - TS^*(\beta, \sigma^\beta_{\mathbf{E}}) > \psi(\sigma^\beta_{\mathbf{J}}) - \psi(\sigma^\beta_{\mathbf{E}})$ and therefore, we additionally require that the effect of the investment of the seller in the social surplus is sufficiently big. Observe that because a lower degree of competition, that is, for $J' \subset J$, the investment of the seller is more inefficient and the investemnt of the buyer stays the same, the level of net social welfare is decreased. Consequently, the maximum is attained at J.

C Appendix

Comparison of the thresholds: In expression (B.4) and (B.6) we have obtained the thresholds below which the buyer invests in the most and the least competitive equilibrium. Here we see under which circumstances the threshold of the least competitive equilibrium is above the most competitive one. We introduce the following lemma that will be useful on the sequel. Hence, by using a similar procedure as in lemma (5) together with what we have proved, we can show that the social increase in welfare coming from the investment of the buyer is larger the higher the investment of the seller:[23]

$$TS^*(\beta, \sigma^\beta_{\prime}) - TS^*(\beta, \sigma^\beta) \geq TS^*(0, \sigma^0_{\prime}) - TS^*(0, \sigma^0).$$

The equilibrium thresholds for the highest and the least degree of competition are respectively:

$$\begin{aligned}
K_{\mathbf{HC}} &= TS^*(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) - \Big(TS^*(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*_{-1}(\beta) - TS^*(0, \sigma^0_{\mathbf{E}}) + TS^*_{-1}(0)\Big) \\
&- \sum_{i \neq 1} \Big[TS^*(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*_{-i}(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) + TS^*_{-i}(0, \sigma^0_{\mathbf{E}})\Big] \\
&= TS^*_{-1}(\beta) - TS^*_{-1}(0) - \sum_{i \neq 1} \Big[TS^*(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*_{-i}(\beta, \sigma^\beta_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) + TS^*_{-i}(0, \sigma^0_{\mathbf{E}})\Big]
\end{aligned}$$

$$\begin{aligned}
K_{\mathbf{LC}} &= TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{LC}}) - \Big(TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - \tilde{TS}^i_{-1}(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{LC}}) + \tilde{TS}^i_{-1}(0, \sigma^0_{\mathbf{LC}})\Big) \\
&- \sum_{i \neq 1} \Big[TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - \tilde{TS}^1_{-i}(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{LC}}) + \tilde{TS}^1_{-i}(0, \sigma^0_{\mathbf{LC}})\Big] \\
&= \tilde{TS}^i_{-1}(\beta, \sigma^\beta_{\mathbf{LC}}) - \tilde{TS}^i_{-1}(0, \sigma^0_{\mathbf{LC}}) - \sum_{i \neq 1} \Big[TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - \tilde{TS}^1_{-i}(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{LC}}) + \tilde{TS}^1_{-i}(0, \sigma^0_{\mathbf{LC}})\Big]
\end{aligned}$$

We define the difference of both thresholds by $\aleph(\cdot) = K_{\mathbf{LC}} - K_{\mathbf{HC}}$

$$\begin{aligned}
&\aleph(\cdot) = -\Big[TS^*_{-1}(\beta) - \tilde{TS}^i_{-1}(\beta, \sigma^\beta_{\mathbf{LC}}) - \Big(TS^*_{-1}(0) - \tilde{TS}^i_{-1}(0, \sigma^0_{\mathbf{LC}})\Big)\Big] \\
&- \sum_{i \neq 1} \Big[\Big(TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(\beta, \sigma^\beta_{\mathbf{E}})\Big) - \big(TS^*(0, \sigma^0_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{E}})\big) + \Big(TS^*_{-i}(\beta, \sigma^\beta_{\mathbf{E}}) - \tilde{TS}^1_{-i}(\beta, \sigma^\beta_{\mathbf{LC}})\Big) - \Big(TS^*_{-i}(0, \sigma^0_{\mathbf{E}}) - \tilde{TS}^1_{-i}(0, \sigma^0_{\mathbf{LC}})\Big)\Big]
\end{aligned}$$

We now analyze the sign of the previous expression. At this regard we consider three different parts:

$$\begin{aligned}
C &:= -\Big[TS^*_{-1}(\beta) - \tilde{TS}^i_{-1}(\beta, \sigma^\beta_{\mathbf{LC}}) - \Big(TS^*_{-1}(0) - \tilde{TS}^i_{-1}(0, \sigma^0_{\mathbf{LC}})\Big)\Big] \\
D &:= -\sum_{i \neq 1} \Big[TS^*(\beta, \sigma^\beta_{\mathbf{LC}}) - TS^*(\beta, \sigma^\beta_{\mathbf{E}}) - \big(TS^*(0, \sigma^0_{\mathbf{LC}}) - TS^*(0, \sigma^0_{\mathbf{E}})\big)\Big] \\
E &:= -\sum_{i \neq 1} \Big[TS^*_{-i}(\beta, \sigma^\beta_{\mathbf{E}}) - \tilde{TS}^1_{-i}(\beta, \sigma^\beta_{\mathbf{LC}}) - \Big(TS^*_{-i}(0, \sigma^0_{\mathbf{E}}) - \tilde{TS}^1_{-i}(0, \sigma^0_{\mathbf{LC}})\Big)\Big]
\end{aligned}$$

Part C represents the gain that the investing seller appropriates coming from the investing of the buyer, and this is decreasing on the level of investment of the seller as his threat of being unilaterally replaced is lower. part D are the

