UNIVERSITAT ROVIRA I VIRGILI

WORKING PAPERS

Col·lecció "DOCUMENTS DE TREBALL DEL DEPARTAMENT D'ECONOMIA"

"Market Competition and Lower Tier Incentives"

Bernd Theilen

Document de treball nº -1- 2007

DEPARTAMENT D'ECONOMIA
Facultat de Ciències Econòmiques i Empresarials

Edita:

Departament d'Economia
http://www.fcee.urv.es/departaments/economia/public_html/index.html
Universitat Rovira i Virgili
Facultat de Ciències Econòmiques i Empresarials
Avgda. de la Universitat, 1
432004 Reus
Tel. +34 977 759 811
Fax +34 977 300 661

Dirigir comentaris al Departament d'Economia.

Dipòsit Legal: T-550- 2007
ISSN 1988 - 0812

DEPARTAMENT D'ECONOMIA
Facultat de Ciències Econòmiques i Empresarials

Market Competition and Lower Tier Incentives

Bernd Theilen*

Department of Economics, Universitat Rovira i Virgili, Avinguda de la Universitat 1, E-43204 Reus, Spain†

March 2007

Abstract

The relationship between competition and performance–related pay has been analyzed in single–principal–single–agent models. While this approach yields good predictions for managerial pay schemes, the predictions fail to apply for employees at lower tiers of a firm's hierarchy. In this paper, a principal–multi–agent model of incentive pay is developed which makes it possible to analyze the effect of changes in the competitiveness of markets on lower tier incentive payment schemes. The results explain why the payment schemes of agents located at low and mid tiers are less sensitive to changes in competition when aggregated firm data is used.

Journal of Economic Literature classification numbers: D82, J21, L13, L22.
Keywords: Cournot Competition, Contract Delegation, Moral Hazard, Entry, Market Size, Wage Cost.

*I would like to thank Carolina Manzano, Antonio Quesada and Cori Vilella for their valuable comments and suggestions. Financial support from the Spanish "Ministerio de Educación y Ciencia" under project SEJ2004-07477 and the "Departament d'Universitats, Recerca i Societat de la Informació de la Generalitat de Catalunya" under project 2005SGR 00949 is gratefully acknowledged.

†Tel.: +34-977-759-850. *E-mail address*: bt@urv.net.

1 Introduction

During the last two decades the relationship between competition and performance–related–pay (PRP) has been investigated both from the theoretical and the empirical perspective. However, most of the literature has focused on the incentive pay of CEOs and little attention has been paid to lower layer employees. An exception in the empirical literature is Cuñat and Guadalupe (2005), who also analyze the effect of changes in competition on worker compensation. However, in the theoretical literature, attention has been paid exclusively to the question of how the design of adequate *individual* incentives is affected by changes in the competitive environment. This has given valuable insights into the relation between a firm's CEO and its owners. However, for low-tier employees the individual–based approach is expected to be too narrow and to give only incomplete insights. Therefore, the purpose of this paper is to develop a simple model which yields testable predictions of how changes in competition affect incentive pay at the lower layers of a firm's hierarchy.

Most of the results in the empirical literature on executive compensation are summarized in Murphy (1999). The main results are that (1) incentive compensation increases with firm size, though the relation between CEO pay and firm size has weakened over time, (2) pay-performance sensitivities vary across industries, and are particularly low in regulated industries, (3) pay-performance sensitivities have become larger in recent years because of the trend towards more competition due to the spread of information technologies, the reduction of barriers to entry, waves of deregulation and the reduction in transport costs. The theoretical literature mainly supports these findings. Schmidt (1997), Raith (2003) and Vives (2004) analyze a two–stage game in which firms consisting of a principal and an agent compete in an imperfect market. The principal uses incentive payments to incite agents to choose an unobservable effort level to reduce production costs. They show that there are two counteracting effects on incentive provision when competition increases. On the one hand, there is a business steeling effect which induces firms to provide higher powered incentive schemes because with more competition any cost advantage more easily attracts business from rival firms. On the other hand, there is a scale effect which induces firms to reduce incentives because with more competition market shares decrease and firms have less to gain from possible cost advantages. Which of the two effects dominates is generally ambiguous, though Vives (2004) finds that for most of the different competitive specifications used in the literature the first effect dominates the second one. Therefore, the theoretical prediction is that more competition increases the steepness of performance-related pay.

Cuñat and Guadalupe (2005) analyze the effect of increasing competition not only on CEO pay but also on the compensation schemes of executives and workers in the UK in the late nineties. They confirm the results for CEO's and find that more competition increases the steepness of performance–related–pay in all layers. However, the effect weakens and is mostly not significant for lower layers in the firm. With the theoretical results concerning the relationship between competition and PRP at hand, this is a surprising result for two reasons. First, Burgess and Metclaf (2000) show, also for a large sample of UK firms, that the adoption of

PRP is greater, especially for low-tier employees, in firms in environments with a low degree of competition than in those operating under a high degree of competition. For example, the number of firms that apply PRP for occupations like sales, skilled manual clerical/secretary and unskilled manual multiplies by eight, five, four and a half and four, respectively, while it only multiplies by three for managers. Second, there is a vast amount of empirical evidence to suggest that PRP at lower layers yields considerable productivity gains.[1] More competition should be expected to increase the pressure to realize potential productivity gains and therefore accelerate the adoption of PRP schemes. Consequently, without further theoretical explanation the results of Cuñat and Guadalupe (2005) can be seen to support the wide-spread view that executive compensation is not intended to be incentive efficient but used by CEOs as a means of self–enrichment where changes in competition serve as a pretext to increase PRP. As a result, the PRP of CEOs increases more than proportionally compared to the PRP of low–tier employees.[2]

The theoretical models on PRP do not specify the kind of agent the principal contracts. Thus, one might expect that the predictions can be applied to employees at any firm layer.[3] However, there are two reasons why the results cannot be applied directly to lower layers of the firm. First, at lower layers team work is a pervasive feature (c.f. Ichniowski and Shaw, 2003). The reward for participating in teams is likely to be some form of group–based pay or joint performance evaluation which has different characteristics to the individual–based schemes we find for CEOs. One major difference is that group–based incentives can be undermined by free riders. Though the free–rider problem can be mitigated by peer pressure (Kandel and Lazear, 1992), decentralized decision making (Baker, 1992) and long-term relationships (Che and Yoo, 2001) it usually results in lower powered incentive schemes than those of individual–based PRP.[4] Therefore, changes in competition should also be expected to have different impacts on low-tier PRP schemes compared to those of CEOs. Especially the question of how changes in competition affect free–riding should be addressed here. Second, in single–principal single–agent models changes in competition have no employment effects but these effects might be considerable for agents at lower layers. For example, Griffith, Harrison and Macartney (2006) find that increased product market competition reduced unemployment in OECD countries in the 1980s and 1990s. Of course, changes in employment might also be expected to affect the wage setting process and thereby the structure of incentive schemes. Therefore, again, the results from single–principal–single–agent models should not be applied straightforwardly to incentive pay at lower layers.

The aim of this paper is to extend former models which analyze the relationship between performance–related–pay and market competition by explicitly including employment effects which changes in competition may cause. It thereby aims to close the gap between studies that

[1] An excellent overview on the results of case studies, intra-industry analyses and national cross-industry studies is Ichniowski and Shaw (2003).

[2] See also Bolton et al. (2006) who develop a model that provides a different persepective on this view.

[3] See, for example, Cuñat and Gaudalupe (2005, p.1060).

[4] See, for example, Jensen and Murphy (1990), Holmstrom and Milgrom (1991, 1994) and Che and Yoo (2001).

analyze the effect of competition on individual–based PRP like Schmidt (1997), Raith (2003) and Vives (2004) and those that analyze group–based PRP from an organizational perspective that excludes explicit modelling of changes in the product market (see Holmstrom and Milgrom (1990), Varian (1990), Ramakrishnan and Thakor (1991), Itoh (1991, 1992, 1993) and Macho-Stadler and Pérez-Castrillo (1993) as examples for this approach). In this paper, product market competition and incentive payments are the result of a multi–stage game in which firms are modeled as a three–tier hierarchy. At the top tier the principal contracts a supervisor and decides the number of employees of her firm. At the second tier a supervisor designs the contracts of third–tier workers. Contract design is complicated by the unobservability of subordinates' effort choices. That is, supervisors and principals in each firm must solve moral hazard problems.

The main difference between the group–based approach and the single–principal single–agent models is that firms can react to changes in competition not only by changing employees' effort but also by adapting the number of employees to new market conditions. However, if the labor inputs of third-tier employees are substitutable and if PRP is group–based this will affect the free–riding of third–tier employees. Therefore, any change in competition which affects firm size will affect third–tier and mid–tier employees differently. For mid-tier employees we assume that labor inputs are not substitutable and that free-riding therefore is a minor issue. Firm size (the number of employees) itself is determined by the costs of entry and labor market conditions. The paper shows that this gives rise to two kinds of heterogeneities. First, PRP sensitivities differ between employees at different hierarchical levels of the firm, or more generally, between employees whose labor inputs are substitutable and whose performance cannot be measured individually and those whose labor inputs are either difficult to substitute or whose performance can be measured individually. Second, PRP sensitivities of third–tier agents differ between firms which react to changes in competition by changing their employees' efforts and those which also change employment, or between firms that operate in closed markets and those that operate in markets with low barriers to entry, or between those which anticipate changes in wage levels due to changes in employment and those which do not. Taken together, the results explain the discrepancy between empirical studies that use aggregate firm data and those that are based on individual or firm data. The former find that incentive pay–performance sensitivities have significantly increased in recent years for CEO's while they have remained nearly unchanged at lower tiers. This is because the aggregation of heterogeneous firm data means that different responses to changes in competition at lower tiers compensate each other while they work in the same direction at mid and top tiers. Firm or case studies however, confirm that firms in many circumstances have substancially increased PRP at lower tiers after a change in the competitive environment. However, the handicap of these studies is that their findings cannot be generalized. Therefore, one of the main insights of the model is that future studies should take into account the kind of heterogeneities which have been identified when firm and individual data is aggregated.

