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Market Competition and Lower Tier Incentives

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Abstract

The relationship between competition and performance–related pay has been analyzed in single–principal–single–agent models. While this approach yields good predictions for managerial pay schemes, the predictions fail to apply for employees at lower tiers of a firm's hierarchy. In this paper, a principal–multi–agent model of incentive pay is developed which makes it possible to analyze the effect of changes in the competitiveness of markets on lower tier incentive payment schemes. The results explain why the payment schemes of agents located at low and mid tiers are less sensitive to changes in competition when aggregated firm data is used.

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1 Introduction

During the last two decades the relationship between competition and performance–related–pay (PRP) has been investigated both from the theoretical and the empirical perspective. However, most of the literature has focused on the incentive pay of CEOs and little attention has been paid to lower layer employees. An exception in the empirical literature is Cuñat and Guadalupe (2005), who also analyze the effect of changes in competition on worker compensation. However, in the theoretical literature, attention has been paid exclusively to the question of how the design of adequate *individual* incentives is affected by changes in the competitive environment. This has given valuable insights into the relation between a firm's CEO and its owners. However, for low-tier employees the individual–based approach is expected to be too narrow and to give only incomplete insights. Therefore, the purpose of this paper is to develop a simple model which yields testable predictions of how changes in competition affect incentive pay at different layers of a firm's hierarchy.

Most of the results in the empirical literature on executive compensation are summarized in Murphy (1999). The main results are that (1) incentive compensation increases with firm size, though the relation between CEO pay and firm size has weakened over time, (2) pay-performance sensitivities vary across industries, and are particularly low in regulated industries, (3) pay-performance sensitivities have become larger in recent years because of the trend towards more competition due to the spread of information technologies, the reduction of barriers to entry, waves of deregulation and the reduction in transport costs. The theoretical literature mainly supports these findings. Schmidt (1997) and Raith (2003) analyze a two–stage game in which firms consisting of a principal and an agent compete in an imperfect market. The principal uses incentive payments to incite agents to choose an unobservable effort level to reduce production costs. They show that there are two counteracting effects on incentive provision when competition increases. On the one hand, there is a business–stealing effect which induces firms to provide higher powered incentive schemes because with more competition any cost advantage more easily attracts business from rival firms. On the other hand, there is a scale effect which induces firms to reduce incentives because with more competition market shares decrease and firms have less to gain from possible cost advantages. Which of the two effects dominates is generally ambiguous, though Vives (forthcoming) finds that for most of the different competitive specifications used in the literature the first effect dominates the second one. Therefore, the theoretical prediction is that more competition increases the steepness of performance-related pay and yields higher effort.

Cuñat and Guadalupe (2005) analyze the effect of increasing competition not only on CEO pay but also on the compensation schemes of executives and workers in the UK in the late nineties. They confirm the results for CEO's and find that more competition increases the steepness of performance–related–pay in all layers. However, the effect weakens and is mostly not significant for lower layers in the firm. With the theoretical results concerning the relationship between competition and PRP at hand, this is a surprising result for two reasons. First, Burgess and Metcalfe (2000) show, also for a large sample of UK firms, that the adoption of

PRP is greater, especially for low-tier employees, in firms in environments with a low degree of competition than in those operating under a high degree of competition. For example, the number of firms that apply PRP for occupations like sales, skilled manual clerical/secretary and unskilled manual multiplies by eight, five, four and a half and four, respectively, while it only multiplies by three for managers. Second, there is a vast amount of empirical evidence to suggest that PRP at lower layers yields considerable productivity gains.[1] More competition should be expected to increase the pressure to realize potential productivity gains and therefore accelerate the adoption of PRP schemes. Consequently, without further theoretical explanation the results of Cuñat and Guadalupe (2005) can be seen to support the wide-spread view that executive compensation is not intended to be incentive efficient but used by CEOs as a means of self–enrichment where changes in competition serve as a pretext to increase PRP. As a result, the PRP of CEOs increases more than proportionally compared to the PRP of low–tier employees.[2]

The theoretical models on PRP do not specify the kind of agent the principal contracts. Thus, one might expect that the predictions can be applied to employees at any firm layer.[3] However, there are two reasons why the results cannot be applied directly to lower layers of the firm. First, at lower layers team work is a pervasive feature (c.f. Ichniowski and Shaw, 2003). The reward for participating in teams is likely to be some form of group–based pay or joint performance evaluation which has different characteristics to the individual–based schemes we find for CEOs. One major difference is that group–based incentives can be undermined by free riders. Though the free–rider problem can be mitigated by peer pressure (Kandel and Lazear, 1992), decentralized decision making (Baker, 1992) and long-term relationships (Che and Yoo, 2001) it usually results in lower powered incentive schemes than those of individual–based PRP.[4] Therefore, changes in competition should also be expected to have different impacts on low-tier PRP schemes compared to those of CEOs. Especially the question of how changes in competition affect free–riding should be addressed here. Second, in single–principal–single–agent models changes in competition have no employment effects but these effects might be considerable for agents at lower layers. For example, Griffith, Harrison and Macartney (2006) find that increased product market competition reduced unemployment in OECD countries in the 1980s and 1990s. Of course, changes in employment might also be expected to affect the structure of incentive schemes. Therefore, again, the results from single–principal–single–agent models should not be applied straightforwardly to incentive pay at lower layers.

The aim of this paper is to extend former models which analyze the relationship between performance–related–pay and market competition by explicitly including employment effects which changes in competition may cause. It thereby aims to close the gap between studies that analyze the effect of competition on individual–based PRP like Schmidt (1997) and Raith (2003)

[1] An excellent overview on the results of case studies, intra-industry analyses and national cross-industry studies is Ichniowski and Shaw (2003).

[2] See also Bolton et al. (2006) who develop a model that provides a different persepective on this view.

[3] See, for example, Cuñat and Gaudalupe (2005, p.1060).

[4] See, for example, Jensen and Murphy (1990), Holmstrom and Milgrom (1991, 1994) and Che and Yoo (2001).

and those that analyze group–based PRP from an organizational perspective that excludes explicit modelling of changes in the product market (see Holmstrom and Milgrom (1990), Varian (1990), Ramakrishnan and Thakor (1991), Itoh (1991, 1992, 1993) and Macho-Stadler and Pérez-Castrillo (1993) as examples for this approach). In this paper, product market competition and incentive payments are the result of a multi–stage game in which firms are modeled as a three–tier hierarchy. At the top tier the principal designs contracts of second and third tier employees. Contract design is complicated by the unobservability of subordinates' effort choices. That is, principals in each firm must solve moral hazard problems. It is assumed that the unique commonly observable and verifiable variable is firm output. Thus, all contracts must be based on this variable. The advantage of the model is that it allows to analyze the effect of changes in competition on different layers of a hierarchy using the *same* unit of incentive measure. This enables us to see if the hierarchical level is responsible for differences in the effect of changes in competition and not the specific choice of the incentive measure.

The main difference between the group–based approach and the single–principal single–agent models is that firms can react to changes in competition not only by changing employees' effort but also by adapting the number of employees to new market conditions. However, if the labor inputs of third-tier employees are substitutable and if PRP is group–based this will affect the free–riding of third–tier employees. Therefore, any change in competition which affects firm size will affect third–tier and mid–tier employees differently. For mid-tier employees we assume that labor inputs are more difficult to substitute and that free-riding therefore is a minor issue. Firm size (the number of employees) itself is determined by the costs of entry. The paper shows that this gives rise to two kinds of heterogeneities. First, PRP sensitivities differ between employees at different hierarchical levels of the firm, or more generally, between employees whose labor inputs are substitutable and whose performance cannot be measured individually and those whose labor inputs are either difficult to substitute or whose performance can be measured individually. Second, PRP sensitivities of third–tier agents differ between firms which react to changes in competition by changing their employees' efforts and those which also change employment, or between firms that operate in closed markets and those that operate in markets with low barriers to entry. Taken together, the results explain the discrepancy between empirical studies that use aggregate firm data and those that are based on individual or firm data. The former find that incentive pay–performance sensitivities have significantly increased in recent years for CEO's while they have remained nearly unchanged at lower tiers. This is because the aggregation of heterogeneous firm data means that different responses to changes in competition at lower tiers compensate each other while they work in the same direction at mid and top tiers. Firm or case studies however, confirm that firms in many circumstances have substantially increased PRP at lower tiers after a change in the competitive environment. However, the handicap of these studies is that their findings cannot be generalized. Therefore, one of the main insights of the model is that future studies should take into account the kind of heterogeneities which have been identified when firm and individual data is aggregated.

