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A REMBRANDT IS A REMBRANDT[1]

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As paintings are assets, we propose to model a painting's price dynamics as a diffusion process, i.e. as the financial literature models share prices, but correcting by size. We show that the influence of size on the artwork price diminishes as the painting gets older because 1) prices incorporate progressively more noise and 2) for high quality artists, the relative importance of size on price decreases as the artist consolidates and authorship gains importance as explanatory variable. Our theoretical results are consistent with data from a sample of 19th- and 20th-century Catalan painters of similar quality. These findings suggest that an artist's quality and antiquity should be taken into account in order to obtain more efficient estimates of parameters in hedonic art market models.

1. INTRODUCTION

In February 2004 three oils on canvas painted by Picasso were auctioned at Sotheby's (London). The most expensive of these was sold for £901,600 pounds and measured 65 x 54 cm. The second most expensive was sold for £812,000 pounds and measured 81 x 65 cm while the cheapest (£543,200) was the largest (81 x 100 cm). Frequently, however, new artists, price their oils on canvas according to their size: the bigger the canvas, the more expensive the picture. Art supermarkets, where, for each artist and technique, works are classified and priced according to their size, very graphically represent the paradigm of this conduct.

Nowadays, the academic literature accepts that size is a positive determinant of a painting's price. To determine the average return of an art investment, the so-called hedonic models estimate art

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price indices by regressing the logarithm of the price of each painting on a set of time-variant and time-invariant characteristics. The size of a painting is a time-invariant variable and is one of the few such variables that appear in the majority of works as significant [see, for example, Anderson (1974), Buelens and Ginsburgh (1993), Agnello and Pierce (1996), Chanel, Gérard-Varet and Ginsburgh (1996) or Czujack (1997)].

The hedonic approach seeks to explain market value from a common set of attributes and, therefore, it considers that a certain homogenization of artworks is possible. As an extreme example, two paintings by the same artist, from the same period, with the same theme, format, technique and support, and sold at the same auction should fetch the same price. The disadvantages of the hedonic approach are well known. The selection of the variables and the functional form, the risk of bias due to systematic movements in the unobserved characteristics of items included in the analysis and the stability of the parameters are problematic. In fact, Anderson (1974) has already suggested that the effect of size on price is not constant over time.

On the one hand, the larger-size-higher-price rule is common when pricing a new artist's works, while size is a price determinant in papers of the hedonic tradition. On the other hand, for proven, high-quality artists, auction price records are full of counterexamples to the larger-size-higher-price rule, as is shown by the Picasso example. Moreover, parameters of hedonic models can be time-variant, and this could be the case for the size of the canvas, i.e. the influence of size on price is affected by time and by the quality of the artist.

When an artist completes a painting, few attributes affect its price. The most important of these are generally the name of the artist, the size of the painting, and the technique and medium used[2]. As the painting gets older, other factors begin to affect its price e.g. its state of conservation, whether it has been restored and how certain its authenticity is. This increases the noise in the art market and, over time and for every quality of artist, the size of the painting determines its price less and less. Also over time, increase the importance of factors such as the name of the artist, if the artist is

[2] Here we do not take into account the circumstances of the sale, which can also affect price.

successful. Consequently, for high quality artists[3], size loses weight as an explanatory factor of price over time[4]. In summary, for a Velázquez or a Rembrandt, size does not matter.

To prove those points, we have modeled the time-series behavior of an artist's work. A work of art—a painting, a drawing, a sculpture, a lithograph—is an asset, i.e. a good that is potentially an object of investment. The price of a painting is modeled using a diffusion process, as is usual in the financial literature to describe the behavior of share prices. Works of art are often partly used for investment purposes and there is evidence that art market prices follow financial markets [Goetzmann (1993), Chanel (1995) or Goetzmann & Spiegel (1995)]. Unlike equities, paintings are not divisible, but the same dynamic behavior that is attributed to share return can be applied to the return from a standard-sized painting by a given artist. We assume that the price of a standard-sized painting follows a geometric Brownian motion, related to the dynamic behavior of the art market, but with a particular drift and with volatility coefficients. For non-standard-sized paintings by the artist we correct the price of a standard-sized work by weighing it with a size factor. This size factor is time-independent since we cannot give any temporal return to size.

We use the model to prove that, since prices incorporate progressively more noise, the weight of size on price falls as a painting gets older. Also, from the random characteristics of an artist's paintings, we define an artist as "of high quality" as one for whom the expected return of his[5] standard work is greater than half of its variance, which is a measure of volatility. We show that for high quality artists, the influence of size on price falls over time as the artist consolidates and authorship gains importance.

On the empirical side, ideally, to show that the passage of time affects the price, we would need for each artist enough observations over a lengthy time interval. Furthermore, we should keep in mind that an artist along his live goes through different periods with distinct market values, so that, in fact, we should consider him, in analytical terms, like different painters. To overcome these difficulties, we test the two propositions from a sample of 19th- and 20th-century Catalan painters of similar high quality so that they can be considered to be a same artist. The sample (auction prices of

[3] In this paper "high quality" should be taken to mean "of high monetary value".
[4] We cannot attribute any defined tendency for the impact of size on price.

oils on canvas of these painters) provides enough observations and extends over a sufficiently long temporary dimension for our analysis. The empirical study must therefore be viewed as an illustrative example. As expected, we found that the influence of size on price decreases with time, i.e., for the less recent artists, the influence of size on price is smaller than it is for the more recent artists.

