UNIVERSITAT ROVIRA I VIRGILI

WORKING PAPERS

Col·lecció "DOCUMENTS DE TREBALL DEL DEPARTAMENT D'ECONOMIA"

"Foundations for Contest Success Functions"

Luis Corchón
Matthias Dahm

Document de treball nº -8- 2008

DEPARTAMENT D'ECONOMIA
Facultat de Ciències Econòmiques i Empresarials

Edita:

Departament d'Economia
http://www.fcee.urv.es/departaments/economia/public_html/index.html
Universitat Rovira i Virgili
Facultat de Ciències Econòmiques i Empresarials
Avgda. de la Universitat, 1
432004 Reus
Tel. +34 977 759 811
Fax +34 977 300 661

Dirigir comentaris al Departament d'Economia.

Dipòsit Legal: T - 1178 - 2008
ISSN 1988 - 0812

DEPARTAMENT D'ECONOMIA
Facultat de Ciències Econòmiques i Empresarials

Foundations for Contest Success Functions*

Luis Corchón† and Matthias Dahm‡

June 11, 2008

Abstract

In the literature the outcome of contests is either interpreted as win probabilities or as shares of the prize. With this in mind, we examine two approaches to contest success functions. In the first we analyze the implications of contestants' incomplete information concerning the 'type' of the contest administrator. While in the case of two contestants this approach can rationalize prominent contest success functions, we show that it runs into difficulties when there are more agents. Our second approach interprets contest success functions as sharing rules and establishes a connection to bargaining and claims problems which is independent of the number of contestants. Both approaches provide foundations for popular contest success functions and guidelines for the definition of new ones.

Keywords: Endogenous Contests, Contest Success Function.

JEL Classification: C72 (Noncooperative Games), D72 (Economic Models of Political Processes: Rent-Seeking, Elections), D74 (Conflict; Conflict Resolution; Alliances).

* This paper is the melting of two independent projects. We are grateful for important input from Nicolás Porteiro and regret his decision not to co-author the more extensive analysis of the present paper. We also wish to thank J. Atsu Amegashie, Miguel A. Ballester, Carmen Beviá, Steven Callander, Caterina Calsamiglia, Juan A. Crespo, Francesco De Sinopoli, Bard Harstad, Sjaak Hurkens, Carolina Manzano, Carlos Maravall, Antonio Quesada, Bernd Theilen, William Thomson and Cori Vilella for helpful comments. The suggestions by the editor C. D. Aliprantis and an anonymous referee substantially improved the analysis. The usual disclaimer applies. This work has been partially carried out while Dahm was visiting Northwestern University (Kellogg School of Management, MEDS, CMS-EMS). The hospitality of this institution and the financial support of the Spanish Secretaría de Estado de Educación y Universidades with co-funding by the European Social Fond is gratefully acknowledged. The first author acknowledges financial support from the Spanish Ministerio de Educación y Ciencia, project SEJ2005-06167/ECON and the second by the Departament d'Universitats, Recerca i Societat de la Informació (Generalitat de Catalunya), project 2005SGR00949 and the Spanish Ministerio de Educación y Ciencia, project SEJ2007-67580-C02-01.

† Departamento de Economía. Universidad Carlos III de Madrid. Calle Madrid, 126. 28903 Getafe (Madrid). Spain. E-mail: lcorchon@eco.uc3m.es. Phone: +34 916 249617. Fax: +34 916 249875.

‡ Departamento de Economía. Universitat Rovira i Virgili. Avenida de la Universitat, 1. 43204 Reus (Tarragona). Spain. E-mail: matthias.dahm@urv.cat. Phone: +34 977 759 850. Fax: +34 977 759 810.

"The strategic approach also seeks to combine axiomatic cooperative solutions and non-cooperative solutions. Roger Myerson recently named this task the 'Nash program'." (Rubinstein (1985), p. 1151)

1 Introduction

A contest is a game in which players exert effort in order to win a certain prize. Contests have been used to analyze a variety of situations including lobbying, rent-seeking and rent-defending contests, advertising, litigation, political campaigns, conflict, patent races, arms races, sports events or R&D competition. A crucial determinant for the equilibrium predictions of contests is the specification of the so-called contest success function (CSF) which relates the players' efforts and win probabilities. Justifications for a particular CSF can be twofold. A justification can be on normative grounds, because it is the unique CSF fulfilling certain axioms, or essential properties. A justification can also be positive when it can be shown that the CSF arises from the strategic interaction of players, thereby yielding a description of situations when it can be expected to be realistic. The purpose of the present paper is to contribute to our understanding of CSFs in both dimensions.

Formally, a contest success function associates, to each vector of efforts $\boldsymbol{G}$, a lottery specifying for each agent a probability p_i of getting the object. That is, $p_i = p_i(\boldsymbol{G})$ is such that, for each contestant $i \in N := \{1, ..., n\}$, $p_i(\boldsymbol{G}) \geq 0$, and $\sum_{i=1}^{n} p_i(\boldsymbol{G}) = 1$.

The canonical example of a contest situation is rent-seeking. In a pioneering paper, Tullock (1980) proposed a special form of the contest success function, namely, given a positive scalar R,

$$p_i = \frac{G_i^R}{\sum_{j=1}^{n} G_j^R}, \text{ for } i = 1, ..., n. \tag{1}$$

Gradstein (1995, 1998) postulated the following variation of this form where, given $q_i > 0$ for all $i \in N$,

$$p_i = \frac{G_i q_i}{\sum_{j=1}^{n} G_j q_j}, \text{ for } i = 1, ..., n. \tag{2}$$

A generalization that comprises both previous functional forms is, given $a_i \geq 0$ for all $i \in N$,

$$p_i = \frac{G_i^R q_i + a_i}{\sum_{j=1}^{n} (G_j^R q_j + a_j)}, \text{ for } i = 1, ..., n. \tag{3}$$

A different functional form, the logit model, was proposed by Hirshleifer (1989) where, given a positive scalar k,

$$p_i = \frac{e^{kG_i}}{\sum_{j=1}^{n} e^{kG_j}}, \text{ for } i = 1, ..., n. \tag{4}$$

Note that the four expressions (1) – (4) are specific instances of the following functional form

$$p_i = \frac{f_i(G_i)}{\sum_{j=1}^{n} f_j(G_j)}, \text{ for } i = 1, ..., n. \tag{5}$$

The so-called effectivity functions f_i are usually interpreted as determining how 'effective' agent i's effort is in affecting the win probability of agent i. Most papers dealing with contest models in the literature analyze a CSF which is a special case of the form in (5) (Nitzan (1994), Konrad (2007)). Consequently, the present paper will be mainly concerned with deriving foundations for CSFs of this form. Notice, for later reference, that in (5) the win probability of any contestant is responsive to changes in the efforts of all other contenders, if the f_i are strictly increasing.