The paper is organized as follows. Section 2 presents the model. Section 3 shows the market equilibrium values. Section 4 analyzes the effects of changes in the competitiveness of markets under alternative assumptions concerning the endogeneity of different variables. Finally, Section 5 concludes. All proofs are confined to the Appendix.

2 The model

1. The firm. There are n identical firms. Each firm i is represented as a three–tier hierarchy. The hierarchy consists of one principal (P_i) at the top of the hierarchy or at layer 1, one agent (A_{2i}) at layer 2 and a team of K_i agents (A_{3ik}, $k = 1, ..., K_i$) at layer 3. The agents in each firm jointly produce a marketable output q_i for the principal. The principal makes all entry, personnel (she chooses the value of K_i), and production decisions. The mid-tier agent, also called supervisor, controls the agents at tier 3, also called workers.

2. Production. The agents contribute to production by choosing a certain level of effort which is unobservable by other agents and the principal. Denote agent A_{2i}'s effort choice by $e_{2i} \in \Re^+$ and agent A_{3ik}'s effort choice by $e_{3ik} \in \Re^+$. The general production technology is linear homogenous in capital and total labor output. The units of capital employed in production are normalized to unity. The total worker's labor output is $\sum_{k=1}^{K_i} e_{3ik}$, that is, a worker's effort is perfectly substitutable, which implies that we assume that workers perform essentially the same tasks without any synergy effects. Supervisors produce "supervision", "control" and "coordination" of the workers' tasks. We assume that total labor output increases with more coordination for any given level of workers' labor output. Concretely, we assume the following production function:

$$q_i(K_i, e_{2i}, e_{3i}) = K_i^{\alpha_1} e_{2i}^{\alpha_2} \left(\sum_{k=1}^{K_i} e_{3ik} \right)^{\alpha_3},$$

where $e_{3i} = e_{3i1}, ...e_{3ik}, ...e_{3iK_i}$, $\alpha_1 > 0$, $\alpha_2 \geq 0$, and $\alpha_3 \geq 0$.[5] Notice that production still increases in K_i when $\alpha_3 = 0$. This guarantees that a firm's output increases with employment, though the unobservable effort of third–tier employees does not increase employment. We can interpret K_i as the part of third–tier agents' labor input that is observable and contractible. This implies that the principal, apart from unobservable effort, contracts one unit of labor from each third-tier agent. To simplify the exposition of the results we make:

Definition 1 $\delta = \alpha_1 + \frac{\alpha_3}{2}$, $\varepsilon = \frac{\alpha_2+\alpha_3}{2}$ *and* $A = \left[2^{-\alpha_3} \alpha_2^{\alpha_2} \alpha_3^{2\alpha_3}\right]^{\frac{1}{2-\alpha_2-\alpha_3}}$.

Finally, the production function displays non-increasing returns to scale, and third-tier labor is more important for production than second–tier labor:

[5] Similar team production functions have been applied by Beckmann (1977), or more recently, by Ferrall and Shearer (1999).

Assumption 1 $\delta \leq 1$, $\varepsilon < 1$, and $\delta - \varepsilon > 0$.

3. Utility and contracts. The fact that individual effort is non-observable to the principal, implies that she faces a moral hazard problem.[6] Furthermore, because individual performance measures are unavailable, this problem can only be solved by designing an appropriate group–incentive scheme. The design of this incentive scheme is delegated by the principal to the supervisor at tier two. We assume that the scheme, which the supervisor offers agent A_{3ik} at tier 3, has the form of a linear transfer $t_{3ik} = \beta_{3i}q_i + \gamma_{3i}$. That is, the contract includes a fixed payment and a variable payment that depends on output.[7] The transfer t_{3i} is chosen by A_{2i} under the restriction that $\beta_{3i} \geq 0$ (penalties are not allowed) and $\gamma_{3i} \geq \gamma_3$. γ_3 represents the industry wage level which is determined by the bargaining power of workers and principals and includes compensation for observable effort. Furthermore, notice that contracts are identical. Because agents are identical this can be interpreted as a non-discrimination restriction imposed by workers.[8] Agent A_{3ik} chooses his effort to maximize his utility $U_{3ik} = t_{3i} - \frac{1}{2}e_{3ik}^2$. Agent A_{2i} is paid by the principal with transfer $t_{2i} = \beta_{2i}q_i + \gamma_{2i}$. His utility is $U_{2i} = t_{2i} - K_i t_{3i} - \frac{K_i}{2}e_{2i}^2 = \widetilde{\beta}_{2i}q_i + \widetilde{\gamma}_{2i} - \frac{K_i}{2}e_{2i}^2$ where $\widetilde{\beta}_{2i} = \beta_{2i} - K_i\beta_{3i}$ and $\widetilde{\gamma}_{2i} = \gamma_{2i} - K_i\gamma_{3i}$ are the net incentive rate and the net fixed wage, respectively.[9] Notice that the supervisor's effort cost increases with the number of subordinates and the effort level chosen. Furthermore, marginal effort increases with the number of subordinates taking into account the fact that coordination and control of tasks become more difficult as the number of subordinates grows. We assume that the principal faces the restrictions $\widetilde{\beta}_{2i} \geq 0$ and $\widetilde{\gamma}_{2i} \geq 0$ when she designs A_{2i}'s contract. The principal's objective is the maximization of the firm's net profits $\pi_i = pq_i - t_{2i} - F$, where p is the market price and F is fixed costs or entry costs.

4. Competition and demand. Competition is for a homogeneous good with inverse demand function $p = a - bq$, $a > 0$, $b > 0$, where q is aggregate output. n identical firms enter the market. For simplicity we assume that n is a continuous variable. With free entry firms enter until profits are zero.

5. Timing. Competition and contracting are defined to be a multi-stage game in which the

[6]Notice that unlike single–principal–single–agent models, we have not included a random term. However, with team production the moral hazard problem stems from the fact that the principal cannot identify individual effort from the observation of output though production is non–random (cf. Espinosa and Macho–Stadler, 2003).

[7]Lazear (2000) shows that this is a realistic assumption concerning the payment schemes applied for low–tier employees in enterprises in industrial sectors. McAfee and McMillan (1991) analyze the conditions under which it is optimal to use linear team contracts.

[8]In fact, β_{3i} is also determined by a bargaining process. However, this process mostly takes place inside the firm. Thus, while the fixed wage is assumed to be the same throughout the industry, performance-related-pay can differ between firms but not between employees of the same firm.

[9]A major difference in models that analyze managerial incentives is that agents are assumed to be risk–neutral. Consequently, there is no trade–off between risk and incentives like in Prendergast (2000) or Raith (2003). However, due to the assumption of limited liability and the fact that the fixed wage is determined exogenously, employees at lower tiers do not support much risk. Therefore, here, incentive pay is basically used to stimulate production and the trade–off is between productivity gains and wage costs.

sequence of events is as follows: At stage 1 the principals of all firms simultaneously decide the size of their firm (K_i) and the remuneration (t_{2i}) of their supervisors. At stage 2, the supervisors determine the contracts with their subordinates (t_{3i}) and choose their effort (e_{2i}). At stage 3 agents A_{3ik} observe their supervisor's effort and, simultaneously, make their own effort choice (e_{3ik}). Then, firms' output and market price are realized, the agents are paid and the principals obtain their profits.

The competitiveness of markets in the model is affected by four parameters: the intercept a and the slope b of the inverse demand, the cost of entry F, and the fixed wage level γ_3. If we assume that consumers are homogeneous, an increase in b can be interpreted as an increase in market size (in the number of consumers). An increase in a indicates an increase in the consumers' valorization of the good. A decrease in a can also be interpreted as the emergence of closer substitutes. We can argue that an increase in market size (lower b) and a decrease in the substitutability of the good (higher a) indicate more competition when the equilibrium number of firms increases and the equilibrium price decreases (cf. Raith, 2003). Finally, following the same argument, a reduction in the cost of entry and a reduction in the fixed wages can also be interpreted as an indicator of more competition.

3 Third–tier contracts

The market game is solved by backward induction. At *stage 3* of the game, when agents A_{3ik} simultaneously choose their efforts, they know the effort choice of their supervisors and the number of workers in their firm. Thus, optimal effort is given as the solution to

$$\max_{e_{3ik}} U_{3ik} = \beta_{3i} q_i(K_i, e_{2i}, e_{3i}) + \gamma_{3i} - \frac{1}{2} e_{3ik}^2. \tag{1}$$

Without further proof we obtain the following stage 3 equilibrium values:

Proposition 1 *For a given incentive rate β_{3i}, given supervisor effort e_{2i} and given firm size K_i in every subgame perfect Nash equilibrium agent A_{3ik} in each firm chooses effort*

$$e_{3ik} = (\alpha_3 \beta_{3i} e_{2i}^{\alpha_2})^{\frac{1}{2-\alpha_3}} K_i^{\frac{\alpha_1+\alpha_3-1}{2-\alpha_3}}. \tag{2}$$

Firm i's third stage equilibrium output is

$$q_i(K_i, e_{2i}, \beta_{3i}) = (\alpha_3 \beta_{3i})^{\frac{\alpha_3}{2-\alpha_3}} K_i^{\frac{2\alpha_1+\alpha_3}{2-\alpha_3}} e_{2i}^{\frac{2\alpha_2}{2-\alpha_3}}. \tag{3}$$

Agent U_{3ik}'s utility is given by

$$U_{3ik} = \left(1 - \frac{\alpha_3}{2K_i}\right) \beta_{3i} q_i + \gamma_{3i}. \tag{4}$$

Equations (2) and (3) imply that A_{3ik}'s effort increases as the incentive rate increases which in turn increases firm i's output. From (4) we see that a worker's utility is strictly positive. This is the consequence of the limited liability assumption which implies that agents receive an informational rent. Furthermore, a worker's utility increases with the incentive rate of his contract. Thus, ceteris paribus, agents at tier 3 prefer higher powered incentives. Notice that workers cannot observe the effort choice of other workers in the firm. Thus, when deciding their own effort choice they face a free–rider problem. More effort yields higher output and therefore higher wages. However, while a worker only receives part of his own effort contribution through a wage increase, his wage also increases when other workers work more without any cost to him. If workers could cooperatively implement their effort choices within a coalition, the effort choice for each individual would be $K_i^{\frac{1}{2-\alpha_3}}$ higher than the effort choice in (2).[10] Therefore, as expected, the efficiency loss due to free–riding increases with firm size.

