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DEPARTAMENT D'ECONOMIA
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Trade Disclosure and Price Dispersion

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Abstract

This paper determines the effects of post-trade opaqueness on market performance. We find that the degree of market transparency has important effects on market equilibria. In particular, we show that dealers operating in a transparent structure set regret-free prices at each period making zero expected profits in each of the two trading rounds, whereas in the opaque market dealers invest in acquiring information at the beginning of the trading day. Moreover, we obtain that if there is no trading activity in the first period, then market makers only change their quotes in the opaque market. Additionally, we show that trade disclosure increases the informational efficiency of transaction prices and reduces volatility. Finally, concerning welfare of market participants, we obtain ambiguous results.

Key words: Market Microstructure, Post-trade Transparency, Price Experimentation and Price Dispersion.

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1 Introduction

As we enter the 21st century the demand for a global equity market seems to be growing. Investing institutions, investment banks and companies are already increasingly global, and technology is pushing in that direction. There is however a possible problem lurking in the wings: regulation. Already regulatory differences are complicating the existence of a single European equity market as requirements on accounting standards, provisions for disclosure or transparency rules all varied hugely between markets. The need to better understand the relationship between market structure and market quality is greater than ever in a EU seeking to construct a single financial market.

One of the challenges regulators face is to agree on the desired level of transparency in stock-exchange dealings.[1] In the United States, Arthur Levitt who was chairman of the Securities and Exchange Commission's (SEC) throughout the Clinton presidency, devoted much time to improving standards of disclosure and transparency in the equity markets. The view of the Commission is straightforward: "The Commission has long believed that transparency –the real time, public dissemination of trade and quotation information– plays a fundamental role in the fairness and efficiency of the secondary markets...transparency helps to link dispersed markets and improves the price discovery, fairness, competitiveness and attractiveness of US markets."[2] In the same vein, SEC also argued "...transparent disclosure of quotes and trades promotes best execution."[3] In contrast, in the UK, the Securities and Investment Board (SIB) has argued that there are important differences between quotation transparency and trade transparency, and that transparency (in the context of prompt publication of large trades) should be restricted if it is necessary to assure adequate liquidity.[4]

The Federation of European Securities Exchanges (FESE) has made clear that real-time reporting is basically state-of-the-art and should be the standard on Europe's financial markets. It claims that reporting of "standard" trades at the end of the day is not deemed sufficient. Nevertheless, markets with low degrees of transparency seem to be doing quite well. In less than forty years, the eurobond market has gone from zero to become the second largest bond market in the world and the largest for corporate bonds. And that happened despite the market having almost none of the characteristics which it is claimed by some are essential for a market which is fair and efficient for all. Compared to equity markets, its transparency has been limited. And yet investors have received good returns over many years and their confidence in the market has been steadfastly maintained. Is there then a basis for imposing post-trade transparency on transactions?

[1] Real world trading systems exhibit considerable heterogeneity in the degree of transparency they offer. Automated limit order book systems as the type used by the Toronto Stock Exchange and the Paris Bourse offer high degrees of transparency. Foreign exchange and corporate junk bond markets offer very little transparency whereas other dealer markets as Nasdaq or the London Stock Exchange offer moderate degrees of transparency (see Madhavan (2000)).

[2] See SEC Market 2000 Study, Chapter IV-1.

[3] See Release No. 34-36310; File No. S7-30-95.

[4] For a further discussion on these issues see Bloomfield and O'Hara (1999).

How transparency affects market behavior is a question addressed by several papers (see, for instance, Biais (1993), Pagano and Roëll (1996), Madhavan (1995,1996), Gemmill (1996), Porter and Weaver (1998), Naik et al. (1999), Bloomfield and O'Hara (1999,2000), Flood et al. (1999), Frutos and Manzano (2002), among others). The numerous works focused on transparency show both the importance and the complexity of this issue. The main contribution of this paper is to deepen in the effects of post-trade transparency in the performance of the market in a model based on Glosten and Milgrom (1985). We think that the study of this issue in a well-known framework may help to understand their implications. More precisely, we model a quote-driven market in which the daily trade takes place in two different intervals of time whereas new information only arrives at the beginning of the day. Using this set-up we compare two market structures: *a post-trade transparent market*, in which trades are made public and *a post-trade opaque market,* in which trades are not disclosed. In both market structures the information contained in customer orders is valuable.[5] To undertake the comparison we first provide an explicit characterization and computation of dealers' equilibrium pricing strategies in the two market structures which allows to understand what are the driving forces behind dealers' behavior.[6]

We show that prices in an opaque market result from the interplay between informational and strategic considerations, whereas in a transparent market prices are only informationally driven. Dealers operating in a transparent market set regret-free prices at each period making zero expected profits in each of the two trading rounds. By contrast, dealers in an opaque market set prices away from the short run equilibrium. We show that in the opaque market structure price-setting dealers invest in acquiring information by setting more attractive prices from investors' viewpoint at the beginning of the trading day. They depart from maximizing current expected profits, in order to produce information that will yield future expected profits.[7] More importantly, in equilibrium, dealers try to attract order flow in both directions, i.e., they try to be competitive on both sides of the market and to do so they set jointly their ask and bid. This result departs from most of the theoretical results in the literature on dealer markets where dealers set their asks and bids independently. Nevertheless, our result is consistent with the findings in the experimental works by Bloomfield and O'Hara (1999,2000).[8]

This paper provides a series of testable predictions on the impact of transparency on

[5]Mr. Steil, an analyst at the Council on Foreign Relations, has argued in an interview with the Economist that NYSE specialists' enviable profitability is linked largely to their knowledge of order flow (see The Economist, May 5th 2001).

[6]The experimental evidence on transparency suggests that dealers behave quite differently in opaque markets than in transparent markets (see, for instance, Bloomfield and O'Hara (1999 and 2000)).

[7]The investment in producing information is also found in Leach and Madhavan (1993) when the market is run by a monopolist. By contrast, these authors show that market makers in a multiple-dealer system do not have incentives to carry out this investment since they cannot profit in future trading. We here show that this is only true when there is post-trade transparency.

[8]The result may seem to be in conflict with the empirical evidence reported in Hansch et al. (1998) that a majority of market makers try to attract order flow primarily in one direction. Nevertheless, it may not be so as our dealers do not care about their inventories, whereas inventories are the driving force of their findings.

price dynamics. Note that the explicit computation of the equilibrium pricing strategies allows us to examine the impact of market opaqueness on metrics of market quality as spreads, volatility and price efficiency. Some of the results we obtain are similar to those delivered by Madhavan (1995) or Bloomfield and O´Hara (2000), among others.[9] Nevertheless, we also offer new predictions that might be useful to econometricians with price data. Among the similar results we also find that post-trade opaqueness has the following effects. 1). It results in a reversal in the normal intraday pattern of the bid-ask spread. 2). It increases price volatility because the differences between the dealers who transact in the two periods are bigger in the opaque market and, this is reflected in prices. 3). It reduces transaction price efficiency because less information is impounded in prices. The main new predictions we offer are may help to reconcile the theoretical results on trade disclosure with the empirical evidence from opaque markets as the FX market or the corporate junk bond markets. In particular, 1). We show that transaction prices in an opaque market do not follow a martingale, and consequently, the first order differences in prices may not be uncorrelated. This topic is of relevance since until now in all the information-based models this property was satisfied.[10] 2). In the opaque market, spreads increase over time even if there was no order in the past.[11] Similarly, we show that if dealers have asymmetric beliefs about the value of the security, then spreads are history dependent. 3). Finally, we show that prices in opaque markets are more spread out. In particular we show that in equilibrium there is price dispersion in the opaque market whereas this is not the case if trades are reported.[12]

The article is organized as follows. In the next section we present a sequential trade model. Section 3 characterizes the equilibrium in the transparent market. Section 4 derives dealers' pricing strategies in an opaque market. Section 5 compares some market indicators corresponding to both market structures. Concluding comments are presented in Section 6. Proofs are included in the Appendix.

[9] Madhavan (1995) examines the issue of post-trade information disclosure in a two-period dynamic model. The model shows that both informed traders and "large" (or strategic) liquidity traders should prefer non-transparency because it facilitates dynamic trading strategies, like "working" a large order over time. Dealers are willing not to disclose trades because they profit from the reduction in price competition. Even though this paper is close to ours there are important differences in the results that each paper deliver. Throughtout the paper we comment some of these differences that are due to the fact that Madhavan's equilibrium in (his) opaque market is not an equilibrium in our set-up.

[10] Note that in many of these models post-trade transparency is either explicitly or implictly assumed. We here find that this serial uncorrelation property depends on the degree of post-trade transparency of the market.

[11] Peng (2001) provides evidence that the bid-ask spreads increase over time when no orders arrive. This empirical finding is supportative of this prediction.

[12] Empirical research on the corporate junk bond market (an opaque market) shows evidence of price dispersion across dealers. See, for instance, Saunders, Srinivasan and Walter (2002) where this finding is present for a sample of bond trades conducted by a major asset manager/dealer in the OTC corporate bond market.

2 The Model

In this section we describe the basic structure of our sequential trade model, which is similar to Glosten and Milgrom (1985) or Easley and O'Hara (1987). We consider an economy with a single risky asset, whose liquidation value is denoted by v. The risky asset can take on two possible values, 0 and 2, both equally likely. Potential buyers and potential sellers trade the risky security with market makers or dealers, who are responsible for supplying liquidity by simultaneously setting prices at which they will buy and sell the asset. We will assume that there are only two dealers indexed by D, dealer X and dealer Y, who are both risk-neutral.

Liquidity is demanded by two possible types of investors: informed traders and uninformed traders.[13] Informed traders perfectly know the liquidation value of the risky asset. If an informed trader observes the high liquidation value, then she will buy the stock if the smallest ask price is below 2; if she has observed the low liquidation value, then she will sell it if the highest bid price is above 0. Uninformed traders do not know the liquidation value and they are hence equally likely potential buyers or potential sellers. They differ in their trading motivations. These may reflect their liquidity needs, their price-sensitivity, or individual specific trading rules. These factors influence the willingness of an uninformed trader to transact. We will here assume that with probability $1 - r$ they decide not to trade.[14]

Trades occur throughout the trading day. We divide the day into two intervals of time, $t = 0$ and $t = 1$. At each time interval, first market makers select ask and bid prices at which they are willing to buy or to sell one unit of the asset. Then, a trader is selected according to a probabilistic arrival process described below, and she decides her order size; i.e., whether to buy one unit at the smallest ask price ($q_t = +1$), to sell one unit at the highest bid price ($q_t = -1$), or not to trade at all ($q_t = 0$).

The probabilistic structure of a trading day is depicted in the tree diagram in Figure 1. The first node of the tree corresponds to nature selecting whether information will be good or bad. This node is only reached at the beginning of the trading day, meaning that new information occurs only between trading days. In the second node, an investor is selected. With probability half, she is an informed trader and with the complementary probability she is an uninformed trader. The third node corresponds to the trading decision each trader will make if given the opportunity to trade. Whether an informed trader buys or sells depends upon the relationship between the value of the risky asset she has observed and the prices. An uninformed trader is equally likely a potential buyer or a potential seller. Moreover, she will trade with probability r, with $1 > r > 0$, and will not trade with the remaining probability $1 - r$. Note that when r approaches zero, an order to trade can

[13] It is well-known that in order to avoid a no-trade equilibrium, at least some traders must transact for non-speculative reasons such as liquidity needs (see Milgrom and Stokey (1982)).