The paper is organized as follows. Section 2 presents the model. Section 3 shows the effort equilibrium values. Section 4 analyzes the effects of changes in the competitiveness of markets under alternative assumptions concerning the endogeneity of different variables. Finally, Section

5 concludes. All proofs are confined to the Appendix.

2 The model

1. The firm. There are n identical firms. Each firm i is represented as a three–tier hierarchy. The hierarchy consists of one principal (P_i) at the top of the hierarchy or at layer 1, one agent (A_{2i}) at layer 2 and a team of K_i agents (A_{3ik}, $k = 1, ..., K_i$) at layer 3. The agents in each firm jointly produce a marketable output q_i for the principal. The principal makes all entry, personnel (she chooses the value of K_i), and production decisions.

2. Production. The agents contribute to production by choosing two levels of effort, one that is observable to the principal and that is non-observable to her. Denote agent A_{2i}'s unobservable effort choice by $e_{2i} \in \Re^+$ and normalize the observable effort to unity. Similarly, agent A_{3ik}'s observable effort choice is denoted by $e_{3ik} \in \Re^+$ and his observable effort choice is normalized to unity. The general production technology is linear homogenous in capital and total labor output. The units of capital employed in production are also normalized to unity. The total worker's unobservable labor output is $e_{3i} = \sum_{k=1}^{K_i} e_{3ik}$ and their total observable labor output is K_i, that is, a worker's effort is perfectly substitutable, which implies that we assume that workers perform essentially the same tasks without any synergy effects. Supervisors produce "supervision", "control" and "coordination" of the workers' tasks. We assume that total labor output increases with more coordination for any given level of workers' labor output. Concretely, we assume the following continuous twice differential production function:

$$q_i = q_i(K_i, e_{2i}, e_{3i})$$

with $\partial q_i/\partial x > 0$, $\partial^2 q_i/\partial x^2 < 0$, $\partial^2 q_i/\partial x \partial y \geq 0$ where $x, y = K_i, e_{2i}, e_{3i}$, $x \neq y$. As a special case of this function

$$q_i = K_i^{\alpha_1} e_{2i}^{\alpha_2} \left(\sum_{k=1}^{K_i} e_{3ik} \right)^{\alpha_3},$$

is used, where $\alpha_1 > 0$, $\alpha_2 \geq 0$, and $\alpha_3 \geq 0$.[5] Notice that when there is no unobservable effort ($\alpha_2 = \alpha_3 = 0$), we have a traditional Cobb-Douglas production function (with capital inputs normalized to unity). To simplify the exposition of the results we make:

Definition 1 $\delta = \alpha_1 + \frac{\alpha_3}{2}$, $\varepsilon = \frac{\alpha_2 + \alpha_3}{2}$ and $A = \left[2^{\alpha_2} (2 - \alpha_3)^{-\alpha_2} \alpha_2^{\alpha_2 - \alpha_3} \alpha_3^{2\alpha_3} \right]^{\frac{1}{2(1-\varepsilon)}}$

Finally, the assumptions concerning the derivatives of q_i are fulfilled if the production function displays non-increasing returns to scale: $2\alpha_1 + \alpha_2 + \alpha_3 \leq 1$.

3. Utility and contracts. The fact that individual effort is non-observable to the principal,

[5] Similar team production functions have been applied by Beckmann (1977), or more recently, by Ferrall and Shearer (1999).

implies that she faces a moral hazard problem.[6] Furthermore, because individual performance measures are unavailable, this problem can only be solved by designing an appropriate group–incentive scheme. All agents are paid with an incentive scheme that has the form of a linear transfer. For agent A_{3ik} the transfer is $t_{3ik} = \beta_{3i}q_i + \gamma_{3i}$ and for agent A_{2i} the transfer is $t_{2i} = \beta_{2i}q_i + \gamma_{2i}$. That is, the contract includes a fixed payment and a variable payment that depends on output.[7] The transfers are chosen by the principal under the restriction that $\gamma_{2i} \geq \gamma_2$ and $\gamma_{3i} \geq \gamma_3$. γ_2 and γ_3 represent industry wage levels which are determined by the bargaining power of workers, supervisors and principals and includes compensation for observable effort. Furthermore, notice that contracts are identical. Because agents are identical this can be interpreted as a non-discrimination restriction imposed by workers.[8] Agent A_{3ik} chooses his effort to maximize his utility $U_{3ik} = t_{3i} - \frac{1}{2}e_{3ik}^2$. Similarly, agent A_{2i} chooses his effort to maximize $U_{2i} = t_{2i} - \frac{1}{2}e_{2i}^2$.[9] Notice that the supervisor's effort cost increases with the number of subordinates and the effort level chosen. Furthermore, marginal effort increases with the number of subordinates taking into account the fact that coordination and control of tasks become more difficult as the number of subordinates grows. The principal's objective is the maximization of the firm's net profits $\pi_i = pq_i - t_{2i} - K_i t_{3i} - F$, where p is the market price and F is fixed costs or entry costs.

4. Competition and demand. Competition is for a homogeneous good with inverse demand function $p = p(q/S)$ where q is aggregated output and S is a parameter that measures market size. An increase in S means that markets become larger. I assume that $p'(q) < 0$ and $p'(q) + p''(q)q/S < 0$.[10] n identical firms enter the market. For simplicity it is assumed that n is a continuous variable. With free entry firms enter until profits are zero. The objective of the paper is to analyze how incentive payments vary for different tiers under various endogeneity assumptions when market size changes.[11]

[6] Notice that unlike single–principal–single–agent models, we have not included a random term. However, with team production the moral hazard problem stems from the fact that the principal cannot identify individual effort from the observation of output though production is non–random (cf. Espinosa and Macho–Stadler, 2003).

[7] Lazear (2000) shows that this is a realistic assumption concerning the payment schemes applied for low–tier employees in enterprises in industrial sectors. McAfee and McMillan (1991) analyze the conditions under which it is optimal to use linear team contracts.

[8] In fact, β_{3i} is also determined by a bargaining process. However, this process mostly takes place inside the firm. Thus, while the fixed wage is assumed to be the same throughout the industry, performance-related-pay can differ between firms but not between employees of the same firm.

[9] A major difference in models that analyze managerial incentives is that agents are assumed to be risk–neutral. Consequently, there is no trade–off between risk and incentives like in Prendergast (2000) or Raith (2003). However, due to the assumption of limited liability and the fact that the fixed wage is determined exogenously, employees at lower tiers do not support much risk. Therefore, here, incentive pay is basically used to stimulate production and the trade–off is between productivity gains and wage costs.

[10] This is a standard assumption for the usual Cournot case of strategic substitutes competition (cf. Vives, 1999, chapter 4).

[11] Notice, that this objective is different to the papers of Schmidt (1997), Raith (2003), Schmutzler (2008) and Vives (forthcoming). These authors analyze the relationship between competition and PRP or R&D effort which requires to define first measures of competitiveness of markets. Vives (forthcoming), for example, considers that with free entry markets are more competitive if market size increases and entry costs fall, and with restricted entry markets are more competitive if the number of competitors increases. The same measures are applied in

5. Timing. Competition and contracting are defined to be a multi-stage game in which the sequence of events is as follows: At stage 1 the principals of all firms simultaneously decide the size of their firm (K_i) and the remunerations (t_{2i} and t_{3i}) of their employees. At stage 2, the supervisors choose their effort (e_{2i}). At stage 3 agents A_{3ik} observe their supervisor's effort and, simultaneously, make their own effort choice (e_{3ik}). Then, firms' output and market price are realized, the agents are paid and the principals obtain their profits.

