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Security Prices and Market Transparency: The Role of Prior Information

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Abstract

This paper analyzes the role of traders' priors (proper versus improper) on the implications of market transparency by comparing a pre-trade transparent market with an opaque market in a set-up based on Madhavan (1996). We show that prices may be more informative in the opaque market, regardless of how priors are modelled. In contrast, the comparison of market liquidity and volatility in the two market structures are affected by prior specification.

Key words: Market Microstructure, Transparency, Prior Information.

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1 Introduction

One of the most surprising phenomena is the heterogeneity in pre-trade transparency exhibited in financial markets.[1] Automated limit-order-book systems such as the Paris Bourse and the Toronto Stock Exchange offer high degrees of pre-trade transparency (not only current quotes but information on limit orders away from the best quote are disseminated). By contrast, in U.S. markets generally only the best bid and the best offer is displayed.[2] Facing this diversity, both academics and practitioners have shown their interest in the economic implications of pre-trade transparency.

There is broad agreement that transparency does matter; it affects the informativeness of the order flow and hence the process of price discovery. Nevertheless, the existing literature shows that the key effects of transparency on security markets are complex and contradictory, since the degree of transparency and the type of transparency (pre-trade or post-trade) affect market outcomes. Changes in transparency regimes alter the information sets of market participants, change their optimal behavior and hence influence the price formation process. Prices, on the other hand, have an impact on not only the fairness and efficiency of the markets, but also on their attractiveness. The economic literature has shown steady interest in the implications of transparency.

Our paper is closely related to the literature on pre-trade transparency. As Madhavan (2000) suggested, pre-trade transparency is the key question of many important issues concerning the design and regulation of securities markets. For instance, it permeates debates on the choice of floor-based or automated trading systems, the willingness of investors to supply liquidity through limit orders, the growth of upstairs trading, and the nature and extent of disclosure of order imbalances at openings or during trading halts.

One of the investigations about the effects of pre-trade transparency on market performance is that of Madhavan (1996). Madhavan studies theoretically the extent to which the dissemination of information about orders affects the performance of a batch auction market. He shows that transparency can exacerbate volatility and decrease liquidity.[3] These results are derived under the proviso that rational investors hold improper or noninformative priors.

A number of criticisms has been raised concerning the use of noninformative priors by Bayesians who ultimately believe only in proper prior, subjective Bayesian analysis as a foundation for statistics. Many of them hinge on a classical quotation by Poincaré (1905):

[1]Pre-trade transparency refers to the wide dissemination of price quotations and orders before the trade.

[2]An exception is the Chicago Board Options Exchange where the book of customer limit orders can be wiewed by traders on the floor.

[3]Madhavan focuses on the availability of information about traders' motives (i.e., whether liquidity traders can be identified). The impact of anonymity on liquidity in an electronic limit order book is also investigated by Foucault, Moinas, and Theissen (2003), while Theissen (2003) examines anonymity in a setting where there is a market maker alongside a limit order book.

"If we were not ignorant there would be no probability, there could only be certainty. But our ignorance cannot be absolute for then there would be no longer any probability at all. Thus the problems of probability may be classed according to the greater or less depth of this ignorance".

However, Berger (1988) argues that noninformative Bayesian analysis is the single most powerful method of statistical analysis, in the sense of being the ad hoc method most likely to yield a sensible answer for a given investment of effort.

We will abstract here from the debate on the convenience of using proper versus improper priors. Rather than that, our purpose is to study the implications of choosing one over the other. To this end, we consider a set-up similar to that in Madhavan (1996), but with one important difference: we model priors in a way which encompasses the two strands of the literature on prior modeling; i.e., proper and improper priors. This modeling strategy allows us to tackle two issues simultaneously. On the one hand, we can study how transparency affects metrics of market quality when investors hold proper priors.[4] On the other hand, our analysis will show which market indicators are affected by prior specification, which may turn out to be useful in empirical investigations on transparency.

We show that results reported in Madhavan (1996) related to trading volume in a transparent versus an opaque market are independent of prior specification. However, we find that the comparison of market liquidity and volatility does depend on prior modelling. In particular, Madhavan (1996) finds that there is an inverse relationship between market depth and volatility. When this is the case, an increase in transparency either stabilize prices and increase market liquidity (both of them suitable properties of a financial market) or increase volatility and reduces liquidity (both of them undesirable for a market). Since preferences for both of these market indicators are aligned, one could conclude that transparency is unambiguously a good or a bad property for a market to have. However, we here show that this inverse relationship between market depth and volatility may not hold if investors have proper or informative priors, and consequently, that trade-offs among these two market indicators can occur. Consequently, a clear ranking between transparent and opaque markets may not exist.

Additionally, we also show that the opaque market may yield more informative prices. The rationale is that in an opaque market (more of) private information is incorporated into prices as this market facilitates camouflage. It is easier for investors to hide their private information which results in more informative equilibrium prices.

Our paper is also related to the literature that has focused on the desirability of pre-announcements of intentions to trade (sunshine trading). Admati and Pfleiderer (1991) analyze some of the implications of preannounced trading for financial market equilib-

[4] It is important to note that most theoretical analysis on transparency set models in which agents hold proper priors. This is so in the seminal papers by Admati and Pfleiderer (1991), Biais (1993), Pagano and Roëll (1996), among others. Consequently, the comparison of results should be made under the same modeling conditions.

rium. They find that the identification of liquidity orders reduces the trading costs of those who preannounce, but its effects on the trading costs and welfare of other traders are ambiguous. They also show that sunshine trading increases the informativeness of the price whereas it reduces the variance of the price change.[5] Admati and Pfleiderer consider a continuum of informed agents, who are price-takers and their motive for trading is information. We here consider a finite number of informed traders, who behave strategically and their motive for trading is information and hedging. Thus, whenever traders hold proper priors, our results assess the impact of relaxing the assumption of a continuum of informed agents in their conclusions. Throughout the paper we discuss how their results are altered when the set of informed traders is finite and they behave strategically.

The remainder of this paper is organized as follows. Section 2 outlines the notation and the hypotheses, which are common for both settings. Section 3 characterizes the unique symmetric linear equilibrium in both market structures. Section 4 examines the economic implications of transparency. Concluding comments are presented in Section 5. Finally, all the proofs are included in the Appendix.

2 The Model

The framework we present is the one proposed by Madhavan (1996). We consider a pure exchange economy where a risky asset, with random liquidation value $\widetilde{v}$, is traded against a riskless bond (taken as the numeraire), whose return is normalized to zero. The risky asset is traded at a market-clearing price $\widetilde{p}$ and thus its return is $\widetilde{v}-\widetilde{p}$. We assume that $\widetilde{v}$ is normally distributed with mean $\overline{v}$ and variance σ_v^2.

There are $N \geq 3$ rational expectations investors in the market,[6] indexed by n. Traders have negative exponential utilities in their terminal wealth, with common risk aversion coefficient ρ, and have an initial wealth W_{0n}. Before trading takes place, each trader receives a private signal conveying information about the liquidation value of the risky asset, and a private endowment for the risky asset. Let $\tilde{s}_n = \widetilde{v} + \widetilde{\varepsilon}_n$ denote the private signal of the n-th speculator, and let $\tilde{y}_n$ denote his shares of the risky asset.

In addition to the demands of the speculative traders, there is another component of order flow, denoted by $\tilde{z}$, which represents the imbalance arising from traders whose demands are price inelastic.

We assume that all random variables $\widetilde{\varepsilon}_1, ..., \widetilde{\varepsilon}_N, \tilde{y}_1, ..., \tilde{y}_N$ and $\tilde{z}$ are normally and independently distributed with a mean normalized to zero and a variance equal to σ_ς^2 for $\varsigma = \varepsilon, y, z$. Furthermore, the joint distribution of these random variables is common

[5] Dia and Pouget (2004) empirically study sunshine trading in the West-African Bourse. In Dia and Pouget (2005), they show that a pre-opening period coupled with a long-term relationship among market participants can constitute a market mechanism to accommodate sunshine trading.