[14] Easley and O'Hara (1992) propose a similar specification. Leach and Madhavan (1993) do also allow the possibility of no trade by an uninformed investor but in their model the probability with which no trade occurs is determined endogenously whereas here is assumed to be an exogenous variable.

only come from an informed trader. When r approaches one, uninformed traders always choose to trade. In the second period the trader selection process is repeated. At the end of this second interval of time the liquidation value of the risky asset is made public and agents consume.

[Insert Figure 1]

Since the problem we are addressing involves a multi-dealer dynamic pricing game of incomplete information, the equilibrium concept we use is that of Perfect Bayesian equilibrium, searching for dealer pricing strategies which hold the typical property found in rational expectations models of incorporating the information the trade itself reveals.

Finally, the liquidity value of the risky asset and the investor's arrival process are assumed independent random variables. The joint distribution of all these random variables will be common knowledge.

3 The Post-trade Transparent Market

In a transparent market, at the end of the first round, all the trading information related to $t = 0$ is publicly disclosed. Both dealers will hold the same information and will hence quote the same bid and ask prices. Further, competition combined with risk neutrality dictates that any rents earned on trades would be bid away. Consequently, prices will equal reservation quotes so that the expected profit on any trade will be zero. The result is due to the price competition among symmetric risk-neutral dealers.

When computing the optimal pricing strategies by the dealers, we consider that an informed trader's strategy is to sell when $v = 0$ and to buy if $v = 2$. We will then show the optimality of this behavior. The following proposition explicitly characterizes the Bayes-Nash equilibrium in a market with post-trade transparency.[15]

Proposition 1 *There exists a unique Bayes-Nash equilibrium in the post-trade transparent market, where the equilibrium price quotation function at* $t = 0$, $P_0(q_0)$, *satisfies*

$$P_0(q_0) = E(v|q_0) = 1 + \beta q_0, \text{ and}$$

the equilibrium price quotation function at $t = 1$, $P_1(q_1; q_0)$, *is given by*

$$P_1(q_1; q_0) = E(v|(q_0, q_1)) = \begin{cases} 1 + \alpha q_1, & \text{for } q_1 = q_0 \neq 0, \\ 1, & \text{for } q_1 \neq q_0 \neq 0, \\ 1 + \beta q_1, & \text{for } q_0 = 0, \end{cases}$$

where $\alpha = \dfrac{2 + 2r}{2 + 2r + r^2}$ *and* $\beta = \dfrac{1}{1 + r}$.

[15] The derivation of the equilibrium in our transparent market is similar to that of Madhavan (1995). However, the equilibria are not identical because of the different models we consider.

The logic behind these expressions is clear. Consider, for instance, a potential sequence of buy orders. Dealers know that this potential sequence of trades could be generated by (1) the independent arrival of informed traders in periods 0 and 1, (2) the arrival of an informed trader in period 0 followed by a liquidity trader in period 1, (3) the arrival of a liquidity trader in period 0 followed by an informed trader in period 1, or (4) the independent arrival of liquidity traders in both periods. In cases (1) through (3) above, there is an investor who is informed, and her order reveals that the value of the risky asset is the high one, whereas in case (4) the potential sequence of trades is not informative about the liquidation value. Denoting by $1-\alpha$ to the conditional probability of case (4) given the potential sequence of trades, it follows that $E\left(v|\left(1,1\right)\right) = 1+\alpha$, where α is derived by applying Bayes' rule. Now, Bertrand competition combined with risk-neutrality implies that $P_1\left(1;1\right) = 1+\alpha$. The expressions for the other prices are obtained similarly.

When the first round ends without any trading activity, dealers do not revise their quotes as they do not observe any new relevant information about v. Otherwise, following a trade, they set new prices since the type of trade has signal value about v. In particular, when there is trade continuation, a sell (buy) in the first period decreases (increases) the bid (ask) price in the second period. Note that $\alpha > \beta$ for all $r > 0$. By contrast, when there is trade reversal, dealers set their ask and their bid in the second period equal to the ex ante expected value of the security. In either case, bids and asks are not symmetric around previous transaction prices. Note that the midquote following a sell (buy) is higher (lower) than the previous transaction prices.

Finally, equilibrium prices lie in the interval $(0,2)$. This ensures that the informed trader's strategy to sell (buy) the asset if she has observed the low (high) value is optimal in both periods.

4 The Post-trade Opaque Market

In an opaque market, the trading information related to $t=0$ is not made public which prevents free-riding from non-trading dealers. A dealer may hence now choose to invest in producing information by pricing more aggressively in the first round so as to use his private knowledge from trade to extract rents in the second round. By doing so, dealers depart from maximizing expected profits in each period to maximize the sum of their profits.[16]

An important feature of our modelling strategy is that the amount of information dealers can acquire in the opaque market depends on whether they choose to be *competitive* on both sides of the market or just on one side. A dealer who is competitive on both sides of the market is perfectly informed about the occurrence and sign of the order at

[16] Any equilibrium strategy in the opaque market must yield zero overall expected profits as there is price competition between risk-neutral dealers who are ex-ante identical. However, contrary to the transparent market, equilibrium strategies do not necessarily yield zero expected profits in each period.

the first round. A dealer who is competitive on one side becomes perfectly informed if he attended the order.[17] Otherwise, he is unable to distinguish between the event of no order arrival and the event of the order was attended by his competitor. We will say that a dealer specializes if he chooses to be competitive only on one side of the market. We will say that a dealer invests in perfect learning if he chooses to be competitive on both sides of the market.

In order to obtain the optimal pricing strategies we solve the model by backward induction under the proviso that an informed trader sells when $v = 0$ and buys if $v = 2$. That is, given the information sets the dealers bring into the second round, we solve for the dealers' optimal pricing strategies at $t = 1$. Given these optimal strategies at $t = 1$, we then solve for the optimal strategies at $t = 0$.

4.1 Optimal Quotes at t=1

In the second period there are two relevant continuation paths to consider depending on dealers' decision of being competitive on both sides of the market or just on one side:

1) The continuation path that follows dealers' specialization at $t = 0$, i.e., a dealer setting the best ask and his competitor setting the best bid.

2) The continuation path that follows investment in perfect learning at $t = 0$, i.e., a dealer setting both the best ask and the best bid.[18]

The key difference between these two paths is the amount of informational asymmetry among dealers. In the second path, we have the competition between a dealer with complete information and a dealer completely ignorant about the order type in the first period. By contrast, in the first path, a non-trading dealer knows at least that the order was not the one for which he was competitive. Thus, in the second path one dealer follows his priors and the other revise his beliefs, whereas in the first one both dealers revise their beliefs but each one incorporates different information.

In either case dealers are not concerned about the learning effect of their actions at $t = 1$ as there is no other period at which profit from the information they may gain. It is hence optimal for them to treat each side of the market (buys and sells) independently. Nevertheless, contrary to the transparent market, these quotes may differ across dealers as they hold different information. In particular, they will start $t = 1$ with different reservation quotes. Note that the definition of regret-free reservation quotes implies

$$P_{r,1}(q_1; \Theta_D) = E(v|q_1, q_0 \in \Theta_D), \quad (1)$$

[17] Throughout this section when we write informed or uniformed referred to a dealer, we are specifying his knowledge, or lack of it, about the order type in the first round, and not the knowledge about the liquidation value of the security.

[18] The continuation path that follows after dealers set equal prices in the first period can be analyzed by using the results for specialization and/or perfect learning. We will further elaborate on this point as we discuss in more detail these paths.

where Θ_D denotes the information set of dealer D at the beginning of $t = 1$.[19] Expression (1) can be expanded to obtain

$$P_{r,1}(q_1; \Theta_D) = 1 + q_1 \Delta_{r,1}(q_1; \Theta_D), \tag{2}$$

where $\Delta_{r,1}(q_1; \Theta_D)$ stands for the reservation fee of dealer D.

The implications of (1) and (2) are several. First, note that the higher the subjective probability of the high value of v, the lower (greater) the willingness to sell (buy). Second, it is easy to see that there exists a relationship between the reservation ask and bid prices. In particular, the following equalities hold:

$$\begin{aligned} \Delta_{r,1}(q_1; \{i\}) &= \Delta_{r,1}(-q_1; \{-i\}), \text{ for } i = 1, 0, -1, \\ \Delta_{r,1}(q_1; \{0, i\}) &= \Delta_{r,1}(-q_1; \{0, -i\}), \text{ for } i = 1, -1, \text{ and} \\ \Delta_{r,1}(q_1; \{1, 0, -1\}) &= \Delta_{r,1}(-q_1; \{1, 0, -1\}). \end{aligned}$$

The intuition behind equalities above is clear. Consider, for instance, the reservation selling fee of a dealer who knows that the order at $t = 0$ was a buy. This dealer assigns a very high probability to the high value of the security as he knows that a sequence of buys is most likely generated by informed trading with good news. His problem when setting his reservation selling fee is isomorphic to the problem faced by a dealer when setting his reservation buying fee, knowing that the past order was a sale. Moreover, since both trading histories ($(1, 1)$ and $(-1, -1)$) are equally likely, then the symmetry of the model and the assumed trading path independence ensure that $\Delta_{r,1}(1; \{1\}) = \Delta_{r,1}(-1; \{-1\})$. All the other equalities follow similarly.

The relationship between the reservation ask and bid prices goes on to the optimal quotes. In particular, the equilibrium price quotations, $P_1(q_1; \Theta_D) = 1 + q_1 \Delta(q_1; \Theta_D)$, are such that $\Delta(q_1; \Theta_D)$ satisfies the same equalities than $\Delta_{r,1}(q_1; \Theta_D)$. One can hence focus on one quote and then apply equalities above to derive the remaining quote. In what follows we will concentrate on the ask prices.

4.1.1 Specialization

There are two continuation paths that involve specialization: dealer X sells and dealer Y buys and its symmetric counterpart. Since both are identical once renaming the dealers, we will here, without loss of generality, only examine the first one.

[19] The information set of dealer D, Θ_D, where

$$\Theta_D \in \{\{1\}, \{0\}, \{-1\}, \{0, 1\}, \{0, -1\}, \{1, 0, -1\}\},$$

reflects all the values of q_0 to which he assigns positive probability. If $\Theta_D = \{i\}$, then he assigns probability one to $q_0 = i$, for $i = 1, 0, -1$. If $\Theta_D = \{0, i\}$, then he assigns positive probability to both $q_0 = 0$ and $q_0 = i$, for $i = 1, -1$. Finally, if $\Theta_D = \{1, 0, -1\}$, then he is completely ignorant about the value of q_0. Further, he assigns to each possible value its prior probability of occurrence.

We first note that specialization gives rise to a game of incomplete information in which each dealer may have two types; i.e., there are four potential players. These are dealer X who observed a buy order, dealer X who did not trade and does not hence know whether $q_0 = 0$ or $q_0 = -1$, dealer Y who observed a sell order and, finally, dealer Y who did not trade and does not hence know whether $q_0 = 0$ or $q_0 = 1$. We will refer to them as $X(1)$, $X(0,-1)$, $Y(-1)$ and $Y(1,0)$, respectively. Note that the realization of q_0 will determine which types are actually present at $t = 1$. So, for instance, if $q_0 = 1$, then the only types that will be in the market next period are $X(1)$ and $Y(1,0)$. Further, dealer $X(1)$ knows that he is competing against a dealer $Y(1,0)$, whereas the latter is uncertain about the type of his opponent. He believes that his opponent is $X(0,-1)$ with probability η ($\eta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0,1\}, q_1 = 1)$) and $X(1)$ with the complementary probability.