3 Effort choice

The market game is solved by backward induction. At *stage 3* of the game, when agents A_{3ik} simultaneously choose their efforts, they know the effort choice of their supervisors and the number of workers in their firm. Thus, optimal effort is given as the solution to

$$\max_{e_{3ik}} U_{3ik} = \beta_{3i} q_i(K_i, e_{2i}, e_{3i}) + \gamma_{3i} - \frac{1}{2} e_{3ik}^2. \tag{1}$$

We get the following result:

Lemma 1 *There exists a unique symmetric subgame perfect Nash equilibrium in which each third tier agent in firm i chooses* $e_{3ik} = e_{3ik}(K_i, e_{2i}, \beta_{3i})$*. For total worker's unobservable labor output* $e_{3i} = K_i e_{3ik}$ *we have:* $de_{3i}/dK_i > 0$*,* $de_{3i}/de_{2i} > 0$ *and* $de_{3i}/d\beta_{3i} > 0$ *and for firm i's third stage equilibrium output* $q_i = q_i(K_i, e_{2i}, \beta_{3i})$ *we have:* $dq_i/dK_i > 0$*,* $dq_i/de_{2i} > 0$ *and* $dq_i/d\beta_{3i} > 0$*.*

For our example we obtain that for a given incentive rate β_{3i}, given supervisor effort e_{2i} and given firm size K_i in every subgame perfect Nash equilibrium agent A_{3ik} in each firm chooses effort

$$e_{3ik} = (\alpha_3 \beta_{3i} e_{2i}^{\alpha_2})^{\frac{1}{2-\alpha_3}} K_i^{\frac{\alpha_1+\alpha_3-1}{2-\alpha_3}}. \tag{2}$$

So, firm i's third stage equilibrium output is

$$q_i(K_i, e_{2i}, \beta_{3i}) = (\alpha_3 \beta_{3i})^{\frac{\alpha_3}{2-\alpha_3}} K_i^{\frac{2\alpha_1+\alpha_3}{2-\alpha_3}} e_{2i}^{\frac{2\alpha_2}{2-\alpha_3}} \tag{3}$$

and agent U_{3ik}'s utility is given by

$$U_{3ik} = \left(1 - \frac{\alpha_3}{2K_i}\right) \beta_{3i} q_i + \gamma_{3i}. \tag{4}$$

Equations (2) and (3) imply that A_{3ik}'s effort increases as the incentive rate increases which in turn increases firm i's output. From (4) we see that a worker's utility is strictly positive.

this paper. Furthermore, the consequences of changes in market size on PRP under restricted entry are analyzed to compare them to those under free entry. However, this is not to say that market size is an adequate measure of the competitiveness of markets under restricted entry.

This is the consequence of the limited liability assumption which implies that agents receive an informational rent. Furthermore, a worker's utility increases with the incentive rate of his contract. Thus, ceteris paribus, agents at tier 3 prefer higher powered incentives. Notice that workers cannot observe the effort choice of other workers in the firm. Thus, when deciding their own effort choice they face a free–rider problem. More effort yields higher output and therefore higher wages. However, while a worker only receives part of his own effort contribution through a wage increase, his wage also increases when other workers work more without any cost to him. If workers could cooperatively implement their effort choices within a coalition, the effort choice for each individual would be $K_i^{\frac{1}{2-\alpha_3}}$ higher than the effort choice in (2).[12] Therefore, as expected, the efficiency loss due to free–riding increases and, as can be seen from equation (2), individual effort decreases with firm size.

At *stage 2*, supervisor A_{2i} chooses his own effort. Formally, he solves the following program $[P_2]$:

$$\max_{e_{2i}} U_{2i} = \beta_{2i} q_i(K_i, e_{2i}, \beta_{3i}) + \gamma_{2i} - \frac{1}{2} e_{2i}^2. \tag{5}$$

We get the following result:

Lemma 2 *Suppose that* $d^2 q_i(K_i, e_{2i}, \beta_{3i})/dx^2 < 0$ *and* $d^2 q_i(K_i, e_{2i}, \beta_{3i})/dxdy \geq 0$ *with* $x, y = K_i, e_{2i}, \beta_{3i}$, $x \neq y$.[13] *Then, for second stage supervisor* A_{2i}*'s equilibrium effort* $e_{2i} = e_{2i}(K_i, \beta_{2i}, \beta_{3i})$ *we have:* $de_{2i}/dK_i > 0$, $de_{2i}/d\beta_{2i} > 0$ *and* $de_{2i}/d\beta_{3i} > 0$ *and for firm i's second stage equilibrium output* $q_i = q_i(K_i, \beta_{2i}, \beta_{3i})$ *we have:* $dq_i/dK_i > 0$, $dq_i/d\beta_{2i} > 0$ *and* $dq_i/d\beta_{3i} > 0$.

For our example we get:

$$e_{2i} = \left(\left(\frac{2\alpha_2}{2-\alpha_3} \right)^{\frac{2-\alpha_3}{2}} \beta_{2i}^{\frac{2-\alpha_3}{2}} (\alpha_3 \beta_{3i})^{\frac{\alpha_3}{2}} K_i^{\delta} \right)^{\frac{1}{2(1-\varepsilon)}} \tag{6}$$

Notice, that the first-order constraint is sufficient for a maximum under assumption 1. Now, firm i's second stage equilibrium output is

$$q_i(K_i, \beta_{2i}, \beta_{3i}) = A \left(\alpha_2^{\alpha_3} \alpha_3^{-\alpha_3} K_i^{2\delta} \beta_{2i}^{\alpha_2} \beta_{3i}^{\alpha_3} \right)^{\frac{1}{2(1-\varepsilon)}}. \tag{7}$$

and agents U_{2i}'s utility is given by

$$U_{2i} = \frac{2(1-\varepsilon)}{2-\alpha_3} \beta_{2i} q_i + \gamma_2. \tag{8}$$

From equation (7) we find that the principal can increase firm output (and profits) by employing more workers (by increasing K_i) or by increasing the PRP of workers and the supervisor

[12] In this case the coaltion would solve $\max_{e_{3i}} \sum_{k=1}^{K_i} U_{3ik} = K_i \beta_{3i} q_i(K_i, e_{2i}, e_{3i}) + K_i \gamma_{3i} - \frac{1}{2} \sum_{k=1}^{K_i} e_{3ik}^2$ and individual effort were $e_{3ik} = (\beta_{3i} \alpha_3 e_{2i}^{\alpha_2})^{\frac{1}{2-\alpha_3}} K_i^{\frac{\alpha_1+\alpha_3}{2-\alpha_3}}$.

[13] Sufficient conditions for this are that the original production function fulfills: $\partial^2 q_i/\partial e_{3i}^2 \cdot e_{3i} + \partial^2 q_i/\partial e_{3i} \partial K_i < 0$ and $sign(\partial^3 q_i/\partial x \partial y \partial z) = sign(\partial^2 q_i/\partial x \partial y) \cdot sign(\partial q_i/\partial z)$.

(by increasing β_{2i} and β_{3i}). However, from (4) and (8) we see that increasing PRP implies that workers and the supervisor obtain higher informational rents which reduces the principal's profits. The principal's problem is to choose the combination of PRP schemes (and firm size) that maximizes firm profits. In the next section this problem is analyzed under different endogeneity assumptions.[14]

4 Market Equilibrium

How firms react to changes in the competitiveness of markets depends on the time they have to react. In the short run, if a firm wants to increase production because market size has increased, for example, it may only want or be able to increase the production effort of its employees. However, in the mid term it might be more efficient to increase employment instead, or both. In the long run, an increase in production may increase industry profits and cause the entry of new firms in industries with low barriers to entry (cf. Raith, 2003). In this case, all decisions concerning employment must also take into account the reactions of potential entrants. In the remainder of this section, the effect of changes in market size on incentive rates under these different settings is analyzed. For this analysis our special case of the production function is applied, because without further assumptions on the production function a comparison of PRP under different endogeneity assumptions is not possible.

4.1 Given firm size and market structure

If firm size cannot be adjusted, at *stage 1*, the principal chooses the wage contracts of her employees to maximize firm profit. Formally, she solves the following program $[P_1]$:

$$\max_{\beta_{2i},\gamma_{2i},\beta_{3i},\gamma_{3i}} \pi_i = p(q/S)q_i - (\beta_{2i}q_i + \gamma_{2i}) - K_i(\beta_{3i}q_i + \gamma_{3i}) - F \quad (9)$$
$$\text{s.t. (2), (6), } U_{2i} \geq 0,\ U_{3ik} \geq 0, \gamma_{2i} \geq \gamma_2 \text{ and } \gamma_{3i} \geq \gamma_3.$$

The principals' maximization problem is subject to incentive, participation and limited liability constraints. The solution of $[P_1]$ yields:

[14]It should be noted, that the assumption that the supervisor and third-tier agents choose their efforts sequentially is not essential for the results derived in the following section. If they would choose their efforts simultaneously, the output were

$$q_i = \widetilde{A}\left(\alpha_2^{\alpha_2}\alpha_3^{-\alpha_3}K_i^{2\delta}\beta_{2i}^{\alpha_2}\beta_{3i}^{\alpha_3}\right)^{\frac{1}{2(1-\varepsilon)}}$$

where $\widetilde{A} = \alpha_3^{\frac{\alpha_3}{1-\varepsilon}}$. This expresion is the same as in equation (7) except that the parameter A is substituted by $\widetilde{A}$. Therefore, considering this parameter change the results in section 4 remain valid.