[6] If $N \leq 2$, a symmetric linear equilibrium fails to exist.

knowledge.

In this set-up, we analyze and contrast two trading mechanisms that differ in their degree of transparency. The first mechanism is opaque in that trading is completely anonymous and traders do not receive any information about the composition of order flow. The second mechanism is transparent so that the realization of $\tilde{z}$ is disclosed to all market participants prior to trading. In what follows, we will use superscripts O and T to refer to the opaque and the transparent markets, respectively.

Priors modelling strategy

Regarding the true value of the asset two possible approaches can be followed. The first one is to assume that there is a noninformative prior with uniform density on $\mathcal{R}^1$. Under this approach the posterior of $\widetilde{v}$ given s_n is normally distributed with mean s_n and variance σ_ε^2. The second one is to assume that the prior of $\widetilde{v}$ is proper, in particular, as is standard in models on market microstructure, that $\widetilde{v}$ is normally distributed with mean $\overline{v}$ and variance σ_v^2.[7]

In this paper, we model the prior information using a specification which encompasses both of the above approaches. By doing so, we are able to determine how the strength of prior information affects the robustness of the results on market metrics provided by Madhavan (1996). In particular, we assume that $\widetilde{v}$ is normally distributed with finite mean $\overline{v}$ and variance σ_v^2. With this specification, the posterior distribution of $\widetilde{v}$, given an observation of the investor n' s private signal s_n, is normally distributed with

$$E(\widetilde{v}|s_n) = \frac{\sigma_\varepsilon^2 \overline{v} + \sigma_v^2 s_n}{\sigma_v^2 + \sigma_\varepsilon^2} \qquad \text{and} \qquad var(\widetilde{v}|s_n) = \frac{\sigma_v^2 \sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}.$$

The prior variance σ_v^2 represents the strength of the prior. If σ_v^2 is large, a relatively large range of values are plausible a priory, representing weak prior information. Conversely, if σ_v^2 is small, a relatively narrow range of values are plausible a priory, representing strong prior information. Whenever $\frac{\sigma_\varepsilon^2}{\sigma_v^2}$ is very large, then prior information is stronger than the data information, and hence, $E(\widetilde{v}|s_n) \simeq \overline{v}$ and $var(\widetilde{v}|s_n) \simeq \sigma_v^2$. Conversely, if $\frac{\sigma_\varepsilon^2}{\sigma_v^2}$ is very low, prior information is much weaker than the data, and hence, $E(\widetilde{v}|s_n) \simeq s_n$ and $var(\widetilde{v}|s_n) \simeq \sigma_\varepsilon^2$, as data information dominates. It is important to point out that the posterior $N(s_n, \sigma_\varepsilon^2)$ cannot be obtained as the posterior distribution of any proper prior distribution. Nevertheless, $N(s_n, \sigma_\varepsilon^2)$ is the posterior distribution for a noninformative prior with uniform density on $\Re$.[8] Even though in this paper we assume that $\widetilde{v}$ has a proper prior, the normality assumption will allow us to get arbitrarily close to a setup with improper priors, as does Madhavan's (1996), by taking a large variance of $\widetilde{v}$.

[7]The first approach corresponding to improper priors is used in Madhavan (1996). The second one is used in most models of market microstructure (see for instance, Hellwig (1980), Diamond and Verrechia (1981), Glosten and Milgrom (1985), Kyle (1985, 1989), Huddart, Hughes and Levine (2001), among others).

[8]For a further discussion on the relationship between proper and improper prior distributions, see Berger (1985) and O'Hagan (1994).

3 The Symmetric Linear Equilibrium

For the tractability of the analysis, we focus on symmetric linear rational expectations equilibria (*SLE*). We hence search for traders' optimal strategies in the market structure M, $M = O, T$, under the proviso that they are identical linear functions, i.e., for all n

$$x_n^M(p; I_n^M) = \mu^M + A^M\left(I_n^M\right) - \gamma^M p,$$

where I_n^M is a vector which collects all the information held by trader n, and μ^M, A^M and γ^M are constants.[9]

Following Kyle (1989), in a *SLE* the optimal demand function for informed trader n is given by

$$x_n^M(p; I_n^M) = \frac{E(\widetilde{v}|p, I_n^M) + \frac{1}{(N-1)\gamma^M} y_n - p}{\frac{1}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M)}. \tag{1}$$

This demand maximizes traders' utility whenever

$$\frac{2}{(N-1)\gamma^M} + \rho var(\widetilde{v}|p, I_n^M) > 0. \tag{2}$$

Since in equilibrium markets must clear, i.e., aggregate demand and supply must coincide, then

$$N\mu^M + A^M\left(\sum_{j=1}^{N} I_j^M\right) - N\gamma^M p + z = \sum_{j=1}^{N} y_j.$$

Note that asset endowments affect both aggregate demand and aggregate supply. Because of this, it is convenient at this stage to disentangle y_n from I_n^M, and rewriting equation above as

$$N\mu^M + \bar{A}^M\left(\sum_{j=1}^{N} \bar{I}_j^M\right) + (\alpha^M - 1)\sum_{j=1}^{N} y_j - N\gamma^M p + z = 0$$

or, equivalently,

$$p = \frac{1}{N\gamma^M}\left(N\mu^M + \bar{A}^M\left(\sum_{j=1}^{N} \bar{I}_n^M\right) + (\alpha^M - 1)\sum_{j=1}^{N} y_j + z\right), \tag{3}$$

where $\bar{I}_n^M$ is the vector I_n^M with the last component (the one corresponding to y_n) equal to zero, analogously for $\bar{A}^M$.

From the distributional assumptions and the previous expression for p, it follows that the vector $\left(p, I_n^M\right)$ is informatively equivalent to the vector $\left(h_n^M, s_n\right)$, where $h_n^M = \bar{A}^M\left(\sum_{j\neq n} \bar{I}_j^M\right) + (\alpha^M - 1)\sum_{j\neq n} y_j + \delta_{\{M=O\}} z$, and $\delta_{\{M=O\}}$ is an indicator function that takes

[9] Note that in the opaque market I_n^M is a vector of private signals observed by trader n, i.e., $I_n^O = (s_n, y_n)$, whereas in the transparent market I_n^M is a vector which contains the private signals observed by trader n, (s_n, y_n), and the public realization of $\tilde{z}$, i.e., $I_n^T = (s_n, y_n, z)$.

value one if $M = O$ and zero if $M = T$. Using the distributional assumptions, it follows that

$$E(\widetilde{v}|p, I_n^M) = E(\widetilde{v}|h_n^M, s_n)$$

and

$$var(\widetilde{v}|p, I_n^M) = var(\widetilde{v}|h_n^M, s_n).$$

Finally, applying standard normal theory, we have

$$E(\widetilde{v}|h_n^M, s_n) = \bar{v} + cov\left(\widetilde{v}, \left(\tilde{h}_n^M, \tilde{s}_n\right)\right) var^{-1}\begin{pmatrix}\tilde{h}_n^M \\ \tilde{s}_n\end{pmatrix}\begin{pmatrix}h_n^M - E\left(\tilde{h}_n^M\right) \\ s_n - E\left(\tilde{s}_n\right)\end{pmatrix}, \tag{4}$$

and

$$var(\widetilde{v}|h_n^M, s_n) = \sigma_v^2 - cov\left(\widetilde{v}, \left(\tilde{h}_n^M, \tilde{s}_n\right)\right) var^{-1}\begin{pmatrix}\tilde{h}_n^M \\ \tilde{s}_n\end{pmatrix} cov\left(\widetilde{v}, \begin{pmatrix}\tilde{h}_n^M \\ \tilde{s}_n\end{pmatrix}\right). \tag{5}$$

In what follows we characterize expressions above for the two market structures considered in this paper.