In section 2 we develop the general model and in section 3 we adapt it to the specific study into the effect of size. In section 4 we present the illustrative example. The final section provides a synthesis of the main contributions.

2. THE MODEL

Before modeling art prices we should make some general comments: 1) a work of art has several singular aspects that make it resemble a monopoly output; 2) two works of art by the same artist, from the same period of time, of the same size, format[6], medium, technique and subject, in the same condition and sold under the same conditions should fetch the same price; 3) in all eras, artists normally use a format, support, technique and subject that varies little from one work to another; 4) on the other hand, although for an artist there is always one size that can be considered as standard, the size of the painting can vary; 5) information about an auction's art prices is common knowledge[7]; 6) art prices are related to other prices of economic goods, particular those of financial markets.

Comments 2-4 suggest that, to model art prices, we can focus on the price of the average-sized standard painting of an artist in a particular period in his life. Let us call this variable S_{it},, where i is the artist in a period of his life and t is the moment of sale[8]. By artist we mean a painter at a particular artistic stage of his life. Comment 6, on the other hand, suggests that the prices of artist i's work are related to the prices of the art market, which is in turn related to the evolution in the

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[5] For reasons of clarity we have chosen to use the masculine form.
[6] By format we mean oblongness measured by the quotient between the longest and the shortest sides.
[7] Auction prices are not secret and are published by numerous papers and web pages.
[8] Different eras of the same artist are treated as if there were different artists.

prices of other economic goods. We assume that these relations can be captured by simple stochastic processes.

Art market prices are related to stock market prices, so it is natural to model the art market with the standard methods for modeling the stock market. Let M_t be the value of the art market price index at moment t. We assume that M_t, follows a geometric Brownian motion, i.e. it evolves according to a diffusion process generated by the stochastic differential equation:

$$dM_t/M_t = \mu_M\, dt + \sigma_M\; dw_{1t} \qquad (1)$$

where μ_M is the instantaneous growth rate of the index, σ_M is the standard deviation by instant of time, and $dw_{1t} = \varepsilon\, (dt)^{1/2}$ is the differential of a Wiener process with mean 0 and variance dt, $\varepsilon \sim N(0, 1)$. Note that, according to several authors, μ_M is lower than that of financial assets[9].

The dependence of the price of artist i's works on the art market can be modeled by assuming that the price of a standard work by artist i is obtained from the art market index and affecting it lineally using a stochastic process that incorporates the artist's specific, idiosyncratic components:

$$S_{it} = X_{it} \cdot M_t \qquad (2)$$

$$dX_{it}/X_{it} = \mu_{Xi}\, dt + \sigma_{Xi}\, dw_{2t} \qquad (3)$$

where μ_{Xi} is the drift of the idiosyncratic component, σ_{Xi} *is* its volatility, and $dw_{2t} = \varepsilon\, (dt)^{1/2}$ is the differential of a Wiener process with mean 0 and variance dt, $\varepsilon \sim N(0, 1)$. Furthermore, w_1 and w_2 are two independent Brownian motions, i.e. $dw_{1t} \cdot dw_{2t} = 0$, in the sense of mean square convergence, which is usual in stochastic calculus (see, for example, Neftci, 1996, p.112-116).

This assumption is not entirely new. In their repeated sales analysis, Goetzman & Spiegel (1995) and Locatelli & Zanola (1999) made a similar one. These authors considered that the growth rate of a painting's price can be decomposed into two parts—one corresponding to the market and one to the

[9] However, the estimated returns on an art holding can depend on the time frame (see, for instance, the review by Ashenfelter and Graddy (2003)).

idiosyncratic factor. However, their analysis is different. They apply the term idiosyncratic to the piece of art while we apply it to the artist.

For a given artist, changes in the size of his work generate changes in its price. Let D_i represent the size of artist i's work and let D_i^* be the mean size of his paintings, $E(D_i)=D_i^*$. We then assume that the price of the painting of size D_i can be expressed as:

$$P_{it} = f_{it}(D_i - D_i^*) \cdot S_{it} \qquad (4)$$

with $f_{it}(0)=1$. As we cannot attribute any return to size, i.e. any defined tendency for the impact of size on price, we consider f_{it} constant, $f_{it} = f_i$. Moreover, from comment 1, it seems appropriate to take a non-linear form for f_i. We adopt the exponential function, which is usual, for example, in hedonic regression (see, for instance, Locatelly and Zanola, 2005), $f_i = e^{\alpha_i(D_i - D^*)}$, where $\alpha_i > 0$ is constant for a given artist but can change from one artist to another. This can be considered as an approximation of a two-parts tariff, which is a typical monopoly practice. Then, for artist i the price is equal to:

$$P_{it} = e^{\alpha_i(D_i - D^*)} \cdot S_{it} \qquad (5)$$

We also assume that size and price, D_i and S_{it}, are affected by different random shocks. In particular, we assume that D_i and lnS_{it} are orthogonal variables, $E((D_i - D_i^*)(\ln S_{it} - E(\ln S_{it}))) = 0$.