However, there are also some CSFs in the literature which are not special cases of the form in (5). The first two consider the case of two contestants and build on the idea that only differences in effort should matter – an idea introduced by Hirshleifer in (4). Baik (1998) proposed the following form, given a positive scalar σ,

$$p_1 = p_1(\sigma G_1 - G_2) \text{ and } p_2 = 1 - p_1. \tag{6}$$

Che and Gale (2000) postulate the following piece-wise linear difference-form

$$p_1 = max\left\{min\left\{\frac{1}{2} + \sigma(G_1 - G_2), 1\right\}, 0\right\} \text{ and } p_2 = 1 - p_1. \tag{7}$$

Recently, Alcalde and Dahm (2007) proposed a CSF in which relative differences matter. Given an ordered vector of efforts such that $G_1 \geq G_2 \geq ... \geq G_n$ and a positive scalar R, the serial contest success function is defined as

$$p_i = \sum_{j=i}^{n} \frac{G_j^R - G_{j+1}^R}{j \cdot G_1^R}, \text{ for } i = 1, ..., n \text{ with } G_{n+1} = 0. \tag{8}$$

In the literature the outcome of contests has been interpreted to capture two different situations: as win probabilities or as shares of the prize.[1] With this in mind, we examine two approaches to contest success functions.

In the first we postulate the existence of a contest administrator who allocates the prize to one of the contestants. However, contestants have incomplete information about the type of the contest administrator. We show that this approach can generate CSFs for any number of contestants. However, while in the case of two contestants this approach can rationalize a large class of contest success functions, we show that it runs into difficulties when there are more agents.

Our second approach interprets contest success functions as sharing rules and establishes a connection to bargaining and claims problems which is independent of the number of contestants. The analysis exploits the observation that these problems are mathematically related – but not equivalent – to the problem of assigning win probabilities in contests. A main result here follows Dagan and Volij (1993) and shows that the class of contest success functions given in (5) can be understood as the weighted Nash bargaining solution where efforts represent the weights of the agents. We turn then to the framework of bargaining with claims (Chun and Thomson (1992))

[1] A prominent example for the latter is Wärneryd (1998). He analyzes a contest among jurisdictions for shares of the GNP and compares different types of jurisdictional organization.

to incorporate explicitly the contestants' efforts in the description of the problem. This allows to associate prominent solution concepts in this framework to the previously mentioned class of contest success functions and to a generalized version of Che and Gale's difference-form contest (7).

Both approaches provide foundations for popular contest success functions and guidelines for the definition of new ones. In our view both types of foundations complement each other nicely. For instance, we show that (7) can be understood, on one hand, as contestants trying to sway away the contest administrator's decision in a setting analogous to the model of a circular city by Salop (1979). On the other, we show that this CSF is also related to the claim-egalitarian solution (Bossert (1993)). Both approaches lend support to an extension of this CSF to three contestants of the following form. Let $G_1 \geq G_2 \geq G_3$ and a and b be positive scalars. If $G_1 - G_3 \geq a$ then $p_3 = 0$ and the other contestants obtain win probabilities as in (7). Otherwise let

$$p_i = \frac{1}{3} + b\left(2G_i - G_j - G_k\right), \text{ for } i = 1, 2, 3 \text{ and } i \neq j, k. \tag{9}$$

However, the requirement that for $n = 2$ the CSF reduces to (7) implies that $(a, b) = ((3\sigma)^{-1}, \sigma/2)$ in the first and $(a, b) = ((2\sigma)^{-1}, 2\sigma/3)$ in the second approach. This underlines that the appropriate extension depends on the application and institutional details the contest model is intended to capture.

Foundations for contest success functions have been reviewed by Garfinkel and Skaperdas (2007) and Konrad (2007). The most systematic approach has been normative and the seminal paper is Skaperdas (1996). He proposed five axioms and showed that they are equivalent to assuming a CSF of the form given in (5) with $f_i(\cdot) = f(\cdot)$ for all $i \in N$, where $f(\cdot)$ is a positive increasing function of its argument. Therefore, for simplicity, we refer in the sequel to (5) as Skaperdas' class of CSFs. Skaperdas also showed that if in addition to the other five axioms the CSF is assumed to be homogeneous of degree zero in $\boldsymbol{G}$ then we obtain (1).[2] Our paper contributes to this literature indirectly by making connections to related problems which are well understood from a normative point of view. For instance, we establish a relationship between Che and Gale's difference-form CSF (7) and the principle of equal sacrifice.

As for the positive approach, we are not aware of any work understanding CSFs as sharing rules as our second approach does.[3] However, our first approach is related to other works. Assume that efforts are a noisy predictor of performance in the contest. When noise enters additively in performance and is distributed as the extreme value distribution, we obtain the logit specification, McFadden (1974). This procedure was generalized by Lazear and Rosen (1981) and Dixit (1987) to general distributions. When noise enters multiplicatively, Hillman

[2] An extension of Skaperdas' result to non-anonymous CSFs is given by Clark and Riis (1998). Skaperdas also axiomatized the logit model (4).

[3] Anbarci, Skaperdas and Syropoulos (2002) present a model in which a two party conflict over a resource can either be settled through bargaining over the resource or through a contest. The contest defines the disagreement point of the bargaining problem to which three different bargaining solutions are applied. See also Esteban and Sákovics (2006). In contrast, in our framework we interpret bargaining to be over win probabilities and derive contest success functions as bargaining rules.

and Riley (1988) derived (1) for the case of two contestants when noise follows an exponential distribution (see also Hirshleifer and Riley (1992)). This was generalized by Jia (2007) to $n > 2$.[4] Our approach differs from these papers by changing performance to the broader concept of utility and using a uniformly distributed and one dimensional random variable.

Epstein and Nitzan (2006) partially rationalize CSFs by analyzing how a contest administrator rationally decides whether to have a contest and if a contest takes place how he chooses among a fixed set of CSFs. In contrast, in our approach the administrator chooses deterministically but the contestants face a CSF because of their uncertainty about the type of the administrator.

2 External Decider

2.1 Two Contenders

Assume that one person has to decide to award a prize to one of two contestants. In the situation we have in mind contestants are uncertain about a characteristic of the decider that is relevant for his decision. So contenders exert effort without knowing the realization of the characteristic and then the decision-maker decides whom to give the prize based both on the contestants' efforts and his type.

Let Θ be the set of states of the world. Let θ be an arbitrary element of Θ. We assume that $\Theta = [0, 1]$ and that θ is uniformly distributed. Let V_i be the decider's payoff if the prize is awarded to contestant $i = 1, 2$. V_i is assumed to depend on the state of the world, i.e. $V_i = V_i(\theta)$. This may reflect the uncertainty in the contestants' minds about the preferences of the decider. We will assume the following single-crossing property.

(SC) $V_1(\theta)$ is decreasing in θ and $V_2(\theta)$ is strictly increasing in θ.

Taking into account efforts, let $U_i(V_i(\theta), G_i)$ be the decider's payoff if the prize is awarded to contestant $i = 1, 2$. This function is assumed to be increasing in both arguments and for simplicity we will write $U_i(\theta, G_i)$. For the sake of interpretation let G_i be interpreted as the level of advertisement (resp. quality) made (resp. provided) by contestant $i = 1, 2$. Let

$$\theta' = \begin{cases} 1 \text{ if } U_1(\theta, G_1) > U_2(\theta, G_2), \forall\theta \in \Theta \\ 0 \text{ if } U_1(\theta, G_1) < U_2(\theta, G_2), \forall\theta \in \Theta \\ \{\theta | U_1(\theta, G_1) = U_2(\theta, G_2)\} \text{ otherwise.} \end{cases} \tag{10}$$

Under our assumptions θ' is well-defined and unique. Moreover, θ' equals p_1, the probability that contestant 1 gets the prize. We now provide several examples in which we solve for p_1 as a function of G_1 and G_2. This way we obtain the contest success function as arising from the maximization of the payoff function of the decider.