3.1 The Opaque Market

When the market is opaque, a rational trader only has two pieces of information: his inventory holdings and his private signal about the true value of the risky asset. Consequently, investor n's *SLE* demand function in an opaque market is given by

$$x_n^O(p; s_n, y_n) = \mu^O + \beta^O s_n + \alpha^O y_n - \gamma^O p, \text{ for all } n. \tag{6}$$

To study the existence of a *SLE*, we first write all coefficients of speculators' demands as functions of the coefficient α^O. This coefficient is then characterized as a root of a polynomial. If such a root exists, then one might conclude that a *SLE* exists. Furthermore, this root lies in a specified interval. These facts are formally stated in the following three results:

Lemma 1: In a SLE,

$$\mu^O = \frac{2(1 - \alpha^O)}{\rho\left(1 + (N-1)\,\alpha^O\right)\sigma_v^2}\bar{v}, \tag{7}$$

$$\beta^O = \frac{1 - \alpha^O}{\rho\sigma_\varepsilon^2}, \textit{ and} \tag{8}$$

$$\gamma^O = \frac{1 - \alpha^O}{\rho\sigma_\varepsilon^2}\left(1 + \frac{2\sigma_\varepsilon^2}{\left(1 + (N-1)\,\alpha^O\right)\sigma_v^2}\right). \tag{9}$$

Lemma 2: In a SLE, it holds that

$$\alpha^O \in \left(\frac{1}{N-1}, \frac{2N-1+\phi}{(N-1)(1+\phi)} \right).$$

In addition, this coefficient is a root of a polynomial of degree three

$$Q(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha - d,$$

whose coefficients are given by $a = (N-1)(\phi+1)$, $b = 3 - 4N + \phi(1-2N)$, $c = 5N - 3 + \phi(N+1) + \varphi$, *and* $d = 2N - 1 + \phi + \frac{1}{(N-1)}\varphi$, *where* $\phi = \rho^2\sigma_\varepsilon^2\sigma_y^2$ *and* $\varphi = \rho^2\sigma_\varepsilon^2\sigma_z^2$.

Lemma above shows that α^O is a function of N and of two payoff unrelated components in the trade: ϕ and φ. Note that $\phi = \rho^2\sigma_y^2\sigma_\varepsilon^2$ is a liquidity component in investors' trade (endowments can be thought of as liquidity shocks), and $\varphi = \rho^2\sigma_\varepsilon^2\sigma_z^2$ is a liquidity component due to noise traders.[10] Furthermore, implicit differentiation shows that α^O is strictly decreasing in both liquidity components. If the order shock is small enough ($\varphi \to 0$), then α^O goes to $\frac{2N-1+\phi}{(N-1)(1+\phi)}$.

Proposition 3: If $N \geq 3$, *then there exists a unique SLE.*

This result shows existence and uniqueness of the symmetric linear equilibrium under general conditions. We only require that $N \geq 3$ as in Kyle (1989). Recall that a *SLE* does not always exist because, with infinitely inelastic demand by noise traders, speculators may have "too much" monopoly power if there are not enough of them.

3.2 The Transparent Market

We here analyze a trading mechanism in which the inelastic demand is displayed to all traders before they submit their demands, so that a trader's information is given by $I_n^T = (s_n, z, y_n)$. In this market structure, the symmetric linear demand strategy can be written as

$$x_n^T(p; s_n, y_n, z) = \mu^T + \beta^T s_n + \alpha^T y_n + \xi^T z - \gamma^T p, \text{ for all } n. \tag{10}$$

The following two results allow us to explicitly characterize the unique *SLE* corresponding to the transparent market.

Lemma 4: In a SLE,

$$\mu^T = \frac{2(1-\alpha^T)}{\rho(1+(N-1)\alpha^T)\sigma_v^2}\bar{v}, \tag{11}$$

[10] A similar decomposition is used by Naik et al. (1999) to characterize the symmetric linear equilibrium in a model with interdealer trading.

$$\beta^T = \frac{1-\alpha^T}{\rho\sigma_\varepsilon^2}, \tag{12}$$

$$\gamma^T = \frac{1-\alpha^T}{\rho\sigma_\varepsilon^2}\left(1+\frac{2\sigma_\varepsilon^2}{\left(1+(N-1)\,\alpha^T\right)\sigma_v^2}\right), \textit{and} \tag{13}$$

$$\xi^T = -\frac{\left(1+(N-1)\,\alpha^T\right)\sigma_v^2+2\sigma_\varepsilon^2}{\left(N\sigma_v^2+\sigma_\varepsilon^2+\phi\left(\sigma_v^2+\sigma_\varepsilon^2\right)\right)\left(1+(N-1)\,\alpha^T\right)}. \tag{14}$$

Notice that equilibrium coefficients are independent of σ_z^2 as liquidity demand is now common knowledge.

Proposition 5: A SLE in a transparent market exists iff

$$N < (N-2)\,\phi\cdot$$

If it exists, then

$$\alpha^T = \frac{2N-1+\phi}{(N-1)(\phi+1)}.$$

Proposition 5 shows that in a transparent market a *SLE* may fail to exist. Notice that existence requires the liquidity shock via endowments to be large enough. The rationale for this inequality is as follows. Whenever the price is not a good estimator of the private information (because either σ_y^2 or σ_ε^2 are high), and agents face a high inventory cost from maintaining the initial holdings of the risky asset (ρ is high), then ϕ will be large enough for speculators to find participating in the market profitable. However when the inequality is not satisfied, so that $\phi \leq \frac{N}{N-2}$ a *SLE* fails to exist. If traders are well informed and there is little endogenous liquidity trading (i.e., ρ and σ_y^2 are low, which gives a low ϕ), then agents are unwilling to reveal their information to others, finding the readjustment of their portfolio very expensive. Consequently, they decide to consume their initial endowment, and a market breakdown follows.

3.3 Equilibrium Comparison

In what follows we study the impact of transparency on the strategic behavior of investors. We here show that it influences not only existence of equilibrium, but also the price intercept and the slope of traders' demands. Next two corollaries state these results formally.

Corollary 6: If a SLE exists for the transparent market, then it exists for the opaque market as well.

This corollary tells us that trading is more robust in the opaque market than in the transparent market. In other words, transparency may induce a form of market failure.[11]

We now compare some equilibrium coefficients of traders' strategies in the two market structures.[12] Recall that the equilibrium demand of trader n in a *SLE* is given by

$$x_n^M(p; I_n^M) = \mu^M + \beta^M s_n + \alpha^M y_n + \xi^T \delta_{\{M=T\}} z - \gamma^M p. \tag{15}$$

Note that the functional form of coefficients μ^M, β^M and γ^M coincide in the two market structures. They are all strictly decreasing functions of α^M. Regarding the coefficient ξ^T, it is negative in equilibrium so that traders' actions partly accommodate the noise shock.

Transparency has two main consequences. First, it makes traders' demands less price responsive. In the transparent market, an increase in the price of the risky asset makes agents more optimistic about its liquidation value, which leads to a smaller reduction in the individual demands as compared to the opaque market. Second, it makes traders' demands more sensitive to liquidity shocks and less sensitive to signals. The rationale is that in the transparent market, there is a higher need for camouflage which makes demands less sensitive to signals. In summary, $\alpha^T > \alpha^O$ whereas $\beta^T < \beta^O$ and $\gamma^T < \gamma^O$.