Proposition 1 *There exists a unique symmetric subgame perfect Nash equilibrium in which each firm chooses* $\beta_{2i} = \beta_2$ *defined by:*

$$\frac{1}{S}p'\left(nq_i/S\right)q_i + p\left(nq_i/S\right) - \frac{2}{\alpha_2}\beta_2 = 0. \tag{10}$$

Third tier incentive rates are

$$\beta_{3i} = \frac{\alpha_3}{\alpha_2 K}\beta_{2i}, \tag{11}$$

and fixed payments are $\gamma_{2i} = \gamma_2$ and $\gamma_{3i} = \gamma_3$.

From proposition 1 we see that incentive rates depend on firm size, market size and the number of firms. For the effect of changes in these variables on incentive rates we have:

Corollary 1 *With given firm size and market structure, incentive rates of workers and supervisors are higher in larger markets* ($\partial\beta_{2i}/\partial S\mid_{K,n}> 0$, $\partial\beta_{3i}/\partial S\mid_{K,n}> 0$)*, lower in markets with more firms* ($\partial\beta_{2i}/\partial n < 0$, $\partial\beta_{3i}/\partial n < 0$) *and lower in larger firms* ($\partial\beta_{2i}/\partial K < 0$, $\partial\beta_{3i}/\partial K < 0$).

In larger markets the market share of firms increases. Therefore, to produce more, it is necessary to incite agents to make more effort. Thus, incentive rates increase. These results are similar to those we can find in single–agent–single–principal models (cf. Raith, 2003 and Schmidt, 1997). Concerning changes in firm size, as we have seen above, in larger firms the free–rider problem is aggravated. So it becomes more costly for the principal to incite a given effort level and from (2) and (11) we find that workers' equilibrium efforts and incentive rates decrease when firm size increases.

4.2 Endogenous firm size and given market structure

If changes in competition are expected to be durable, the principal might want to adopt firm size also. With endogenous firm size, at *stage 1* the principal solves the program $[P_1']$:

$$\begin{aligned} \max_{K_i,\beta_{2i},\gamma_{2i},\beta_{3i},\gamma_{3i}} \pi_i &= p(q/S)q_i - (\beta_{2i}q_i + \gamma_{2i}) - K_i\left(\beta_{3i}q_i + \gamma_{3i}\right) - F \\ &\text{s.t. (2), (6), } U_{2i} \geq 0, U_{3ik} \geq 0, \gamma_{2i} \geq \gamma_2 \text{ and } \gamma_{3i} \geq \gamma_3. \end{aligned} \tag{12}$$

The only difference with respect to program P_1 is that now the principal also maximizes with respect to K_i. The solution of $[P_1']$ yields:

Proposition 2 *There exists a unique symmetric subgame perfect Nash equilibrium in which each firm chooses size* $K^* = K(S,n)$*, where* K^* *is defined by*

$$\frac{K^*\gamma_3}{\alpha_1 q_i} - \frac{1}{S}p'\left(nq_i/S\right)q_i - p\left(nq_i/S\right) = 0 \tag{13}$$

with $q_i = A\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{\varepsilon} {K^*}^{\varepsilon+\alpha_1}$. *Incentive rates are*

$$\beta_{2i} = \left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon} {K^*}^{1-\varepsilon-\alpha_1} \qquad \text{and} \qquad \beta_{3i} = \frac{\alpha_3}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon} {K^*}^{-\varepsilon-\alpha_1} \tag{14}$$

and fixed payments are $\gamma_{2i} = \gamma_2$ and $\gamma_{3i} = \gamma_3$.

The main characteristic of the agent's incentive scheme is that it depends on two variables: firm size and the fixed wage level. Furthermore incentive rates depend particularly on the relative contributions of supervisors and workers to production. This is of practical importance when we compare the structure of incentive payments in different industries or when changes in competition lead to changes in task assignment and organizational structure. To see how changes in different measures of competition affect the incentive rates of supervisors and workers, we start by analyzing the effect of changes in the competitiveness of markets on firm size.

Corollary 2 *Equilibrium firm size is higher in larger markets ($\partial K^*/\partial S > 0$) and in more concentrated markets ($\partial K^*/\partial n < 0$).*

The intuition of this result is straightforward. In larger markets or in more concentrated markets firms have a larger market share and produce more. Therefore, employment is increased. Together, Corollary 1 and 2 imply that changes in the degree of competition have counteracting effects on incentive rates when firm size is an endogenous variable. On the one hand, with an increase in a firm's market share the principal can increase production by increasing production effort, which requires higher powered incentive schemes. On the other hand, she can increase production by employing more agents, which aggravates free–riding and induces lower incentive rates for workers but not for supervisors. From (11) we see that for workers the second effect dominates the first one. In larger markets and in more concentrated markets firm size increases and the workers' incentive rates decrease. For supervisors both effects work in the same direction. Therefore, in larger markets and in more concentrated markets their incentive rate increases. The following result resumes these findings:

Corollary 3 *With endogenous firm size and given market structure workers' incentive rates are lower in larger markets ($\partial\beta_{3i}/\partial S\mid_n < 0$) and in more concentrated markets ($\partial\beta_{3i}/\partial n > 0$). Supervisors' incentive rates are higher in larger markets ($\partial\beta_{2i}/\partial S\mid_n > 0$) and in more concentrated markets ($\partial\beta_{2i}/\partial n < 0$).*

The major difference between this result and Corollary 1 is that with endogenous firm size changes in the competitiveness of markets can have opposite effects on the incentive rates of supervisors and workers. For example, the entry of firms in the market reduces the market share of incumbent firms and therefore firm production. This reduction is achieved by reducing firm size. However, the reduction in firm size means that the supervisor's effort and incentive rates are reduced while workers' effort and incentive rates increase. This result therefore gives an intuitive explanation for the results of Cuñat and Guadalupe (2005). Changes in the competitiveness of markets affect CEO and executive pay–performance–sensitivities in the same direction in firms that respond to these changes with an increase in effort and those which also increase employment, while workers' pay-performance sensitivities are affected in opposite directions in these two kinds of firms. Therefore, when firms with different responses to changes in the competitive environment are aggregated, the effect on PRP sensitivities of workers are less significant than those of CEO's and executives.

4.3 Endogenous firm size and market structure

When market structure is endogenous, the number of firms in the market is determined by the cost of entry. This affects the forgoing analysis in two ways. On the one hand, a change in market size will now also cause the entry or exit of firms. On the other hand, a variation in the costs of entry affects the variables of interest in the model only through changes in the number of firms whose implications we have already discussed. From substitution of equations (14) into (7) we see that the equilibrium output is

$$q_i = A\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\varepsilon+\alpha_1}} \tag{15}$$

and from substitution of equations (14) and (15) into (12) we find that firm i's equilibrium profits are

$$\pi_i = p\left(nq_i/S\right)q_i - \frac{\alpha_1+\varepsilon}{\alpha_1}K^*\gamma_3 - \gamma_2 - F. \tag{16}$$

In the long–run firms enter until profits are zero and changes in the competitiveness of markets affect the number of firms and firm size simultaneously. Then, the number and size of firms in equilibrium is determined by equation (13) and the zero profit condition in (16). This yields:

Proposition 3 *There exists a unique symmetric subgame perfect Nash equilibrium in which firms enter until profits are zero and where firm size $K^* = K(S,F)$ and the number of firms in the market $n^* = n(S,F)$ are defined by*

$$\frac{K^*\gamma_3}{\alpha_1 q_i} - \frac{1}{S}p'\left(nq_i/S\right)q_i - p\left(nq_i/S\right) = 0 \tag{17}$$

$$p\left(nq_i/S\right)q_i - \frac{\alpha_1+\varepsilon}{\alpha_1}K^*\gamma_3 - \gamma_2 - F = 0 \tag{18}$$

with $q_i = A\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{\varepsilon} K^{*^{\varepsilon+\alpha_1}}$. Incentive rates are

$$\beta_{2i} = \left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\varepsilon-\alpha_1}} \quad \textit{and} \quad \beta_{3i} = \frac{\alpha_3}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{-\varepsilon-\alpha_1}} \tag{19}$$

and fixed payments are $\gamma_{2i} = \gamma_2$ and *$\gamma_{3i} = \gamma_3$.*

Regarding changes in the competitiveness of markets because of variations in market size or entry costs we get the following result:

Corollary 4 *An increase in market size increases firm size and causes entry ($\partial K^*/\partial S > 0$, $\partial n^*/\partial S > 0$). An increase in entry costs increases firm size and causes exit ($\partial K^*/\partial F > 0$, $\partial n^*/\partial F < 0$).*

This result comes from two counteracting effects on firm size. The first effect, as we have seen in Corollary 3, is that in larger markets firm size will be larger. However, profits will also be larger and so the second effect is that firms will enter into the market, which decreases firm size. Corollary 4 indicates that if market size changes the first effect dominates the second. As expected, we find that the equilibrium number of firms is larger in larger markets and markets with lower entry costs.