[4] In related work Fullerton and McAfee (1999) and Baye and Hoppe (2003) offer micro-foundations for a subset of CSFs of the form in (1) in the context of innovation tournaments and patent races following an analogous procedure.

In these examples $V_i(\theta)$ enters either additively (in the spirit of McFadden (1974)) or multiplicatively (as in Hillman and Riley (1988)). In Examples 1 and 2 the effect of a contestant's advertisement is completely separated from the decider's bias. The function $U_i(\theta, G_i)$ is additively separable in both arguments. Here, the merit of an alternative in the decider's eyes might be positive even when advertising is zero, and vice versa. Moreover, the marginal product of advertising is independent of the decider's bias. This contrasts with the multiplicative form of Example 3 in which (i) a prerequisite for the merit of an alternative is both that the decider likes it (at least a little) and that advertising is positive; and (ii) an increase of the decider's bias raises the marginal product of advertising. Example 4 is a combination of these two extreme cases in the sense that for one contestant the relationship is multiplicative, while for the other the effect of advertising is independent of the bias.

Example 1 *Let* $U_1(\theta, G_1) = V_1(\theta) + a_1 G_1$ *and* $U_2(\theta, G_2) = V_2(\theta) + a_2 G_2$, *where* $a_1, a_2 > 0$. *Thus,* $a_1 G_1 - a_2 G_2 = V_2(\theta) - V_1(\theta) \equiv z(\theta)$, *say. Since* $z(\cdot)$ *is invertible we get,* $p_1 = z^{-1}(a_1 G_1 - a_2 G_2)$ *which is the form in (6) considered by Baik (1998).*[5] *Notice that this procedure is identical to the one used in models of spatial differentiation in order to obtain the demand function (see Hotelling (1929)).*

Example 2 *Let* $U_1(\theta, G_1) = \theta + 2\sigma G_1 - 1/2$ *and* $U_2(\theta, G_2) = -\theta + 2\sigma G_2 + 1/2$, *where* σ *is a positive scalar. In this case, it is easily calculated that* $p_1 = max\left\{min\left\{1/2 + \sigma(G_1 - G_2), 1\right\}, 0\right\}$. *We obtain (7) the family of difference-form contest success functions analyzed by Che and Gale (2000).*

Example 3 *Let* $U_1(\theta, G_1) = (1-\theta) f_1(G_1)$ *and* $U_2(\theta, G_2) = \theta f_2(G_2)$. *Here we obtain* $p_1 = f_1(G_1)/(f_1(G_1) + f_2(G_2))$. *This is Skaperdas' class of CSFs (5) for* $n = 2$.

Example 4 *Let* $U_1(\theta, G_1) = f_1(G_1)$ *and* $U_2(\theta, G_2) = 2\theta f_2(G_2)$ *if* $\theta \leq 1/2$ *and* $U_2(\theta, G_2) = f_2(G_2)/(2(1-\theta))$ *if* $1/2 \leq \theta < 1$. *Analogous reasoning as before yields* $p_1 = f_1(G_1)/(2f_2(G_2))$ *if* $f_1(G_1) \leq f_2(G_2)$ *and* $p_1 = 1 - f_2(G_2)/(2f_1(G_1))$ *otherwise. This expression is a generalization of the family of serial contests in (8) analyzed in Alcalde and Dahm (2007).*

In order to derive a general result concerning what kind of CSFs can be derived from the maximization of the payoffs of the decider we will now consider the class of CSF which are $\mathbb{C}^1$ in $\mathbb{R}^n_{++}$. This leaves outside our study CSFs like (7) but includes (8) when $n = 2$.

A difficulty in our study is that many well-known CSFs fail to be continuous when $G_i = 0$ all i and constant in its own effort when $G_j = 0$ all $j \neq i$, e.g. (1). A way to solve these problems is to stay away from the troublesome boundaries of $\mathbb{R}^n_+$ as we do in Definitions 2.1 and 2.2.

Definition 2.1 $p_i = p_i(\mathbf{G})$ *is regular if for all* $\mathbf{G} \in \mathbb{R}^n_{++}$, $\partial p_i(\mathbf{G})/\partial G_i > 0$ *and* $\partial p_i(\mathbf{G})/\partial G_j < 0$ *for all* $j \neq i$.

[5] Alternatively, we may assume that the payoff function of the decider is $U_i = V_i(\theta) - a_j G_j$, $i \neq j$, reflecting the disutility received from the effort made by contestant 2, if the prize is awarded to contender 1. The same applies to Example 2 and to Example 3 by taking $U_1 = (1-\theta)/f_2(G_2)$ and $U_2 = \theta/f_1(G_1)$.

Notice that the CSFs in (1) – (4) and (6) are regular. The one in (5) is regular if we assume, as in Szidarovsky and Okuguchi (1997), that $f_i^{'}(G_i) > 0$ and $f_i(0) = 0$ for all $i \in N$. The CSF given in (8) is regular if $n = 2$.

Definition 2.2 *The contest success function* $\{p_1(\boldsymbol{G}), p_2(\boldsymbol{G}), ..., p_n(\boldsymbol{G})\}$ *is rationalizable if there is a list of payoff functions* $U_i(\theta, G_i)$ *strictly increasing on* G_i, $i = 1, 2, ..., n$ *such that for any* $\hat{\boldsymbol{G}} \in \mathbb{R}^n_{++}$,

$$p_i(\hat{\boldsymbol{G}}) = probability\{U_i(\theta, \hat{G}_i) > U_j(\theta, \hat{G}_j), \forall j \neq i\}, \text{ for } i = 1, ..., n.$$

We need the following assumption:

Assumption 1: $p_i \to 1$ when $G_i \to \infty$ and $p_i \to 0$ when $G_i \to 0$.