The qualitative results discussed so far are robust to prior specification. In particular, $\alpha_P^M = \alpha_I^M$ for any M, where the subscript P stands for proper prior and the subscript I for improper prior, and consequently, $\beta_P^M = \beta_I^M$. Regarding the coefficient ξ^T, since $\alpha^T = \frac{2N-1+\phi}{(N-1)(\phi+1)}$, then (14) gives $\xi^T = -\frac{(N-1)\alpha^T-1}{(N-1)(1+(N-1)\alpha^T)}$, which implies that $\xi_P^T = \xi_I^T$. Furthermore, $\mu_I^M = 0$ will coincide with μ_P^M if $\bar{v} = 0$. Nevertheless, in either market structure, the slopes of demand curves are sensitive to prior information. Specifically, if priors are non-informative, then $\gamma_I^M < \gamma_P^M$ and $\gamma_I^M = \beta_I^M$. The next corollary summarizes some of these results.

Corollary 7: a) Traders' demands are less price-sensitive in a transparent market. Moreover, price sensitivity is smaller if priors are uninformative.

b) Transparency reduces the expected volume of portfolio hedging trade, given by[13]

$$E(\left|\left(\alpha^M - 1\right) \sum_{j=1}^{N} \widetilde{y}_j\right|).$$

Finally, we show that prior specification does not affect informed traders' volume.

[11] Note that this form of maket failure refers to the non-existence of the type of equilibria we focus on, that is, symmetric linear equilibria.

[12] For this comparison to be meaningful, we restrict our attention to parameter values that satisfy $N < (N-2)\,\phi$.

[13] The portfolio hedging component of the individual net demand is given by $\left(\alpha^M - 1\right) \widetilde{y}_i$.

Corollary 8: Equilibrium informed trading volume does not depend on prior specification.

4 Transparency and Market Quality

In this section we examine the economic implications of transparency. Specifically, we analyze the differences between the opaque and the transparent markets in terms of some measures of market quality.

- **Market Liquidity**

In order to measure the impact of transparency on market liquidity, we now compare the market depth in the two market structures. The market depth is defined as the quantity of noise trading required to induce the price of the risky asset to rise by one unit.

Proposition 9: If $N\sigma_{\varepsilon}^2(1+\phi)+\sigma_v^2\left(N^2-\phi(N^2-4N+2)\right)<0,$ *then the transparent market is deeper. Otherwise, the same result holds if* σ_z^2 is sufficiently small.

Concerning the impact that changes in the noise demand have on market price, notice that from Equation (1) and the market clearing condition, it follows that

$$p=\frac{\sum_{n=1}^{N}E(\widetilde{v}|h_n^M,s_n)+\left(\frac{1}{(N-1)\gamma^M}+\rho var(\widetilde{v}|h_n^M,s_n)\right)z-\left(\sum_{n=1}^{N}y_n\right)\rho var(\widetilde{v}|h_n^M,s_n)}{N}.$$

This expression shows that noise trading affects market price through three channels: *an adverse-selection effect*, a *strategic-behavior effect* and a *risk-bearing effect.*

The *adverse-selection effect* is captured by $\frac{\sum_{n=1}^{N}E(\widetilde{v}|h_n^M,s_n)}{N}$ by the way of h_n^M. Note that in the opaque market an increase in the noise demand makes speculator n realize that h_n^O increases. Recall that $h_n^O=\beta^O\sum_{j\neq n}s_j+\left(\alpha^O-1\right)\sum_{j\neq n}y_j+z$. He, not knowing the source of this increase, assumes that it may be due to his competitors receiving favorable signals about the payoff of the risky asset. Each speculator therefore adjusts his forecast upward, and this in turn causes the price to increase. In contrast, in the transparent market this effect is not present as noise demand is displayed.

The *strategic-behavior effect* is measured by $\dfrac{1}{(N-1)N\gamma^M}z$. Since traders' strategies are less price-sensitive in the transparent market, this second effect is more significant for this market structure.

Finally, the *risk-bearing effect*, captured by $\frac{\left(z-\sum_{n=1}^{N}y_n\right)\rho var(\widetilde{v}|h_n^M,s_n)}{N}$, is due to the fact that an order by a liquidity trader forces speculators to take the other side of the market.

Price must hence adjust to induce risk-averse speculators to bear this risk. As speculators are better informed in the transparent market, this risk-bearing effect is more significant in the opaque market.

To understand the inequality given in the statement of Proposition 9, notice that when N converges to infinity, the strategic-behavior effect vanishes. Therefore, we unambiguously obtain that the equilibrium price is more sensitive to changes in z in the opaque market, or equivalently, that the transparent market is deeper. In the opposite case, for instance when N is small ($N = 3$) and σ_z^2 is large enough, the strategic-behavior effect dominates and leads to the conclusion that transparency reduces market liquidity.

Regarding the role of prior specification, it can be easily studied by focusing again on Proposition 9. In an economy in which σ_z^2 is low enough, prior specification does not affect the comparison between the two markets. In contrast, if σ_z^2 is large enough, then prior specification has a bite. To clarify this assertion, let us rewrite the inequality in Proposition 9 as follows:

$$N\frac{\sigma_\varepsilon^2}{\sigma_v^2}(1+\phi) + \left(N^2 - \phi(N^2 - 4N + 2)\right) < 0.$$

If σ_v^2 goes to infinity, the transparent market is deeper whenever $N^2 - \phi(N^2 - 4N + 2) < 0$. Nevertheless, even when this inequality holds, the opaque market may be deeper provided that σ_v^2 is low enough, as $N\frac{\sigma_\varepsilon^2}{\sigma_v^2}(1+\phi)$ is always strictly positive. These arguments allow us to derive the next result.

Corollary 10: The comparison of the market liquidity between the two market structures is ambiguous and depends on the specification of priors.

The ambiguity of the effect of changes in pre-trade transparency on liquidity is consistent with the empirical evidence.[14]

- **Expected trading costs of liquidity traders**

The expected trading costs of liquidity traders are given by

$$\begin{aligned} C^O &= E\left(\tilde{z}\left(\tilde{p}^O - \tilde{v}\right)\right) = \frac{1}{N\gamma^O}\sigma_z^2 \text{ and} \\ C^T &= E\left(\tilde{z}\left(\tilde{p}^T - \tilde{v}\right)\right) = \frac{N\xi^T + 1}{N\gamma^T}\sigma_z^2. \end{aligned}$$

Consequently, $C^O > C^T$ if and only if $\frac{1}{N\gamma^O} > \frac{N\xi^T+1}{N\gamma^T}$, which is equivalent to say $\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} < \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1}$. The combination of this inequality and Corollary 10 allows us to conclude that the comparison of expected trading costs does also depend on the specification of priors.

[14]Madhavan, Porter and Weaver (2005) find that an increase in pre-trade transparency in the Toronto Stock Exchange led to lower depth. This result contrasts with the empirical investigation of pre-trade transparency at the NYSE conducted by Boehmer, Saar and Yu (2004).

When the number of speculators goes to infinity, the expected trading costs of the announcers are strictly lower when preannouncement takes place than when it does not, which is consistent with Proposition 1 in Admati and Pfleiderer (1991). Note that this result does not hold in general in our model.