To analyze this situation, we start by deriving the reservation ask prices. Similarly, $X(0,-1)$ believes his opponent is $Y(1,0)$ with probability θ, $\theta = \Pr(q_0 = 0, q_1 = 1 | q_0 \in \{0,-1\}, q_1 = 1)$, and $X(1)$ with the complementary probability. Let $A^D_{r,1}(\Theta_D)$ and $a^D_{r,1}(\Theta_D)$ denote dealer D's reservation ask price and reservation selling fee, respectively, at $t = 1$.

Lemma 2 *If at $t = 0$ dealer X set the best ask and dealer Y set the best bid, then the reservation selling fees at $t = 1$ are the following:*

$$a^X_{r,1}(\{1\}) = \alpha, \quad a^Y_{r,1}(\{0,1\}) = \eta\beta + (1-\eta)\alpha, \quad a^X_{r,1}(\{0,-1\}) = \theta\beta, \ \textit{and} \ \ a^Y_{r,1}(\{-1\}) = 0.$$

Consequently,

$$A^X_{r,1}(\{1\}) > A^Y_{r,1}(\{1,0\}) > A^X_{r,1}(\{0,-1\}) > A^Y_{r,1}(\{-1\}).$$

In the opaque market reservation quotes differ across dealers. Those with complete information ($X(1)$ and $Y(-1)$) have the same reservation quotes than in the transparent market. However, those with partial information have reservation quotes that are linear combinations between the reservation quotes of perfectly informed dealers. So, for instance, $\eta\beta + (1-\eta)\alpha$ is a linear combination between the reservation selling fees of a $D(0)$ and a $D(1)$ dealer. Those with pessimistic information about the value of the security, can use this information to undercut the price of their competitors in order to make extra profits. This undercutting generates a situation similar to the Edgeworth cycle that results in the non-existence of a pure strategy equilibrium.

In equilibrium, $X(1)$, whenever present in the market, will set an ask price equal to his reservation ask price as he is the dealer with the most optimistic information about the true value. Dealer $Y(1,0)$ will also set this ask price unless he is winning against both of his (potential) opponents. Assume first that this is the case. Since his ask price cannot be below his reservation ask and since $A^Y_{r,1}(\{1,0\}) > A^X_{r,1}(\{0,-1\})$, dealer $X(0,-1)$ gains by slightly undercutting this price contradicting that $Y(1,0)$ wins over his two opponents. Assume then that $Y(1,0)$ only wins against $X(1)$. If this is the case, then in equilibrium he must set a price no smaller than the reservation ask of dealer $X(1)$. Consider now the best response by $X(0,-1)$. To maximize his profits, he will slightly undercut the price set by $Y(1,0)$. But, given this strategy, $Y(1,0)$ finds profitable to deviate by slightly undercutting this price and winning no matter his opponent's type, a contradiction. Dealers' best replies generate a cycle with no end.[20]

Proposition 3 *Under specialization there is no equilibrium continuation in which dealers use pure price strategies.*

Nevertheless there exists an equilibrium in which dealers randomize as shown in next proposition.

Proposition 4 *Under specialization, the equilibrium ask quotation at $t=1$ is given by $A_1(\Theta_D) = 1 + a_1(\Theta_D)$, where $a_1(\Theta_D)$ is set according to the following mixed-strategies:*
- A dealer who knows that the past order was a buy sets his reservation selling fee, α.
- A dealer who knows that the past order was a sale randomizes in the interval $[s,z]$ according to the probability distribution

$$H(a) = \frac{1}{1-\theta}\left(\frac{a-s}{a}\right).$$

- A dealer with information set $\{0,1\}$ plays a mixed strategy with support $[z,\alpha]$. He assigns probability according to the distribution F which has a mass point at α, where

$$F(a) = \frac{a-z}{a-\beta}.$$

- A dealer with information set $\{0,-1\}$ randomizes in the interval $[s,\alpha)$ according to the probability distribution G which has a kink at z. Further,

$$G(a) = \begin{cases} (1-\theta)H(a), & \text{if } s \le a \le z, \\ 1-\theta+\theta\frac{F(a)}{F(\alpha)}, & \text{if } z \le a < \alpha. \end{cases}$$

Finally, the values of z and s are given by

$$\begin{aligned} z &= \frac{\eta\theta\beta + (1-\eta)\,\alpha}{\eta\theta + 1 - \eta} = 2\frac{r^3 - r^2 + 2r + 4}{3r^4 - 4r^2 + 8r + 8}, \text{ and} \\ s &= \theta z = \frac{4(1-r^2)\,(r^3 - r^2 + 2r + 4)}{(2+2r-r^2)\,(3r^4 - 4r^2 + 8r + 8)}. \end{aligned}$$

[20] The intuition is similar to that in Dennert (1993) where no equilibrium in pure price strategies exists. There is a discontinuity of the payoffs such that even a slight change of prices produces a discontinuous shift in the expected market maker's clientele.

The properties of the mixed strategies equilibrium deserve some comments. In this equilibrium a dealer informed about a previous purchase, $X(1)$, never wins. In equilibrium, he sets an ask price equal to his reservation ask price. Further, the two possible types of dealer Y randomize over linked pairs of prices.[21] In particular, $Y(-1)$ randomizes in the interval $[s, z]$, whereas $Y(1, 0)$ randomizes in $[z, \alpha)$. The relationship between s and z is clear: at $t = 1$ dealer $Y(-1)$ trades a smaller but certain wealth, $(1+s-E(v|(-1, 1)) = s$, against a larger but less likely wealth, he gets z with probability θ. The expected profit that $Y(-1)$ derives from any of his price strategies is the same. Hence, $s = \theta z$. Dealer $Y(1, 0)$ gets zero expected profit as he may share the market with dealer $X(1)$. Both types of dealer Y may have $X(0, -1)$ as their opponent. Because of this, $X(0, -1)$ randomizes over the union of the asks set by his two potential opponents, i.e., over the interval $[s, \alpha)$. Further, he derives a positive expected profit from any of the ask prices he sets.

Note that as r approaches zero all ask prices approach 2 with probability one ($s \to z \to \alpha \to 1$). The intuition is clear: as r approaches zero, then either there is an order coming from an informed investor or there is no trade. Consequently, in equilibrium, dealers perfectly anticipate that a buy order implies $v = 2$.

In the limit, when $r = 1$, the opaque market becomes informationally equivalent to the transparent market. Note that in the first period each dealer was competitive in one side of the market. Consequently, a dealer who did not trade can correctly infer that the order type was the one for which he was not competitive. The equilibrium strategies at $t = 1$ converge to the equilibrium strategies in the transparent market.

The equilibrium strategies can be depicted by means of a box, shown in Figure 2. In the x-axis we arrange dealers depending of his willingness to transact. In the y-axis, we plot prices. In this box, when analyzing the ask side, we consider the southwest corner as the origin. In contrast, we use the northwest corner as the origin when we study the bid side.

[Insert Figure 2]

Both dealers' expected profits at $t = 1$ coincide. We can hence focus, w.l.o.g., in dealer X. First note that he can only benefit from his private information in case of reversal. If he attended a buy order at $t = 0$, then he only makes profits in the second period if there is a sell order, in which case, he makes profits of s. Consequently, his expected profits are ms, where m is the probability of reversal from a buy to a sell. If he did not trade, $q_0 \neq 1$, then he will only make profits if at $t = 1$ the order is a purchase, $q_1 = 1$. The profits he will make equal $(s - \theta\beta)$. Consequently, his expected profits are $n(s - \theta\beta)$, where $n = \Pr(q_1 = 1 | q_0 \in \{0, -1\})$ is the probability that $X(0, -1)$ assigns to a future buy. Adding up, using Bayes' rule, the overall expected profits from trading at $t = 1$ are

$$\frac{1}{16}\left[\left(2r + r^2\right)s + \left(2 + 2r - r^2\right)(s - \theta\beta)\right].$$

[21]Randomization over linked pair of prices is also found in the dynamic auction analized by De Frutos and Rosenthal (1998).

4.1.2 Investment in Perfect Learning

As with specialization, investment in perfect learning can induce two symmetric continuation paths. We here only analyze the continuation path in which dealer Y is competitive on both sides of the market. Note that if the competitive dealer did not trade in the first period, then he correctly infers that $q_0 = 0$. Thus, the potential players (or types of players) at $t = 1$ are: dealer Y who observed a buy order, dealer Y who knows that $q_0 = 0$, dealer Y who observed a sell order, and dealer X who is completely uninformed. We will refer to them as $Y(1)$, $Y(0)$, $Y(-1)$ and $X(1,0,-1)$, respectively. The following lemma derives their reservation selling fees.

Lemma 5 *If at $t = 0$ dealer Y was competitive in both sides of the market, then the reservation selling fees at $t = 1$ are the following:*

$$a^Y_{r,1}(\{1\}) = \alpha, \quad a^Y_{r,1}\{-1\}) = 0, \text{ and } \quad a^Y_{r,1}(\{0\}) = a^X_{r,1}(\{1,0,-1\}) = \beta.$$

Consequently,

$$A^Y_{r,1}(\{1\}) > A^Y_{r,1}(\{0\}) = A^X_{r,1}(\{1,0,-1\}) > A^Y_{r,1}(\{-1\}).$$

As in the previous path, dealers' information determines their willingness to sell. Obviously, the informed dealer has the same reservation quotes than if he were in a transparent market, whereas the uninformed dealer holds in both periods the same reservation quotes. The informed dealer has a double advantage over his competitor, if there was trading activity at $t = 0$. On the one hand, he has a more accurate estimate of the liquidation value of the risky asset. On the other hand, he knows precisely what information his competitor possesses. This double advantage will have an impact not only on profits but also on the pricing strategies of both dealers.

In equilibrium, the dealer with the smallest reservation ask, $Y(-1)$, will always undercut the ask set by his competitor. Dealer X accounts for this fact, and hence he optimally revises his beliefs about the true value, increasing his reservation ask price up to the reservation ask of dealer $X(0,1)$. This increase makes both $Y(-1)$ and $Y(0)$ increase the price they set to undercut the new (higher) reservation price of X. But then, dealer X only wins when meeting a dealer $Y(1)$. He accounts for this fact, and hence he optimally increases his reservation ask price up to the reservation ask of a dealer $Y(1)$, making both $Y(-1)$ and $Y(0)$ further increase the quotes they offer up to just slightly below the reservation ask of a dealer $Y(1)$. But then X can decrease his quote up to slightly above his starting reservation quote winning to any of his opponents while making profits. This will activate a behavior as the one described above leading to a cycle with no end which results in the non-existence of a pure strategy equilibrium. The mixed-strategies equilibrium is given in the next proposition.