It is easy to check that Tullock's CSF (1) satisfies Assumption 1 (A.1 in the sequel). Also Skaperdas' class of CSFs (5) satisfies A.1 when $f_i(G_i)$ are strictly positive for strictly positive values of efforts, $f_i \to \infty$ when $G_i \to \infty$ and $f_i \to 0$ when $G_i \to 0$. It is fulfilled by the serial CSF in (8) and the form in (6) includes cases where A.1 is satisfied. Now we can prove the following:

Proposition 2.1 *If A.1 holds and* $p_1(G_1, G_2)$ *is regular, it is rationalizable by a pair of payoff functions fulfilling the single crossing condition. If* $p_1(G_1, G_2)$ *is rationalizable by a pair of payoff functions fulfilling the single crossing condition and* $\partial p_i(\mathbf{G})/\partial G_j \neq 0$ *for all* i, j*, it is regular.*

Proof. Suppose $p_1(G_1, G_2)$ is regular. Notice that this implies that for any $\boldsymbol{G} \in \mathbb{R}^2_{++}$, $p_i \in (0, 1)$. Let $f(p_1, G_1, G_2) \equiv p_1 - p_1(G_1, G_2)$. Fix p_1 and G_2, say $\bar{p}_1$ and $\bar{G}_2$. By A.1 we have that $f(\bar{p}_1, G_1, \bar{G}_2) < 0$ for G_1 sufficiently large and $f(\bar{p}_1, G_1, \bar{G}_2) > 0$ for G_1 sufficiently close to zero. By the intermediate value theorem, there is a G_1 such that $f(\bar{p}_1, G_1, \bar{G}_2) = 0$. By the definition of a regular CSF this value of G_1, say $\bar{G}_1$, is unique. This means that there is a unique function H such that $G_1 = H(p_1, G_2)$. Since $\partial f(p_1, G_1, G_2)/\partial G_1 < 0$, by the implicit function theorem H is continuous in a neighborhood of $(\bar{p}_1, \bar{G}_2)$. Since this point is arbitrary, H is continuous for all (p_1, G_2). Let $U_1 = G_1$ and $U_2 = H(\theta, G_2)$. Because $p_1(G_1, G_2)$ is regular, H is strictly increasing on θ and G_2. Also U_1 is strictly increasing on G_1 and constant on θ, so the SC assumption holds. By construction, θ' (as defined in equation (10)) equals p_1, thus $p_1(G_1, G_2)$ is rationalizable.

Assume now that $p_1(G_1, G_2)$ is rationalizable by a list of payoff functions fulfilling the single crossing condition (SC). Rationalizability implies that for any $(\hat{G}_1, \hat{G}_2)$ we have $p_1(\hat{G}_1, \hat{G}_2) = \theta'$ (as defined in equation (10)). Moreover, as U_1 is strictly increasing on G_1 and by the single crossing condition (SC) U_2 is strictly increasing on θ, we have that p_1 is strictly increasing in G_1. The opposite holds when G_2 is increased, so the result follows from $\partial p_i(\mathbf{G})/\partial G_j \neq 0$. ∎

We show now that the condition that the partial derivatives do not vanish cannot be dispensed with.

Example 5 *Consider the following smooth difference-form contest between two contenders:*

$$p_1 = \begin{cases} 1 & \textit{if} \quad G_1 - G_2 \geq 1 \\ \frac{1}{2} + \frac{1}{2} e^{\left\{\frac{-(G_1-G_2-1)^2}{1-(G_1-G_2-1)^2}\right\}} & \textit{if} \quad 1 > G_1 - G_2 \geq 0 \\ \frac{1}{2} e^{\left\{\frac{-(G_1-G_2)^2}{1-(G_1-G_2)^2}\right\}} & \textit{if} \quad 0 \geq G_1 - G_2 > -1 \\ 0 & \textit{if} \quad -1 \geq G_1 - G_2 \end{cases} \quad \textit{and } p_2 = 1 - p_1. \tag{11}$$

As in (7), the win probability might be zero–even for positive effort. Contrary to (7) it is $\mathbb{C}^1$. Notice that for $|G_1 - G_2| \leq 1$, p_1 is strictly monotonic. However, when $G_1 = G_2$ the derivative vanishes. So, this CSF is not regular. Define $U_1 = G_1 + \sqrt{(-\ln x)/(1 - \ln x)} - a$, where $(x, a) = (2\theta, 0)$ if $0 < \theta \leq 1/2$ and $(x, a) = (2\theta - 1, 1)$ if $1/2 < \theta \leq 1$.[6] *Let $U_2 = G_2$. Notice that SC holds. Straightforward manipulations show that this pair of utility functions rationalizes the smooth difference-form contest in (11).*

2.2 More than Two Contenders

In the case of three contenders the previous argument does not yield microfoundations for the class of contest success functions axiomatized by Skaperdas. There are two reasons for that which are explained in Propositions 2.2 and 2.3 below. The first result shows that it might be impossible to partition Θ in n non-empty intervals which is what is implied by the SC assumption. The second result shows that even if such a partition is assumed, the win probability of a given contestant might not be responsive to changes in the efforts of all other contenders, as in (5). First, we need the following assumption:

Assumption 2: $U_i(\theta, G_i)$ are continuous and $U_i(\theta, G_i) \to \infty$ when $G_i \to \infty$, $i = 1, 2, ..., n$.

This assumption (A.2 in the sequel) is fulfilled in the payoff functions used in Examples 1 and 2 above. In the case of Example 3 and 4 this assumption is fulfilled if $f_i(G_i) \to \infty$ when $G_i \to \infty$ which is the case in (1). Thus, it looks like a pretty harmless assumption. However, its consequences are not.

Proposition 2.2 *Under Assumption A.2, and when $n = 3$, Skaperdas' class of CSFs (5) cannot be obtained from payoff maximization when SC holds for players 1 and 2.*

Proof. Let $U_3'(G_3) = max\, U_3(\theta, G_3)$, $\theta \in \Theta$. The maximum exists and varies continuously with G_3 (by Berge's maximum theorem). By taking G_1 and G_2 large enough, say G_1' and G_2', the property (SC) and A.2 imply that there is a $\bar{\theta}$, such that

$$\begin{aligned} U_1(\theta, G_1') &> U_3'(G_3), \forall \theta \in [0, \bar{\theta}) \\ U_2(\theta, G_2') &> U_3'(G_3), \forall \theta \in (\bar{\theta}, 1]. \end{aligned}$$

[6] One might also define $U_1 = G_1 + 1$, when $\theta = 0$.

Thus, player 3 never obtains the prize. Moreover, because $U_3'(\cdot)$ is continuous in G_3, small variations in G_3 do not affect neither p_1 nor p_2, thus the result. ■

Similar results can be obtained for $n > 3$ by extending suitably the SC condition. However, as the next result shows, even weak generalizations of the SC condition cause lack of rationalizability of Skaperdas' class of CSFs (5) even if Assumption A.2 is not postulated. First let us consider the following generalization of SC.

Definition 2.3 *A collection of payoff functions $U_i(\theta, G_i)$ $i = 1, 2, ..., n$ satisfies the Generalized Single Crossing (GSC) condition when for all $\mathbf{G}$, there is a permutation in the set of agents $i, j, ..., k$ and a partition of Θ, $(\Theta_i, \Theta_{ij}, \Theta_j, ...\Theta_r, \Theta_{rk}, \Theta_k)$ such that $\Theta_s = \{\theta \mid U_s(\theta, G_s) > U_r(\theta, G_r),\ \forall r \neq s\}$, $s = i, j, ..., k$, $\Theta_{sh} = \{\theta \mid U_s(\theta, G_s) = U_h(\theta, G_h)\}$, with all Θ_{sh} singletons for $s, h = i, j, ..., k$.*

Notice that, when $n = 2$, GSC is implied by SC.