- **Price Informativeness**

Concerning the revelation of information about the payoff of the risky asset through the price, as measured by $\frac{1}{var(\widetilde{v}|p^M)}$, the next proposition shows that prices are not always more informative in the transparent market.[15]

Proposition 11. a) Prices are more informative in the transparent market iff the following inequality holds

$$\alpha^O > C \text{ where } C = \frac{\phi\,(\phi + 4N - 1 - N^2) + N^2}{(N-1)\,(\phi+1)\,\phi}.$$

Moreover, if $\phi\,(2 + N^2 - 4N) - N^2 > 0$, *then prices are always more informative in the transparent market.*[16] *Otherwise, this result holds provided that* σ_z^2 *is low enough.*[17]

b) At the time the trade is made and z *is realized, pre-trade transparency unambiguously increases the informativeness of the price, that is,*

$$var^{-1}(\widetilde{v}|p^O) < var^{-1}\left(\widetilde{v}|p^T, z\right).$$

In the transparent market, the information held by agents is more precise than in the opaque market, that is, $var^{-1}(\widetilde{v}|p^T, s_n, y_n, z) > var^{-1}(\widetilde{v}|p^O, s_n, y_n)$. However, we show that prices may not contribute to this greater precision. This puzzling result is obtained when $\phi\,(2 + N^2 - 4N) - N^2 < 0$, and σ_z^2 is large enough. If this were the case there are pairs (ϕ, φ), with $\varphi > \phi$, for which prices are more informative in the opaque market. Note that if σ_y^2 is small, compared to σ_z^2, it is hard for investors to hide their private information in a transparent market, and consequently, in equilibrium, their demands will not be responsive to private signals. By contrast, in the opaque market (more of) private

[15] Our measure of informational efficiency differs from that in Madhavan (1996). He considers the precision of the information held by informed traders, whereas we use the informational content of the equilibrium price.

[16] A particular case in which prices are more informative in the transparent market is when N converges to infinity. This result is consistent with Admati and Pfleiderer (1991) where it is shown that the equilibrium price with preannouncement is more informative about the true value than the equilibrium price without preannouncement.

[17] Baruch (2005) develops a model to address the welfare implications of making a limit order book visible to the market. He finds that, on average, prices are more informative in the open book environment. This result differs from ours. This is due to the fact that the two papers make different assumptions. In Baruch, all market participants are risk-neutral, there is a single informed trader who trades for informational reasons, and who sets market orders. Here we have several risk-averse informed traders who have hedging motives to trade.

information is incorporated into prices as this market facilitates camouflage whenever σ_z^2 is large. In particular, if σ_z^2 goes to infinity, so that $\varphi > \phi$ trivially holds, then $\alpha^O = \frac{1}{N-1} < C$ and prices will be more informative in an opaque market.

Corollary 12: Results related to price informativeness are robust to prior specification.

- **Volatility**

The volatility of equilibrium prices, measured by $var(\widetilde{v} - \widetilde{p}^M)$, is given by[18]

$$var(\widetilde{v} - \widetilde{p}^O) = \left(1 - \frac{\beta^O}{\gamma^O}\right)^2 \sigma_v^2 + \left(\frac{\beta^O}{N\gamma^O}\right)^2 N\sigma_\varepsilon^2 + \left(\frac{1-\alpha^O}{N\gamma^O}\right)^2 N\sigma_y^2 + \left(\frac{1}{N\gamma^O}\right)^2 \sigma_z^2, \text{ and}$$

$$var(\widetilde{v} - \widetilde{p}^T) = \left(1 - \frac{\beta^T}{\gamma^T}\right)^2 \sigma_v^2 + \left(\frac{\beta^T}{N\gamma^T}\right)^2 N\sigma_\varepsilon^2 + \left(\frac{1-\alpha^T}{N\gamma^T}\right)^2 N\sigma_y^2 + \left(\frac{1+N\xi^T}{N\gamma^T}\right)^2 \sigma_z^2.$$

Taking into account Lemmas 1 and 4, we have

$$var(\widetilde{v} - \widetilde{p}^O) = g(\alpha^O, \sigma_v^2) + \left(\frac{\partial \widetilde{p}^O}{\partial \widetilde{z}}\right)^2 \sigma_z^2, \text{ and}$$

$$var(\widetilde{v} - \widetilde{p}^T) = g(\alpha^T, \sigma_v^2) + \left(\frac{\partial \widetilde{p}^T}{\partial \widetilde{z}}\right)^2 \sigma_z^2,$$

with

$$g(\alpha, \sigma_v^2) = \frac{\sigma_\varepsilon^2 \sigma_v^2 \left(4N\sigma_\varepsilon^2 + (\phi+1)\left(1+(N-1)\alpha\right)^2 \sigma_v^2\right)}{N\left(\sigma_v^2\left(1+(N-1)\alpha\right) + 2\sigma_\varepsilon^2\right)^2}.$$

The difference in volatility between the two market structures, $var(\widetilde{v} - \widetilde{p}^O) - var(\widetilde{v} - \widetilde{p}^T)$, becomes

$$var(\widetilde{v} - \widetilde{p}^O) - var(\widetilde{v} - \widetilde{p}^T) =$$
$$g(\alpha^O, \sigma_v^2) - g(\alpha^T, \sigma_v^2) - \left(\left(\frac{\partial \widetilde{p}^O}{\partial \widetilde{z}}\right)^{-1} - \left(\frac{\partial \widetilde{p}^T}{\partial \widetilde{z}}\right)^{-1}\right) \frac{\partial \widetilde{p}^O}{\partial \widetilde{z}} \frac{\partial \widetilde{p}^T}{\partial \widetilde{z}} \left(\frac{\partial \widetilde{p}^O}{\partial \widetilde{z}} + \frac{\partial \widetilde{p}^T}{\partial \widetilde{z}}\right) \sigma_z^2.$$

If σ_v^2 converges to infinity, then $g(\alpha^O, \sigma_v^2) = g(\alpha^T, \sigma_v^2)$. Consequently,

$$sign\left(var(\widetilde{v} - \widetilde{p}^O) - var(\widetilde{v} - \widetilde{p}^T)\right) = -sign\left(\left(\frac{\partial \widetilde{p}^O}{\partial \widetilde{z}}\right)^{-1} - \left(\frac{\partial \widetilde{p}^T}{\partial \widetilde{z}}\right)^{-1}\right),$$

which implies the existence of an inverse relationship between market depth and volatility.

If σ_v^2 is finite, then $g(\alpha^O, \sigma_v^2) \neq g(\alpha^T, \sigma_v^2)$, so that market depth and volatility may be aligned.

Corollary 13: Under improper priors there is an inverse relationship between market depth and volatility. This result may no longer hold if priors are proper unless N is large enough.

[18] Formulae below can be easily derived from Expressions (25) and (26) in the Appendix.

5 Conclusions

This paper has examined the robustness of Madhavan's (1996) results on the economic implications of market transparency. We have shown that the following results are robust to prior specification: market transparency may induce a form of market failure, always increases the precision of traders' prediction, and can exacerbate price volatility. However, the inverse relationship between price volatility and market liquidity obtained in Madhavan (1996) may not hold with proper priors. The practical implication of this point is that a change in transparency that lowers price volatility does not always reduce the execution costs of liquidity traders. Similarly, the comparison on market liquidity across market structures depends on prior specification.

Various difficulties accompany the use of improper priors. On one hand, there is the failure of the law of conditional expectations and "integrating-out." Decision rules may not be admissible with improper priors, as a proper posterior cannot always be derived via Bayes' theorem from an improper prior. On the other hand, the so-called marginalization paradoxes can only affect improper priors. From our viewpoint, introducing improper priors can only be justified when assuming extreme uncertainty is sensible. When it comes to modelling agents that possess diverse private information about the payoff of assets and that actively participate in a financial market, the assumption of complete ignorance (ex-ante) does not seem realistic. We have hence investigated in this paper the economic implications of transparency assuming proper priors.

Pre-trade transparency increases the informativeness of the price at the time the trade is made. This implies that it would reduce the incentives to gather costly private information. But less information acquisition leads to an increase in the risk-bearing cost borne by liquidity traders since the risky asset would be more uncertain. A more complex scenario is needed to analyze the effect of pre-trade transparency on the expected trading costs of liquidity traders when private information is costly. This is left to future research.