Proposition 6 *Under perfect learning, the equilibrium ask quotation at $t = 1$ is given by $A_1(\Theta_D) = 1 + a_1(\Theta_D)$, where $a_1(\Theta_D)$ is set according to the following mixed-strategies:*

-A dealer who knows that there was a buy order sets his reservation selling fee, α.
-A dealer who knows that there was no trade randomizes by setting fees in the interval $[\eta\beta + (1-\eta)\alpha, \alpha)$. He assigns probability according to the distribution K, where

$$K(a) = 1 - \frac{\varphi_1(\alpha - a)}{\varphi_2(a - \beta)}.$$

-A dealer who knows that there was a sell order randomizes in the interval $[\beta, \eta\beta + (1-\eta)\alpha]$. He assigns probability according to the distribution J, where

$$J(a) = \frac{1}{\varphi_3}\left(\frac{a - \beta}{a}\right).$$

-The uninformed dealer randomizes in the interval $[\beta, \alpha]$ according to the probability distribution L, which has a mass point at α, where

$$L(a) = \begin{cases} \varphi_3 J(a), & \text{for all } \beta \le a \le \eta\beta + (1-\eta)\alpha, \text{ and} \\ \varphi_3 + \varphi_2 K(a), & \text{for all } \eta\beta + (1-\eta)\alpha \le a < \alpha, \end{cases}$$

with

$$\varphi_1 = \Pr(q_0 = 1 | q_1 = 1), \ \varphi_2 = \Pr(q_0 = 0 | q_1 = 1) \ \text{ and } \varphi_3 = \Pr(q_0 = -1 | q_1 = 1).$$

In equilibrium, the potential types of the informed dealer randomize over linked intervals of prices.[22] Type $Y(-1)$ randomizes over the interval $[\beta, \eta\beta + (1-\eta)\alpha]$, type $Y(0)$ randomizes over $[\eta\beta + (1-\eta)\alpha, \alpha)$ and the type with the most positive information about the true value, $Y(1)$, sets his reservation selling fee, α. The uninformed dealer randomizes in the convex hull of the interval of fees used by the potential types of his competitor, i.e., in the interval $[\beta, \alpha]$. He makes zero expected profits with any of the pure strategies he uses.

The equilibrium strategies of this continuation path are shown in the Figure 3.

[Insert Figure 3]

In the limit, when $r = 0$, all dealers set an ask price equal to 2 as there will be common knowledge that any buy order comes from an informed investor. For any other r in equilibrium there is price dispersion as both dealers must randomize. When $r = 1$, the uninformed dealer randomizes by setting ask prices belonging to the interval $\left[\frac{3}{2}, \frac{9}{5}\right]$, dealer $Y(-1)$ randomizes in the interval $\left[\frac{3}{2}, \frac{9}{5}\right)$ and $Y(1)$ sets an ask price equal to $9/5$.

As in the previous continuation path, the informed dealer only expects positive profits in case of reversals. So, dealer $Y(-1)$ only expects positive profits if there is a buy order

[22] If both dealers set the same prices at $t = 0$ then the equilibrium continuation is similar to the one described above. A dealer who knows that there was a buy order sets his reservation selling fee, α. A dealer who knows that there was a sell order plays a mixed strategy with support $[\beta, s]$. He assigns probability according to a distribution M. The uninformed dealers randomizes in the interval $[\beta, s]$ according to the probability distribution L, $L(a) = 1 - \frac{\beta}{a}$, which has a mass point at s, $s \le \eta\beta + (1-\eta)\alpha$. Finally, s and $M(a)$ are such that the uninformed dealers always break even.

in which case he makes profits of β. Consequently, his expected profits are $\varphi_3\beta$, where, recall, φ_3 is the probability of a reversal from a sell to a buy. By symmetry, $\varphi_3\beta$ are also the expected profits of dealer $Y(1)$. Finally, dealer $Y(0)$ is indifferent between $q_1 = 1$ and $q_1 = -1$ since his expected profits in the two possible events are equal. In either case, he plays a mixed strategy with support $[\eta\beta + (1-\eta)\alpha, \alpha)$, which yields expected profits of $\frac{1}{2}(1+r)\varphi_1(\alpha - \beta)$. Direct computations yield the following expected profits from trading at $t = 1$ for the informed dealer:

$$E\left(\Pi_1^*\left(a_1\left(\Theta_Y\right), b_1\left(\Theta_Y\right)\right)\right) = \varphi_3\frac{3-r}{4} = \left(\frac{2r+r^2}{1+r}\right)\frac{3-r}{16}.$$ [23]

The comparison between the expected profits under perfect learning about the order type and the expected profits under specialization shows that (private) information is valuable. Further, it is interesting to note (see figure below) the sharp difference between the expected profits in the two scenarios when $r = 1$.[24] The rationale is as follows: if dealers specialize, then their private information can be perfectly inferred and they hence do not earn any informational rents. In contrast, under perfect learning the expected profits are strictly increasing in r, and therefore, they are positive when $r = 1$. To understand this result notice that there are two opposite effects. On one hand, the signal they gain from the order type is less informative as it is less likely that it will come from an informed investor. Consequently, their informational rents decrease as their private information is less valuable. But, on the other hand, an increase in r reduces the probability of no trading, and hence, the types $Y(1)$ and $Y(-1)$, which are the types of dealers with a double informational advantage with respect to his competitor, are more likely. The second effect dominates making expected profits under perfect learning strictly increasing in r as shown in the next plot.

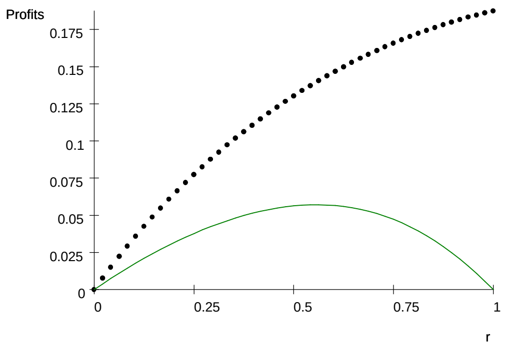

Figure 4. Expected profits at t=1 in the two possible continuation paths. We have used the continuous line for specialization.

[23] Note that they are strictly larger than the expected profits if prices were equal at $t = 0$ which equal $\frac{\left(2r+r^2\right)}{(1+r)}\frac{1}{32}$.

[24] When $r = 0$ profits are zero in either scenario as any order comes from an informed investor.

4.2 Optimal Quotes at $t = 0$

Given the optimal responses corresponding to $t = 1$, we now calculate the Perfect Bayesian equilibrium in the opaque market. It is important to point out that opaqueness in the first round may generate equilibrium prices that depart from the independence property. In most microstructure models [see, for instance, Glosten and Milgrom (1985), Easley and O'Hara (1987), Dennert (1993), and Leach and Madhavan (1993)], each market maker faces two independent bidding problems: one for the ask and other for the bid side of the market. In our model, at $t = 0$, each dealer realizes that when setting his ask price, he needs to consider the relative position of his bid with respect to his competitor's bid, to deduce his expected profits, and vice versa.

Using the equilibrium continuation derived before, the overall expected profits accruing to dealer D if he sets prices $1 + a_0^D$ and $1 - b_0^D$ while his competitor sets prices $1 + a_0^{D'}$ and $1 - b_0^{D'}$, are as follows:

- If he is competitive in both sides of the market ($a_0^D < a_0^{D'}$ and $b_0^D < b_0^{D'}$), then[25]

$$E\left[\Pi^D\right] = \frac{1}{4}\left((1+r)\left(a_0^D + b_0^D - 2\beta\right) + \varphi_3(3-r)\right);$$

- If he is only competitive in the ask side ($a_0^D < a_0^{D'}$ and $b_0^D > b_0^{D'}$), then

$$E\left[\Pi^D\right] = \frac{1}{16}\left(4(1+r)\left(a_0^D - \beta\right) + \left(2r + r^2\right)s + \left(2 + 2r - r^2\right)(s - \theta\beta)\right);$$

- If he is only competitive in the bid side ($a_0^D > a_0^{D'}$ and $b_0^D < b_0^{D'}$), then

$$E\left[\Pi^D\right] = \frac{1}{16}\left(4(1+r)\left(b_0^D - \beta\right) + \left(2r + r^2\right)s + \left(2 + 2r - r^2\right)(s - \theta\beta)\right);$$

- If he is not competitive in any side of the market ($a_0^D > a_0^{D'}$ and $b_0^D > b_0^{D'}$), then

$$E\left[\Pi^D\right] = 0.$$

To better understand expected profits above, consider first the case in which a dealer is competitive on both sides of the market and, further, he chooses the same selling and buying fees. Let Δ_0^{TS} be the fee that makes null the expected profits of attending both sells and buys. Similarly, consider the case in which a dealer is competitive on just one side of the market. Let Δ_0^{OS} denote the fee that makes null the expected profits from attending either a buy or a sell. Using this notation we can rewrite dealer D's expected profits as follows:

[25] It is easy to see that perfect learning dominates any continuation path in which there are equal prices either in one side of the market or in both sides. We will hence consider here that dealers set different prices. This restriction is satisfied in equilibrium as we will later show.

Event	Expected profits
$a_0^D < a_0^{D'}$ and $b_0^D < b_0^{D'}$	$\frac{1}{4}(1+r)\left(a_0^D + b_0^D - 2\Delta_0^{TS}\right)$
$a_0^D < a_0^{D'}$ and $b_0^D > b_0^{D'}$	$\frac{1}{4}(1+r)\left(a_0^D - \Delta_0^{OS}\right)$
$a_0^D > a_0^{D'}$ and $b_0^D < b_0^{D'}$	$\frac{1}{4}(1+r)\left(b_0^D - \Delta_0^{OS}\right)$
$a_0^D > a_0^{D'}$ and $b_0^D > b_0^{D'}$	0

Notice that for any given pair of prices set by dealer D', dealer D prefers to attend both sides of the market instead of specializing on the bid side if and only if $a_0^D \geq 2\Delta_0^{TS} - \Delta_0^{OS}$. Similarly, he prefers to attend both sides instead of specializing on the ask side if and only if $b_0^D \geq 2\Delta_0^{TS} - \Delta_0^{OS}$. Next proposition shows that in an opaque market one dealer (let us say, w.l.o.g., dealer Y) will be competitive on both sides of the market and will hence gain perfect information about the order flow. Consequently, in equilibrium there will be investment in perfect learning.[26] Next proposition summarizes the strategic behavior of dealers in an opaque market.

Proposition 7 *The following set of strategies constitutes a Perfect Bayesian equilibrium for the opaque market:*

At $t = 0$ *:*

-Dealer Y *sets the following ask and bid:* $1 + \Delta_0^{TS}$ *and* $1 - \Delta_0^{TS}$*, and*

-Dealer X *randomizes by setting prices* (A, B) *such that*

$$\left\{(B, A) \in \left[1 - \Delta_0^{OS}, 1 - \Delta_0^{TS}\right] \times \left[1 + \Delta_0^{TS}, 1 + \Delta_0^{OS}\right] : A - B = \Delta_0^{OS} + \Delta_0^{TS}\right\}, \qquad (3)$$

where

$$\Delta_0^{TS} = \frac{8 + 2r - r^2 + r^3}{8(1+r)^2}, \text{ and}$$
$$\Delta_0^{OS} = \frac{3r^7 - 7r^6 + 4r^5 + 50r^4 - 4r^3 - 28r^2 + 40r + 32}{2(1+r)(8 - 4r^2 + 3r^4 + 8r)(2 + 2r - r^2)}.$$

He chooses among the prices that satisfy (3) according to a uniform distribution probability.

At $t = 1$*, dealers follow the equilibrium strategies described in Proposition 6.*[27]

[26] In Leach and Madhavan (1993) it is shown that a specialist may experiment with prices to induce more informative order flow whereas in multiple-dealer markets there is no experimentation (investment in producing information) because of free-riding problems. We here show that investment in producing information is also present in markets with multiple dealers when trade disclosure is not mandatory.

[27] It is important to point out that the proposed strategies in Madhavan (1995) do not constitute an equilibrium in our framework. His equilibrium is as follows. At $t = 0$ both dealers set the same prices. At $t = 1$, the uninformed dealer quotes a price equal to the expected value of the security given that a continuation occurs, whereas the informed dealer (marginally) improves these quotes if and only if there is a reversal. Note that at the second round they play a pure strategies equilibrium while we have shown that in (our) opaque market there is no continuation equilibrium in pure strategies.