To analyze the effect of changes in competition on incentive rates, notice from (19) that market size and entry cost affect incentive rates only indirectly through a change in firm size. Therefore, the results in Corollary 4 mean:

Corollary 5 *With endogenous firm size and market structure workers' incentive rates are lower in larger markets ($\partial \beta_{3i}/\partial S < 0$) and in markets with higher entry costs ($\partial \beta_{3i}/\partial F < 0$). Supervisors' incentive rates are higher in larger markets ($\partial \beta_{2i}/\partial S > 0$) and in markets with higher entry costs ($\partial \beta_{2i}/\partial F > 0$).*

As far as supervisor incentive rates are concerned, this result is very similar to the findings of Raith (2003). He also finds that incentive rates are higher in larger markets and lower in markets with lower entry costs. The main interest in Corollary 5 is how changes in competition affect workers' PRP. Changes in market size and entry cost have just the opposite effect on workers' incentive rates compared to those of supervisors. This difference can be explained as follows. When market size increases firms want to increase production. In single–principal–single–agent models like that of Raith (2003) this increase in production can only be achieved by increasing the agent's effort, which aggravates the moral hazard problem and requires stronger incentives. Here production can also be increased by employing more agents and this is what happens when market structure is exogenous. However, when more agents are employed the free–rider problem becomes more severe. Therefore, performance–related–pay is reduced and workers make less effort. This implies that firms produce with higher unit costs per output. So, new entrants, which are smaller in size, will have a competitive advantage and the incumbents must reduce firm size to initial levels to be competitive. Thus, the free-rider effect, which is not present in single–principal–single–agent models, is the driving force behind the difference in the results we obtain. Again, as in the case for markets without entry, we can explain why Cuñat and Guadalupe (2005) cannot find significant effects of changes in competition on the incentives rates of workers (for whom there is a free–rider problem) while they find a positive and significant effect on those of CEO's (without a free–rider problem) in the same firms.

5 Conclusion

Empirical evidence suggests that performance–related–pay sensitivities increase significantly with product market competition for CEO's. However, for workers, the effects of competition are much weaker and mostly insignificant (cf. Cuñat and Guadalupe (2005), p.1060). This is at odds with theoretical results that suggest that, though the amount of incentive payments should decrease with the hierarchical levels inside a firm, PRP sensitivities should increase equally at all levels with changes in competition. In this paper, a simple model of PRP in hierarchical agency is developed. The three–tier hierarchy includes a principal at the top, a supervisor on the mid tier and workers on the bottom tier. Incentive payments are used in the model because of moral hazard problems concerning supervisor's and workers' effort levels, which determine joint production.

The main result of the paper is that PRP for lower tiers of a firm generates two kind of heterogeneities. First, changes in the competitiveness of markets can have different impacts on mid-tier and low–tier agents. The reason for this is that when firms apply group–incentive schemes and react to changes in competition by adjusting employment, this adjustment affects the free–rider problem at lower tiers. Second, while a firm's reaction to changes in competition concerning the PRP of mid-tier agents is similar under different endogeneity assumptions, it is very sensitive to these assumptions when we consider workers' PRP. Firms that react to changes in competition by adjusting only their production effort and firms that also adjust employment will apply different changes to the intensity of workers' PRP. The same is true for firms that operate in closed markets and firms that operate in markets with low barriers to entry. Together, these two kinds of heterogeneities explain the observation that the pay-performance sensitivities in worker contracts are significantly lower than those in managerial contracts when aggregated firm data is used.

Of course, this is not to say that free-riding is the only explanation for differences in PRP in different hierarchical levels. Schmidt (1997) suggests that fear of liquidation (and a costly search for a new job) can act as a spur in a more competitive market. Nalebuff and Stiglitz (1983) and Hart (1983) suggest that more competitors or more similar firms might provide better benchmarks and so it is easier to write high–powered incentives because one can filter out common components of performance. If these benchmarks already exist, as for example for salesmen in a sales–force or managers of stores in large chains, one can expect that additional competition might have a smaller effect on the power of performance contracts. This also can explain a more muted effect on lower tier employees than CEOs.

Though the results have been obtained under very specific assumptions, they can be generalized for several reasons. First, the results depend on the assumption that worker's labor inputs are perfect substitutes. However, this is just the contrary assumption to Raith (2003) where the assumption that the results also apply to lower levels of the firm hierarchy implicitly implies that workers' contributions are completely independent. Thus, with less substitutability, the effects found in this paper will also be present while the impact of changes in competition on firm size will be lower and the results concerning effort and incentive rates will be closer to those of Raith (2003). Second, we have used firm output as a common unit of incentive measurement. However, concerning top-tier agents we get similar results regarding the effects of changes in competition on PRP as in models which use other units of measurement, like profits or cost reductions. The effect of changes in competition on different layers of a hierarchy using the other units of incentive measure should therefore not be expected to depend on the specific unit of measurement. Finally, the results depend on the assumption that changes in competition do not affect the task and job assignment within firms. However, a major effect of changes in competition may merely lead to a restructuring of these conditions (cf. Ichniowski and Shaw, 2003). In this case, a change in the PRP sensitivities is the consequence of the fact that workers perform different tasks with different responsibilities. This analysis is beyond the scope of this paper.

6 Appendix

PROOF OF LEMMA 1:

The first-order condition is

$$\beta_{3i}\frac{\partial q_i(K_i, e_{2i}, e_{3i})}{\partial e_{3i}} - e_{3ik} = 0 \quad \forall k. \tag{20}$$

Denote agent A_{3ik} reaction function by $e_{3ik} = \Psi_{3ik}(\sum_{j\neq k} e_{3ij}, K_i, e_{2i}, \beta_{3i})$. From the implicit function theorem we have

$$\frac{\partial \Psi_{3ik}}{\partial \sum_{j\neq k} e_{3ij}} = -\frac{\beta_{3i}\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}^2}}{\beta_{3i}\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}^2} - 1}. \tag{21}$$

Thus, $-1 < \frac{\partial \Psi_{3ik}}{\partial \sum_{j\neq k} e_{3ij}} < 0$ which implies existence and uniqueness of a Nash equilibrium applying theorems 2.7 and 2.8 in Vives (1999, pp. 42). Summation of equations (20) for all k yields:

$$K_i\beta_{3i}\frac{\partial q_i(K_i, e_{2i}, e_{3i})}{\partial e_{3i}} - e_{3i} = 0. \tag{22}$$

From the implicit function theorem we get:

$$\frac{de_{3i}}{dK_i} = -\beta_{3i}\frac{\frac{\partial q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}} + K_i\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}\partial K_i}}{K_i\beta_{3i}\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}^2} - 1} > 0, \tag{23}$$

$$\frac{de_{3i}}{de_{2i}} = -\beta_{3i}\frac{K_i\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}\partial e_{2i}}}{K_i\beta_{3i}\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}^2} - 1} \geq 0, \tag{24}$$

$$\frac{de_{3i}}{d\beta_{3i}} = -\frac{K_i\frac{\partial q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}}}{K_i\beta_{3i}\frac{\partial^2 q_i(K_i,e_{2i},e_{3i})}{\partial e_{3i}^2} - 1} > 0. \tag{25}$$

Regarding the production function we get: $\frac{dq_i}{dK_i} = \frac{\partial q_i}{\partial K_i} + \frac{\partial q_i}{\partial e_{3i}}\frac{de_{3i}}{dK_i} > 0$, $\frac{dq_i}{de_{2i}} = \frac{\partial q_i}{\partial e_{2i}} + \frac{\partial q_i}{\partial e_{3i}}\frac{de_{3i}}{de_{2i}} > 0$, and $\frac{dq_i}{d\beta_{3i}} = \frac{\partial q_i}{\partial e_{3i}}\frac{de_{3i}}{d\beta_{3i}} > 0$.