We can extend the basic model presented so far to include variations in format from the standard oblongness of the artist's frame. Let F_i be the format of artist i's paintings and let F_i^* be its mean. The price can then be expressed as:

$$P_{it} = g_{it}(F_i - F_i^*)\, f_{it}(D_i - D_i^*) \cdot S_{it}$$

with $g_{it}(0)=1$. However, to simplify we will omit variations in format, i.e. we will consider their influence on price to be irrelevant (either because they are too small ($F_i - F_i^* \cong 0$) or because the art market does not appreciate them ($g_{it} \cong 1,\ \forall F_i - F_i^*$)).

Finally, we could also incorporate the influence of other factors on the final price of a work of art. For a painting by artist I that does not have the usual technique, support, subject or state of conservation, the final price, π_{it}, could be found by modeling each of these variables using a stochastic process generated from independent Wiener processes. In particular, the price for a painting with a different technique can be affected by a process, Q_{it} that reflects differences in price due to differences in technique. Similarly, if the support of the painting is different, the process N_{it} will reflect the differences in price due to this support. The final market price can be expressed as:

$$\pi_{it} = P_{it} \, Q_{it} \cdot N_{it} \qquad (6)$$

where:

$$dQ_{it}/Q_{it} = \mu_{Qi}\, dt + \sigma_{Qi}\, dw_{3\,t}$$

$$dN_{it}/N_{it} = \mu_{Ni}\, dt + \sigma_{Ni}\, dw_{4\,t}$$

In both expressions μ_{Qi} and μ_{Ni} are drifts, σ_{Qi} and σ_{Ni} are standard deviations, $dw_{jt} = \varepsilon\, (dt)^{1/2}$, $j=3, 4$ is the differential of a Wiener process with mean 0 and variance dt, $\varepsilon \sim N(0, 1)$, and, as before, w_3 and w_4 are Wiener processes that are independent of each other and of w_1 and w_2.

3. THE PRICE OF A PAINTING

In this section we develop the model presented in section two in order to obtain an expression of the price of a painting by artist i in t as a function of its size. To simplify the notation we will omit the subindex i. To analyze the impact of size on price, we will consider variables other than size as standard for a given artist—as we mentioned in section two, including these variables, though possible, would complicate the analysis. Price will therefore be modeled from expressions (1) to (5).

To determine the characteristics of P_t, we shall look at those of S_t, the price of a standard painting of artist i. As is usual in the financial literature, we find the behavior of S_t from Ito's lemma, which provides the differential of a function that depends on Ito stochastic processes, a general case of expressions (2) and (4), and on time. Applying Ito's lemma we get (see Annex A.1.):

$$dS_t \,/\, S_t = \mu\, dt + \sigma\, dz_t \qquad (7)$$

where $\mu = \mu_M + \mu_X$ and $\sigma\, dz_t = \sigma_M\, dw_{1t} + \sigma_X\, dw_{2t}$

Therefore, we have that the price of a standard painting follows a geometric Brownian motion, $dS_t/S_t \sim N(\mu\, dt,\ \sigma\sqrt{dt})$, with $\sigma\sqrt{dt} = \sigma_M\sqrt{dt} + \sigma_X\sqrt{dt}$. Now solving the stochastic differential equation (7) again using Ito's lemma (see Annex A.2.), we get:

$$\ln S_t = \ln S_0 + (\mu - \sigma^2/2)\, t + \sigma z_t \qquad (8)$$

where $z_t = \varepsilon\, t^{1/2}$..

So, S_t is lognormal distributed, $\ln S_t \sim N(\ln S_0 + (\mu - \sigma^2/2)\, t,\ \ \sigma\, t^{1/2})$, which prevents the existence of negative prices that would be possible if it were normally distributed.

The main results of this paper stem from expressions (5) and (8). For nonstandard-size paintings, the logarithm of the price of a painting by artist i is equal to:

$$\ln P_t = -\alpha D^* + \alpha D + \ln S_0 + (\mu - \sigma^2/2)\, t + \sigma z_t \qquad (9)$$

Then, given t, as D and $\ln S_t$ are orthogonal random variables, $E((D - D^*)(\ln S_t - E(\ln S_t))) = 0$, the expected value of $\ln P_t$ conditional on D is equal to:

$$E(\ln P_t / D) = -\alpha D^* + \ln S_0 + (\mu - \sigma^2/2)\, t + \alpha D \qquad (10)$$

$$= \beta_0 + \beta_1 D$$

where β_0 and β_1 are specific to the artist because $-\alpha$, D^*, μ and σ are specific to the artist.