[23] This can also be easily shown by differentiation.

gains that all sellers appropriate from a higher investment of the buyer and part E is the extra gains appropriated by the none investing seller when there is either collective replacement or unilateral replacement and the higher is the investment of the seller in the last case the higher is this part.
To obtain the sign of this expression, we know that by lemma (7) part C is always negative. By lemma (4) we know that D is also negative. However, point E will be either negative or positive depending on how much investment differs in both equilibrium. Therefore, if the equilibrium investment is not much different, we have that sign[E] $\approx$ sign[C] and $D \approx 0$. Therefore, we obtain that the threshold in the highest competitive equilibrium is above the one in the least competitive, i.e. $\aleph(\cdot) < 0$ and $K_{\mathbf{LC}} < K_{\mathbf{HC}}$. However, as long as the investment differs point E might be positive. This implies that unilateral replacement by the investing seller becomes a higher threat that optimal collective replacement. When this last part is sufficiently large then we have that the difference in thresholds is positive. Therefore, all this depends on the primitive of the economy. The difference on the investment threshold for any type of equilibrium with respect to the most competitive equal to:

$$\begin{aligned}\aleph^J(\cdot) = &-\Big[TS^*_{-1}(\beta) - \tilde{TS}^J_{-1}(\beta,\sigma^\beta_{\mathbf{J}}) - \Big(TS^*_{-1}(0) - \tilde{TS}^J_{-1}(0,\sigma^0_{\mathbf{J}})\Big)\Big] \\ &- \sum_{i\neq 1}\Big[TS^*(\beta,\sigma^\beta_{\mathbf{J}}) - TS^*(\beta,\sigma^\beta_{\mathbf{E}}) - \big(TS^*(0,\sigma^0_{\mathbf{J}}) - TS^*(0,\sigma^0_{\mathbf{E}})\big)\Big] \\ &- \sum_{i\neq 1}\Big[TS^*_{-i}(\beta,\sigma^\beta_{\mathbf{E}}) - \tilde{TS}^J_{-i}(\beta,\sigma^\beta_{\mathbf{J}}) - \Big(TS^*_{-i}(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}^J_{-i}(0,\sigma^0_{\mathbf{J}})\Big)\Big]\end{aligned} \tag{C.1}$$

Again the first and the second part are always negative while this is not the case for the last part. Again this comes from the fact that the higher the investment of the seller in equilibrium, the cheapest might be the replacement undertaken by of a subset $J' \subset J$. Again this depends on the trading externality to be sufficiently big such that investment by the seller in a "tacitly" coordinating equilibrium is larger.

Out of equilibrium contracts that sustain the equilibrium transfer : In Chiesa and Denicoló (2009) they show what type of menu of contracts sustain an equilibrium with unilateral replacement. The menus offered by the different sellers are

$$M_1 = \Big\{m_1^*, m_1^0, \tilde{m}_1^i\Big\}; \quad M_2 = \big\{m_2^*, m_2^0, \tilde{m}_2^1\big\}; \quad M_3 = \big\{m_3^*, m_3^0\big\}, \cdots, M_N = \big\{m_N^*, m_N^0\big\}$$

where $T_i^1 = U(X^*) - \big(V(X^*_{1,i}) + C(x_1^*)\big)$ and

$$\tilde{m}_1 = \big(x', U(x') - \Pi\big)$$

if the conditions

$$V_i(x') - U(x') \leq \pi_1; \quad 2U(x') - U(2x') \geq \Pi$$

hold, they show that

$$\Pi(\tilde{m}_1, m_2^0, m_3^0, ..., m_N^0) \geq \Pi(\text{any other combination of trading contracts})$$

Hence, by using the same procedure we can show that for $J = N \setminus \{i\}$ and with the menus

$$M_1 = \Big\{m_1^*, m_1^0, \tilde{m}_1^{N-1\{i\}}\Big\}; \quad M_2 = \Big\{m_2^*, m_2^0, \tilde{m}_2^{N-1\{i\}}, \tilde{m}_2^{N-1\{1\}}\Big\}; \quad M_3 = \Big\{m_3^*, m_3^0, \tilde{m}_3^{N-1\{i\}}, \tilde{m}_3^{N-1\{1\}}\Big\}, \cdots, M_N = \Big\{m_N^*, m_N^0, \tilde{m}_N^{N-1\{i\}}, \tilde{m}_N^{N-1\{1\}}\Big\}$$

where $T_i^{N-1} = U(X^*) - \left(V(0) + \sum_{j \neq i} C(x_1^*)\right)$ and

$$\tilde{m}_1^{N-1\{i\}} = \left(\frac{\sum_{h \neq 1} x'}{N-1}, \frac{U(\sum_{h \neq 1} x') - \Pi}{N-1}\right)$$
$$\tilde{m}_2^{N-1\{i\}} = \left(\frac{\sum_{h \neq 1} x'}{N-1}, \frac{U(\sum_{h \neq 1} x') - \Pi}{N-1}\right)$$
$$...$$
$$\tilde{m}_N^{N-1\{i\}} = \left(\frac{\sum_{h \neq 1} x'}{N-1}, \frac{U(\sum_{h \neq 1} x') - \Pi}{N-1}\right)$$

similar conditions hold, then he buyer cannot do better with any combination of the trading contracts.

$$\Pi(\tilde{m}_1, m_i^0, \left\{\mathbf{m}_j^{N-1\{i\}}\right\}_{j \neq 1, i}) \geq \Pi(\text{any other combination of trading contracts})$$

And a similar argument can be used for any J.