PROOF OF LEMMA 2:

The first-order condition is

$$\beta_{2i}\frac{dq_i(K_i, e_{2i}, \beta_{3i})}{de_{2i}} - e_{2i} = 0. \tag{26}$$

From the implicit function theorem we get:

$$\frac{de_{2i}}{dK_i} = -\frac{\beta_{2i}\frac{d^2q_i}{de_{2i}dK_i}}{\beta_{2i}\frac{d^2q_i}{de_{2i}^2} - 1} \geq 0, \tag{27}$$

$$\frac{de_{2i}}{d\beta_{2i}} = -\frac{\frac{dq_i}{de_{2i}}}{\beta_{2i}\frac{d^2q_i}{de_{2i}^2} - 1} > 0, \tag{28}$$

$$\frac{de_{2i}}{d\beta_{3i}} = -\frac{\beta_{2i}\frac{d^2q_i}{de_{2i}d\beta_{3i}}}{\beta_{2i}\frac{d^2q_i}{de_{2i}^2} - 1} \geq 0. \tag{29}$$

Regarding the production function we get: $\frac{dq_i}{dK_i} = \frac{\partial q_i}{\partial K_i} + \frac{\partial q_i}{\partial e_{2i}}\frac{de_{2i}}{dK_i} > 0$, $\frac{dq_i}{d\beta_{2i}} = \frac{de_{2i}}{d\beta_{2i}} > 0$, and $\frac{dq_i}{d\beta_{3i}} = \frac{\partial q_i}{\partial e_{2i}}\frac{de_{2i}}{d\beta_{3i}} + \frac{\partial q_i}{d\beta_{3i}} > 0$.

PROOF OF PROPOSITION 1:

From problem $[P_1]$ we see that it is clearly optimal to set $\gamma_{2i} = \gamma_2$, and $\gamma_{3i} = \gamma_3$. Now, equations (4) and (8) imply that $U_{3i} \geq 0$ and $U_{2i} \geq 0$ if $\beta_{3i} \geq 0$, $\beta_{2i} \geq 0$ and $q_i \geq 0$. Assuming this (we will check later that the conditions are fulfilled), the maximization problem can be rewritten as:

$$\max_{\beta_{2i},\beta_{3i}} \pi_i = [p(q/S) - \beta_{2i} - K_i\beta_{3i}]\, q_i - \gamma_{2i} - K_i\gamma_{3i} - F \tag{30}$$

where $q_i = q_i(K_i, \beta_{2i}, \beta_{3i}) = A\left(\alpha_2^{\alpha_3}\alpha_3^{-\alpha_3}K_i^{2\delta}\beta_{2i}^{\alpha_2}\beta_{3i}^{\alpha_3}\right)^{\frac{1}{2(1-\varepsilon)}}$. The first-order conditions are

$$\frac{\partial \pi_i}{\partial \beta_{2i}} = \left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K_i\beta_{3i}\right)\frac{\partial q_i}{\partial \beta_{2i}} - q_i = 0 \quad \forall i. \tag{31}$$

$$\frac{\partial \pi_i}{\partial \beta_{3i}} = \left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K_i\beta_{3i}\right)\frac{\partial q_i}{\partial \beta_{3i}} - K_iq_i = 0 \quad \forall i. \tag{32}$$

Using the symmetry assumption, $K_i = K$, and substituting $\frac{\partial q_i}{\partial \beta_{2i}}$ and q_i we get:

$$\left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K\beta_{3i}\right)\frac{\alpha_2}{2\,(1-\varepsilon)}q_i\beta_{2i}^{-1} - q_i = 0 \quad \forall i, \tag{33}$$

$$\left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K\beta_{3i}\right)\frac{\alpha_3}{2\,(1-\varepsilon)}q_i\beta_{3i}^{-1} - Kq_i = 0 \quad \forall i. \tag{34}$$

This yields

$$\beta_{3i} = \frac{\alpha_3}{\alpha_2 K}\beta_{2i}, \tag{35}$$

$$\frac{1}{S}p'(q/S)q_i + p(q/S) = \frac{2}{\alpha_2}\beta_{2i}. \tag{36}$$

In equilibrium, $\beta_{2i} = \beta_2$. Thus, β_2 is implicitly defined by

$$\frac{1}{S}p'\left(\frac{n}{S}q_i\right)q_i + p\left(\frac{n}{S}q_i\right) = \frac{2}{\alpha_2}\beta_2 \tag{37}$$

where $q_i = A\left(K^{2\delta-\alpha_3}\beta_2^{\alpha_2+\alpha_3}\right)^{\frac{1}{2(1-\varepsilon)}}$. Notice that in equilibrium $\beta_2 > 0$ because the left hand-side of (37) decreases in β_2 and is positive for $\beta_2 = 0$ and the right hand-side increases in β_2 and is 0 for $\beta_2 = 0$. Then, from (35) $\beta_{3i} > 0$. Therefore, $q_i > 0$ and the participation constraints are satisfied ($U_{3i} \geq 0$ and $U_{2i} \geq 0$).

PROOF OF COROLLARY 1:

Applying the implicit function theorem, from (10) we get:

$$\frac{\partial\beta_2}{\partial S} = -\frac{-\left[n\left(p''\frac{q_i}{S} + p'\right) + p'\right]\frac{q_i}{S^2}}{\frac{1}{S}\left[np''\frac{q_i}{S} + (n+1)\,p'\right]\frac{dq_i}{d\beta_2} - \frac{2}{\alpha_2}\beta_2} > 0, \tag{38}$$

$$\frac{\partial\beta_2}{\partial n} = -\frac{\left[p''\frac{q_i}{S} + p'\right]\frac{q_i}{S}}{\frac{1}{S}\left[np''\frac{q_i}{S} + (n+1)\,p'\right]\frac{dq_i}{d\beta_2} - \frac{2}{\alpha_2}\beta_2} < 0, \tag{39}$$

$$\frac{\partial\beta_2}{\partial K} = -\frac{\frac{1}{S}\left[n\left(p''\frac{q_i}{S} + p'\right) + p'\right]\frac{dq_i}{dK}}{\frac{1}{S}\left[np''\frac{q_i}{S} + (n+1)\,p'\right]\frac{dq_i}{d\beta_2} - \frac{2}{\alpha_2}\beta_2} < 0. \tag{40}$$

From (11) we get that $sign\left(\frac{\partial\beta_{3i}}{\partial x}\right) = sign\left(\frac{\partial\beta_2}{\partial x}\right)$, for $x = S, n, K$.

PROOF OF PROPOSITION 2:

From problem $[P_1']$ we see that it is clearly optimal to set $\gamma_{2i} = \gamma_2$, and $\gamma_{3i} = \gamma_3$. Now, equations (4) and (8) imply that $U_{3ik} \geq 0$ and $U_{2i} \geq 0$ if $\beta_{3i} \geq 0$, $\beta_{2i} \geq 0$ and $q_i \geq 0$. Assuming this (we will check later that the conditions are fulfilled), the maximization problem can be rewritten as:

$$\max_{K_i,\beta_{2i},\beta_{3i}} \pi_i = (p(q/S) - \beta_{2i} - K_i\beta_{3i})\,q_i - \gamma_2 - K_i\gamma_3 - F. \tag{41}$$

where $q_i(K_i, \beta_{2i}, \beta_{3i}) = A\left(\alpha_2^{\alpha_3}\alpha_3^{-\alpha_3}K_i^{2\delta}\beta_{2i}^{\alpha_2}\beta_{3i}^{\alpha_3}\right)^{\frac{1}{2(1-\varepsilon)}}$. The first-order conditions are

$$\frac{\partial \pi_i}{\partial K_i} = \left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K_i\beta_{3i}\right)\frac{\partial q_i}{\partial K_i} - \beta_{3i}q_i - \gamma_3 = 0, \tag{42}$$

$$\frac{\partial \pi_i}{\partial \beta_{2i}} = \left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K_i\beta_{3i}\right)\frac{\partial q_i}{\partial \beta_{2i}} - q_i = 0, \tag{43}$$

$$\frac{\partial \pi_i}{\partial \beta_{3i}} = \left(\frac{1}{S}p'(q/S)q_i + p(q/S) - \beta_{2i} - K_i\beta_{3i}\right)\frac{\partial q_i}{\partial \beta_{3i}} - K_iq_i = 0, \tag{44}$$

where

$$\frac{\partial q_i}{\partial K_i} = \frac{\delta}{1-\varepsilon}q_iK_i^{-1} > 0, \tag{45}$$

$$\frac{\partial q_i}{\partial \beta_{2i}} = \frac{\alpha_2}{2(1-\varepsilon)}q_i\beta_{2i}^{-1} > 0, \tag{46}$$

$$\frac{\partial q_i}{\partial \beta_{3i}} = \frac{\alpha_3}{2(1-\varepsilon)}q_i\beta_{3i}^{-1} > 0. \tag{47}$$