As, for a given t, $\ln P_t$ is a linear function of the size D, the conditional expectation $E(\ln P_t / D)$ is the regression, the least squares projection of $\ln P_t$ on D, because it minimizes $E((\ln P_t - \beta_0 + \beta_1 D)^2)$. Therefore, the usual expressions for the coefficients in regression analysis of $\beta_1 = Cov(D, \ln P_t)/Var(D)$ and $\beta_0 = E(\ln P_t) - \beta_1 E(D)$ apply [10].

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[10] Although $\ln P_t$ is not a linear function of D, $E(\ln P_t / D)$ is likewise the least squares projection of $\ln P_t$ on D if the variables $(\ln P_t, D)$ follow a normal multivariable probability distribution, which is surely the most usual case, from the characteristics of the probability distribution of S_t.

On the other hand, expression (10) shows that the value of β_0 depends on t, the interval of time elapsed since the first sale of the artist's work. We make explicit this dependence, $\beta_0 = \beta_0(t)$, because it enables us to clarify the analysis that follows. On the other hand, β_1 does not depend on t because of the assumption that the value the market confers on size is not affected by the passage of time.

As a result, t affects the relative importance of β_0 and $\beta_1 D$ to explain lnP_t, which depends on $\beta_0'(t) = \mu - \sigma^2/2$, i.e. on the sign of the difference between the drift minus half of the variance coefficients of the geometric Brownian motion that describes the price of a standard painting by the artist.

If an artist is rated of high quality according to the market, the prices of his paintings should, on average and in the long run, show an upward drift that is greater than their volatility. In terms of the geometric Brownian motion model, the instantaneous expected growth rate should be higher than half of the variance for any instant of time:

Definition: An artist is considered of high quality if $\mu - \sigma^2/2>0$.

Proposition 1. If an artist is rated of high quality, the importance of $\beta_1 D$ for explaining lnP_t decreases, relative to the importance of β_0, the more time that has passed since the first sale of the artist's paintings, i.e. for higher values of t.

Proof. The relative importance of β_0 and $\beta_1 D$ for explaining lnP_t,, starting from expression (10) for $E(lnP_t\ /D)$, is given by:

$$1 = \frac{\beta_0(t)}{\beta_0(t)+\beta_1 D} + \frac{\beta_1 D}{\beta_0(t)+\beta_1 D} \qquad (11)$$

For a high quality artist we have $\mu - \sigma^2/2>0$. Then, when t increases, the first quotient goes to 1 and the second quotient goes to 0. ♦

For a high quality artist, when we take higher values of t, the part of the price due to size decreases because the intercept increases with t, while the slope does not change. On the other hand, for nonhigh quality artists according to the previous definition, size explains a greater proportion of the price as time goes by.

If we consider two artists who are equally rated, use frames of the same size and display the same coefficient β_1, the paintings of the older artists must have the highest values.

However, time has another effect on the declining ability of size to explain the price, and this effect is independent of the quality of the artist.

Proposition 2. As we take higher values of t, the regression $E(lnP_t\ /D)$ for explaining lnP_t becomes less accurate.

Proof. If we subtract expression (9) from expression (10), i.e. $lnP_t = -\alpha D^* + \alpha D + ln\ S_0 + (\mu - \sigma^2/2)\ t + \sigma z_t$, from $E(lnP_t) = \beta_0 + \beta_1 D^*$, we obtain $lnP_t - E(lnP_t) = \beta_1(D\text{-}D^*) + u$, where $u = \sigma z_t = \sigma\ \varepsilon\ t^{1/2}$ with $\varepsilon \sim N\ (0, 1)$. As D and lnS_t are independent random variables, so $E((D - D^*)(lnS_t - ElnS_t))=0$, D and u are also orthogonal, $E((D-D^*)(u-Eu))=0$, and we can do the decomposition of the sum of squares in the analysis of variance:

$$E(lnP_t - E(lnP_t))^2 = \beta_1^2\ E(D\text{-}D^*)^2 + Eu^2$$

$$= \beta_1^2\ VarD + \sigma^2 t$$

The analysis of variance of lnP_t shows that if t increases, the variance of the process also increases. This increase is completely due to the non-explained variance of the regression, i.e. when t increases, there is only an increase in the sum of the squares of the residuals.

In proportional terms:

$$1 = \frac{\beta_1^2 VarD}{\beta_1^2 VarD + t} + \frac{\sigma^2 t}{\beta_1^2 VarD + t} \qquad (12)$$

The last equation shows that if t tends to infinity, the determination coefficient, R^2, tends to 0. On the other hand, the proportion of the changes in lnP_t that are not explained by the regression tends to one. This does not depend on the quality of the artist; it occurs whether $\mu - \sigma^2/2$ is superior or inferior to *0*. In other words, it occurs whatever the value of the coefficient $\beta_0 = -\alpha D^* + \ln S_0 + (\mu - \sigma^2 / 2)t$ is. ♦

Therefore, for older artists, regardless of their quality, size explains a smaller part of the price, in the sense that the regression of lnP_t on D becomes less accurate, since the regression errors become greater.

From propositions 1 and 2, as the results are accumulative, we can conclude that for a high quality old artist, size is not a very explicative factor for the price of his paintings. For such an artist, the intercept, β_0, is high compared to the slope, β_1. The main part of the price is due to the artist's reputation and only a small part is due to size (Proposition 1). On the other hand, time also implies a higher level of noise, i.e. it makes observations get progressively more dispersed around the regression line, so size becomes even less explanatory of price (Proposition 2).