Proposition 2.3 *When the utility functions satisfy the GSC and are continuous, Skaperdas' class of CSFs (5) cannot be obtained from payoff maximization.*

Proof. We will prove the result for $n = 3$. The extension to $n > 3$ is straightforward. Without loss of generality let the permutation of N be $1, 2, 3$. Then,

$$\begin{aligned} U_1(\theta, G_1) &> U_j(\theta, G_j), j = 2, 3, \forall \theta \in \Theta_1 \\ U_2(\theta, G_2) &> U_j(\theta, G_j), j = 1, 3, \forall \theta \in \Theta_2 \\ U_3(\theta, G_3) &> U_j(\theta, G_j), j = 1, 2, \forall \theta \in \Theta_3. \end{aligned}$$

Thus, $p_1 = \textit{length}\ \Theta_1$, $p_2 = \textit{length}\ \Theta_2$ and $p_3 = \textit{length}\ \Theta_3$. It is clear that p_1 (resp. p_3) does not depend on G_3 (resp. G_1) for small variations of this variable. Thus, the required functional form can not be obtained in this case. ■

Notice that the results in Propositions 2.2 and 2.3 do not depend on $F(\theta)$ being uniform. The reason is that given an interval $[a, b]$ different distributions assign different probability mass $F(b) - F(a)$. However, in these results it is crucial that the delimiters a and b do not depend on the effort of one contender.

Albeit this difficulty in deriving the class of functions axiomatized by Skaperdas for more than three contestants, contestants' uncertainty about the type of the contest administrator seems to be a reasonable approach to CSFs. Therefore, it is an important research program to find contest success functions that are rationalizable according to Definition 2.2 above and to work out the consequences of these new functional forms on equilibrium, comparative statics, etc. We show now that although this route appears to be promising, it is not free from difficulties. We will work out two examples and we will show that in both cases:[7]

[7] This may also happen for $n = 2$, see Che and Gale (2000).

- Contest success functions are neither differentiable nor concave.

- Despite the symmetric nature of basic data, no symmetric Nash equilibrium exists.

Example 6 *Let $U_1(\theta, G_1) = (1-\theta)G_1$, $U_2(\theta, G_2) = G_2 2/3$ and $U_3(\theta, G_3) = \theta G_3$. Notice that if $G_1 = G_2 = G_3$, $p_1 = p_2 = p_3 = 1/3$. We will compute the best reply of contestant 1.*
If $G_2 2/3 < G_3$ we have two cases: First, if $G_1 < G_2 2/3$, then $p_1 = 0$. Second, if $G_1 \geq G_2 2/3$, then

$$p_1 = \begin{cases} (G_1 - G_2 2/3)/G_1 & \text{if } G_1 < (G_3 G_2 2/3)/(G_3 - G_2 2/3) \\ G_1/(G_1 + G_3) & \text{otherwise.} \end{cases}$$

If $G_2 2/3 \geq G_3$ we have again two cases

$$p_1 = \begin{cases} 0 & \text{if } G_1 < G_2 2/3 \\ (G_1 - G_2 2/3)/G_1 & \text{otherwise.} \end{cases}$$

In a symmetric equilibrium $\hat{\mathbf{G}}$ we have $G_1 \geq G_2 2/3$ and $G_1 < (G_3 G_2 2/3)/(G_1 - G_2 2/3)$. Thus, contender 1 maximizes $V(G_1 - G_2 2/3)/G_1 - G_1$, where V is the value of the prize. If the equilibrium is symmetric it must be at positive level of effort. Thus, the maximum is interior and the first order condition yields the best reply, namely $G_1 = (V G_2 2/3)^{1/2}$.
For $\hat{G}_1 = \hat{G}_2$ this yields $\hat{G}_1 = V\,2/3$. We now have to make sure that this payoff is larger than the payoff associated to $G_1 = 0$ (yielding a p_1 and a payoff equal to 0). This is equivalent to $\hat{G}_2 \leq V\,27/100$, which contradicts $\hat{G}_1 = \hat{G}_2 = V\,2/3$.

Example 6 can be criticized because the existence of endpoints (0 and 1) makes contenders non-symmetric. For instance, if $G_1 = G_2 = G_3$, a variation of G_2 affects p_1 and p_3, but a variation of G_1 only affects p_2. Thus, we now adapt the model of Salop (1979) of a circular city to our framework. Here symmetry of the effects of efforts is restored since each contender affects the win probability of all other contenders.

Example 7 *Suppose that three contenders are symmetrically distributed at locations $(l_1, l_2, l_3) = (0, 1/3, 2/3)$ on the unit circle, which is now our set of states of the world. Assume that $U_i(\theta, G_i) = u - k\,|l_i - \theta| + G_i^\alpha$, where u, k and α are positive scalars and $\alpha \leq 1$. Notice that when effort levels are similar, the relevant competition is pairwise: 1 competes only with 2 (resp. 3) for $\theta \in [0, 1/3]$ (resp. $\theta \in [2/3, 1]$), while only 2 and 3 compete for $\theta \in [1/3, 2/3]$. Thus, the state of the world for which, given efforts, the decider is indifferent between candidates 1 and 2 is*

$$\theta_{12} = \frac{1}{6} + \frac{1}{2k}\left(G_1^\alpha - G_2^\alpha\right).$$

A similar reasoning in the case of 1 and 3 yields

$$\theta_{13} = \frac{5}{6} + \frac{1}{2k}\left(G_3^\alpha - G_1^\alpha\right).$$

This implies that $p_1 = \theta_{12} + 1 - \theta_{13}$. In order to determine the CSF in general, suppose without loss of generality that $G_1 \geq G_2 \geq G_3$. If $G_1^\alpha - G_3^\alpha \geq k/3$, then we obtain a generalized version *of Che and Gale's 2-player contest (given in (7))*

$$p_1 = \min\left\{\frac{1}{2} + \frac{1}{k}(G_1^\alpha - G_2^\alpha), 1\right\},\ p_2 = 1 - p_1 \text{ and } p_3 = 0;$$

and otherwise

$$p_i = \frac{1}{3} + \frac{1}{2k}\left(2G_i^\alpha - G_j^\alpha - G_k^\alpha\right),\ \text{for } i = 1, 2, 3 \text{ and } i \neq j, k.$$

Assume $\alpha < 1$. A symmetric equilibrium $\hat{\mathbf{G}}$ requires that $\hat{G}_1$ maximizes 1's payoffs, given $\hat{G}_2$ and $\hat{G}_3$ and that $\hat{G}_1 = \hat{G}_2 = \hat{G}_3$. Thus, $\hat{G}_1$ maximizes $p_1V - G_1$, where V is the value of the prize. If the maximum is interior, $\hat{G}_1 = (\alpha V/k)^{1/(1-\alpha)}$. Thus if payoffs for 1 for this value of efforts are negative, 0 effort is the best reply and no symmetric equilibrium exists.

Note that it is straightforward to extend the last example to more than three contestants. The so derived CSF can be seen as an extension of Che and Gale's linear difference-form (given in (7)) to more than two contestants (see (9)).

2.3 An Alternative Notion of Rationalizability

The simple setting considered so far might be adapted in several ways in order to yield Skaperdas' class of CSFs (5) when there are more than three contestants: (i) The type of the contest administrator might be multidimensional; (ii) the distribution function might be non-uniform; (iii) the rationalizability notion might be different. Given that (i) and (ii) have already be explored (e.g. in Hillman and Riley (1988)), we pursue now (iii).