At *stage 2*, supervisor A_{2i} determines the wage contract of the third–tier agents he supervises and chooses his own effort. The fact that supervisors determine the contracts of their subordinates is equivalent to assuming that workers are paid by the principal and that these payments are authorized by the supervisor who informs the principal.[11] Formally, agent A_{2i} solves the following program $[P_2]$:

$$\max_{\beta_{3i},\gamma_{3i},e_{2i}} U_{2i} = (\beta_{2i} - K_i\beta_{3i})\, q_i(K_i, e_{2i}, \beta_{3i}) + \gamma_{2i} - K_i\gamma_{3i} - \frac{K_i}{2}e_{2i}^2 \quad (5)$$
$$\text{s.t. (2), } U_{3ik} \geq 0,\ \gamma_{3i} \geq \gamma_3, \text{ and } \beta_{3i} \geq 0.$$

Supervisors maximize their objective function subject to three type of constraints for each of their subordinates: Incentive constraints (2) due to the supervisor's moral hazard problems, participation constraints which guarantee each worker a minimum utility level (here normalized to zero) and limited liability constraints. Solving problem $[P_2]$ we obtain:

Proposition 2 *For a given incentive rate β_{2i} and given firm size K_i in every subgame perfect Nash equilibrium agent A_{2i} in each firm chooses an incentive rate of*

$$\beta_{3i} = \frac{\alpha_3}{2K_i}\beta_{2i}, \quad (6)$$

and fixed payments $\gamma_{3i} = \gamma_3$. Efforts are

$$e_{2i} = (\alpha_2 A)^{\frac{1}{2}} \left(\beta_{2i} K_i^{\delta-1}\right)^{\frac{1}{2(1-\varepsilon)}} \quad \text{and} \quad e_{3ik} = \left(\frac{\alpha_3^2}{2}A\right)^{\frac{1}{2}} \left(\beta_{2i} K_i^{\varepsilon+\delta-2}\right)^{\frac{1}{2(1-\varepsilon)}}. \quad (7)$$

Firm i's second stage equilibrium output is

$$q_i(K_i, \beta_{2i}) = A\left(\beta_{2i}^{\varepsilon} K_i^{\delta-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \quad (8)$$

[10] In this case the coaltion would solve $\max_{e_{3i}} \sum_{k=1}^{K_i} U_{3ik} = K_i\beta_{3i}q_i(K_i, e_{2i}, e_{3i}) + K_i\gamma_{3i} - \frac{1}{2}\sum_{k=1}^{K_i} e_{3ik}^2$.
[11] See Mookherjee and Reichelstein (2001) for this interpretation.

Agents' utilities are given by:

$$U_{2i} = (1-\varepsilon)\, A\beta_{2i}^{\frac{1}{1-\varepsilon}} K_i^{\frac{\delta-\varepsilon}{1-\varepsilon}} + \gamma_{2i} - K_i\gamma_3 \qquad \textit{and} \qquad U_{3ik} = \left(1 - \frac{\alpha_3}{2K_i}\right)\frac{\alpha_3}{2K_i} A\beta_{2i}^{\frac{1}{1-\varepsilon}} K_i^{\frac{\delta-\varepsilon}{1-\varepsilon}} + \gamma_3. \quad (9)$$

From Proposition 2 we find that a worker's wage contract has the following characteristics: (a) His incentive rate increases with the incentive rate of his supervisor. When the supervisor has higher incentives to increase output, he is also interested in making his subordinates work harder. Therefore, workers' incentive rates increase. (b) The incentive rate decreases with the number of third–tier employees. As we have seen before, with more workers in the firm the free-rider problem is aggravated. Therefore, as can be seen from (7) an increase in the number of employees means that each worker will work less in equilibrium and, consequently, workers' incentive rates decrease. However, from (7) we see that total worker effort ($\sum e_{3ik}$) increases with K_i which means that the reduction in individual effort is more than compensated by the increase in size. Furthermore, supervisors work less which is due to the fact that their disutility of effort increases to higher power in firm size than their marginal remuneration. Therefore, with comparable incentive rates supervisors in larger firms work less. (c) Finally, the incentive rate increases with α_3. When the unobservable effort of third tier agents becomes more important for production, their remuneration increases.

Another characteristic of A_{3ik}'s incentive scheme is that part of his rent is extracted by his supervisor. To see this *rent-extraction effect* consider the case that $\alpha_2 = 0$. So supervisors do not contribute to production. From (7) we see that their effort choice is indeed zero in this case. Therefore, it is not necessary to incite A_{2i} to choose an efficient second-best effort level. Thus, we would expect that $K_i\beta_{3i} = \beta_{2i}$. However, from equation (6) we see that $K_i\beta_{3i} < \beta_{2i}$ which reflects the fact that A_{2i} retains part of the incentive payment designated to his subordinates even when he does not contribute anything to production. This effect is well-known and in the literature is also called the *double marginalization of rents effect* (cf. McAfee and McMillan, 1995).[12]

4 Market Equilibrium

How firms react to changes in the competitiveness of markets depends on the time they have to react. In the short run, if a firm wants to increase production because market size has increased, for example, it may only want or be able to increase the production effort of its employees. However, in the mid term it might be more efficient to increase employment instead, or both. In the long run, an increase in production may increase industry profits and cause the entry of new firms in industries with low barriers to entry (cf. Raith, 2003). In this case, all decisions concerning employment must also take into account the reactions of potential entrants. Finally,

[12] However, Holmstrom (1982) has shown that there is also a positive role for a supervisor (or principal): administering incentive schemes that are not budget–balancing.

changes in market structure and employment may also affect the wage bargaining process which in turn affects incentive provision within firms. In the remainder of this section, the effect of changes in competition on incentive rates under these different settings is analyzed.

4.1 Given firm size and market structure

If firm size cannot be adjusted, at *stage 1*, the principal chooses the wage contract of her supervisor to maximize firm profit. Formally, she solves the following program $[P_1]$:

$$\max_{\beta_{2i},\gamma_{2i}} \pi_i = (a - bq)\, q_i - \beta_{2i} q_i - \gamma_{2i} - F \tag{10}$$
$$\text{s.t. } (6),\ (7),\ U_{2i} \geq 0,\ \gamma_{2i} \geq K_i \gamma_{3i}, \text{ and } \beta_{2i} \geq 0.$$

The principals' maximization problem is subject to incentive, participation and limited liability constraints. Now, the incentive constraints, (6) and (7), imply that the principal due to the unobservability of the supervisor's effort and the ex-post choice of third-tier incentive rates must take into account the supervisor's incentive problem $[P_2]$ when she herself decides on second–tier incentive rates and firm size. The solution of $[P_1]$ yields:

Proposition 3 *There exists a unique symmetric subgame perfect Nash equilibrium in which each firm chooses $\beta_{2i} = \beta_2$ defined by:*

$$a - b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}} - \frac{1}{\varepsilon}\beta_2 = 0. \tag{11}$$

Incentive rates are

$$\widetilde{\beta}_{2i} = \left(1 - \frac{\alpha_3}{2}\right)\beta_2 \quad \text{and} \quad \beta_{3i} = \frac{\alpha_3}{2K}\beta_2, \tag{12}$$

and fixed payments are $\gamma_{2i} = K\gamma_3$. Firm i's first stage output is given by (8), efforts are given by (7) and utilities are given by (9).

From proposition 3 we see that incentive rates depend on firm size, market size, the substitutability of products and the number of firms while they are independent from the fixed wage cost. For the effect of changes in these variables on incentive rates we have:

Corollary 4 *Incentive rates are higher in larger markets, markets with less substitutable products and more concentrated markets, and lower in larger firms. Incentive rates are independent of fixed wage costs.*

In larger markets, markets with less substitutable products and more concentrated markets, the market share of firms increases. Therefore, to produce more, it is necessary to incite agents to make more effort. Thus, incentive rates increase. These results are similar to those we can find in single–agent–single–principal models (cf. Raith, 2003 and Schmidt, 1997). Concerning changes in firm size, as we have seen above, in larger firms the free–rider problem is aggravated. So it becomes more costly for the principal to incite a given effort level and from (7) and (12) we find that equilibrium efforts and incentive rates decrease when firm size increases.