Note that, in equilibrium, for any price set by dealer X his opponent is better off by attending both sides of the market as it is satisfied that $a_0^D = \Delta_0^{TS} \geq 2\Delta_0^{TS} - \Delta_0^{OS}$. Our prediction is supported by laboratory experiments conducted by Bloomfield and O'Hara (1999,2000). These authors call this behavior as *capturing early order flow.* Consequently, the second round of trade in the opaque market will be characterized by the interplay between a perfectly informed dealer and a completely uninformed dealer. The asymmetry between the dealers gives rise to an equilibrium in the second round in mixed strategies and, consequently, to price dispersion.[28] Moreover, Proposition 7 provides an interesting contrast to the finding of Madhavan (1995) and Wu and Zhang (2002), which both of them examine the effect of informed disclosure on securities market performance. In our equilibrium the uninformed dealer may attend the order even in a reversal.

Next corollary characterizes the corresponding equilibrium price quotation functions.

Corollary 8 *There exists a Perfect Bayesian equilibrium in the post-trade opaque market, where the equilibrium price quotation function at $t = 0$, $P_0(q_0)$, satisfies*

$$P_0(q_0) = 1 + \Delta_0^{TS} q_0, \text{ and}$$

the equilibrium price quotation at $t = 1$, $P_1(q_1; q_0)$, is given by

$$P_1(q_1; q_0) = 1 + \min\{a_1(q_0 \times q_1), a_1(1, 0, -1)\} \times q_1.$$

Both dealers make overall expected profits equal to zero. Nevertheless, the dealer that chooses to be competitive on both sides of the market makes negative expected profits in the initial period and positive expected profits in the final period. Just note that

$$P_0(q_0) = 1 + \Delta_0^{TS} q_0 = 1 + \beta q_0 + (\Delta_0^{TS} - \beta) q_0 = E[v|q_0] - \pi q_0,$$

where $\pi > 0$, whenever $r \neq 0$. Notice that π reflects the order flow payment that a dealer assumes in order to gain monopoly power over information by capturing order flow that need not be disclosed.

5 Comparison across Market Structures

This section is devoted to the comparison between the two market structures. To this end, the superscript $T(O)$ in a variable means that it corresponds to the post-trade transparent (opaque) market. We will first describe some characteristics of the sequences of prices generated in the two markets to examine the main differences in price dynamics between them. We will then study the implications of structure for metrics of market quality such as spreads, volatility and price efficiency. Finally, we study the impact of transparency on the welfare of market participants.

[28] In the dealer market for corporate bonds (a low transparent market), Gehr et al. (1992), using lower-frequency corporate bond quote data, find that bid-ask spreads between dealers do often not intersect, a finding that is consistent with our equilibrium characterization.

5.1 Price Dynamics

Equilibrium price quotations in the two market structures are given by:

$$-P_0^T(q_0) = 1+\beta q_0, \text{ and } P_1^T(q_1;q_0) = \begin{cases} 1+\alpha q_1, \text{ for } q_1 = q_0 \neq 0, \\ 1, \text{ for } q_1 \neq q_0 \neq 0, \\ 1+\beta q_1, \text{ for } q_0 = 0, \end{cases}$$

$$-P_0^O(q_0) = 1+\Delta_0^{TS} q_0, \text{ and } P_1^O(q_1;q_0) = 1+\min\{a_1(q_0\times q_1), a_1(1,0,-1)\}\times q_1.$$

Prices in the first round are more attractive in the opaque market as the competition between dealers to secure an informational advantage makes them set higher bids and lower asks. The price difference between the two structures $(\Delta_0^{TS}-\beta)$ is significative, and it increases with r as shown in next figure.

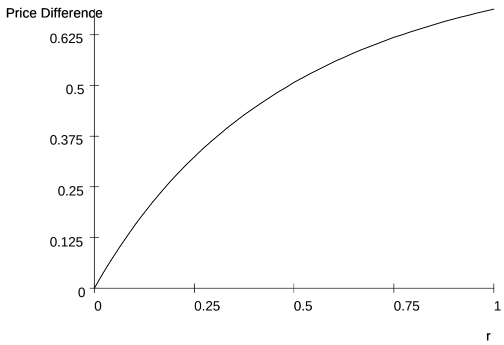

Figure 5. The difference between the first round asks in the two markets.

Consider now the prices at the second round of trade. If there is no trading activity in the first period, then market makers only change their quotes in the opaque market. In the transparent market, dealers do not observe new relevant information and consequently, they do not revise their quotes.[29] By contrast, in the opaque market dealers set different prices across periods even when no transaction in the first period occurs. Recall that the most competitive dealer sets prices at $t=0$ that yield negative current expected profits. Consequently, if $q_0 = 0$, then he must unwind his position to avoid losses in period 1. This dynamic strategy results in smaller bids and higher asks in the opaque market when there is no trade in the first round.

If there is trading activity at both rounds, prices in the second round are more attractive for investors in the opaque market in the event of a continuation whereas the opposite holds in reversals. In a continuation, a sequence of buys (sells) generates an increasing (decreasing) sequence of prices in either market structure. In either period, the opaque

[29] Easley and O'Hara (1992) obtain that the spreads will decrease over time when no orders arrive in a post-trade transparent market. Their result is due to the fact that the absence of trades may imply that no news arrive, i.e. no informational event takes place, and therefore the likelihood of informed trading decreases. By contrast, in our model, it is assumed that there is always new information.

market provides a smaller ask and a larger bid (in expected terms). In a reversal from a buy to a sell the second period bid is smaller in the opaque market.

Another important difference between the two market structures is that in the opaque market quotes corresponding to the second round of trade are uncertain (as dealers play mixed strategies) despite there is no new shock in the fundamentals. Also note that under opaqueness the probability with which the two dealers set the same quotes in either round is zero, whereas it is one in the transparent market.

5.1.1 Spreads

Consider now the impact of post-trade opaqueness on spreads.

Corollary 9 *a) The intraday patterns in bid-ask spreads are opposite under both market structures. While in the transparent mechanism spreads decrease from period 0 to period 1, in the opaque market structure, they increase.*

b) Expected spreads are decreasing in r in both market structures. The difference between expected spreads (in absolute value) is also decreasing in r in the transparent market structure. By contrast, in the opaque mechanism, it is increasing in r unless r is large enough.

In the transparent market, at $t = 0$ dealers are unwilling to improve competitive prices because their competitors will free-ride on the information gained by them by observing the trading history. Further, the expected bid-ask spread narrows over the day because the adverse selection faced by the market makers is mitigated as they learn from order flow. In the opaque market, by contrast, a completely different temporal pattern emerge. At the first period, the most competitive dealer quotes a tight spread to acquire private information.[30] This creates a double winner's curse problem in his competitor: i). with respect to the informed investor, and ii). with respect to the informed dealer. This relatively severe adverse selection problems widens his spreads and, by extension, the spreads of the informed dealer who needs to cash his investment in information acquisition.

Expected spreads are decreasing in r as the adverse selection problem faced by market makers is mitigated when the probability that an uninformed trader transacts increases. By the same token, an increase in r leads to a less informative order flow, and therefore, market makers have a more similar information across periods of trade. In the transparent market, this is all that counts when setting prices; consequently, the difference between expected spreads is decreasing in r. In the opaque market, by contrast, as r increases the winner's curse with respect to the informed investor does also decrease but the one with respect to the informed dealer increases (recall that the expected profits from trading at

[30] This could provide an interpretation for the findings by Madhavan, Porter and Weaver (2000) that an increase in ex-ante transparency on the Toronto Stock Exchange in 1990 led to an increase in spreads.

$t = 1$ of the perfectly informed dealer are increasing in r). When r is small the second effect dominates and, consequently, the difference in spreads (in absolute value) becomes increasing in r.

5.1.2 Volatility

Suppose now that trading activity has occurred at $t = 0$. The possible absolute price changes in the two structures are summarized in next table:

	Transparent	Opaque (Expected values)
Continuation	$\alpha - \beta = \frac{r(2+r)}{(2+2r+r^2)(1+r)}$	$\frac{(r+2)r(3-r)}{8(1+r)^2} + \frac{\ln\left(\frac{4(r+1)}{4+2r-r^2}\right)}{r+1} + \frac{r(r+2)\ln\left(\frac{4+2r-r^2}{2+2r+r^2}\right)}{4(r+1)^2}$
Reversal	$\beta = \frac{1}{1+r}$	$\frac{24+18r-r^2+r^3}{8(1+r)^2} + \frac{\left(r^2-2r-4\right)\ln\left(\frac{4(1+r)}{-r^2+2r+4}\right)}{(r+2)r(1+r)}$

Price volatility is due to differences between the prices set by the dealers who accommodate the order in the two periods. In the transparent market, the only difference between them is the information about v. By contrast, in the opaque market there are strategic considerations that influence price quotations. To understand this point, consider a continuation in buy orders. In the opaque market, the dealers that will attend the buys are dealer Y in the first period, and dealer $X(1, 0, -1)$ in the second one. Both dealers have the same information about v. However, they differ in the strategies they follow (dynamic versus static) and in the degree of uncertainty about the competitor's type (while at $t = 0$ there is no such uncertainty, at $t = 1$ dealer $X(1, 0, -1)$ does not know his competitor's type). The strategic differences in the opaque market have a larger impact than the informational differences in the transparent market resulting in a larger price change in the opaque market.

Consider now a reversal from a buy to a sell. In the transparent market it induces a large price change, in comparison with that of a continuation, as beliefs revert. In the opaque market, this informational difference reinforces the strategic difference, resulting in an absolute price change which exceeds that of a continuation. Consequently, in either case, $\left|P_1^T - P_0^T\right| < \left|E(P_1^O) - P_0^O\right|$ holds. As the probabilities of continuations and reversals are equal in both market structures, we obtain the following result:

Corollary 10 *Price volatility, measured by the expected absolute change in price is higher in the opaque market, i.e.,* $\left(E\left(\left|P_1^T - P_0^T\right|\right)\right) < \left(E\left(\left|E(P_1^O) - P_0^O\right|\right)\right)$ *for all* r.

5.1.3 Efficiency

In what follows we use the concepts of efficiency proposed by Roberts (1967). We will say that prices are strong-form efficient if they reflect all private information, semi-strong-form efficient if they reflect all publicly available information, and weak-form efficient if they reflect the information in their own past values. Note that whenever some traders have superior information, prices will not exhibit strong-form efficiency. In our setup, this holds whatever $r \neq 0$. In the standard sequential trade model proposed by Glosten and Milgrom (1985), the sequence of transaction prices follows a martingale with respect to the sequence of trades. This property implies that prices are semi-strong-form efficient in the sense that they reflect all the information available to the market makers. Since these authors model a post-trade transparent market, it is not surprising that our transaction prices corresponding to the transparent market hold the same property. However, in the post-trade opaque market, transaction prices are not a martingale with respect to past prices and so they are not weak-form efficient. To understand this result, notice that at $t = 0$ when setting an ask price, dealers bear in mind that this price corresponds to a purchase. In the opaque market this is not made public, and consequently, prices at $t = 1$ do not necessarily incorporate this information. All these results are summarized in the following corollary.

Corollary 11 *In equilibrium, transaction prices are semi-strong-form efficient in the transparent market and they are not weak-form efficient in the opaque market.*

5.2 Welfare

Now we turn our attention to the effect of transparency on the welfare of market participants.