6 Appendix

Proof of Lemma 1: When the market is opaque Equations (4) and (5) become

$$E(\widetilde{v}|h_n^O, s_n) = \bar{v} + cov\left(\widetilde{v}, \left(\tilde{h}_n^O, \tilde{s}_n\right)\right) var^{-1}\begin{pmatrix}\tilde{h}_n^O \\ \tilde{s}_n\end{pmatrix}\begin{pmatrix}h_n^O - \beta^O(N-1)\bar{v} \\ s_n - \bar{v}\end{pmatrix},$$

and

$$var(\widetilde{v}|h_n^O, s_n) = \sigma_v^2 - cov\left(\widetilde{v}, \left(\tilde{h}_n^O, \tilde{s}_n\right)\right) var^{-1}\begin{pmatrix}\tilde{h}_n^O \\ \tilde{s}_n\end{pmatrix} cov\left(\widetilde{v}, \begin{pmatrix}\tilde{h}_n^O \\ \tilde{s}_n\end{pmatrix}\right),$$

where, $\tilde{h}_n^O$ can be rewritten as $\tilde{h}_n^O = \beta^O \sum_{j\neq n} \tilde{s}_j + \left(\alpha^O - 1\right) \sum_{j\neq n} \tilde{y}_j + \tilde{z}$, because of (6).

Using the expressions of $\tilde{h}_n^O$ and $\tilde{s}_n$, we first derive the variance and covariance matrixes in expressions above, which are given by

$$cov\left(\widetilde{v},\left(\tilde{h}_n^O,\tilde{s}_n\right)\right)=\left(\beta^O\left(N-1\right)\sigma_v^2,\sigma_v^2\right),\text{ and}$$

$$var\begin{pmatrix}\tilde{h}_n^O\\ \tilde{s}_n\end{pmatrix}=\begin{pmatrix}\Sigma_{11}^O & \beta^O\left(N-1\right)\sigma_v^2\\ \beta^O\left(N-1\right)\sigma_v^2 & \sigma_v^2+\sigma_\varepsilon^2\end{pmatrix},$$

where

$$\Sigma_{11}^O=\left(\beta^O\right)^2\left(N-1\right)\left(\left(N-1\right)\sigma_v^2+\sigma_\varepsilon^2\right)+\left(\alpha^O-1\right)^2\left(N-1\right)\sigma_y^2+\sigma_z^2.$$

Note that the variance matrix is nonsingular. Its determinant, D^O, is strictly positive, with

$$D^O=\left(\beta^O\right)^2\left(N-1\right)\sigma_\varepsilon^2\left(N\sigma_v^2+\sigma_\varepsilon^2\right)+\left(\left(\alpha^O-1\right)^2\left(N-1\right)\sigma_y^2+\sigma_z^2\right)\left(\sigma_v^2+\sigma_\varepsilon^2\right).$$

Substituting these values into the expressions for the posterior mean and variance, and undertaking straightforward computations, we get

$$E(\widetilde{v}|p,I_n^O)=\bar{v}+a_1\left(N\gamma^Op-\beta^Os_n+\left(1-\alpha^O\right)y_n-N\mu^O-\beta^O(N-1)\bar{v}\right)+a_2\left(s_n-\bar{v}\right),$$

and

$$var(\widetilde{v}|p,I_n^O)=a_2\sigma_\varepsilon^2,$$

where

$$\begin{aligned}a_1&=\frac{\sigma_\varepsilon^2\sigma_v^2\beta^O\left(N-1\right)}{D^O},\text{ and}\\ a_2&=\frac{\sigma_v^2\left(\left(\beta^O\right)^2\left(N-1\right)\sigma_\varepsilon^2+(\alpha^O-1)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{D^O}.\end{aligned}$$

Plugging the expression for $E(\widetilde{v}|p,I_n^O)$ obtained above into (1), and equating coefficients according to (6), it follows that the coefficients of speculators' demands corresponding to a *SLE* are the solutions to the following system of equations:

$$\mu^O=\frac{\bar{v}(1-a_2)-a_1\left(\beta^O(N-1)\bar{v}+N\mu^O\right)}{\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)},\tag{16}$$

$$\beta^O=\frac{a_2-a_1\beta^O}{\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)},\tag{17}$$

$$\alpha^O=\frac{a_1(1-\alpha^O)+\frac{1}{(N-1)\gamma^O}}{\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)},\text{ and}\tag{18}$$

$$\gamma^O=\frac{1-a_1N\gamma^O}{\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)}.\tag{19}$$

From this system we obtain all coefficients as functions of α^O. To do so, we derive two auxiliary equations that will turn out to be very useful in proving this lemma. The first one is obtained after dividing (18) by (19) and performing simple manipulations, which gives

$$a_1\gamma^O\left(1+(N-1)\,\alpha^O\right)=\alpha^O-\frac{1}{N-1}\cdot \tag{20}$$

The second one follows from rearranging (19), and combining it with (20), which results in

$$\rho var(\widetilde{v}|p,I_n^O)=\frac{2(1-\alpha^O)}{\gamma^O\left(1+(N-1)\,\alpha^O\right)}\cdot \tag{21}$$

Now, to get β^O, notice that plugging the values of a_1 and a_2 into (17), we have

$$\beta^O=\frac{\sigma_v^2\left((\alpha^O-1)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{\left(\frac{1}{(N-1)\gamma^O}+\rho var(\widetilde{v}|p,I_n^O)\right)D^O}\cdot \tag{22}$$

Then, substituting $\rho var(\widetilde{v}|p,I_n^O)$ by the value in (21), it follows that

$$\beta^O=\frac{\left(1+(N-1)\,\alpha^O\right)(N-1)\,\gamma^O\sigma_v^2\left((\alpha^O-1)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{\left(2N-1-(N-1)\,\alpha^O\right)D^O}\cdot$$

Multiplying both sides of equality above by $\beta^O\sigma_\varepsilon^2$, and recalling that $a_1=(N-1)\,\beta^O\sigma_\varepsilon^2\sigma_v^2/D^O$, we get

$$\left(\beta^O\right)^2\sigma_\varepsilon^2=\frac{\left(1+(N-1)\,\alpha^O\right)\gamma^O a_1\left((\alpha^O-1)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{2N-1-(N-1)\,\alpha^O}\cdot$$

Plugging (20) into the resulting equation gives

$$\left(\beta^O\right)^2\sigma_\varepsilon^2=\frac{\left((N-1)\alpha^O-1\right)\left((\alpha^O-1)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{\left(2N-1-(N-1)\,\alpha^O\right)(N-1)}\cdot \tag{23}$$

In addition, using (19) and (20), it follows that

$$\left((N-1)\,\alpha^O-1\right)\rho var(\widetilde{v}|p,I_n^O)=2\,(N-1)\,a_1\left(1-\alpha^O\right)\cdot$$

Since $var(\widetilde{v}|p,I_n^O)=a_2\sigma_\varepsilon^2$, plugging the values of a_1 and a_2 into the previous expression, and substituting $\left(\beta^O\right)^2\sigma_\varepsilon^2$ by the value in (23), straightforward computations give

$$\rho\frac{\left((N-1)\,\alpha^O-1\right)\left(\left(\alpha^O-1\right)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{2N-1-(N-1)\,\alpha^O}=\beta^O\,(N-1)\left(1-\alpha^O\right)\cdot \tag{24}$$

Notice that (23) and (24) imply that $\left(\beta^O\right)^2\sigma_\varepsilon^2=\frac{\beta^O\left(1-\alpha^O\right)}{\rho}$, or equivalently, the equality given in (8), since (22) guarantees that $\beta^O\neq 0$.

To derive the expression for γ^O, we first compute $var(\widetilde{v}|p, I_n^O)$, using (23), which gives

$$var(\widetilde{v}|p, I_n^O) = \frac{2\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2\left(1+(N-1)\alpha^O\right)+2\sigma_\varepsilon^2}.$$

Substituting the expression above into (21), and operating, the expression for γ^O given in (9) is derived.