4.2 Endogenous firm size and given market structure

If changes in competition are expected to be durable, the principal might want to adopt firm size also. With endogenous firm size, at *stage 1* the principal solves the program $[P_1']$:

$$\max_{K_i,\beta_{2i},\gamma_{2i}} \pi_i = (a - bq)\, q_i - \beta_{2i} q_i - \gamma_{2i} - F \quad \text{s.t. (6), (7), } U_{2i} \geq 0,\ \gamma_{2i} \geq K_i \gamma_{3i}, \text{ and } \beta_{2i} \geq 0. \tag{13}$$

The only difference with respect to program P_1 is that now the principal also maximizes with respect to K_i. The solution of $[P_1']$ yields:

Proposition 5 *There exists a unique symmetric subgame perfect Nash equilibrium in which each firm chooses size* $K^* = K(a, b, \gamma_3, n)$*, where* K^* *is defined by*

$$a - b(1+n)\, A \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}} - \frac{1}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\delta}} = 0. \tag{14}$$

Incentive rates are

$$\widetilde{\beta}_{2i} = \left(1 - \frac{\alpha_3}{2}\right)\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\delta}} \quad \text{and} \quad \beta_{3i} = \frac{\alpha_3}{2}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{-\delta}} \tag{15}$$

and fixed payments are $\gamma_{2i} = K^*\gamma_3$*. Firm i's first stage output is*

$$q_i = A\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}}, \tag{16}$$

and firm i's profits are

$$\pi_i = \frac{1-\delta}{\delta-\varepsilon}\gamma_3 K^* + bA^2\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon} K^{*^{2\delta}} - F. \tag{17}$$

Agents' efforts are

$$e_{2i} = \left(\frac{\alpha_2\varepsilon\gamma_3}{(\delta-\varepsilon)}\right)^{\frac{1}{2}} \quad \text{and} \quad e_{3ik} = \left(\frac{\alpha_3^2\varepsilon\gamma_3}{2(\delta-\varepsilon)K^*}\right)^{\frac{1}{2}}, \tag{18}$$

and agents' utilities are

$$U_{2i} = \frac{(1-\varepsilon)\,\varepsilon}{(\delta-\varepsilon)}\gamma_3 K^* \quad \text{and} \quad U_{3ik} = \left(\left(1 - \frac{\alpha_3}{2K^*}\right)\frac{\alpha_3\varepsilon}{2(\delta-\varepsilon)} + 1\right)\gamma_3. \tag{19}$$

The main characteristic of the agent's incentive scheme is that it depends on two variables: firm size and the fixed wage level. Furthermore incentive rates depend particularly on the relative contributions of supervisors and workers to production. This is of practical importance when we compare the structure of incentive payments in different industries or when changes in competition lead to changes in task assignment and organizational structure. Interestingly, from (15) we also find that the incentive rates of supervisors are always greater than those of their subordinates, irrespectively of their contributions. The model therefore yields an alternative explanation for the fact that remuneration in firms increases with the hierachical level independently of considerations such as qualifications and task assignment.

To see how changes in different measures of competition affect the incentive rates of supervisors and workers, we start by analyzing the effect of changes in the competitiveness of markets on firm size.

Corollary 6 *Equilibrium firm size and production are higher in larger markets, markets with less substitutable products, more concentrated markets and markets with lower fixed wages.*

The intuition of this result is straightforward. In markets with less substitutable products, larger markets and more concentrated markets, firms have a larger market share. Therefore, employment and, as we see from (16), firm production are increased. A reduction in the fixed wage yields a reallocation of work. Firms employ more workers who exert less effort.

Together, Corollary 4 and 6 imply that changes in the degree of competition have counteracting effects on incentive rates when firm size is an endogenous variable. On the one hand, with an increase in a firm's market share the principal can increase production by increasing production effort, which requires higher powered incentive schemes. On the other hand, she can increase production by employing more agents, which aggravates free–riding and induces lower incentive rates for supervisors and workers. However, from (12) we see that the reduction in workers' incentive rates is higher than that in supervisors' incentive rates when K increases. Therefore, if a firm's market share increases after a change in the competitive environment the distance between supervisors' and workers' incentive rates will increase. We find that with a change in market size, the substitutability of products or market concentration, the first effect dominates the second for supervisors while for workers the first effect is dominated by the second. If the fixed wage cost is changed, we have seen that the first effect is null. Hence, the incentive rate of supervisors and workers decreases when the fixed wage cost decreases. The following result resumes these findings:

Corollary 7 *Supervisors' incentive rates are higher in larger markets, markets with less substitutable products, more concentrated markets and markets with higher fixed wages. Workers' incentive rates are lower in larger markets, markets with less substitutable products and more concentrated markets, and higher in markets with higher fixed wages.*

The major difference between this result and Corollary 4 is that endogenous firm size changes in the competitiveness of markets can have opposite effects on the incentive rates of supervisors and workers. For example, the entry of firms in the market reduces the market share of incumbent firms and therefore firm production. This reduction is achieved by reducing firm size. However, the reduction in firm size means that the supervisor's effort and incentive rates are reduced while workers' effort and incentive rates increase. This result therefore gives an intuitive explanation for the results of Cuñat and Guadalupe (2000). Changes in the competitiveness of markets affect CEO and executive pay–performance–sensitivities in the same direction in firms that respond to these changes with an increase in effort and those which also increase employment, while workers' pay-performance sensitivities are affected in opposite directions in these two kinds of firms. Therefore, when firms with different responses to changes in the competitive environment are aggregated, the effect on PRP sensitivities of workers are less significant than those of CEO's and executives.

4.3 Endogenous firm size and market structure

When market structure is endogenous, the number of firms in the market is determined by the cost of entry. This affects the forgoing analysis in two ways. On the one hand, a change in market size, the substitutability of products or the fixed wage cost will now also cause the entry or exit of firms. On the other hand, a variation in the costs of entry affects the variables of interest in the model only through changes in the number of firms whose implications we have already discussed. From equation (17), we see that profits increase with firm size, and from Corollary 6 we know that firm size decreases with n. Thus, in the long–run firms enter until profits are zero. Furthermore, in the long-run, changes in the competitiveness of markets affect the number of firms and firm size simultaneously. We get:

Proposition 8 *An increase in market size increases firm size and causes entry. An increase in the substitutability of products does not alter firm size and causes exit. An increase in entry costs increases firm size and causes exit. An increase in the fixed wage cost reduces firm size and causes exit (entry) if the number of firms in equilibrium is small (large).*

Proposition 8 is the result of two counteracting effects on firm size. The first effect, as we have seen in Corollary 6, is that in larger markets, markets with less substitutable products and markets with lower fixed wages, firm size will be larger. However, profits will also be larger and so the second effect is that firms will enter into the market, which decreases firm size. Proposition 8 indicates that if market size or the fixed wage cost changes the first effect dominates the second, while if the substitutability of products changes the two effects cancel each other out. As expected, we find that the equilibrium number of firms is larger in larger markets, markets with less substitutable products and markets with lower entry costs. Surprisingly, a reduction in the fixed wage cost may also cause exit in less concentrated markets. However, this stems from the fact that the gains from increasing the firm size of incumbent firms in unconcentrated markets (where firms are small and which causes exit) more than compensate for the gains for potential entrants. Therefore, the net effect is that firm size increases and the number of firms decreases when the fixed wage level decreases in concentrated markets.

To analyze the effect of changes in competition on incentive rates, notice from (15) that market size, the substitutability of products and entry cost affect incentive rates only indirectly through a change in firm size. A change in the fixed wage level has a direct substitution effect, which is positive for supervisors and workers, and an indirect effect through firm size, which is negative for supervisors and positive for workers. Therefore, the results in Proposition 8 mean:

Corollary 9 *Supervisors' and workers' incentive rates are independent of changes in the substitutability of products and increasing in fixed wage costs. Supervisors' incentive rates are increasing in market size and entry costs and workers' incentive rates are decreasing in market size and entry costs.*

As far as supervisor incentive rates are concerned, this result is very similar to the findings of Raith (2003). He also finds that incentive rates are higher in larger markets and lower in markets with lower entry costs. However, in markets with more substitutable products he finds that incentive rates increase (cf. Raith, 2003, proposition 5). The main interest in Corollary 9 is how changes in competition affect workers' PRP. Changes in market size and entry cost have just the opposite effect on workers' incentive rates compared to those of supervisors. This difference can be explained as follows. When market size increases firms want to increase production. In single–principal–single–agent models like those of Raith (2003) and Vives (2005) this increase in production can only be achieved by increasing the agent's effort, which aggravates the moral hazard problem and requires stronger incentives. Here production can also be increased by employing more agents and this is what happens when market structure is exogenous. However, when more agents are employed the free–rider problem becomes more severe. Therefore, performance–related–pay is reduced and workers make less effort. This implies that firms produce with higher unit costs per output. So, new entrants, which are smaller in size, will have a competitive advantage and the incumbents must reduce firm size to initial levels to be competitive. Thus, the free-rider effect, which is not present in single–principal–single–agent models, is the driving force behind the difference in the results we obtain. Again, as in the case for markets without entry, we can explain why Cuñat and Guadalupe (2005) cannot find significant effects of changes in competition on the incentives rates of workers (for whom there is a free–rider problem) while they find a positive and significant effect on those of CEO's (without a free–rider problem) in the same firms.

4.4 Endogenous firm size, market structure and wage costs

As we have seen above, changes in the competitiveness of markets have employment effects. For example, if industry employment increases, unemployment in the industry is reduced and the workers' bargaining power increases which means that if these changes are permanent the fixed wage cost γ_3 increases. Indeed, Cuñat and Guadalupe (2005) report that in the UK an increase in competition leads to an increase in the fixed component of workers' pay. As far as the effects of changes in market size and entry costs on employment are concerned we find:

Corollary 10 *Employment is larger in markets with less substitutable products and in larger markets, and decreases (increases) with entry costs when there are few (many) firms in the market.*

An increase in market size increases profits which induces entry and increases firm size. A decrease in the substitutability of products increases the number of firms while firm size does not change. Therefore, in both cases industry employment is increased. Furthermore, a decrease in the entry cost F reduces firm size and leads to the entry of new firms. The total effect on employment depends on which of these two factors dominates. Corollary 10 indicates that employment increases if the entry cost decreases and if there are few firms in the market. Otherwise, employment decreases.