Corollary 12 *a) The investors who transact in the first period prefer the opaque market.*

b) If there is no trading activity in the first period, then investors prefer the transparent market. Otherwise, those investors whose order type coincides (differs) with the previous one are better (worse) off in the opaque market.

This result tells us that the lack of transparency benefits investors who transact at $t = 0$ at the expense of traders who arrive later. The rationale is as follows. Opaqueness increases competition among dealers to attract valuable order flow, leading to better prices for all period 0 investors. On the other hand, concerning the later period, in case of a continuation all period 1 investors prefer the opaque market.[31] An informed trader prefers the opaque market because it provides her more camouflage. An uninformed trader prefers the opaque market because he will transact with the uninformed dealer, whereas in the transparent market she will transact with an informed dealer. By contrast, in case of no trading in the first round or a reversal any investor prefers the transparent market. Notice that, in this case the opaqueness reduces price competition because the informed dealer can undercut his competitor's quote while still earning positive profits.

[31] This result is consistent with Madhavan (1995) that shows that large traders are better off in an opaque market because this structure allows them to break their orders over time without attracting too much attention of the market, and hence, reducing the corresponding price impact.

6 Conclusions

Using a two-stage model of trading in a dealer market, we have here analyzed the effects of post-trade transparency on the performance of two market structures that differ in the amount of information that is publicly available at the end of the first stage. The main distinctive characteristic of our model is that the lack of transparency forces market makers to consider if they prefer to specialize on one side of the market or to jointly determine their asks or bids to become competitive in both sides of the market.

We can summarize our results as follows. In the opaque market dealers may be able to offer better prices because they can profit in subsequent trading from the private information they infer from the current trades. The upshot of this is that the investors that transact in the first period are better off in the opaque market, but this comes at the expense of traders who transact in the second period. Moreover, it is shown that if there is no trading activity in the first period, then market makers only change their quotes in the opaque market. Additionally, we obtain that the lack of transparency provokes several price distortions: it exacerbates price volatility, creating price dispersion, and it also reduces price efficiency.

There are several extensions of the model that could be explored. Notice that the risk-neutrality assumption makes the dealers' inventory positions not to play any role. However, it seems likely that both information and inventory positions are relevant in real financial markets. Consequently, a richer model with risk-aversion is required to examine both aspects and the interaction between them.

Another relevant extension could allow multiple trading periods. We have shown that in the opaque market prices do not exhibit the Martingale property. We suspect that in this new dynamic context the first-order price differences could not be uncorrelated. This topic is of relevance since until now in all the information-based models this property is satisfied. Finally, notice that in our model the probability that an uninformed trader transacts is exogenous. In a more complex analysis it could be endogenous, and in this case the informativeness of the order flow would depend on dealers' behavior.

7 Appendix

Since throughout the paper we must determine how the market makers' beliefs evolve over the trading day, we first state the probabilities of the events $v = 2$ and $v = 0$ conditional on a sequence of trades. Using Bayes' rule, it follows that

$$\Pr\left(v=2|(q_0,q_1)\right)=\begin{cases}\frac{1+\alpha q_0}{2}, \text{ if } (q_0,q_1)\in\{(1,1),(-1,-1)\}\\ \frac{1+\beta}{2}, \text{ if } (q_0,q_1)\in\{(1,0),(0,1)\}\\ \frac{1-\beta}{2}, \text{ if } (q_0,q_1)\in\{(-1,0),(0,-1)\}\\ \frac{1}{2}, \text{ otherwise,}\end{cases}$$

where $\alpha = \dfrac{2+2r}{2+2r+r^2}$ and $\beta = \frac{1}{1+r}$. Finally, the conditional probability of the event $v = 0$ is derived as follows:

$$\Pr\left(v = 0|(q_0, q_1)\right) = 1 - \Pr\left(v = 2|(q_0, q_1)\right), \text{ for any } (q_0, q_1).$$

Proof of Proposition 1: In the post-trade transparent market, the transaction price must equal the expected value of the asset conditional upon the trading history and the incoming order. Hence, $P_1(q_1; q_0) = E\left(v|\left(q_0, q_1\right)\right)$. Using the previous conditional probabilities of v, we obtain the desired expressions.

Similarly, $P_0(1) = E(v|q_0 = 1) = \Pr(q_1 = 1|q_0 = 1)E(v|(1,1)) +$

$$+\Pr(q_1 = 0|q_0 = 1)E(v|(1,0)) + \Pr(q_1 = -1|q_0 = 1)E(v|(1,-1)).$$

Using Bayes' rule and the equilibrium quotation functions at $t = 1$, the expression for $P_0(1)$ is obtained. Similar computations provide the expression for $P_0(-1)$. ■

Proof of Lemma 2: Appealing to the symmetry in both the model and the strategies, we only need to compute dealer X's reservation ask price. If he executed the order in the first stage, then at $t = 1$, then he has the same information as in the transparent market structure, and consequently, applying Proposition 1 his reservation ask price is derived. If he did not execute the order at $t = 0$, he deduces that either $q_0 = 0$ or $q_0 = -1$, as he was competitive on buys. Therefore, $A^X_{r,1}\left(\{0,-1\}\right) = E\left(v|q_0 \in \{0,-1\}, q_1 = 1\right) = \theta E\left(v|\left(0,1\right)\right) + \left(1-\theta\right) E\left(v|\left(-1,1\right)\right)$, where using Bayes' rule

$$\theta = \Pr(q_0 = 0, q_1 = 1|q_0 \in \{0,-1\}, q_1 = 1) = \frac{2(1-r^2)}{2+2r-r^2} \tag{4}$$

Finally, using Proposition 1 the desired expression for this reservation ask price is obtained. ■

Proof of Proposition 4: If there was a sell order in $t = 0$, then dealer Y knows that his opponent's information set is $\{0,-1\}$. We claim that he randomizes in the interval $[s, z]$. By setting an ask price equal to $1 + s$, he wins with probability one achieving expected profits from trading $\Pi_1\left(Y(-1); s\right) = s$. By playing $1+z$, he wins with probability $1-G(z)$ and he loses with the complementary probability. Consequently, $\Pi_1\left(Y(-1); z\right) = (1 - G(z))z$.
Since in a mixed strategy equilibrium expected profits from playing any selling fee in the support must be equal, $\Pi_1\left(Y(-1); s\right) = \Pi_1\left(Y(-1); z\right)$ yields

$$G(z) = \frac{z-s}{z}. \tag{5}$$

Consider now dealer X with information set $\{0,-1\}$. This player did not trade in the first round and, hence, he does not know the identity of his opponent. He believes his opponent is $Y(0,1)$ with probability θ, whose expression is given in (4). We claim that he randomizes in the interval $[s, \alpha)$, with $s < z < \alpha$. By playing s, he wins with probability one, and then, his expected profits from trading are given by

$$\Pi_1(X(0,-1); s) = 1 + s - E(v|q_0 \in \{0,-1\}, q_1 = 1). \tag{6}$$

By playing z, with probability one he wins if his opponent is $Y(0,1)$, whereas with probability one he loses if his opponent is $Y(-1)$. Since in the first case dealer X deduces that $q_0 = 0$, we have

$$\Pi_1(X(0,-1); z) = \theta\left(1 + z - E(v|q_0 = 0, q_1 = 1)\right). \tag{7}$$

Finally, when playing $a \to \alpha$, he wins with probability $1 - F(\alpha)$ if his opponent is $Y(0,1)$, and he always loses if his opponent is $Y(-1)$. Expected profits converge to

$$\theta\left(1 - F(\alpha)\right)\left(1 + \alpha - E(v|q_0 = 0, q_1 = 1)\right). \tag{8}$$

In a mixed strategy equilibrium, (6) = (7) and (6) = (8) or, using the results in Proposition 1 and Lemma 2,

$$\begin{aligned} s &= \theta z, \text{ and} &(9)\\ s - \theta\beta &= \theta\left(1 - F(\alpha)\right)(\alpha - \beta). &(10)\end{aligned}$$

Consider now that Y did not trade in $t = 0$ and does not hence know the identity of his opponent. He believes his opponent is $X(0,-1)$ with probability η, where $\eta = \Pr(q_0 = 0, q_1 = 1|q_0 \in \{0,1\}, q_1 = 1)$. We claim that he randomizes in the interval $[z, \alpha]$. By playing z, he wins with probability $1 - G(z)$ if his opponent is $X(0,-1)$ and with probability one if his opponent is $X(1)$. The expected profits from playing this strategy are

$$\eta\left(1 - G(z)\right)\left(1 + z - E(v|q_0 = 0, q_1 = 1)\right) + (1 - \eta)\left(1 + z - E(v|q_0 = 1, q_1 = 1)\right).$$

Finally, when playing α he loses with probability one if his opponent is $X(0,-1)$ and he obtains zero expected profits if his opponent is $X(1)$ since in this case he deduces that $q_0 = 1$ and $1 + \alpha = E(v|q_0 = 1, q_1 = 1)$. Thus, $\Pi_1(Y(0,1); \alpha) = 0$.
Equalizing the expected profits from playing these two strategies, we get

$$\eta\left(1 - G(z)\right)(z - \beta) + (1 - \eta)(z - \alpha) = 0. \tag{11}$$

Using (5), equations (9) and (11) define a system of two equations and two unknowns whose solution is $z = \frac{\eta\theta\beta + (1-\eta)\alpha}{\eta\theta + 1 - \eta}$ and $s = \theta\frac{\eta\theta\beta + (1-\eta)\alpha}{\eta\theta + 1 - \eta}$, which are both larger than the reservation selling fees of players $Y(0,1)$, $X(0,-1)$ and $Y(-1)$. Finally, by substituting the value of s into (10), we obtain that $F(\alpha) = \eta\frac{\theta}{\eta\theta + 1 - \eta}$.

Once the limits of the supports are derived, we can now proceed to compute the distribution functions.

In order to get F we first compute the expected profits of dealer $X(0,-1)$ when choosing $a > z$, which are $\theta(1 - F(a))(a - \beta)$. Differentiating with respect to a and equating the derivative to zero, it follows that $f(a)(a-\beta) = 1 - F(a)$. This f.o.c. generates a differential equation which boundary condition is $F(z) = 0$. The solution to the differential equation yields $F(a) = \frac{a-z}{a-\beta}$.

Similarly, to obtain H we differentiate the expected profits of dealer $X(0,-1)$ with respect to a. This allows us to obtain a differential equation which boundary condition is $H(s) = 0$. The solution to the differential equation yields $H(a) = \frac{1}{1-\theta}\left(\frac{a-s}{a}\right)$.