Finally, regarding the expression of μ^O given in (7), observe that dividing (16) by (19), we have

$$\frac{\bar{v}(1-a_2)-a_1\left(\beta^O(N-1)\bar{v}+N\mu^O\right)}{1-a_1N\gamma^O}=\frac{\mu^O}{\gamma^O}.$$

Taking into account that $var(\widetilde{v}|p, I_n^O)=\sigma_v^2(1-a_1\beta^O(N-1)-a_2)$ and Equation (21), (7) is obtained. ■

Proof of Lemma 2: From (22) and (2), we know that $\beta^O>0$, and hence, $\alpha^O<1$ and $\gamma^O>0$ because of (8) and (9). In addition, the inequality $\alpha^O>1/(N-1)$ follows from (20) and the positive sign of γ^O and β^O.

The exact value of α^O is easily obtained from (24) and (8), which provides an equation whose unique unknown is α^O. In particular, we have

$$\frac{\left((N-1)\,\alpha^O-1\right)\left(\left(\alpha^O-1\right)^2(N-1)\sigma_y^2+\sigma_z^2\right)}{2N-1-(N-1)\,\alpha^O}=\frac{(N-1)\left(1-\alpha^O\right)^2}{\rho^2\sigma_\varepsilon^2}.$$

Equation above shows that α^O is a solution of the polynomial stated in the statement of this lemma. Finally, notice that this polynomial can be rewritten as

$$(N-1)\,(1+\phi)\,(\alpha-1)^2\left(\alpha-\frac{2N-1+\phi}{(N-1)\,(1+\phi)}\right)+\varphi\left(\alpha-\frac{1}{N-1}\right),$$

which implies that $\alpha^O<\frac{2N-1+\phi}{(N-1)(1+\phi)}$. ■

Proof of Proposition 3: By virtue of Lemmas 1 and 2, the study of the existence of a *SLE* is reduced to the analysis of the roots of $Q(\alpha)$ that belong to $\left(\frac{1}{N-1},\frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$. Evaluating this polynomial in the extremes of the interval,[19] we have

$$Q(\frac{1}{N-1})=-2\frac{(N-2)^2}{N-1}<0 \text{ and } Q\left(\frac{2N-1+\phi}{(N-1)\,(1+\phi)}\right)=\frac{2\varphi}{1+\phi}>0.$$

[19] Rearranging (19), we have

$$\frac{N-2}{N-1}=\gamma^O\left(\rho var(\widetilde{v}|p,s_n,y_n)+a_1N\right),$$

which implies that in a *SLE* it is required that $N>2$, because of the positiveness of γ^O and a_1.

Next, we distinguish two cases:

Case 1. $\phi(N-2)-N>0$. In this case $Q(\alpha)$ is strictly concave in $\left(\frac{1}{N-1},\frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$, which tells us that $Q'(\alpha)$ is strictly decreasing. Since $Q'(\frac{2N-1+\phi}{(N-1)(1+\phi)})>0$, we obtain that $Q(\alpha)$ is strictly increasing in $\left(\frac{1}{N-1},\frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$. Hence, we can ensure that this polynomial has a unique zero in this interval.

Case 2. $\phi(N-2)-N\leq 0$. Notice that $Q'(\alpha)$ can be rewritten as

$$Q'(\alpha)=3(N-1)(1+\phi)(\alpha-1)\left(\alpha-\frac{-3+5N+\phi+\phi N}{3(N-1)(1+\phi)}\right)+\varphi.$$

From this expression, it follows that $Q'(\alpha^O)>0$ since in this case $\frac{-3+5N+\phi+\phi N}{3(N-1)(1+\phi)}\geq 1$. Finally, the property that the root of $Q(\alpha)$ in $\left(\frac{1}{N-1},\frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$ satisfies that $Q'(\alpha)>0$ guarantees the uniqueness of such a root.

Finally, since this polynomial has a unique zero in $\left(\frac{1}{N-1},\frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$, we can ensure that there exists a unique *SLE*. ■

Proof of Lemma 4: Recall that in the transparent market, $h_n^T=\beta^T\sum_{j\neq n}s_j+\left(\alpha^T-1\right)\sum_{j\neq n}y_j$, and $I_n^T=(s_n,z,y_n)$. Moreover, $E(\widetilde{v}|p,I_n^T)=E(\widetilde{v}|h_n^T,s_n)$ and $var(\widetilde{v}|p,I_n^T)=var(\widetilde{v}|h_n^T,s_n)$. Computations similar to the ones employed to show Lemma 1 give:

$$E(\widetilde{v}|p,I_n^T)=\bar{v}+b_2(s_n-\bar{v})+$$

$$b_1\left(N\gamma^Tp-\beta^Ts_n+\left(1-\alpha^T\right)y_n-\left(N\xi^T+1\right)z-N\mu^T-b_1\beta^T(N-1)\right),$$

and

$$var(\widetilde{v}|p,I_n^T)=b_2\sigma_\varepsilon^2,$$

where

$$\begin{aligned}
b_1 &= \frac{\beta^T\sigma_\varepsilon^2\sigma_v^2(N-1)}{D^T},\text{ and}\\
b_2 &= \frac{\sigma_v^2(N-1)\left(\left(\beta^T\right)^2\sigma_\varepsilon^2+(\alpha^T-1)^2\sigma_y^2\right)}{D^T},
\end{aligned}$$

with

$$D^T=(N-1)\left(\left(\beta^T\right)^2\sigma_\varepsilon^2\left(N\sigma_v^2+\sigma_\varepsilon^2\right)+\left(\alpha^T-1\right)^2\sigma_y^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\right).$$

Plugging $E(\widetilde{v}|p,I_n^T)$ into (1), and equating coefficients according to (10), it follows that the coefficients of speculators' demands corresponding to a *SLE* are the solutions to the following system of equations:

$$\begin{aligned}
\mu^T &= \frac{\bar{v}(1 - b_1\beta^T(N-1) - b_2) - b_1 N\mu^T}{\frac{1}{(N-1)\gamma^T} + \rho var(\widetilde{v}|p, I_n^T)}, \\
\beta^T &= \frac{b_2 - b_1\beta^T}{\frac{1}{(N-1)\gamma^T} + \rho var(\widetilde{v}|p, I_n^T)}, \\
\alpha^T &= \frac{b_1(1-\alpha^T) + \frac{1}{(N-1)\gamma^T}}{\frac{1}{(N-1)\gamma^T} + \rho var(\widetilde{v}|p, I_n^T)}, \\
\gamma^T &= \frac{1 - b_1 N\gamma^T}{\frac{1}{(N-1)\gamma^T} + \rho var(\widetilde{v}|p, I_n^T)}, \text{ and} \\
\xi^T &= \frac{-b_1\left(N\xi^T + 1\right)}{\frac{1}{(N-1)\gamma^T} + \rho var(\widetilde{v}|p, I_n^T)}.
\end{aligned}$$

Performing similar computations as in the proof of Lemma 1, we derive the expressions of β^T, μ^T and γ^T given in this lemma. We hence omit this part of the proof. The expression of ξ^T follows from dividing the last two equations in the system above and operating which gives $\xi^T = -b_1\gamma^T$, which is equivalent to (14) when plugging the values of β^T and γ^T in the expression of b_1. ■

Proof of Proposition 5: For an opaque market, we showed in the proof of Lemma 2 that α^O satisfies

$$\frac{\left((N-1)\alpha^O - 1\right)\left(\left(\alpha^O - 1\right)^2 (N-1)\sigma_y^2 + \sigma_z^2\right)}{2N - 1 - (N-1)\alpha^O} = \frac{(N-1)\left(1-\alpha^O\right)^2}{\rho^2\sigma_\varepsilon^2}.$$

When the market is transparent a similar equation must hold with $\sigma_z^2 = 0$. In particular, we obtain

$$\frac{\left((N-1)\alpha^T - 1\right)\left(1-\alpha^T\right)^2 (N-1)\sigma_y^2}{2N - 1 - (N-1)\alpha^T} = \frac{(N-1)\left(1-\alpha^T\right)^2}{\rho^2\sigma_\varepsilon^2}.$$