4. THE EFFECT OF TIME ON THE VARIABILITY IN THE PRICE OF A PAINTING: AN EMPIRICAL APPROACH

In this section we present an empirical study to illustrate that, over time, size explains a smaller part of the price of a picture. We have taken the auction prices of paintings by consolidated 19th and 20th century artists of similar quality, so that they can be considered a same artist. For each artist we have adjusted a multiple regression of size on price in which the conditions of the sale were controlled. As expected, the data show that the model suits the more recent painters better. This result provides examples for the propositions in the preceding section for similar quality artists.

4.a. Data and Model:

To include artists of as similar quality as possible, we restricted our selection to consolidated artists from the same school (the Catalan school), whose works are mainly sold in Spain[11] and for whom we have enough observations. The advantage of taking painters from a local school is that they are

[11] This strategy restricts our attention to "artists not of the first order ". The advantage of this is that it avoids artists who are known on their own because of the characteristic traits of their paintings, which would make

fairly homogeneous since what gives them economic value is the fact that they belong to that school[12]. The main disadvantage of this strategy is that it is not easy to generalize the results.

We identified 237 oils on canvas from five Catalan painters sold in Spanish auction houses between 1984 and 1996 and included in the annual catalogues published by *Antiquaria*. The information available on each painting (artist, size, and hammer price, plus date and auction house of sale) was recorded. Only the pieces catalogued with the artist's name and surname were included, since this indicates that the auction house guarantees authenticity. We restricted our study to works painted on the same support and by the same technique and selected only recognized auction houses (from the criteria of the catalogues). This reduced the number of observations so, to have enough observations, we took data from several years.

The selected artists are: Josep Amat (Barcelona, 1901–1991), Francesc Gimeno (Tortosa, 1858–Barcelona, 1927), Eliseu Meifren (Barcelona, 1859–1940), Joaquim Mir (Barcelona, 1873–1940) and Modest Urgell (Barcelona, 1839–1919). These artists are similar. In Spain their paintings have the same public and are sold in Spain. The price of their paintings is quite similar so, in this sense, the artists are of a similar quality. However, they are not of the same generation: between the oldest and the youngest there are six or seven decades.

For each artist the following model was adjusted using multiple regression analysis:

$$lnP_i = a + bD_i + g_1C_i + g_2M_i + e_i$$

where lnP is the logarithm of the price of a painting, D is its size, C is the city where the sale took place, M is the state of the market, a is the intercept, b, g_1, g_2 are the estimated coefficients and e is the error term.

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them a less homogeneous group.

[12] The school due to the place where the artist works is very general and can last for several decades.

Prices were deflated by the official index drawn up by the Spanish Institute of Statistics without commissions and log transformed, as is usual in the available literature [see, for instance, Anderson (1974), Stein (1977), Buelens i Ginsburgh (1993), Chanel (1995) and Gerard-Varet (1995)].

Size is expressed as the length of the longest side of the stretcher bar. This is not the most common way express size in the literature. Area, on its own or together with other variables, is more common. However, in the Spanish art market, the measurement often used is points. This is a categorical measure that attaches more importance to the longest side. The disadvantage of using area is not only that canvases with the same number of points have different areas, but also that the order of the paintings according to area may be different from the order in terms of points[13].

Sizes differs between artists. The means of the stretcher bars of the longest size range from 42 cm to 121 cm. Formats (shortest side/longest side) are quite similar: the mean of the format by painter ranges from 0.74 to 0.81, except for Urgell who used oblong frames (Urgell's format=0.57).

Our model assumes that the relationship between the log of the price and the dimension is linear for non-extreme sizes. It is agreed that extreme sizes can have a negative effect on the number of potential buyers and, therefore, on price. Extreme-sized paintings have been excluded from our analysis. The paintings we analyzed had between 1 and 120 points and 3 paintings were excluded.

The city and the market are included in the model as control variables. The city is a binary variable that indicates where the sale was held. Madrid was coded as *0* and Barcelona was coded as *1*. Only two of the thirteen auction houses included in the model are in Barcelona and the rest are in Madrid. Differences in prices according to place of sale have been described in the literature [see Pesando (1993) and Pesando and Shum (1996), or Schanapper (1999) for eighteenth-century French auction houses].

[13] For example, a 12-point frame corresponds to a painting whose longest side is 61 cm, while the shortest side can measure 50 cm if it is a frame for portraits, 46 cm for landscapes and 38 cm for seascapes. The areas are 3,050 cm^2, 2,806 cm^2 and 2,318 cm^2, respectively. A 10-point portrait frame measures 55x46 cm^2 (2,530 cm^2), which is greater than the area of a 12-point seascape.