Finally, to derive G we first study the problem faced by $Y(-1)$. His expected profits from trading become $(1-G(a))a$,with $a < z$. Differentiating with respect to a, we obtain a f.o.c. which defines a differential equation whose boundary condition is $G(s) = 0$. The solution to this differential equation yields $G(a) = \frac{a-s}{a}$. Notice that $G(z) = \frac{z-s}{z}$, which is consistent with (5). Similarly, to derive the final part of G, i.e., when $a > z$, we consider the maximization problem faced by $Y(0,1)$. His expected profits from trading will be:

$$\eta\left(1-G(a)\right)(a-\beta) + (1-\eta)(a-\alpha).$$

Differentiating with respect to a, we obtain a f.o.c. which defines a differential equation with boundary condition $G(z) = \frac{z-s}{z}$. The solution to the differential equation yields:

$$G(a) = \frac{a - (\eta\beta + (1-\eta)\alpha)}{\eta(a-\beta)},$$

with $G(\alpha) = 1$ as claimed. ■

Proof of Lemma 5: It is omitted since it is similar to the proof of Lemma 2. ■

Proof of Proposition 6: We first note that dealer X makes zero expected profits from trading at $t = 1$ when playing the pure strategies: $a^X_{r,1}(\{1,0,-1\})$, $a^X_{r,1}(\{1,0\})$ and $a^X_{r,1}(\{1\})$. Let φ_1, φ_2 and φ_3 denote the probabilities assigned by this dealer to the events that his opponent is $Y(1)$, $Y(0)$ and $Y(-1)$, respectively. Thus,

$$\varphi_1 = \Pr(q_0 = 1|q_1 = 1), \varphi_2 = \Pr(q_0 = 0|q_1 = 1), \text{ and } \varphi_3 = \Pr(q_0 = -1|q_1 = 1).$$

Bayes' rule yields the expressions for these probabilities. If dealer X sets a such that $\beta < a < \eta\beta + (1-\eta)\alpha$, then his expected profits from trading are:

$$\varphi_1(a-\alpha) + \varphi_2(a-\beta) + \varphi_3(1-J(a))a.$$

Since these expected profits must be zero, it follows that $J(a) = \frac{1}{\varphi_3}\left(\frac{a-\beta}{a}\right)$. Similarly, if dealer X plays a, with $\eta\beta + (1-\eta)\alpha < a < \alpha$, then his expected profits from trading are:

$$\varphi_1(a-\alpha) + \varphi_2(1-K(a))(a-\beta).$$

Since these expected profits must also be zero, we have $K(a) = 1 - \frac{\varphi_1(\alpha-a)}{\varphi_2(a-\beta)}$.

Consider now player $Y(-1)$. His expected profits from trading at $t = 1$ when setting his opponent reservation ask are β. Similarly, by quoting $\eta\beta + (1-\eta)\alpha$, his expected profits from trading are $(1 - L(\eta\beta + (1-\eta)\alpha))(\eta\beta + (1-\eta)\alpha)$. Since these expected profits must be equal, we must have

$$L(\eta\beta + (1-\eta)\alpha) = \frac{2r + r^2}{4 + 4r} = \varphi_3.$$

Consider now player $Y(0)$. His expected profits from trading playing $\eta\beta + (1-\eta)\alpha$ are $(1 - L(\eta\beta + (1-\eta)\alpha))(1-\eta)(\alpha - \beta)\cdot$ By setting a selling fee $a \to \alpha$, his expected profits from trading would converge to $(1 - L(\alpha))(\alpha - \beta)$. Equating these two values and operating, we obtain $L(\alpha) = \varphi_2 + \varphi_3$.

To end this proof, we now proceed to derive the expression of the distribution function L. We first compute the expected profits of dealer $Y(-1)$ when playing a selling fee $a > \beta$. Differentiating the expected profits with respect to a, the f.o.c. generates a differential equation which boundary condition is $L(\eta\beta + (1-\eta)\alpha) = \varphi_3$. The solution to the differential equation yields:

$$L(a) = \frac{a - \beta}{a}, \text{ for all } \beta \leq a \leq \eta\beta + (1-\eta)\alpha\cdot$$

Similarly, to obtain the last part of L we differentiate dealer $Y(0)$'s expected profits from trading with respect to a for $a > \eta\beta + (1-\eta)\alpha$ to obtain a differential equation which boundary condition is $L(\eta\beta + (1-\eta)\alpha) = \varphi_3$. The solution to this differential equation yields: $L(a) = 1 - \frac{(1-\varphi_3)(1-\eta)(\alpha-\beta)}{a-\beta}$,for all $\eta\beta + (1-\eta)\alpha \leq a < \alpha$. Finally, from direct computations, we rewrite the expression of L as in the statement of this proof. ■

Proof of Proposition 7: By following the purported equilibrium strategies Y gets zero expected profits. To study the profitability of potential deviations we first note that we can restrict our attention to deviations that involve setting prices (A, B) such that $(B, A) \in \left[1 - \Delta_0^{OS}, 1 - \Delta_0^{TS}\right] \times \left[1 + \Delta_0^{TS}, 1 + \Delta_0^{OS}\right]$ as any strategy that implies setting a price outside these ranges is weekly dominated. Note that any $B < 1 - \Delta_0^{OS}$ is payoff equivalent to the strategy $B = 1 - \Delta_0^{OS}$. Similarly, $B > 1 - \Delta_0^{TS}$ yields strictly smaller payoffs than the strategy $B = 1 - \Delta_0^{TS}$, for any A and any (A_0^X, B_0^X).

Given X's equilibrium behavior, dealer Y's expected profits from a strategy (A, B) are given by

$$\begin{aligned}&\frac{1+r}{4}\left\{\Pr\left(A < A_0^X,\, B > B_0^X\right)\left[A - B - 2\Delta_0^{TS}\right] + \right.\\&\left.\Pr\left(A < A_0^X,\, B < B_0^X\right)\left[A - 1 - \Delta_0^{OS}\right] + \Pr\left(A > A_0^X,\, B > B_0^X\right)\left[1 - \Delta_0^{OS} - B\right]\right\}.\end{aligned}$$

Since $B_0^X = A_0^X - \left(\Delta_0^{OS} + \Delta_0^{TS}\right)$ expected profits above simplify to:

$$\begin{aligned}&\frac{1+r}{4}\left\{\Pr\left(A < A_0^X < B + \Delta_0^{OS} + \Delta_0^{TS}\right)\left[A - B - 2\Delta_0^{TS}\right] + \right.\\&\Pr\left(A_0^X > \max\left\{A,\ B + \Delta_0^{OS} + \Delta_0^{TS}\right\}\right)\left[A - 1 - \Delta_0^{OS}\right] + \\&\left.\Pr\left(A_0^X < \min\left\{A,\ B + \Delta_0^{OS} + \Delta_0^{TS}\right\}\right)\left[1 - \Delta_0^{OS} - B\right]\right\}.\end{aligned}$$

To compute these expected profits we cluster all possible deviations into two groups:

1. Deviations that involve price strategies (A, B) such that $A - B \leq \Delta_0^{OS} + \Delta_0^{TS}$, and
2. Deviations that involve price strategies (A, B) such that $A - B > \Delta_0^{OS} + \Delta_0^{TS}$.

If (1) holds, then expected profits simplify to

$$\frac{1}{4}\frac{(1+r)}{\Delta_0^{OS} - \Delta_0^{TS}}\left\{\left(B + \Delta_0^{OS} + \Delta_0^{TS} - A\right)\left[A - B - 2\Delta_0^{TS}\right] + \left(1 - \Delta_0^{TS} - B\right)\left(A - 1 - \Delta_0^{OS}\right) + \left(A - 1 - \Delta_0^{TS}\right)\left(1 - \Delta_0^{OS} - B\right)\right\}.$$

Differentiating expression above with respect to A and B yields

$\frac{\partial E(\Pi T)}{\partial A} = \frac{1}{2}(1+r)\left(\frac{1}{\Delta_0^{OS} - \Delta_0^{TS}}\right)\left(1 + \Delta_0^{TS} - A\right) < 0$, for all $A > 1 + \Delta_0^{TS}$, and

$\frac{\partial E(\Pi T)}{\partial B} = \frac{1}{2}(1+r)\left(\frac{1}{\Delta_0^{OS} - \Delta_0^{TS}}\right)\left(1 - \Delta_0^{TS} - B\right) > 0$, for all $B < 1 - \Delta_0^{TS}$.

Expected profits are maximized at dealer Y's equilibrium strategy. Hence, no deviation as the one proposed above is profitable.

If (2) holds, then the probability of attending both sides of the market is zero. Consequently, expected profits simplify to

$$-\frac{1}{4}\frac{(1+r)}{\Delta_0^{OS} - \Delta_0^{TS}}\left\{\left(A - 1 - \Delta_0^{OS}\right)^2 + \left(1 - \Delta_0^{OS} - B\right)^2\right\},$$

which is always negative.

Consider now dealer X. By following the purported equilibrium strategies he gets zero expected profits and he does not attend any side of the market. To attend it, he must set $A_0^X < 1 + \Delta_0^{TS}$ and/or $B_0^X > 1 - \Delta_0^{TS}$. If he improves only one price, for instance the ask price, then he obtains negative expected profits since $A_0^X < 1 + \Delta_0^{OS}$. Otherwise, he also obtains negative expected profits because of $A_0^X - B_0^X < 2\Delta_0^{TS}$. ■

Proof of Corollary 9: Let $S_t^T\left(S_t^O\right)$ denote the expected spread at t in the post-trade transparent (opaque) market, with $t = 0, 1$. On the one hand, Proposition 1 implies that $S_0^T = \frac{2}{1+r}$, and $S_1^T = \frac{3+3r+2r^2}{(2+2r+r^2)(1+r)}$. Hence, $S_0^T - S_1^T$ is a positive and decreasing in r function. On the other hand, from Propositions 7 and 8, we have

$$S_0^O = \frac{8 + 2r - r^2 + r^3}{4(1+r)^2}, \text{ and}$$

$$S_1^O = \frac{10 + 14r + r^2 - r^3}{4(1+r)^2} - \frac{2}{(2+r)r(1+r)}\ln\left(\frac{4(r+1)}{4+2r-r^2}\right) - \frac{(2+r)r}{8(1+r)^3}\ln\left(\frac{4+2r-r^2}{2+2r+r^2}\right).$$

From a simple study of the difference between spreads as a function of r, we obtain that $S_0^O - S_1^O < 0$, for all $r \in (0, 1]$ and it is null when $r = 0$. Further, $S_1^O - S_0^O$ is a concave in r function which attains a maximum at $r = 0.87$. Hence for $r \geq 0.87$ the differences in expected spreads decreases with r. ■

Proof of Corollary 11: Consider the transparent market. The martingale property of transaction prices with respect to public information directly follows from the law of iterated expectations. The martingale property dictates that prices are semi-strong-form efficient.

Consider now the opaque market, and let $a_1^U = a_1(\{1, 0, -1\})$, for short. Prices are given by

$$\begin{aligned} P_0^O(q_0) &= 1 + \Delta_0^{TS} \times q_0, \text{ and} \\ P_1^O(q_1, q_0) &= 1 + \min\left\{a_1(\{q_0 \times q_1\}), a_1^U\right\} \times q_1. \end{aligned}$$

Since there is one-to-one mapping between P_0^O and q_0, it follows that

$$E(P_1^O | P_0^O) = E(P_1^O | q_0).^{32}$$

Suppose now that $q_0 = 1$. Notice that

$$A_1^O(1) = 1 + a_1^U, \text{ and } B_1^O(1) = 1 - \min\left\{a_1(-1), a_1^U\right\}.$$

Then,

$$E(P_1^O | q_0 = 1) = 1 + \varsigma E(a_1^U) - (1 - \varsigma) E\left(\min\left\{a_1(\{-1\}), a_1^U\right\}\right), \tag{12}$$

where

$$\varsigma = \Pr(q_1 = 1 | q_0 = 1, q_1 \neq 0) = \frac{1}{2}\left(\frac{2 + 2r + r^2}{1 + 2r + r^2}\right). \tag{13}$$

Using Proposition 6, it follows that

$$E\left(a_1^U\right) = \frac{1}{1+r}\left(1 + \ln\left(\frac{4(1+r)}{4 + 2r - r^2}\right) + \frac{r(r+2)}{4(1+r)} \ln\left(\frac{4 + 2r - r^2}{2 + 2r + r^2}\right)\right), \text{ and}$$

$$E\left(\min\left\{a_1(\{-1\}), a_1^U\right\}\right) = \frac{1}{1+r}\left(2 + \frac{(r^2 - 2r - 4)}{(r+2)r} \ln\left(\frac{4(1+r)}{4 + 2r - r^2}\right)\right).$$

Let $T = \ln\left(\frac{4(1+r)}{4+2r-r^2}\right)$ and $Z = \ln\left(\frac{4+2r-r^2}{2+2r+r^2}\right)$. Plugging expressions above into (12), using (13) and operating, if follows that

$$E(P_1^O | q_0 = 1) = 1 + \Phi,$$

where

$$\Phi = -\frac{-2 + 2r + r^2}{2(1+r)^3} + \frac{3 + 2r}{(1+r)^3} T + \frac{(2 + 2r + r^2)(2r + r^2)}{8(1+r)^4} Z.$$

[32] It is important to notice that transaction prices arise when traders choose to negotiate. Therefore, when computing these expectations we assume that there is trading activity in both periods, that is, $q_t \neq 0$, $t = 0, 1$.