Solving this equation, the expression for the coefficient α^T given in the statement of this lemma is derived. Note that $\alpha^T = 1$ also solves the equation. Nevertheless, this solution is ruled out as $\alpha^T = 1$ implies $\gamma^T = 0$, which prevents existence of an equilibrium (note that equation (3) only has sense for $\gamma^T \neq 0$). The condition $N < (N-2)\phi$ follows from imposing that the value of α^T ($\alpha^T = \frac{2N-1+\phi}{(N-1)(\phi+1)}$) be smaller than one. Substituting this value in the expressions of the other parameters, we can explicitly characterize the unique *SLE* in this market structure. ■

Proof of Corollary 6: The proof directly follows from the fact that $N < (N-2)\phi$ requires that $N > 2$. Since N is a natural number, this implies that $N \geq 3$. ■

Proof of Corollary 7: a) Lemma 2 states that $\alpha^O \in \left(\frac{1}{N-1}, \frac{2N-1+\phi}{(N-1)(1+\phi)}\right)$, which implies that $\alpha^O < \alpha^T$. The Expressions (9) and (13) tell us that the coefficient associated with the price of the risky asset (as a function of the coefficient corresponding to the investor's endowment) coincides in both market structures. Moreover, it is easy to see that γ^M is strictly decreasing in α^M. Hence, $\gamma^O > \gamma^T$ as $\alpha^O < \alpha^T$. In addition, $\gamma_I^M < \gamma_P^M$ directly follows from Expressions (9) and (13).

b) Recall that the portfolio hedging component of the individual net demand is given by $\left(\alpha^M - 1\right)\widetilde{y}_i$. The expected volume of portfolio hedging is given by $E(\left|\left(\alpha^M - 1\right)\sum_{j=1}^{N}\widetilde{y}_j\right|)$, which can be written as $\left(1-\alpha^M\right)E(\left|\sum_{j=1}^{N}\widetilde{y}_j\right|)$. Therefore, the result directly follows from $\alpha^O < \alpha^T$. ■

Proof of Corollary 8: According to (15) and the market clearing condition, it follows that the equilibrium trading volume for informed trader n is $\beta^M\left(s_n - \frac{\sum_{j=1}^{N}s_j}{N}\right) + \left(\alpha^M - 1\right)\left(y_n - \frac{\sum_{j=1}^{N}y_j}{N}\right) - \frac{z}{N}$. Using $\alpha_P^M = \alpha_I^M$ and $\beta_P^M = \beta_I^M$ in the previous expression, the result directly follows. ■

Proof of Proposition 9: Market depth is defined as $\left(\frac{\partial \tilde{p}^M}{\partial \tilde{z}}\right)^{-1}$. By virtue of (6) and (10), the market clearing condition implies that

$$\tilde{p}^O = \frac{1}{N\gamma^O}\left(N\mu^O + \beta^O\sum_{j=1}^{N}\tilde{s}_j + \left(\alpha^O - 1\right)\sum_{j=1}^{N}\tilde{y}_j + \tilde{z}\right) \tag{25}$$

and

$$\tilde{p}^T = \frac{1}{N\gamma^T}\left(N\mu^T + \beta^T\sum_{j=1}^{N}\tilde{s}_j + \left(\alpha^T - 1\right)\sum_{j=1}^{N}\tilde{y}_j + \left(N\xi^T + 1\right)\tilde{z}\right). \tag{26}$$

Hence,

$$\begin{aligned}\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} &= N\gamma^O, \text{ and}\\ \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1} &= \frac{N\gamma^T}{N\xi^T + 1}.\end{aligned}$$

Substituting coefficients above by their equilibrium values, it follows that $\left(\frac{\partial \tilde{p}^O}{\partial \tilde{z}}\right)^{-1} - \left(\frac{\partial \tilde{p}^T}{\partial \tilde{z}}\right)^{-1}$ is a strictly decreasing in α^O function. Moreover, when $\alpha^O = \alpha^T$, its value is negative and when $\alpha^O = \frac{1}{N-1}$ its value is

$$\frac{N}{\rho\sigma_\varepsilon^2\sigma_v^2}\left(\frac{N\sigma_\varepsilon^2 + N^2\sigma_v^2 - \phi\left(\sigma_v^2\left(2 - 4N + N^2\right) - N\sigma_\varepsilon^2\right)}{\left(1+\phi\right)\left(N-1\right)\phi}\right).$$

Therefore, if $\sigma_\varepsilon^2 N(1+\phi)+\sigma_v^2\left(N^2-\phi\left(2-4N+N^2\right)\right)<0$, then we know that $\left(\frac{\partial p^O}{\partial z}\right)^{-1}<\left(\frac{\partial p^T}{\partial z}\right)^{-1}$, which means that the transparent market is deeper. Otherwise, this result follows when σ_z^2 is low enough as, in this case, α^O is close enough to α^T. ■

Proof of Proposition 11: a) Using the normality assumption, we have that

$$var^{-1}\left(\widetilde{v}|p^M\right)=\left(\sigma_v^2-\frac{cov^2(\tilde{v},\tilde{p}^M)}{var(\tilde{p}^M)}\right)^{-1}, \text{ for } M=O,T.$$

From (8), (12), (25) and (26), we have

$$var^{-1}\left(\widetilde{v}|p^O\right)=\frac{1}{\sigma_v^2\sigma_\varepsilon^2}\left(\sigma_\varepsilon^2+\frac{N\sigma_v^2}{\phi+1+\frac{1}{N\left(1-\alpha^O\right)^2}\varphi}\right)$$

and

$$var^{-1}(\widetilde{v}|p^T)=\frac{1}{\sigma_v^2\sigma_\varepsilon^2}\left(\sigma_\varepsilon^2+\frac{N\sigma_v^2}{\phi+1+\frac{\left(N\xi^T+1\right)^2}{N\left(1-\alpha^T\right)^2}\varphi}\right).$$

Notice that $var^{-1}\left(\widetilde{v}|p^O\right)<var^{-1}(\widetilde{v}|p^T)$ is equivalent to $\frac{1}{1-\alpha^O}>\frac{N\xi^T+1}{1-\alpha^T}\cdot$ Substituting α^T and ξ^T by their values, this inequality simplifies to

$$\alpha^O>C, \text{ with } C=\frac{\phi\left(\phi+4N-1-N^2\right)+N^2}{(N-1)\left(\phi+1\right)\phi}.$$

It is easy to see that $C<\frac{1}{N-1}$ iff $\phi\left(2+N^2-4N\right)-N^2>0$. As $\alpha^O>\frac{1}{N-1}$, then $\alpha^O>C$ holds in this case, and consequently, prices are more informative in the transparent market. Finally, this result is also satisfied provided that α^O is large enough, which is equivalent to saying that σ_z^2 is low enough. Just note that $C<\alpha^T$ and σ_z^2 low enough guarantees that α^O gets close to α^T.

b) The proof of this part is omitted since it is similar to the previous one. ■

Proof of Corollary 12: This result directly follows from the fact that all the conditions stated in Proposition 11 are independent of σ_v^2. ■

Proof of Corollary 13: When N converges to infinity, we have that both α^T and α^O converge to $\frac{2}{\phi+1}$. This implies $g(\alpha^O,\sigma_v^2)=g(\alpha^T,\sigma_v^2)$, and, consequently,

$$sign\left(var(\widetilde{v}-\widetilde{p}^O)-var(\widetilde{v}-\widetilde{p}^T)\right)=-sign\left(\left(\frac{\partial\tilde{p}^O}{\partial\tilde{z}}\right)^{-1}-\left(\frac{\partial\tilde{p}^T}{\partial\tilde{z}}\right)^{-1}\right). \blacksquare$$

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