Using theses expressions, the system of equations (42) - (44) can be rewritten as:

$$\frac{1}{S}p'(q/S)q_i + p(q/S) = \frac{2}{\alpha_2}\beta_{2i} \quad \forall i, \tag{48}$$

$$\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K_i^{1-\varepsilon-\alpha_1} = \beta_{2i} \quad \forall i, \quad \text{and,} \tag{49}$$

$$\frac{\alpha_3}{\alpha_2K_i}\beta_{2i} = \beta_{3i} \quad \forall i. \tag{50}$$

Substituting (49) and (50) into $q_i(K_i, \beta_{2i}, \beta_{3i})$ we get:

$$q_i = A\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{\varepsilon}K_i^{\alpha_1+\varepsilon}. \tag{51}$$

Substituting (49) and (51) into (48) we get:

$$\frac{1}{S}p'(nq_i/S)q_i + p(nq_i/S) = \frac{2}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K_i^{1-\varepsilon-\alpha_1} \quad \forall i. \tag{52}$$

Because of symmetry, $K_i = K$, $\forall i$. Then, in equilibrium we get that optimal firm size K^* is implicitly defined by

$$G(S, n, K^*) = 0, \tag{53}$$

where

$$G(S,n,K) = \frac{2}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{1-\varepsilon-\alpha_1} - \frac{1}{S}p'(nq_i/S)q_i - p(nq_i/S) \tag{54}$$

where $q_i = A\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{\varepsilon}K^{\alpha_1+\varepsilon}$. *Existence* and *uniqueness* are guaranteed because the left-hand side in (52) is strictly decreasing in K_i and positive for $K_i = 0$, while the right-hand side is strictly increasing and 0 for $K_i = 0$. Therefore, in equilibrium $K_i > 0$, and from (49), (50) and (51) $\beta_{2i} > 0$, $\beta_{3i} > 0$ and $q_i > 0$ such that the participation constraints are satisfied ($U_{3i} \geq 0$ and $U_{2i} \geq 0$). Now, (53), (54) and (51) yield (13). Substitution of (49) into (50) gives (14). To guarantee *sufficiency* we obtain the second–order conditions:

$$\frac{\partial^2 \pi_i}{\partial K_i^2} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial K_i^2} + \left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \left[\frac{\partial q_i}{\partial K_i}\right]^2 - 2\beta_{3i}\frac{\partial q_i}{\partial K_i}, \quad (55)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}^2} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial \beta_{2i}^2} + \left[\left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \frac{\partial q_i}{\partial \beta_{2i}} - 2\right] \frac{\partial q_i}{\partial \beta_{2i}}, \quad (56)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{3i}^2} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial \beta_{3i}^2} + \left[\left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \frac{\partial q_i}{\partial \beta_{3i}} - 2K_i\right] \frac{\partial q_i}{\partial \beta_{3i}}, \quad (57)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial \beta_{2i}\partial K_i} + \left[\left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \frac{\partial q_i}{\partial \beta_{2i}} - 1\right] \frac{\partial q_i}{\partial K_i} - \beta_{3i}\frac{\partial q_i}{\partial \beta_{2i}}, \quad (58)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{3i}\partial K_i} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial \beta_{3i}\partial K_i} + \left[\left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \frac{\partial q_i}{\partial \beta_{3i}} - K_i\right] \frac{\partial q_i}{\partial K_i} - q_i - \beta_{3i}\frac{\partial q_i}{\partial \beta_{3i}} \quad (59)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial \beta_{3i}} = \left(\frac{1}{S}p' q_i + p - \beta_{2i} - K_i \beta_{3i}\right) \frac{\partial^2 q_i}{\partial \beta_{2i}\partial \beta_{3i}} + \left[\left(\frac{1}{S^2}p'' q_i + \frac{2}{S}p'\right) \frac{\partial q_i}{\partial \beta_{2i}} - 1\right] \frac{\partial q_i}{\partial \beta_{3i}} - K_i\frac{\partial q_i}{\partial \beta_{2i}} \quad (60)$$

where

$$\frac{\partial^2 q_i}{\partial K_i^2} = \left(\frac{\delta}{(1-\varepsilon)} - 1\right) \frac{\delta}{1-\varepsilon} q_i K_i^{-2} < 0, \quad (61)$$

$$\frac{\partial^2 q_i}{\partial \beta_{2i}^2} = \left(\frac{\alpha_2}{2(1-\varepsilon)} - 1\right) \frac{\alpha_2}{2(1-\varepsilon)} q_i \beta_{2i}^{-2} < 0, \quad (62)$$

$$\frac{\partial^2 q_i}{\partial \beta_{3i}^2} = \left(\frac{\alpha_3}{2(1-\varepsilon)} - 1\right) \frac{\alpha_3}{2(1-\varepsilon)} q_i \beta_{3i}^{-2} < 0, \quad (63)$$

$$\frac{\partial^2 q_i}{\partial \beta_{2i}\partial K_i} = \frac{\alpha_2 \delta}{2(1-\varepsilon)^2} q_i \beta_{2i}^{-1} K_i^{-1} > 0, \quad (64)$$

$$\frac{\partial^2 q_i}{\partial \beta_{3i}\partial K_i} = \frac{\alpha_3 \delta}{2(1-\varepsilon)^2} q_i \beta_{3i}^{-1} K_i^{-1} > 0, \quad (65)$$

$$\frac{\partial^2 q_i}{\partial \beta_{2i}\beta_{3i}} = \frac{\alpha_2 \alpha_3}{4(1-\varepsilon)^2} q_i \beta_{2i}^{-1} \beta_{3i}^{-1} > 0. \quad (66)$$

At the extremum the second-order conditions are:

$$\frac{\partial^2 \pi_i}{\partial K_i^2} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} + \frac{\delta - \alpha_3 - 1 + \varepsilon}{\delta}\right]\frac{2\delta^2}{\alpha_2(1-\varepsilon)}q_i\beta_{2i}K_i^{-2} < 0, \quad (67)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}^2} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - 1 - \frac{2(1-\varepsilon)}{\alpha_2}\right]\frac{\alpha_2}{2(1-\varepsilon)}q_i\beta_{2i}^{-1} < 0, \quad (68)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{3i}^2} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - 1 - \frac{2(1-\varepsilon)}{\alpha_3}\right]\frac{\alpha_2}{2(1-\varepsilon)}K_i^2 q_i\beta_{2i}^{-1} < 0, \quad (69)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - \frac{\alpha_3}{2\delta}\right]\frac{\delta}{1-\varepsilon}q_i K_i^{-1} < 0, \quad (70)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{3i}\partial K_i} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - \frac{1-\varepsilon}{\delta} - \frac{\alpha_3}{2\delta}\right]\frac{\delta}{1-\varepsilon}q_i < 0, \quad (71)$$

$$\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial \beta_{3i}} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - 1\right]\frac{\alpha_2}{2(1-\varepsilon)}K_i q_i\beta_{2i}^{-1} < 0. \quad (72)$$

Sufficiency is guaranteed by uniqueness of the extremum and since the Hessian matrix H is negative definite at the extremum:

$$H_1 = \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2} = \left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - 1 - \frac{2(1-\varepsilon)}{\alpha_2}\right]\frac{\alpha_2}{2(1-\varepsilon)}q_i\beta_{2i}^{-1} < 0, \quad (73)$$

$$\begin{aligned} H_2 &= \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2}\frac{\partial^2 \pi_i}{\partial \beta_{3i}^2} - \left(\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial \beta_{3i}}\right)^2 \\ &= \left(-\frac{\varepsilon}{1-\varepsilon}\left[\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} - 1\right] + 1\right)\frac{\alpha_2}{\alpha_3}K_i^2 q_i^2\beta_{2i}^{-2} > 0, \quad (74) \end{aligned}$$

$$\begin{aligned} H_3 &= \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2}\frac{\partial^2 \pi_i}{\partial \beta_{3i}^2}\frac{\partial^2 \pi_i}{\partial K_i^2} + 2\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial \beta_{3i}}\frac{\partial^2 \pi_i}{\partial \beta_{3i}\partial K_i}\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i} \\ &\quad - \frac{\partial^2 \pi_i}{\partial \beta_{2i}^2}\left[\frac{\partial^2 \pi_i}{\partial \beta_{3i}\partial K_i}\right]^2 - \frac{\partial^2 \pi_i}{\partial \beta_{3i}^2}\left[\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial K_i}\right]^2 - \frac{\partial^2 \pi_i}{\partial K_i^2}\left[\frac{\partial^2 \pi_i}{\partial \beta_{2i}\partial \beta_{3i}}\right]^2 \\ &= \left[\begin{array}{c} +(2\alpha_1 + 2\varepsilon)(1-\varepsilon)\left(\frac{1}{S^2}p''q_i + \frac{2}{S}p'\right)\frac{\partial q_i}{\partial \beta_{2i}} \\ +(2\alpha_1 + 2\varepsilon - 2) \end{array}\right]\frac{(2\delta - \alpha_3)}{2\alpha_3(1-\varepsilon)^2}q_i^3\beta_{2i}^{-1} < 0 \quad (75) \end{aligned}$$

where H_i, $i = 1, 2, 3$ are the successive principal minors of H.