Market state reflects whether the price was low (1986 and 1993–1996; coded as *0*), moderate (1987–1988 and 1992; coded as *1*) or high (1989–1991; coded as *2*) in the year of sale. This coding is consistent with the evolution of prices for the Catalan school [Portús, 1996] and with the evolution of the annual mean of prices for the whole set of painters. Unfortunately, no good Spanish art index is available to deflate the data, while using an index from other countries may be inappropriate since the boom in the Spanish art market took place later than in other countries. While the decline in prices at Christie's and Sotheby's was observed as early as 1990 (see, for instance, Pesando and Shum, 1999), in Spain neither prices nor volume of sales decreased until 1991.

4.b. Results and discussion

The adjusted model shows that, as expected, size has a positive effect on price. This effect of size on the logarithm of the price is statistically significant for every artist (see table I). Regression coefficients are specific for each artist. The state of the market is always positive and significant, while, surprisingly, except for those by Amat, there are no major differences between the prices fetched in Madrid and Barcelona by the paintings of these artists.

Table I: Regressions results

Painters		AMAT	GIMENO	MEIFREN	MIR	URGELL
b (t-Student)		0.0317 (6.3)	0.0207 (3.3)	0.0115 (3.2)	0.0217 (5.3)	0.0113 (3.9)
$g1$ (t-Student)		1.513 (5.3)	0.107 (0.4)*	6.05 (0.28)*	0.257 (0.9)*	0.0625 (0.18)*
$g2$ (t-Student)		0.43 (4.5)	0.771 (5)	0.58 (4.7)	0.77 (4.8)	0.524 (2.78)
Intercept		10.39	12.64	12.93	12.47	12.34
IC (*95%*)	b	0.022 - 0.042	0.008 - 0.033	0.004 - 0.019	0.014 - 0.03	0,006 - 0,017
	g_1	0.935 - 2.09	-0.419 - 0.633	-0.377 - 0.498	-0.321 - 0.836	-0,638 - 0,763
	g_2	0.239 - 0.621	0.458 – 1.084	0.334 - 0.828	0.446 - 1.094	0,141 - 0,907
R^2 (%)		75.4	48.4	36.5	70.8	53.9
F		44.04	10.31	12.08	31.56	14.04
N		47	37	67	43	40

b, $g1$ and $g2$ are regression coefficients for size, city and market, respectively. IC (*95%*) confidence intervals of coefficients ($\alpha=0,05$). *F*: F-Snedecor. *n*: number of observations. *: not statistically significant, otherwise, $p<0,05$.

We used these results to check *proposition 1*. Since we studied similar-quality artists and works painted with the same technique, support and format (except for Urgell) but of different sizes, we analyzed an expression that is analogous to expression 11. We substituted the size of each painting by the mean for the artist and expected elasticity (the coefficient $\frac{a}{a+bD^*}$) to be lower for more recent artists.

Figure 1 shows these coefficients for each artist (on the Y-axis). Each artist is scaled by the median year of his life according to two different measurements of size: the longest side of the stretcher bar and the shortest one. When we took the longest side, Urgell displayed a lower coefficient than expected because of his use of the oblong canvas. This effect disappeared when we took the shortest side, *ceteris paribus*. Therefore, the less recent artists showed higher coefficients than the more recent artists. The expressions $\frac{a}{a+bD^*}$ were high for Gimeno, Meifren and Urgell, average for Mir and low for Amat. Our data corroborated proposition 1: for artists of similar quality, the relative importance of size for explaining the price of a painting decreases over time.