The fact that changes in market size and entry cost can affect industry employment means that in the long–run the wage level γ_3 in the industry may change. To get an idea of how these changes affect incentive rates, consider the following example:

Example 1 *Let $\delta = 1$ and $\varepsilon = \frac{1}{2}$. Then we get from equation (14) and from setting the right-hand side in (17) to zero:*

$$K^* = \sqrt{\frac{F}{bA\gamma_3}} \qquad \textit{and } n^* = \frac{a - 2\sqrt{\frac{\gamma_3}{A}}}{\sqrt{bF}} - 1. \tag{20}$$

Thus, the incentive rates in (15) become:

$$\widetilde{\beta}_{2i} = \left(1 - \frac{\alpha_3}{2}\right)\sqrt{\frac{\gamma_3}{A}} \qquad \textit{and} \qquad \beta_{3i} = \frac{1}{2}\alpha_3\gamma_3\sqrt{\frac{b}{F}}. \tag{21}$$

Workers' incentive rates decrease with the cost of entry and decrease with market size. Supervisors' incentive rates are independent of changes in market size and the entry cost.

From equation (21) we see that changes in competition may have further effects on incentive rates if the fixed wage level itself is endogenous. To analyze this, assume that wages in the *long run are determined by a bargaining process between policy makers who want to maximize employment*

$$n^* K^* = \frac{1}{b} \left(\frac{\left(a - \sqrt{bF}\right)}{\sqrt{A}} \gamma_3^{-\frac{1}{2}} - \frac{2}{A} \right) \tag{22}$$

and unions who try to maximize affiliated workers' rents.[13] *Consider a bargaining process between the two parties in which the interval between offers and counteroffers shrinks to zero. Then, the bargaining process has a unique limiting subgame perfect equilibrium, which approximates the Nash bargaining solution (cf. Binmore et al., 1986). Furthermore, if both parties have the same bargaining power, the wage level is given by:*

$$\gamma_3 \in \arg\max \left[n^* K^* + \frac{2}{bA} \right]^{1/2} \cdot \left[U_{3ik} \right]^{1/2}. \tag{23}$$

The solution of (23) is:

$$\gamma_3 = \frac{(\alpha_3 + 2)^2 F}{\alpha_3^4 b A}. \tag{24}$$

So firm size is given by:

$$K^* = \frac{\alpha_3^2}{\alpha_3 + 2}, \tag{25}$$

and incentive rates are:

$$\widetilde{\beta}_{2i} = \frac{(2 - \alpha_3)(\alpha_3 + 2)}{2\alpha_3^2 A} \sqrt{\frac{F}{b}} \quad \text{and} \quad \beta_{3i} = \frac{(\alpha_3 + 2)^2}{2\alpha_3^3 A} \sqrt{\frac{F}{b}}. \tag{26}$$

A comparison of equations (21) and (26) shows that, incentive rates now increase in the entry cost and in market size. Thus, in case of workers' incentive rates we get the opposite effect to the case where the wage cost was exogenous.

The reason for this result is that an increase in market size has two opposite effects. On the one hand, firm size will increase and therefore firms use *lower incentives rates* for workers because the *free–rider problem* is aggravated. On the other hand, from Corollary 10 we find that employment increases in this case. Consequently, the fixed wage cost will increase. If firms anticipate this increase in fixed wages, due to the *substitution effect* firms employ fewer workers who make greater effort because of higher incentive rates. Thus, firms use *higher incentive rates* for their workers. In the above example, the second effect dominates the first. Therefore, when the fixed wage cost is endogenous, workers' incentive rates decrease in b, while they increase in b when γ_3 is exogenous.

[13] Firms do not take part in the bargaining process because with free entry firm profit shrinks to zero. Therefore, when unions and policy makers internalize this fact, the wage setting process is restricted by entry and exit itself.

We know from Corollary 9 that supervisors' incentive rates will be higher when b is reduced. Therefore, both effects work in the same direction and supervisors' incentive rates decrease in b irrespectively of whether the fixed wage is determined endogenously or exogenously.[14] Similar considerations apply when we consider changes in the cost of entry.

The example shows that when changes in the degree of competitiveness of markets have employment effects which are anticipated by firms we should expect higher variations in incentive rates of mid–tier employees than in those of low–tier employees. Therefore, the example gives another explanation for the observation that low–tier PRP schemes are less sensitive to changes in competition than CEOs' or directors' PRP schemes (cf. Cuñat and Guadalupe, 2005).

5 Conclusion

Empirical evidence suggests that performance–related–pay sensitivities increase significantly with product market competition for CEO's. However, for workers, the effects of competition are much weaker and mostly insignificant (cf. Cuñat and Guadalupe (2005), p.1060). This is at odds with theoretical results that suggest that, though the amount of incentive payments should decrease with the hierarchical levels inside a firm, PRP sensitivities should increase equally at all levels with changes in competition. In this paper, a simple model of PRP in hierarchical agency is developed. The three–tier hierarchy includes a principal at the top, a supervisor on the mid tier and workers on the bottom tier. Incentive payments are used in the model because of moral hazard problems concerning supervisor's and workers' effort levels, which determine joint production.

The main results of the paper are displayed in Figure 1. We find that PRP on the lower tiers of a firm generates two kind of heterogeneities. First, changes in the competitiveness of markets can have different impacts on mid-tier and low–tier agents. The reason for this is that when firms apply group–incentive schemes and react to changes in competition by adjusting employment, this adjustment affects the free–rider problem at lower tiers. Second, while a firm's reaction to changes in competition concerning the PRP of mid-tier agents is similar under different endogeneity assumptions, it is very sensitive to these assumptions when we consider workers' PRP. Firms that react to changes in competition by adjusting only their production effort and firms that also adjust employment will apply different changes to the intensity of workers' PRP. The same is true for firms that operate in closed markets and firms that operate in markets with low barriers to entry. Finally, when changes in competition cause permanent changes in employment, the wage level in the industry might be affected. Firms that anticipate these changes might also react differently to firms that do not anticipate these changes when they redesign workers' PRP schemes. Together, these two kinds of heterogeneities explain the

[14] To be precise, for $\delta = 1$ the first effect is zero. However, from the proof of Corollary 9 we find that this is an exceptional case.

observation that the pay-performance sensitivities in worker contracts are significantly lower than those in managerial contracts when aggregated firm data is used.

Though the results have been obtained under very specific assumptions, they can be generalized for several reasons. First, the results depend on the assumption that worker's labor inputs are perfect substitutes. However, this is just the contrary assumption to Raith (2003) and Vives (2005) where the assumption that the results also apply to lower levels of the firm hierarchy implicitly implies that workers' contributions are completely independent. Thus, with less substitutability, the effects found in this paper will also be present while the impact of changes in competition on firm size will be lower and the results concerning effort and incentive rates will be closer to those of Raith (2003) and Vives (2005). Finally, the results depend on the assumption that changes in competition do not affect the task and job assignment within firms. However, a mayor effect of changes in competition may merely lead to a restructuring of these conditions (cf. Ichniowski and Shaw, 2003). In this case, a change in the PRP sensitivities is the consequence of the fact that workers perform different tasks with different responsibilities. This analysis is beyond the scope of this paper.

6 Appendix

PROOF OF PROPOSITION 2:

From problem $[P_2]$ we see that it is clearly optimal to set $\gamma_{3ik} = \gamma_3$, $\forall k$. Then, equation (4) implies that $U_{3ik} \geq 0$ when (2) is fulfilled. Now, using (3), the maximization problem can be rewritten as:

$$\max_{\beta_{3ik} \geq 0, e_{2i}} U_{2i} = (\beta_{2i} - K_i \beta_{3i})\, q_i(K_i, e_{2i}, \beta_{3i}) + \gamma_{2i} - K_i \gamma_3 - \frac{K_i}{2} e_{2i}^2 \tag{27}$$

where $q_i = q_i(K_i, e_{2i}, \beta_{3i}) = (\alpha_3 \beta_{3i})^{\frac{\alpha_3}{2-\alpha_3}} K_i^{\frac{2\alpha_1+\alpha_3}{2-\alpha_3}} e_{2i}^{\frac{2\alpha_2}{2-\alpha_3}}$. The first-order constraints are:

$$\begin{aligned} \frac{\partial U_{2i}}{\partial e_{2i}} &= (\beta_{2i} - K_i \beta_{3i}) \frac{2\alpha_2}{2-\alpha_3} q_i e_{2i}^{-1} - K_i e_{2i} = 0, && (28) \\ \frac{\partial U_{2i}}{\partial \beta_{3ik}} &= \left[(\beta_{2i} - K_i \beta_{3i}) \frac{\alpha_3}{2-\alpha_3} \beta_{3i}^{-1} - K_i \right] q_i = 0. && (29) \end{aligned}$$

From (29) we get:

$$\beta_{3i} = \frac{\alpha_3}{2K_i} \beta_{2i}. \tag{30}$$

Then, substituting (30) into (28) yields:

$$e_{2i} = \left(2^{-\frac{\alpha_3}{2}} \alpha_2^{\frac{2-\alpha_3}{2}} \alpha_3^{\alpha_3} \beta_{2i} K_i^{\frac{\alpha_3+2\alpha_1-2}{2}} \right)^{\frac{1}{(2-\alpha_3-\alpha_2)}}. \tag{31}$$