PROOF OF COROLLARY 2:

From (53) and (54) we get:

$$\frac{\partial K^*}{\partial S} = -\frac{\frac{\partial G}{\partial S}}{\frac{\partial G}{\partial K}} = -\frac{\frac{n}{S^3}p''(nq_i/S)q_i^2 + \frac{(n+1)}{S^2}p'(nq_i/S)q_i}{\frac{2(1-\varepsilon-\alpha_1)}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{-\varepsilon-\alpha_1} - \frac{n}{S^2}p''(nq_i/S)q_i\frac{\partial q_i}{\partial K_i} - \frac{n+1}{S}p'(nq_i/S)\frac{\partial q_i}{\partial K_i}} > 0,$$

$$\frac{\partial K^*}{\partial n} = -\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial K}} = -\frac{-\frac{1}{S^2}p''(nq_i/S)q_i^2 - \frac{1}{S}p'(nq_i/S)q_i}{\frac{2(1-\varepsilon-\alpha_1)}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{-\varepsilon-\alpha_1} - \frac{n}{S^2}p''(nq_i/S)q_i\frac{\partial q_i}{\partial K_i} - \frac{n+1}{S}p'(nq_i/S)\frac{\partial q_i}{\partial K_i}} < 0.$$

PROOF OF PROPOSITION 3:

When market structure is endogenous firms enter until profits are zero. Therefore, n and K are simultaneously determined by equation (54) and by setting the right-hand side of equation (16) equal to zero:

$$G(S,F,K^*,n^*) \equiv \frac{2}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon} K^{*^{1-\varepsilon-\alpha_1}} - \frac{1}{S}p'(nq_i/S)q_i - p(nq_i/S) = 0 \tag{76}$$

$$\pi_i(S,F,K^*,n^*) \equiv p\left(nq_i/S\right)q_i - \frac{\alpha_1+\varepsilon}{\alpha_1}K^*\gamma_3 - \gamma_2 - F = 0. \tag{77}$$

This yields

$$K^* = K(S,F), \tag{78}$$

$$n^* = n(S,F). \tag{79}$$

Existence is guaranteed for $p(0) > 0$ in (76) and because from Corollary 2 we have that K^* is decreasing in n^*, which implies that profits decrease when n increases in (77).

PROOF OF COROLLARY 4:

Notice that (78) and (79) are continuously differentiable. Then, from the implicit function theorem we get:

$$\begin{bmatrix} \frac{\partial K^*}{\partial S} & \frac{\partial K^*}{\partial F} \\ \frac{\partial n^*}{\partial S} & \frac{\partial n^*}{\partial F} \end{bmatrix} = -\begin{bmatrix} \frac{\partial G}{\partial K^*} & \frac{\partial G}{\partial n^*} \\ \frac{\partial \pi_i}{\partial K^*} & \frac{\partial \pi_i}{\partial n^*} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial G}{\partial S} & \frac{\partial G}{\partial F} \\ \frac{\partial \pi_i}{\partial S} & \frac{\partial \pi_i}{\partial F} \end{bmatrix} = \frac{\begin{bmatrix} \frac{\partial \pi_i}{\partial n^*}\frac{\partial G}{\partial S} - \frac{\partial \pi_i}{\partial S}\frac{\partial G}{\partial n^*} & \frac{\partial G}{\partial n^*} \\ -\frac{\partial \pi_i}{\partial K^*}\frac{\partial G}{\partial S} + \frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial S} & -\frac{\partial G}{\partial K^*} \end{bmatrix}}{\frac{\partial G}{\partial n^*}\frac{\partial \pi_i}{\partial K^*} - \frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial n^*}} \tag{80}$$

because from (76) $\frac{\partial G}{\partial F} = 0$, and from (77) $\frac{\partial \pi_i}{\partial F} = -1$. Now, using

$$\frac{\partial G}{\partial S} = \left(n\left(p''\frac{q_i}{S} + p'\right) + p'\right)\frac{q_i}{S^2} < 0, \tag{81}$$

$$\frac{\partial G}{\partial K^*} = \frac{(1-\alpha_1-\varepsilon)}{\alpha_1}\gamma_3 q_i^{-1} - \left(n\left(p''\frac{q_i}{S} + p'\right) + p'\right)(\alpha_1+\varepsilon)\frac{q_i}{S}K^{*^{-1}} > 0, \tag{82}$$

$$\frac{\partial G}{\partial n^*} = -\left(p''\frac{q_i}{S} + p'\right)\frac{q_i}{S} > 0, \tag{83}$$

$$\frac{\partial \pi_i}{\partial S} = -\frac{nq_i}{S^2}p'q_i > 0, \tag{84}$$

$$\frac{\partial \pi_i}{\partial K^*} = \left(np'\frac{q_i}{S} + p\right)(\varepsilon+\alpha_1)\,q_i K^{*^{-1}} - \left(\frac{\alpha_1+\varepsilon}{\alpha_1}\right)\gamma_3 < 0, \text{ and} \tag{85}$$

$$\frac{\partial \pi_i}{\partial n^*} = \frac{1}{S}p'q_i^2 < 0, \tag{86}$$

we get:

$$\frac{\partial G}{\partial n^*}\frac{\partial \pi_i}{\partial K^*}-\frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial n^*} = \left(\left(p''\frac{q_i}{S}+p'\right)+p'\right)p'\frac{(\alpha_1+\varepsilon)\,q_i^3}{S^2K^*}-\frac{1-\alpha_1-\varepsilon}{\alpha_1}p'\frac{\gamma_3 q_i}{S}>0, \tag{87}$$

$$\frac{\partial \pi_i}{\partial n^*}\frac{\partial G}{\partial S}-\frac{\partial \pi_i}{\partial S}\frac{\partial G}{\partial n^*} = \frac{1}{S^3}p'^2q_i^3>0, \tag{88}$$

$$-\frac{\partial \pi_i}{\partial K^*}\frac{\partial G}{\partial S}+\frac{\partial G}{\partial K^*}\frac{\partial \pi_i}{\partial S} = \left(n\left(p''\frac{q_i}{S}+p'\right)+p'\right)p'\frac{(\alpha_1+\varepsilon)\,q_i^3}{S^3K^*}-\frac{1-\alpha_1-\varepsilon}{\alpha_1}p'\frac{\gamma_3 n q_i}{S^2}>0. \tag{89}$$

Thus, we have:

$$\frac{\partial K^*}{\partial S}>0,\ \frac{\partial K^*}{\partial F}>0,\ \frac{\partial n^*}{\partial F}<0,\text{ and }\frac{\partial n^*}{\partial S}>0.$$

PROOF OF COROLLARY 5:

The long-run equilibrium effects of changes in S and F on incentive rates are:

$$\frac{d\beta_{2i}}{dS} = (1-\varepsilon-\alpha_1)\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{*^{-\varepsilon-\alpha_1}}\frac{\partial K^*}{\partial S}>0, \tag{90}$$

$$\frac{d\beta_{2i}}{dF} = (1-\varepsilon-\alpha_1)\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{*^{-\varepsilon-\alpha_1}}\frac{\partial K^*}{\partial F}>0, \tag{91}$$

$$\frac{d\beta_{3i}}{dS} = (-\varepsilon-\alpha_1)\frac{\alpha_3}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{*^{-\varepsilon-\alpha_1-1}}\frac{\partial K^*}{\partial S}<0,\ \text{and} \tag{92}$$

$$\frac{d\beta_{3i}}{dF} = (-\varepsilon-\alpha_1)\frac{\alpha_3}{\alpha_2}\left(\frac{\alpha_2}{2\alpha_1}\frac{\gamma_3}{A}\right)^{1-\varepsilon}K^{*^{-\varepsilon-\alpha_1-1}}\frac{\partial K^*}{\partial F}<0. \tag{93}$$

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