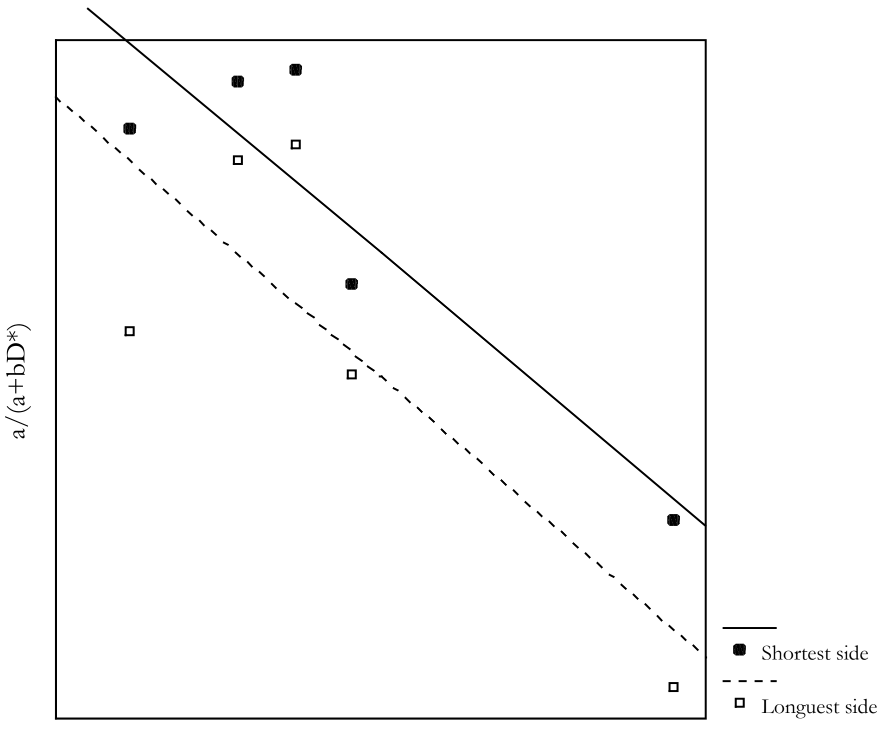

Figure I: *a/(a+bD*)* for each artist according to the median year of his life

The explained variability of each regression is inversely ordered with the year corresponding to the median of the life of the artists. As we can see in figure 2, the older painters (Gimeno, Meifren and Urgell) have small determination coefficients (R^2), Mir has an average one and Amat has a high one. This corroborates *proposition 2*, which states that $E(lnP_t/D)$ is less accurate over time, other factors of heterogeneity being equal.

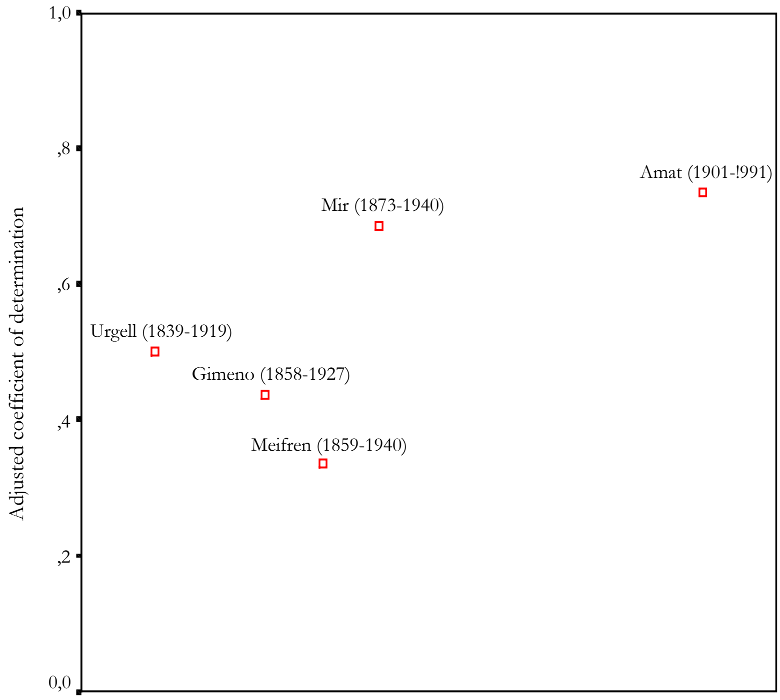

Figure II: R^2 of each regression according to the median year of the life of the artist.

Our results indicate that the time elapsed since a work was painted can be a relevant factor for explaining variability in price: the more time elapsed, the more variable the price. Does this affect the art market? As prices of paintings by older artists are more uncertain, if we estimate them only using easily available information, we would expect uninformed buyers not to enter the market, which would be a place for experts. This is consistent with the description by Keen as early as 1971 of the different types of buyers that frequent Sotheby's in London.

5. CONCLUSIONS

The relationship between the size of a painting and its price disappears over time, i.e. size progressively loses weight as an explanatory factor of the price of a work of art, particularly for those by high-quality artists. This is due to 1) a relative decrease in the importance of size as a factor that adds value to the painting compared to authorship, and 2) an increase in the noise of the data. The first reason is restricted to high-quality artists according to the market, but the second one applies to every artist.

With time, authorship, unlike size, incorporates a return. Authorship has an implicit price dynamics with a drift coefficient that increases as the quality of the artist increases. For very high quality artists, these dynamics account for almost the entire price of a painting (Proposition 1). Moreover, observations are more dispersed around the regression line and size becomes even less explanatory of price (Proposition 2). With time, other factors begin to make the price of a painting more unpredictable e.g. whether it has been damaged and/or restored (or, as in the case of Velázquez's *Hilanderas*, whether it has undergone changes in size or format), if it has been stolen, and the owners' reputation).

These two causes are natural outcomes of our model. We have modeled the time series behavior of an artist's standard size painting using diffusion processes, as is usual in the financial literature for modeling the dynamics of share prices. For a non-standard size painting, we corrected the previous price by a time-invariant size factor because we cannot reasonably attribute any tendency to it.

The results of our model are consistent with data findings. The two propositions have been corroborated empirically by the auction prices of paintings by 19^{th}-and 20^{th}-century Catalan artists. By taking a sample of Catalan painters of similar quality, we attempted to avoid the problems of limited duration and the scarcity of observations associated with each of an artist's vital eras. To use local data is restricting. Nevertheless, our findings avoid having to use an extensive sample of observations that would be too noisy to reflect the fall in the influence of size on prices. In this sense, local studies are useful for studying certain aspects of art market dynamics in detail. Our theoretical work shows that the constant of the regression line is more relevant for explaining the prices of paintings by older painters (Proposition 1). We also find that the variability not explained by the model is smaller for the more recent artists (Proposition 2).