Substitution of (31) into (3) and (27) yields the second stage equilibrium values. To guarantee *sufficiency,* we obtain the second–order conditions:

$$\begin{aligned} \frac{\partial^2 U_{2i}}{\partial e_{2i}^2} &= (\beta_{2i} - K_i \beta_{3i}) \frac{2\alpha_2}{2-\alpha_3} \left(\frac{2\alpha_2}{2-\alpha_3} - 1 \right) q_i e_{2i}^{-2} - K_i, && (32) \\ \frac{\partial^2 U_{2i}}{\partial \beta_{3i}^2} &= \left[(\beta_{2i} - K_i \beta_{3i}) \left(\frac{\alpha_3}{2-\alpha_3} - 1 \right) \beta_{3ik}^{-1} - 2K_i \right] \frac{\alpha_3}{2-\alpha_3} q_i \beta_{3ik}^{-1}, && (33) \\ \frac{\partial^2 U_{2i}}{\partial \beta_{3i} \partial e_{2i}} &= \left[(\beta_{2i} - K_i \beta_{3i}) \frac{\alpha_3}{2-\alpha_3} \beta_{3ik}^{-1} - K_i \right] \frac{2\alpha_2}{2-\alpha_3} q_i e_{2i}^{-1}. && (34) \end{aligned}$$

Sufficiency is guaranteed by uniqueness of the extremum and because the Hessian matrix H is negative definite at the extremum:

$$\begin{aligned} H_1 &= \frac{\partial^2 U_{2i}}{\partial e_{2i}^2} = -2 \frac{2-\alpha_2-\alpha_3}{2-\alpha_3} K_i < 0 && (35) \\ H_2 &= \frac{\partial^2 U_{2i}}{\partial e_{2i}^2} \frac{\partial^2 U_{2i}}{\partial \beta_{3i}^2} - \left(\frac{\partial^2 U_{2i}}{\partial \beta_{3i} \partial e_{2i}} \right)^2 = 8 \frac{2-\alpha_2-\alpha_3}{\alpha_3 (2-\alpha_3)^2} \beta_{2i}^{\frac{2\varepsilon-1}{1-\varepsilon}} A K_i^{\frac{\delta-\varepsilon}{1-\varepsilon}+3} > 0. && (36) \end{aligned}$$

where H_i, $i = 1, 2$, are the successive principal minors of H.

PROOF OF PROPOSITION 3:

From problem $[P_1]$ we see that it is clearly optimal to set $\gamma_{2i} = K_i \gamma_3$. Now, equations (6) and (7) imply that $U_{2i} \geq 0$, as can be seen from (9). Then, using (8) the maximization problem can be rewritten as:

$$\max_{\beta_{2i}} \pi_i = \left(a - b \sum_{i=1}^{n} q_i - \beta_{2i} \right) q_i - K_i \gamma_3 - F \tag{37}$$

where $q_i = q_i(K_i, \beta_{2i}) = A \left(\beta_{2i}^{\varepsilon} K_i^{(\delta - \varepsilon)} \right)^{\frac{1}{1-\varepsilon}}$. The first-order condition is

$$\frac{\partial \pi_i}{\partial \beta_{2i}} = \left(a - b \sum_{i=1}^{n} q_i - b q_i - \beta_{2i} \right) \frac{\partial q_i}{\partial \beta_{2i}} - q_i = 0 \quad \forall i. \tag{38}$$

Using the symmetry assumption, $K_i = K$, and substituting $\frac{\partial q_i}{\partial \beta_{2i}}$ and q_i we get:

$$a - bAK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \left(\sum_{i=1}^{n} \beta_{2i}^{\frac{\varepsilon}{1-\varepsilon}} + \beta_{2i}^{\frac{\varepsilon}{1-\varepsilon}} \right) = \frac{1}{\varepsilon} \beta_{2i} \quad \forall i. \tag{39}$$

In equilibrium, $\beta_{2i} = \beta_2$. Thus, β_2 is implicitly defined by

$$a - b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}} = \frac{1}{\varepsilon} \beta_2. \tag{40}$$

PROOF OF COROLLARY 4:

Applying the implicit function theorem, from (11) we get:

$$\frac{\partial \beta_2}{\partial a} = \frac{1}{\frac{1}{\varepsilon} + \frac{\varepsilon}{1-\varepsilon} b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}-1}} > 0, \tag{41}$$

$$\frac{\partial \beta_2}{\partial b} = -\frac{(n+1)\, AK_i^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_{2i}^{\frac{\varepsilon}{1-\varepsilon}}}{\frac{1}{\varepsilon} + \frac{\varepsilon}{1-\varepsilon} b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}-1}} < 0, \tag{42}$$

$$\frac{\partial \beta_2}{\partial n} = -\frac{bAK_i^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_{2i}^{\frac{\varepsilon}{1-\varepsilon}}}{\frac{1}{\varepsilon} + \frac{\varepsilon}{1-\varepsilon} b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}-1}} < 0, \tag{43}$$

$$\frac{\partial \beta_2}{\partial K} = -\frac{\frac{\delta-\varepsilon}{1-\varepsilon} b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}-1} \beta_2^{\frac{\varepsilon}{1-\varepsilon}}}{\frac{1}{\varepsilon} + \frac{\varepsilon}{1-\varepsilon} b\,(n+1)\, AK^{\frac{\delta-\varepsilon}{1-\varepsilon}} \beta_2^{\frac{\varepsilon}{1-\varepsilon}-1}} < 0, \tag{44}$$

$$\frac{\partial \beta_2}{\partial \gamma_3} = 0. \tag{45}$$

From (12) we get that $sign\left(\frac{\partial \widetilde{\beta}_{2i}}{\partial x}\right) = sign\left(\frac{\partial \beta_{3i}}{\partial x}\right) = sign\left(\frac{\partial \beta_2}{\partial x}\right)$, for $x = a, b, n, K, \gamma_3$.

PROOF OF PROPOSITION 5:

From problem $[P_1']$ we see that it is clearly optimal to set $\gamma_{2i} = K_i\gamma_3$. Now, equations (6) and (7) imply that $U_{2i} \geq 0$, as can be seen from (9). Then, using (8) the maximization problem can be rewritten as:

$$\max_{K_i,\beta_{2i}} \pi_i = \left(a - b\sum_{i=1}^{n} q_i - \beta_{2i}\right) q_i - K_i\gamma_3 - F. \tag{46}$$

where $q_i = q_i(K_i, \beta_{2i}) = A\left(\beta_{2i}^{\varepsilon} K_i^{(\delta-\varepsilon)}\right)^{\frac{1}{1-\varepsilon}}$. The first-order conditions are

$$\frac{\partial \pi_i}{\partial \beta_{2i}} = \left(a - b\sum_{i=1}^{n} q_i - bq_i - \beta_{2i}\right)\frac{\partial q_i}{\partial \beta_{2i}} - q_i = 0, \tag{47}$$

$$\frac{\partial \pi_i}{\partial K_i} = \left(a - b\sum_{i=1}^{n} q_i - bq_i - \beta_{2i}\right)\frac{\partial q_i}{\partial K_i} - \gamma_3 = 0, \tag{48}$$

where

$$\frac{\partial q_i}{\partial \beta_{2i}} = \frac{\varepsilon}{1-\varepsilon} q_i \beta_{2i}^{-1} > 0, \tag{49}$$

$$\frac{\partial q_i}{\partial K_i} = \frac{(\delta-\varepsilon)}{1-\varepsilon} q_i K_i^{-1} > 0. \tag{50}$$

Rewriting (47) and substituting (48) into (47) we get

$$a - b\sum_{i=1}^{n} q_i(K_i, \beta_{2i}) - bq_i(K_i, \beta_{2i}) = \frac{1}{\varepsilon}\beta_{2i} \quad \forall i, \quad \text{and} \tag{51}$$

$$\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K_i^{1-\delta} = \beta_{2i} \quad \forall i. \tag{52}$$

Now, substituting (52) in (51) we get:

$$a - bA\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon}\left(\sum_{i=1}^{n} K_i^{\delta} + K_i^{\delta}\right) - \frac{1}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K_i^{1-\delta} = 0 \quad \forall i. \tag{53}$$

Because of symmetry, $K_i = K$, $\forall i$. Then, in equilibrium we get that optimal firm size K^* is implicitly defined by

$$G\left(a, b, \gamma_3, n, K^*\right) = 0, \tag{54}$$

where

$$G(a, b, \gamma_3, n, K) = \frac{1}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{1-\delta} + b\,(n+1)\,A\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{\delta} - a. \tag{55}$$

Existence and *uniqueness* are guaranteed because the left-hand side in (53) is decreasing in K_i. Now, (54) and (52) yield (14). Substitution of (52) into (6) and using $\widetilde{\beta}_{2i} = \beta_{2i} - K_i\beta_{3i}$

yields (15). Substitution of (52) into (8) gives (16). From (14)-(16) we get (17). Substitution of (52) into (7) gives the first expression in (18) which when substituted together with (15) into (2) yields the second expression in (18). Finally, (4), (9) and (15) give (19). To guarantee *sufficiency* we obtain the second–order conditions:

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}^2} = \left(a - b\sum_{i=1}^{n} q_i - bq_i - \beta_{2i}\right)\frac{\partial^2 q_i}{\partial \beta_{2i}^2} - 2\left[b\frac{\partial q_i}{\partial \beta_{2i}} + 1\right]\frac{\partial q_i}{\partial \beta_{2i}}, \tag{56}$$

$$\frac{\partial^2 \pi_i}{\partial K_i^2} = \left(a - b\sum_{i=1}^{n} q_i - bq_i - \beta_{2i}\right)\frac{\partial^2 q_i}{\partial K_i^2} - 2b\left[\frac{\partial q_i}{\partial K_i}\right]^2, \tag{57}$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i} = \left(a - b\sum_{i=1}^{n} q_i - bq_i - \beta_{2i}\right)\frac{\partial^2 q_i}{\partial \beta_{2i}\partial K_i} - \left[2b\frac{\partial q_i}{\partial \beta_{2i}} + 1\right]\frac{\partial q_i}{\partial K_i}, \tag{58}$$

where

$$\frac{\partial^2 q_i}{\partial \beta_{2i}^2} = \frac{\varepsilon}{1-\varepsilon}\left(\frac{\varepsilon}{1-\varepsilon} - 1\right) q_i \beta_{2i}^{-2} < 0, \tag{59}$$

$$\frac{\partial^2 q_i}{\partial K_i^2} = \frac{(\delta-\varepsilon)}{1-\varepsilon}\left(\frac{(\delta-\varepsilon)}{1-\varepsilon} - 1\right) q_i K_i^{-2} < 0, \tag{60}$$

$$\frac{\partial^2 q_i}{\partial \beta_{2i}\partial K_i} = \frac{(\delta-\varepsilon)\,\varepsilon}{(1-\varepsilon)^2} q_i K_i^{-1}\beta_{2i}^{-1} > 0. \tag{61}$$