Consider a situation where a contest administrator cares not only about the effort of the winner of the contest but also about the effort of others. One might think of the promotion of workers in a firm based on their performance or of firms competing for a research prize based on R&D investment which generates new knowledge. In such a situation the type of the decider represents how much he values the effort of a particular contestant relative to the others. We present an example yielding a special case of Skaperdas' class of CSFs (5) for three contestants. This example can easily be extended to more agents and to more general effectivity functions.

Example 8 *Let $U_1 = (1-\theta)G_1 - \theta(G_2 + G_3)$, $U_3 = \theta G_3 - (1-\theta)(G_1 + G_2)$ and normalize $U_2 = 0$. We have that*

$$\begin{aligned} U_1 &\geq U_2 \Leftrightarrow \theta \leq \theta_{12} \equiv \frac{G_1}{G_1 + G_2 + G_3}, \\ U_1 &\geq U_3 \Leftrightarrow \theta \leq \theta_{13} \equiv \frac{2G_1 + G_2}{2(G_1 + G_2 + G_3)}, \\ U_3 &\geq U_2 \Leftrightarrow \theta \geq \theta_{23} \equiv \frac{G_1 + G_2}{G_1 + G_2 + G_3}. \end{aligned}$$

This yields

$$p_1 = \theta_{12} = \frac{G_1}{G_1 + G_2 + G_3},\ p_2 = \theta_{23} - \theta_{12} = \frac{G_2}{G_1 + G_2 + G_3} \text{ and } p_3 = 1 - \theta_{23} = \frac{G_3}{G_1 + G_2 + G_3}.$$

3 Contest Success Functions as Sharing Rules

Inspired by the second interpretation of the outcome of a contest as shares of the prize we establish now a connection to bargaining and claims problems. This can be interpreted as contestants bargaining over all possible assignments of win probabilities or over shares. If no agreement is reached, all win probabilities are zero. In our approach, a variation in effort only affects the share of the prize. A more complete theory might consider that the size of the prize is also affected. This allows taking into account the opportunity cost of effort (see Anbarci, Skaperdas and Syropoulos (2002) and Garfinkel and Skaperdas (2007)).

3.1 'Classical' Bargaining

A contest problem is a vector $\boldsymbol{f}(\boldsymbol{G}) = (f_1(G_1), ..., f_n(G_n))$ with at least two entries each of which strictly positive.[8] Since we consider a fixed vector of efforts $\boldsymbol{G}$, we will simply use the notation f_i instead of $f_i(G_i)$ and $\boldsymbol{f}$ instead of $\boldsymbol{f}(\boldsymbol{G})$. An allocation in a contest problem is a n-tuple $\boldsymbol{p} = (p_1, ..., p_n) \in \mathbb{R}^n$ with $0 \leq p_i \leq 1$ and $\sum_{i=1}^n p_i = 1$. A contest success function is a function that assigns a unique allocation to each contest problem.

We define now a bargaining problem associated with each contest problem. A bargaining problem is a pair $(S, \boldsymbol{d})$ where $S \subset \mathbb{R}^n$ is a compact convex set, $\boldsymbol{d} \in S$ and there exists $\boldsymbol{s} \in S$ such that $s_i > d_i, i = 1, ..., n$. The set S, the feasible set, consists of all utility vectors attainable by the n contestants through unanimous agreement. The disagreement point $\boldsymbol{d}$ is the utility vector obtained if there is no agreement. In our context it seems natural to define

$$S = \left\{ \boldsymbol{p} \in \mathbb{R}^n \,\middle|\, 0 \leq p_i \leq 1 \text{ and } \sum_{i=1}^n p_i \leq 1 \right\} \text{ and } \boldsymbol{d} = \boldsymbol{0}.$$

A bargaining solution is a function ψ assigning to each bargaining problem $(S, \boldsymbol{d})$ a unique element in S. We are interested in the weighted Nash solution with weights $\boldsymbol{\alpha}$.

Definition 3.1 *Let $\alpha_i > 0$ for all $i = 1, ..., n$. The α-asymmetric Nash solution is defined as*

$$\psi^\alpha = \arg\max_{p \in S} \Pi_{i=1}^n (p_i - d_i)^{\alpha_i}.$$

In this framework it is natural that the effort of a contestant determines his bargaining position. Suppose that efforts affect the exponents of the weighted Nash bargaining solution as defined above. For simplicity, let $\boldsymbol{\alpha} = \boldsymbol{f}$. The next result is parallel to one obtained by Dagan and Volij (1993) in a different framework.[9]

Proposition 3.1 *The α-asymmetric Nash solution for $\boldsymbol{\alpha} = \boldsymbol{f}$ induces Skaperdas' class of CSFs (5).*

[8] If $f_i(G_i) = 0$ for some contestant i, assign zero win probability to this agent and consider the reduced vector in which the entry corresponding to agent i is missing.

[9] In the literature the weighted Nash solution has also been interpreted as a decider maximizing a payoff function. This is another example of the connections between the approaches taken in Section 2 and here.

Proof. Let $\boldsymbol{f}$ be a contest problem, consider the associated bargaining problem and let $\psi^{\alpha} = \boldsymbol{p}^*$. The first-order conditions of the maximization problem defining the asymmetric Nash solution with $\boldsymbol{d} = \boldsymbol{0}$ imply that

$$p_j^* = \frac{\alpha_j}{\alpha_i} p_i^*, \text{ for all } i, j \in N.$$

Given the Pareto optimality of the asymmetric Nash solution we have that $\sum_{j=1}^{n} p_j = 1$. This implies $p_i^* = \alpha_i / \sum_{j=1}^{n} \alpha_j$. ■

Since the preceding result sheets light on the class of contest success functions axiomatized by Skaperdas from a very different angle than the approach of the previous section, it is of interest in its own right. However, it also opens the door to understand CSFs as the outcome of strategic bargaining models based on Rubinstein's alternating offers game. Since it is well known that under certain conditions the asymmetric Nash solution can be supported by such a game, it follows that alternative conditions thought to reflect reasonable properties of underlying institutional details can yield alternative CSFs.

3.2 Bargaining with Claims

It might seem odd that, while the effort vector $\boldsymbol{f}$ defines a contest problem, this information is not used in the description of the associated bargaining problem $(S, \boldsymbol{d})$. If we want to incorporate this information in the description of the problem, the relevant framework is the one of bargaining problems with claims (Chun and Thomson (1992)).[10] A contest bargaining problem is then a triple $(S, \boldsymbol{d}, \boldsymbol{f})$ with the following interpretation: Contestants bargain over all possible assignments of win probabilities. The contestants' effectivity functions translate individual effort into an 'aspiration point' $\boldsymbol{f}$. Thus, $\boldsymbol{f}(\boldsymbol{G})$ measures the social merit that society or the decider awards to the vector of efforts $\boldsymbol{G}$.

If no unanimous agreement is reached, all win probabilities are zero. A contest bargaining solution ϕ assigns to each such triple a unique element in S. A maximal point $\boldsymbol{p}$ of S is a point such that $\sum_{j=1}^{n} p_j = 1$. The proportional solution is defined as follows.