Finally, our results suggest that the hedonic approach is unsuitable for predicting the prices of paintings by individual artists because, when averaging coefficients that differ between artists, we lost relevant information at an individual level. Since each artist is different, we expect the size coefficient and the ordinate at the origin to be different between artists. We therefore suggest segmenting the market in function of the artist's quality and antiquity in order to obtain more efficient and less risky estimates of parameters in hedonic models by limiting the risk of biased results due to movements in the data characteristics of the objects sold. These findings suggest that more specific and less general empirical studies of the art market are needed.

6. ANNEX

A. 1. Ito's Lemma provides a tool for obtaining stochastic differentials for functions of random processes. For a function of two stochastic process, Ito's lemma states:

$$dS_t = \frac{1}{2}\left[\frac{\partial^2 S}{\partial M^2} dM^2 + \frac{\partial^2 S}{\partial X^2} dX^2 + \frac{\partial^2 S}{\partial M \partial X} dMdX\right] + \frac{\partial S}{\partial M} dM + \frac{\partial S}{\partial X} dX$$

Applying Ito's lemma to expression (2), we find that:

$$dS_t = 0 + X_t\, Mt\, (\mu_M \; dt + \sigma_M \, dw_{1t}) + X_t\, Mt\, (\mu_X \, dt + \sigma_X \, dw_{2t})$$

and then:

$$dS_t \,/\, S_t = \mu \, dt + \sigma \, dz_t$$

where $\mu = \mu_M + \mu_X$ and $\sigma \, dz_t = \sigma_M \, dw_{1t} + \sigma_X \, dw_{2t}$

We find that the price of a standard painting follows a geometric Brownian motion i.e.

$$dS_t / S_t \sim N \, (\mu \, dt, \, \sigma \sqrt{dt})$$

with $\sigma \sqrt{dt} = \sigma_M \sqrt{dt} + \sigma_X \sqrt{dt}$.

A. 2. To find the probability distribution of S_t, we will again use Ito's lemma to solve the stochastic differential equation (7). The lemma for a function that depends on an Ito's stochastic process and on time, $h(S_t, t)$, states that the differential of the function is equal to:

$$dh = \frac{\partial h}{\partial t} dt + \frac{\partial h}{\partial S} dS + \frac{\partial^2 h}{\partial S^2} \frac{\sigma^2 S^2 dt}{2}$$

In particular, if $h(S_t, t)=ln(S)$, we have $h'(S)=1/S$ and $h''(S)=-1/S^2$, so that:

$$d\, ln\, S_t = \frac{dS_t}{S_t} - \frac{\sigma^2 dt}{2}$$

$$= (\mu - \sigma^2/2)\, dt + \sigma\, dz_t$$

The integral between $t_0=0$, when the price of the painting is S_0, and t, using τ as an integration variable,

is: $$ln\, S_t = ln\, S_0 + \int_{t_0}^{t} (\mu - \sigma^2/2) d\tau + \int_{t_0}^{t} \sigma dz_\tau$$

$$= ln\, S_0 + (\mu - \sigma^2/2)\, t + \sigma z_t$$

where $\Delta t = t$, $\Delta z = z_t - 0 = \varepsilon\, t^{1/2}$, as $w_{10}=w_{20}=0$ for any Wiener process.

Then,

$ln\, S_t \sim N(ln\, S_0 + (\mu - \sigma^2/2)\, t,\ \sigma\, t^{1/2})$

S_t is lognormal distributed. Finally, S_t is equal to:

$$S_t = S_0\, e^{(\mu - \sigma^2/2)t + \sigma z_t}$$

which has an expected value equal to:

$$E(S_t) = S_0 \int_{-\infty}^{\infty} e^{(\mu - \sigma^2/2)\Delta t}\ e^{\sigma \varepsilon \sqrt{\Delta t}}\ \frac{1}{\sqrt{2\pi}} e^{(-\varepsilon^2/2)} d\varepsilon$$

$$E(S_t) = S_0\ e^{\mu\Delta t} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\ dx \right]$$

where $x = \varepsilon - \sigma\sqrt{\Delta t}$

As the expression in brackets equals 1, $E(S_t) = S_0\ e^{\mu\Delta t}$

Similarly, we can compute the variance of S_t. The square of the price is equal to $S_t^2 = S_0^2\ e^{(2\mu-\sigma^2)\Delta t + 2\sigma\varepsilon\sqrt{\Delta t}}$, so we find $E[S_t^2]$:

$$\mathrm{E}(S_t^2) = S_0^2 \int_{-\infty}^{\infty} e^{2\mu\Delta t - \sigma^2\Delta t + 2\sigma\varepsilon\sqrt{\Delta t}}\ \frac{1}{\sqrt{2\pi}} e^{-\varepsilon^2/2} d\varepsilon = S_0^2 e^{2\mu\Delta t} e^{\sigma^2\Delta t} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(\varepsilon - 2\sigma\sqrt{\Delta t})^2/2} d\varepsilon \right]$$

The expression in brackets is equal to 1, and so,

$$Var(S_t) = E(S_t^2) - E(S_t)^2 = S_0^2\ e^{2\mu\Delta t}\ (e^{\sigma^2\Delta t} - 1)$$

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