Sufficiency is guaranteed by uniqueness of the extremum and since the Hessian matrix H is negative definite at the extremum:

$$H_1 = \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2} = -\left(1 + b\frac{(2\varepsilon)^2}{2-2\varepsilon} q_i \beta_{2i}^{-1}\right)\frac{2}{2-2\varepsilon} q_i \beta_{2i}^{-1} < 0, \tag{62}$$

$$H_2 = \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2}\frac{\partial^2 \pi_i}{\partial K_i^2} - \left(\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i}\right)^2 \tag{63}$$

$$= \left[\frac{1-\delta}{\varepsilon} + 2\delta b q_i \beta_{2i}^{-1}\right]\frac{\delta-\varepsilon}{(1-\varepsilon)^2} K_i^{-2} q_i^2 > 0. \tag{64}$$

where H_i, $i = 1, 2$ are the successive principal minors of H.

PROOF OF COROLLARY 6:

From (54) and (55) we get:

$$\frac{\partial K^*}{\partial a} = -\frac{\frac{\partial G}{\partial a}}{\frac{\partial G}{\partial K}} = \left(\frac{\partial G}{\partial K}\right)^{-1} > 0, \tag{65}$$

$$\frac{\partial K^*}{\partial b} = -\frac{\frac{\partial G}{\partial b}}{\frac{\partial G}{\partial K}} = -(n+1)\, q_i \left(\frac{\partial G}{\partial K}\right)^{-1} < 0, \tag{66}$$

$$\frac{\partial K^*}{\partial \gamma_3} = -\frac{\frac{\partial G}{\partial \gamma_3}}{\frac{\partial G}{\partial K}} = -\left(\frac{1-\varepsilon}{\varepsilon}\beta_{2i} + \varepsilon b\,(n+1)\,q_i\right)\gamma_3^{-1}\left(\frac{\partial G}{\partial K}\right)^{-1} < 0, \tag{67}$$

$$\frac{\partial K^*}{\partial n} = -\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial K}} = -bq_i\left(\frac{\partial G}{\partial K}\right)^{-1} < 0, \tag{68}$$

where

$$\frac{\partial G}{\partial K} = \left(\frac{1-\delta}{\varepsilon}\beta_{2i} + \delta b\,(n+1)\,q_i\right)K^{*^{-1}} > 0 \tag{69}$$

and β_{2i} and q_i are given by (15) and (16).

PROOF OF COROLLARY 7:

The statements concerning a, b and n follow directly from proposition 3. To prove the statements concerning γ_3 from (67) and (69) we get:

$$\frac{\partial \widetilde{\beta}_{2i}}{\partial \gamma_3} = \frac{(2-\alpha_3)}{2}\varepsilon b\,(n+1)\left(\frac{\partial G}{\partial K}\right)^{-1} > 0, \text{ and} \tag{70}$$

$$\frac{\partial \beta_{3i}}{\partial \gamma_3} = \frac{\alpha_3\varepsilon}{2\,(\delta-\varepsilon)}\left(\frac{1-\varepsilon}{\varepsilon}\frac{\beta_{2i}}{q_i} + \delta b\,(n+1)\right)K^{*-1}\left(\frac{\partial G}{\partial K}\right)^{-1} > 0. \tag{71}$$

PROOF OF PROPOSITION 8:

When market structure is endogenous firms enter until profits are zero. Therefore, n and K are simultaneously determined by equation (14) and by setting the right-hand side of equation (17) equal to zero:

$$G(a,b,\gamma_3,F,K^*,n^*) \equiv \frac{1}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{*^{1-\delta}} + b\,(n^*+1)\,A\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon}K^{*^{\delta}} - a = 0, \tag{72}$$

$$\pi_i(a,b,\gamma_3,F,K^*,n^*) \equiv \frac{1-\delta}{\delta-\varepsilon}\gamma_3K^* + bA^2\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon}K^{*^{2\delta}} - F = 0. \tag{73}$$

This yields

$$K^* = K(a,b,\gamma_3,F). \tag{74}$$

$$n^* = n(a,b,\gamma_3,F). \tag{75}$$

Existence is guaranteed for sufficiently large a in (72) and because from Corollary 4 we have that K^* is decreasing in n^*, which implies that profits decrease when n increases in (73). Furthermore, notice that (74) and (75) are continuously differentiable. Then, from the implicit function theorem we get:

$$\begin{bmatrix} \frac{\partial K^*}{\partial a} & \frac{\partial K^*}{\partial b} & \frac{\partial K^*}{\partial \gamma_3} & \frac{\partial K^*}{\partial F} \\ \frac{\partial n^*}{\partial a} & \frac{\partial n^*}{\partial b} & \frac{\partial n^*}{\partial \gamma_3} & \frac{\partial n^*}{\partial F} \end{bmatrix} = -\begin{bmatrix} \frac{\partial G}{\partial K^*} & \frac{\partial G}{\partial n^*} \\ \frac{\partial \pi_i}{\partial K^*} & \frac{\partial \pi_i}{\partial n^*} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial G}{\partial a} & \frac{\partial G}{\partial b} & \frac{\partial G}{\partial \gamma_3} & \frac{\partial G}{\partial F} \\ \frac{\partial \pi_i}{\partial a} & \frac{\partial \pi_i}{\partial b} & \frac{\partial \pi_i}{\partial \gamma_3} & \frac{\partial \pi_i}{\partial F} \end{bmatrix}$$

$$
= \frac{\begin{bmatrix} 0 & -\frac{\partial \pi_i}{\partial b}\frac{\partial G}{\partial n^*} & -\frac{\partial \pi_i}{\partial \gamma_3}\frac{\partial G}{\partial n^*} & \frac{\partial G}{\partial n^*} \\ \frac{\partial \pi_i}{\partial K^*} & -\frac{\partial \pi_i}{\partial K^*}\frac{\partial G}{\partial b} + \frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial b} & -\frac{\partial \pi_i}{\partial K^*}\frac{\partial G}{\partial \gamma_3} + \frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial \gamma_3} & -\frac{\partial G}{\partial K^*} \end{bmatrix}}{\frac{\partial G}{\partial n^*}\frac{\partial \pi_i}{\partial K^*}} \tag{76}
$$

because from (72) $\frac{\partial G}{\partial a} = -1$, and $\frac{\partial G}{\partial F} = 0$, and from (73) $\frac{\partial \pi_i}{\partial a} = 0$, $\frac{\partial \pi_i}{\partial F} = -1$, and $\frac{\partial \pi_i}{\partial n^*} = 0$. Now, using

$$
\begin{aligned}
\frac{\partial G}{\partial b} &= (n^*+1)\, A \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}} > 0, && (77)\\
\frac{\partial G}{\partial \gamma_3} &= \left(\frac{1-\varepsilon}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\delta}} + \varepsilon b\,(n^*+1)\, A \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}}\right)\gamma_3^{-1} > 0, && (78)\\
\frac{\partial G}{\partial K^*} &= \frac{1-\delta}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{-\delta}} + \delta b\,(n^*+1)\, A \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta-1}} > 0, && (79)\\
\frac{\partial G}{\partial n^*} &= bA \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}} > 0, && (80)\\
\frac{\partial \pi_i}{\partial b} &= A^2 \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon} K^{*^{2\delta}} > 0, && (81)\\
\frac{\partial \pi_i}{\partial \gamma_3} &= \frac{1-\delta}{\delta-\varepsilon}K^* + 2\varepsilon b A^2 \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon} K^{*^{2\delta}}\gamma_3^{-1} > 0, \text{ and} && (82)\\
\frac{\partial \pi_i}{\partial K^*} &= \frac{1-\delta}{\delta-\varepsilon}\gamma_3 + 2\delta b A^2 \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon} K^{*^{2\delta-1}} > 0 && (83)
\end{aligned}
$$

we have:

$$
\begin{aligned}
\frac{\partial K^*}{\partial a} &= 0,\ \frac{\partial K^*}{\partial b} < 0,\ \frac{\partial K^*}{\partial \gamma_3} < 0,\ \frac{\partial K^*}{\partial F} > 0,\ \frac{\partial n^*}{\partial a} > 0,\ \frac{\partial n^*}{\partial F} < 0,\\
sign\frac{\partial n^*}{\partial b} &= sign\left[-\frac{1-\delta}{\delta-\varepsilon}\gamma_3 n^* - \delta b\,(n^*+1)\, A^2 \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon} K^{*^{2\delta-1}}\right] < 0, \text{ and}\\
sign\frac{\partial n^*}{\partial \gamma_3} &= sign\begin{bmatrix} -\frac{(1-\delta)}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\delta}} \\ +\left[(1-\delta)(n^*+1)-2\right] bA \left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\delta}} \end{bmatrix}. && (84)
\end{aligned}
$$

The sign of the last expression cannot be determined in general. However, for $n = 1$ the sign is negative, while for large n the sign is positive if δ is small.