Definition 3.2 *The proportional solution ϕ^P is defined as the maximal point $\boldsymbol{p}$ of S on the segment connecting the disagreement point $\boldsymbol{d}$ and the aspiration point $\boldsymbol{f}$.*

Proposition 3.2 *The proportional solution induces Skaperdas' class of CSFs (5).*

Proof. Let $\boldsymbol{f}$ be a contest problem, consider the associated bargaining problem with claims and let $\phi^P = \boldsymbol{p}^*$. The line which passes through the two points $\boldsymbol{d}$ and $\boldsymbol{f}$ is the set of vectors $\boldsymbol{x}$ of the form $\boldsymbol{x} = (1-t)\boldsymbol{d} + t\boldsymbol{f}$, with $t \in \mathbb{R}$. Since $\boldsymbol{d} = \boldsymbol{0}$, $\boldsymbol{x} = t\boldsymbol{f}$. Given that $\boldsymbol{p}^*$ is a maximal point, we have that $t = 1/\sum_{j=1}^{n} f_j$. This implies $p_i^* = f_i / \sum_{j=1}^{n} f_j$. ■

[10] Notice that a contest problem is not equivalent to a bargaining problem with claims. One important difference is that in contest problems there is no upper bound on individual effort levels, that is, $\boldsymbol{f}$.

The richer description of bargaining problems with claims has allowed to define an alternative solution that also explicitly builds on the aspiration point $\boldsymbol{f}$. Bossert (1993) analyzes the claim-egalitarian solution. For the purpose of the next proposition it suffices to consider the case of two contestants. The following definition is adapted to our context because in contest problems there is no upper bound on individual effort levels, that is, $\boldsymbol{f}$.

Definition 3.3 *Let $n = 2$ and $f_h \geq f_l$, $h, l = 1, 2$. The claim-egalitarian solution ϕ^E is defined as the maximal point $\boldsymbol{p}$ of S such that $f_h - p_h = f_l - p_l$ if $f_h - f_l \leq 1$. Otherwise $p_h = 1$ and $p_l = 0$.*

The claim-egalitarian solution selects a point on the Pareto frontier of S such that the loss of each contestant compared with his aspiration level is the same for all agents (if such a point exists). This is an egalitarian solution in the sense that the absolute amount each agent has to give up is equalized across contestants. The next proposition says that this idea is the same as saying that only differences in effort matter.

Proposition 3.3 *For $n = 2$, the claim-egalitarian solution induces a generalization of Che and Gale's difference-form contest success function, that is,*

$$\phi_i^E = p_i^{CG'}(\boldsymbol{G}) = \max\left\{\min\left\{\frac{1}{2} + \frac{1}{2}(f_i - f_j), 1\right\}, 0\right\} \text{ for } i = 1, 2.$$

Proof. The fact that if $|f_i - f_j| \geq 1$ then $\phi_i^E = p_i^{CG'}(\boldsymbol{G})$ is obvious. Suppose $|f_i - f_j| \leq 1$. Since $p_j = 1 - p_i$, we have $f_i - p_i = f_j - (1 - p_i)$. Rearranging yields the desired expression. ■

Notice that when $f_i(G_i) = 2\sigma G_i$ for $i = 1, 2$ where σ is a positive scalar, we obtain (7), the class of linear difference-form functions analyzed in Che and Gale (2000). Notice that it is straightforward to extend the last result to more than three contestants (see (9)).[11] Interestingly, this recommendation differs in the minimal effort necessary to obtain a non-zero share and in the marginal effect of effort from the one based on Example 7.

Definition 3.3 equalizes losses based on absolute claims. This creates the 'kink' and the non-responsiveness of Che and Gale's CSF to effort when the difference in aspiration levels is high enough. Considering relative claims this can be avoided. Notice that f_i/f_h (for $i = 1, ..., n$) indicates the percentage contestant i's aspiration level f_i constitutes of the highest level f_h.

Definition 3.4 *Let $n = 2$ and w.l.o.g. denote $f_h = \max\{f_1, f_2\}$. The relative claim-egalitarian solution ϕ^{RE} is defined as the maximal point $\boldsymbol{p}$ of S such that $f_1/f_h - p_1 = f_2/f_h - p_2$.*

[11] For $n = 3$ and $f_1 \geq f_2 \geq f_3$, it is natural to require the following. If $f_1 - f_2 \geq 1$, then $p_1 = 1$ and $p_2 = p_3 = 0$. If $f_1 - f_3 \geq 1 > f_1 - f_2$, then ϕ^E is the maximal point $\boldsymbol{p}$ of S such that $p_3 = 0$ and $f_1 - p_1 = f_2 - p_2$. Lastly, when $f_1 - f_3 < 1$, then ϕ^E is the maximal point $\boldsymbol{p}$ of S such that $f_1 - p_1 = f_2 - p_2 = f_3 - p_3$.

The relative claim-egalitarian solution selects a point on the Pareto frontier of S such that the loss of each contestant compared with this 'relative claim point' is the same for all agents. The next proposition relates this idea to the serial CSF.[12]

Proposition 3.4 *For $n = 2$ and $f_1 \geq f_2$, the relative claim-egalitarian solution induces a generalization of the serial contest success function, that is,*

$$\phi_i^{RE} = p_i^{S'}(\boldsymbol{G}) = \sum_{j=i}^{2} \frac{f_j - f_{j+1}}{j \cdot f_h} \textit{ for } i = 1, 2 \textit{ and } f_3 = 0.$$

Proof. W.l.o.g. assume $f_1 \geq f_2$. We have that $1 - p_1 = f_2/f_1 - p_2 = f_2/f_1 - 1 + p_1$. This can be rewritten as $p_1 = 1 - f_2/(2f_1) = (f_1 - f_2)/f_1 + f_2/(2f_1)$. Since ϕ^{RE} must be a maximal point, we obtain $p_2 = f_2/(2f_1)$. ■

4 Concluding Remarks

In line with two prominent interpretations of the outcome of contests, this paper has investigated foundations for prominent contest success functions based on two different approaches. The first analyzes the implications of contestants' incomplete information concerning the 'type' of the contest administrator. The second understands CSFs as sharing rules and makes a connection to bargaining and claims problems. Both approaches provide foundations for popular contest success functions and guidelines for the definition of new ones. The results of this paper suggest two lines for future research on contest success functions.

On the normative side, the implications of linking the problem of assigning win probabilities in contests to bargaining, claims and taxation problems are twofold. On one hand, this connection might yield an improved understanding of existing contest success functions, while, on the other hand, it suggests guidelines for the definition of new ones. As for the former, for instance, proportionality principles have been defended at least since the philosophers of ancient Greece. Therefore, it seems possible to obtain different characterizations of the class of contest success functions axiomatized by Skaperdas using ideas of characterizations of proportionality stressed in these related problems.[13] As for the latter, different normative principles might lead to the formulation of different classes of contest success functions. A case in point here is the claim-egalitarian solution that gives a recommendation how to extend the difference-form functions analyzed in Che and Gale (2000) to more than two contestants.