Moreover, $\Phi - \Delta_0^{TS}$ as a function of r is strictly decreasing in the interval $[0, 1]$ and in $r = 0$ is equal to 0. Hence, $\Phi < \Delta_0^{TS}$, and hence, $E(P_1^O|P_0^O) < P_0^O$ if $q_0 = 1$. Analogously, we obtain that $E(P_1^O|P_0^O) > P_0^O$, when $q_0 = -1$. Consequently, one concludes that $E(P_1^O|P_0^O) \neq P_0^O$. ■

Proof of Corollary 12: This proof directly follows from the comparison of prices corresponding to both market structures. ■

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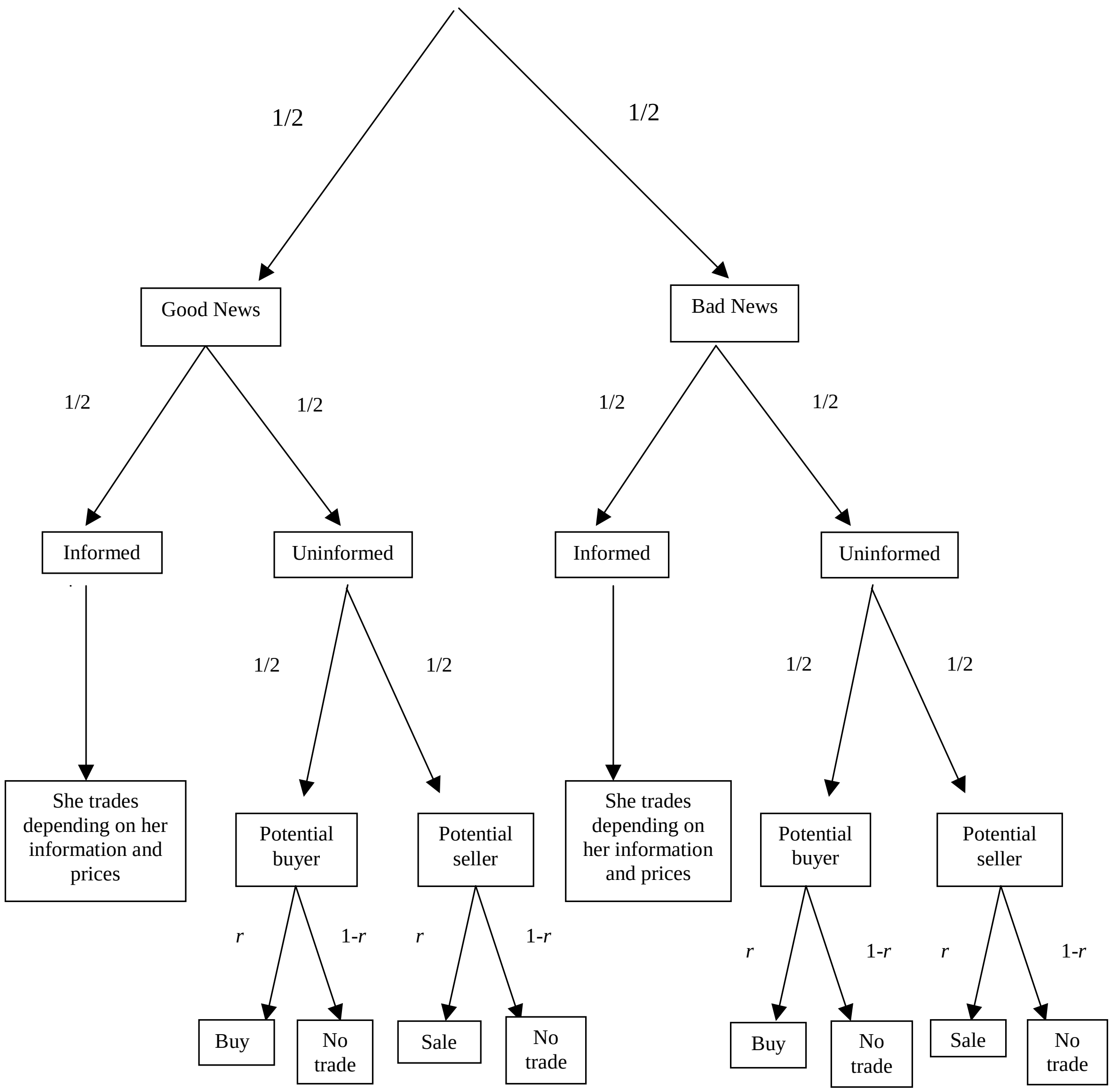

Figure 1. The Probabilistic Structure of Trade.

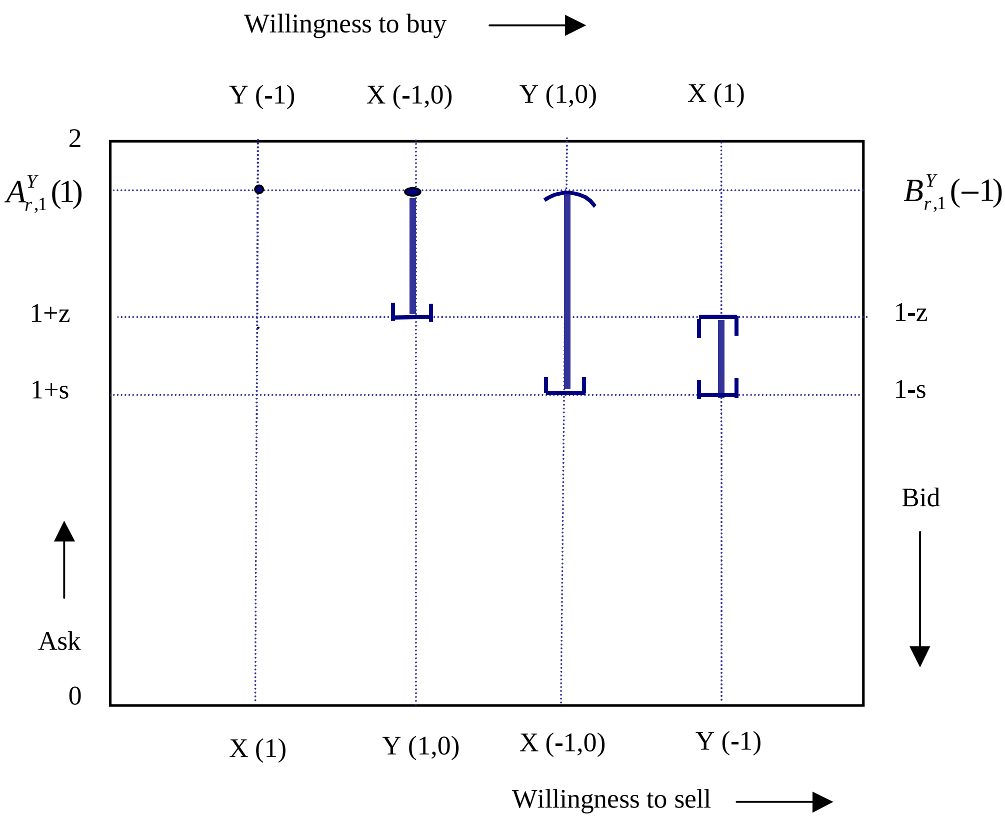

Figure 2. Range of prices in the mixed strategies equilibrium.

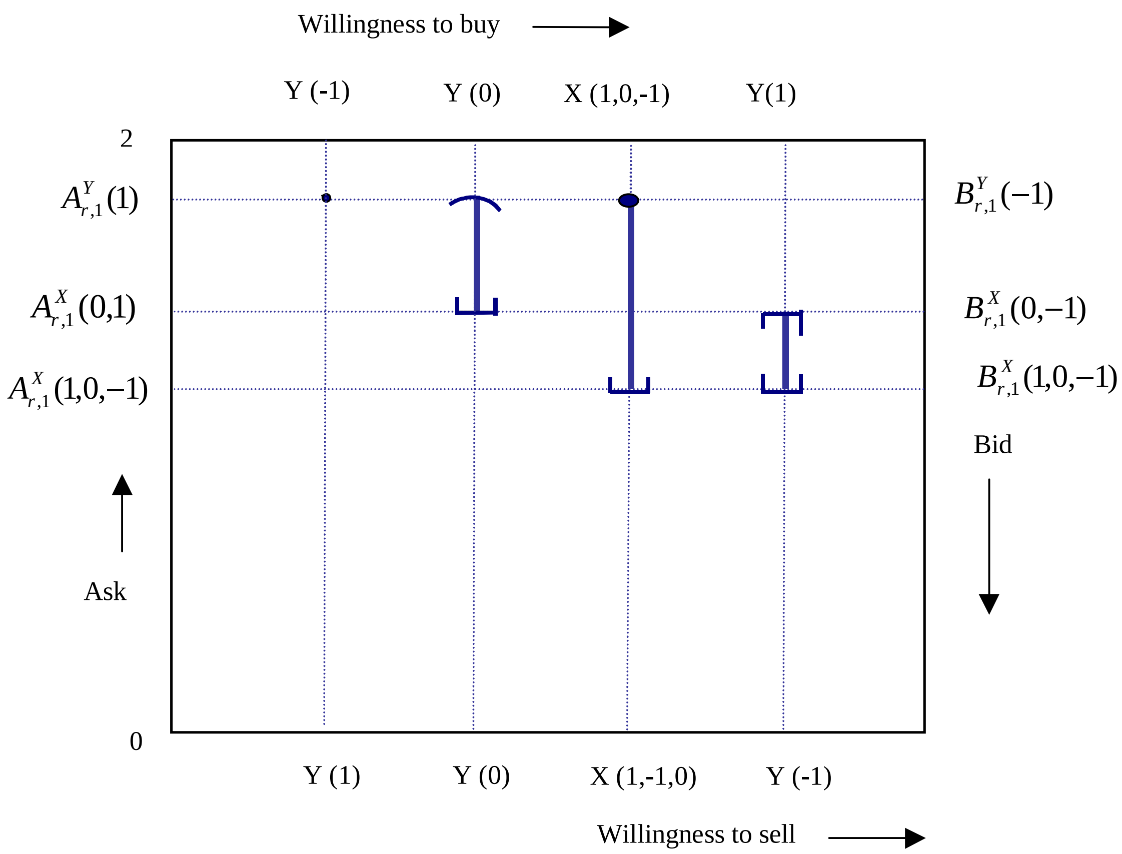

Figure 3. Range of prices in the mixed strategies equilibrium.