PROOF OF COROLLARY 9:

The long-run equilibrium effect of changes in a, b, γ_3, and F on incentive rates is:

$$
\frac{d\widetilde{\beta}_{2i}}{da} = (1-\delta)\left(1-\frac{\alpha_3}{2}\right)\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{-\delta}}\frac{\partial K^*}{\partial a} = 0, \tag{85}
$$

$$\frac{d\widetilde{\beta}_{2i}}{db} = (1-\delta)\left(1-\frac{\alpha_3}{2}\right)\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*-\delta}\frac{\partial K^*}{\partial b} \leq 0, \tag{86}$$

$$\frac{d\widetilde{\beta}_{2i}}{dF} = (1-\delta)\left(1-\frac{\alpha_3}{2}\right)\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*-\delta}\frac{\partial K^*}{\partial F} \geq 0, \tag{87}$$

$$\frac{d\widetilde{\beta}_{2i}}{d\gamma_3} = \frac{\left(1-\frac{\alpha_3}{2}\right)}{\frac{\partial\pi_i}{\partial K^*}}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon}\left[(1-\delta)K^{*1-\delta} + 2b\varepsilon A\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{2\varepsilon-1} K^{*\delta}\right] > 0, \tag{88}$$

$$\frac{d\beta_{3i}}{da} = -\delta\frac{\alpha_3}{2}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*-\delta-1}\frac{\partial K^*}{\partial a} = 0, \tag{89}$$

$$\frac{d\beta_{3i}}{db} = -\delta\frac{\alpha_3}{2}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*-\delta-1}\frac{\partial K^*}{\partial b} > 0, \tag{90}$$

$$\frac{d\beta_{3i}}{dF} = -\delta\frac{\alpha_3}{2}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*-\delta-1}\frac{\partial K^*}{\partial F} < 0, \text{ and} \tag{91}$$

$$\frac{d\beta_{3i}}{d\gamma_3} = \left[(1-\varepsilon)\frac{\alpha_3}{2}K^{*-\delta}\gamma_3^{-1} - \delta\frac{\alpha_3}{2}K^{*-\delta-1}\frac{\partial K^*}{\partial\gamma_3}\right]\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} > 0. \tag{92}$$

PROOF OF COROLLARY 10:

The long-run equilibrium effect of changes in a on total employment is:

$$\frac{\partial nK}{\partial a} = \frac{\partial n}{\partial a}K + \frac{\partial K}{\partial a}n = \frac{\partial n}{\partial a}K > 0 \tag{93}$$

(from (84)). The long-run equilibrium effect of changes in a on total employment is:

$$\frac{\partial nK}{\partial b} = \frac{\partial n}{\partial b}K + \frac{\partial K}{\partial b}n < 0 \tag{94}$$

(from (84)). The long-run equilibrium effect of changes in F on total employment is:

$$\begin{aligned}\frac{\partial nK}{\partial F} &= \frac{\partial n}{\partial F}K + \frac{\partial K}{\partial F}n = \frac{-\frac{\partial G}{\partial K^*}K + \frac{\partial G}{\partial n^*}n}{\frac{\partial G}{\partial n^*}\frac{\partial \pi_i}{\partial K^*}} \\ &= \left[\begin{array}{c} -\frac{1-\delta}{\varepsilon}\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*1-\delta} \\ +\left(n^*-\delta\left(n^*+1\right)\right)bA\left(\frac{\varepsilon}{(\delta-\varepsilon)}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*\delta}\end{array}\right]\left(\frac{\partial G}{\partial n^*}\frac{\partial\pi_i}{\partial K^*}\right)^{-1}. \end{aligned} \tag{95}$$

The sign of this expression cannot be determined in general. However, the expression is negative for $n^* = 1$ and $\delta > 1/2$, and positive for large n^*.

References

[1] Baker, G. (1992): Incentive Contracts and Performance Measurement, *Journal of Political Economy* **100(2)**: 598-614.

[2] Beckmann, M. (1977): Management Production Functions and the Theory of the Firm, *Journal of Economic Theory* **14**:1-18.

[3] Binmore, K.G., Rubinstein, A. and Wolinsky, A. (1986): The Nash Bargaining Solution in Economic Modelling, *RAND Journal of Economics* **17**, 176-188.

[4] Bolton, P., Scheinkman, J. and Xiong, W. (2006): Executive Compensation and Short–Terminist Behaviour in Speculative Markets, *Review of Economic Studies* **73**: 577-610.

[5] Burgess, S. and Metcalfe, P. (2000): Incentive Pay and Product Market Competition, CMPO Working Paper, University of Bristol, No. 00/28.

[6] Che,Y.-K. and Yoo, S.-W. (2001): Optimal Incentives for Teams, *American Economic Review* **91(3)**: 525-541.

[7] Cuñat, V. and Guadalupe, M. (2005): How does Product Market Competition Shape Incentive Contracts?, *Journal of the European Economic Association* **3(5)**: 1058-1082.

[8] Espinosa M.P.and Macho–Stadler, I. (2003): Endogenous Formation of Partnerships with Moral Hazard, *Games and Economic Behavior* **44(1)**: 183-194.

[9] Farrell, C. and Shearer, B. (1999): Incentives and Transactions Costs Within the Firm: Estimating an Agency Model Using Payroll Records, *Review of Economic Studies* **66**: 309-338.

[10] Griffith, R., Harrison, R. and Macartney, G. (2006): Product Market Reforms, Labour Market Institutions and Unemployment, *CEPR Discussion Paper* No. 5599.

[11] Holmstrom, B. (1982): Moral hazard in Teams, *The Bell Journal of Economics* **13(2)**: 324-340.

[12] Holmstrom, B. and Milgrom, P. (1990): Regulating Trade among Agents, *Journal of Institutional and Theoretical Economics* **146(1)**: 169-182.

[13] Holmstrom, B. and Milgrom, P. (1991): Multi-task Principal–Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design, *Journal of Law, Economics, and Organization* **7(S)**: 24-52.

[14] Holmstrom, B. and Milgrom, P. (1994): The Firm as an Incentive System, *American Economic Review* **84(4)**: 972-991.

[15] Ichniowski, C. and Shaw, K. (2003): Beyond Incentive Pay: Insiders' Estimates of the Value of Complementary Human Resource Management Practices, *Journal of Economic Perspectives* **17(1)**: 155-180.

[16] Jensen, M.C. and Murphy, K.J. (1990): Performance Pay and Top-Management Incentives, *Journal of Political Economy* **98(2)**: 225-284.

[17] Itoh, H. (1991): Incentives to Help in Multi-agent Situations, *Econometrica* **59(3)**: 611-636.

[18] Itoh, H. (1992): Cooperation in Hierarchical Organizations: An Incentive Perspective, *Journal of Law, Economics, and Organization* **8(2)**: 321-345.

[19] Itoh, H. (1993): Coalitions, Incentives, and Risk Sharing, *Journal of Economic Theory* **60(2)**: 410-427.

[20] Kandel, E. and Lazear, E. (1992): Peer Pressure and Partnerships, *Journal of Political Economy* **100(4):** 801-817.

[21] Lazear, E. (2000): Performance Pay and Productivity, *American Economic Review* **90(5):** 1346-1361.

[22] Macho-Stadler, I. and Pérez-Castrillo, D. (1993): Moral Hazard with Several Agents: The Gains from Cooperation, *International Journal of Industrial Organization* **11(1)**: 73-100.

[23] McAfee, P. and McMillan, J. (1991): Optimal Contracts for Teams, *International Economic Review* **32(2)**: 561-577.

[24] McAfee, P. and McMillan, J. (1995): Organizational Diseconomies of Scale, *Journal of Economics and Management Strategy* **4(3)**: 399-426.

[25] Mookherjee, D and Reichelstein, S. (2001): Incentives and Coordination in Hierarchies, *Advances in Theoretical Economics* **1(1)**.

[26] Murphy, K.J. (1999): Executive Compensation, In *Handbook of Labor Economics*, Vol 3B, Ashanfetter, O. and Carlo, D.E. (eds.), Elsevier Science, North-Holland.

[27] Prendergast, C. (2002): The Tenuous Trade–Off between Risk and Incentives, *Journal of Political Economy* **110(5)**: 1071-1102.

[28] Raith, M. (2003): Competition, Risk and Managerial Incentives, *American Economic Review* **93**: 1425-1436.

[29] Ramakrishnan, R.T.S. and Thakor, A.V. (1991): Cooperation versus Competition in Agency, *Journal of law, Economics, and Organization* **7(2)**: 248-283.

[30] Schmidt, K. (1997): Managerial Incentives and Product Market Competition, *Review of Economics Studies* **64**: 191-213.

[31] Varian, H.R. (1990): Monitoring Agents with Other Agents, *Journal of Institutional and Theoretical Economics* **146(1)**: 153-174.

[32] Vives, X. (2004): Innovation and Competitive Pressure, *CEPR Discussion Paper* No. 4369.

endogenous variables	influence of exogenous variables on β_{2i}					
	a	b	F	γ_3	n	K
none	$+$	$-$	0	0	$-$	$-$
K	$+$	$-$	0	$+$	$-$	
K, n	0	$-$	$+$	$+$		
K, n, γ_3 $\left(\delta = 1,\, \varepsilon = \frac{1}{2}\right)$	0	$-$	$+$			

endogenous variables	influence of exogenous variables on β_{3i}					
	a	b	F	γ_3	n	K
none	$+$	$-$	0	0	$-$	$-$
K	$-$	$+$	0	$+$	$+$	
K, n	0	$+$	$-$	$+$		
K, n, γ_3 $\left(\delta = 1,\, \varepsilon = \frac{1}{2}\right)$	0	$-$	$+$			

Figure 1. The influence of different exogenous variables on incentive rates under alternative endogeniety assumptions.