[12] This reasoning can easily be extended to more contestants. However, the requirement that $f_i/f_h - p_i = f_{i+1}/f_h - p_{i+1}$ for all $i = 1, ..., n-1$ does not always yield well defined win probabilities. A way out is the following. Consider an ordered vector $f_1 \geq f_2 \geq ... \geq f_n$ and rescale the 'relative claim point' in order to make the pairwise comparisons $f_i/(i \cdot f_h) - p_i = f_{i+1}/(i \cdot f_h) - p_{i+1}$ for all $i = 1, ..., n-1$. This coincides with Definition 3.4 when there are two agents and yields a generalization of the serial contest success function for any number of contestants.

[13] Note that the class of problems in which win probabilities are assigned has a particularly simple structure. This implies that a characterization of a solution for a larger class of problems does not need to characterize a solution for contests.

On the positive side, the implications for future research parallel the normative ones. On one hand, strategic foundations of solution concepts in bargaining, claims and taxation problems that can be related to popular contest success functions might yield rationales for the latter. An example is to link contests with the Bilateral Principle that has proved a fruitful way to incorporate Luce's Choice Axiom into game theory. Dagan *et al.* (1997) have provided a game form capturing the non-cooperative dimension of the consistency property of bankruptcy rules.[14] An adaptation of their result in our framework shows that Skaperdas' class of CSFs (5), can be supported by a pure strategy subgame perfect equilibrium of a certain non-cooperative game.

On the other hand, by incorporating realistic details of contest situations novel contest success functions can be derived. Examples are the recommendation of the circular model in Example 7 how to extend Che and Gale's difference-form function to more than two contestants or the effects of modifying Rubinstein's alternating offers bargaining game.

References

[1] Alcalde, José and Matthias Dahm (2007), "Tullock and Hirshleifer: A Meeting of the Minds," *Review of Economic Design* **11**, 101–124.

[2] Anbarci, Nejat, Stergios Skaperdas and Constantinos Syropoulos (2002), "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects," *Journal of Economic Theory* **106**, 1–16.

[3] D'Aspremont, Claude, Jean Jaskold Gabszewicz and Jacques-François Thisse (1979), "On Hotelling's Stability in Competition," *Econometrica* **47**, 1145–1150.

[4] Baik, Kyung Hwan (1998), "Difference-Form Contest Success Functions and Effort Levels in Contests," *European Journal of Political Economy* **14**, 685–701.

[5] Baye, Michael R. and Heidrun C. Hoppe (2003), "The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games," *Games and Economic Behavior* **44**, 217–226.

[6] Bossert, Walter (1993), "An Alternative Solution to Bargaining Problems with Claims", *Mathematical Social Sciences* **25**, 205–220.

[7] Che, Yeon-Koo and Ian Gale (2000), "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior* **30**, 22–43.

[8] Chun, Youngsub and William Thomson (1992), "Bargaining Problems with Claims", *Mathematical Social Sciences* **24**, 19–33.

[14] Notice that a contest problem is not equivalent to a bankruptcy problem in which the estate is equal to one, since in contest problems there is no lower bound on the sum of individual effort levels, that is, $\sum_{j=1}^{n} f_j$.

[9] Clark, Derek and Christian Riis (1998), “Contest Success Functions: an Extension,” *Economic Theory* **11**, 201–204.

[10] Corchón, Luis (2000), “The Allocative Effects of Rent-Seeking,” *Journal of Public Economic Theory* **2** (4), 483–491.

[11] Dagan, Nir, Roberto Serrano and Oscar Volij (1997), “A noncooperative view of consistent bankruptcy rules,” *Games and Economic Behavior* **18**, 55–72.

[12] Dagan, Nir and Oscar Volij (1993), “The bankruptcy problem: a cooperative bargaining approach,” *Mathematical Social Sciences* **26**, 287–297.

[13] Dixit, Avinash K. (1987), “Strategic Behavior in Contests,” *The American Economic Review* **77** (5), 891–898.

[14] Epstein, Gil S. and Shmuel Nitzan (2006), “The Politics of Randomness,” *Social Choice and Welfare* **27**, 423–433.

[15] Esteban, Joan and Jozsef Sákovics (2006), “A Theory of Agreements in the Shadow of Conflict,” unpublished manuscript.

[16] Fullerton, Richard L. and R. Preston McAfee (1999), “Auctioning Entry into Tournaments”, *Journal of Political Economy* **107**, 573–605.

[17] Garfinkel, Michelle R. and Stergios Skaperdas (2007), “Economics of Conflict: An Overview,” in T. Sandler and K. Hartley (eds.), Handbook of Defense Economics, Vol. 2 chapter 22, Amsterdam: North Holland, 649–709.

[18] Gradstein, Mark (1998), “Optimal Contest Design: Volume and Timing of Rent Seeking in Contests,” *European Journal of Political Economy* **14** (4), 575–585.

[19] Gradstein, Mark, (1995), “Intensity of Competition, Entry and Entry Deterrence in Rent-Seeking Contests”, *Economics and Politics* **7**, 79-91.

[20] Hillman, Arye L. and John G. Riley (1988), “Politically Contestable Rents and Transfers,” UCLA Department of Economics Working Paper #452.

[21] Hirshleifer, Jack (1989), “Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success,” *Public Choice* **63**, 101–12.

[22] Hirshleifer, Jack and John Riley (1992). *The Analytics of Uncertainty and Information.* Cambridge University Press, New York.

[23] Hotelling, Harold (1929), “Stability in Competition,” *Economic Journal* **39**, 41–57.

[24] Jia, Hao (2007), “A Stochastic Derivation of Contest Success Functions,” *Public Choice*, forthcoming.

[25] Konrad, Kai A. (2007), "Strategy in Contests – An Introduction," WZB Discussion Paper SP II 2007–01.

[26] Lazear, Edward P. and Shervin Rosen (1981), "Rank-Order Tournaments and Optimum Labor Contracts," *The Journal of Political Economy*, **89** (5), 841–864.

[27] McFadden, Daniel (1974), "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka (ed.) *Frontiers in Econometrics*, 105–142. Academic Press, New York.

[28] Nitzan, Shmuel (1994), "Modelling Rent-Seeking Contests," *European Journal of Political Economy* **10**, 41–60.

[29] Rubinstein, Ariel (1985), "A Bargaining Model with Incomplete Information About Time Preferences," *Econometrica* **53** (5), 1151–1172.

[30] Salop, Steven C. 1979. "Monopolistic Competition with Outside Goods," *Bell Journal of Economics* **10**, 141–156.

[31] Skaperdas, Stergios (1996), "Contest Success Functions," *Economic Theory* **7**, 283–290.

[32] Szidarovszky, Ferenc and Koji Okuguchi (1997), "On the Existence and Uniqueness of Pure Nash Equilibrium in Rent-Seeking Games", *Games and Economic Behavior*, **18**, 135–140.

[33] Tullock, Gordon (1980). Efficient rent seeking. In J. Buchanan, R. Tollison, and G. Tullock (Eds.), Toward a theory of the rentseeking society. College Station: Texas A&M University Press, 97–112.

[34] Wärneryd, Karl (1998), "Distributional conflict and jurisdictional organization," *Journal of Public Economics* **69**, 435–450.