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"Scheduled Service Versus Personal Transportation: The Role of Distance"

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Scheduled service versus personal transportation: the role of distance[*]

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Abstract

This paper presents a theoretical and empirical analysis of the relationship between frequency of scheduled transportation services and their substitutability with personal transportation (using distance as a proxy). We study the interaction between a monopoly firm providing a high-speed scheduled service and private transportation (i.e., car). Interestingly, the carrier chooses to increase the frequency of service on longer routes *when competing with personal transportation* because by providing higher frequency (at extra cost) it can also charge higher fares which can boost its profits. However, in line with the results of earlier studies, frequency decreases for longer flights *when driving is not a viable option*. An empirical application of our analysis to the European airline industry confirms the predictions of our theoretical model.

Keywords: short-haul routes; long-haul routes; flight frequency; distance

JEL Classification Numbers: L13; L2; L93

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1 Introduction

A problem faced by companies providing scheduled transportation (airlines, railway companies, etc.) is that it is impossible to achieve the level of mobility offered by the use of a private vehicle. Customers traveling by car do not have to bear a schedule delay cost inherent to the limited choice of departure times that characterizes scheduled services. However, providers of scheduled transportation can make their product more attractive when competing with consumers' personal vehicles by offering high-frequency, high-speed services. There are also cases in which driving is not a relevant alternative (when the distance between the endpoints is particularly long), and in this case a scheduled carrier has to make its service attractive with respect to the option of not traveling at all. These two problems may have different solutions; however, no differentiation between them has been offered in the literature.

This paper fills this void by examining fare and frequency choices of a monopoly provider of scheduled transportation services. We compare the case where the customer's next best available alternative is driving with the scenario where driving is not a relevant option (i.e., it is a dominated alternative). The model yields testable predictions regarding frequency-distance relationships, which we put to the test using data from the European airline industry (where services are provided by a single carrier on over sixty percent of airport-pair markets).

In the theoretical part, we model a carrier (which we will consider to be an airline, although the analysis is easily applicable to high-speed rail as well) choosing fares and frequency of services, given that it enjoys an exogenous advantage in terms of higher speed of service than the private vehicle. This part builds on Brueckner (2004), Brueckner and Flores-Fillol (2007) and Bilotkach (2006). Brueckner (2004) considers a monopoly airline's network choice, incorporating decisions concerning frequency in the model. Brueckner and Flores-Fillol (2007) use this framework to analyze fare and frequency choices in duopoly markets. Finally, Bilotkach (2006) introduces a valuation of time similar to the one we use here in a model of airlines' network choice.

We find that the monopolist's choice crucially depends on whether driving is a dominated option or not. The carrier will reduce the frequency of service for longer trips when driving is dominated but, more interestingly, the relationship between frequency and distance may reverse when driving is not dominated and carriers compete with personal transportation.

Our result is explained by a trade-off between two forces. First, the provider of scheduled transportation services will always incur an extra cost when increasing frequency. Indeed, higher frequency implies additional fixed costs and reduces the opportunity of exploiting density economies which, in the case of airlines, arise from the use of bigger aircraft at high load factors. Second, an increase in distance may boost the demand for high-speed scheduled transportation services on short-haul routes where the use of personal vehicles is a relevant option for travelers. This is because an increase in distance makes the high-speed transportation mode more competitive and so providers of scheduled transportation services are able to increase frequency and charge higher fares.

On short-haul routes, our theoretical model shows that the positive effect of distance on frequency derived from charging higher fares outweighs the negative effect derived from incurring extra costs. This explains the main result of our analysis: the positive relationship between frequency and distance on routes where personal transportation is a relevant option. However, on long-haul routes where driving is a dominated option, an increase in distance does not necessarily imply an increase in the demand for scheduled services. Hence, we can expect a negative relationship between frequency and distance since the provider of scheduled services tries to minimize costs (and avoid potential travelers staying at home). Finally, from the perspective of the social optimum, we find that a monopoly carrier provides lower frequency of service than is socially optimal and the number of passengers making use of the scheduled transportation services is inefficiently low (as is usual for models of this kind).

Our model relates to the issue of intermodal competition and choice of transport mode. In this vein, Combes and Linnemer (2000) consider a model à la Hotelling in which two transportation modes compete (car and airplane) when a new infrastructure is built. More recently, Cantos-Sánchez *et al.* (2008) study alternative regulatory regimes in a model of intermodal competition and suggest an empirical application to the Spanish market. The issue of mode substitution and its effects has also been discussed, for example by Bel (1997), González-Savignat (2004), Janic (2003) and López-Pita and Robusté (2004). Some studies on choice of transport mode conclude that commuters mostly consider frequency of service (and more generally convenience of service) as one of the factors determining their elasticity (Voith, 1997 and Asensio, 2002); or the impact of urban transit projects (Baum-Snow and Kahn, 2000). Since with longer distances scheduled services become more attractive than personal transportation, due to their higher speed, we study how the monopolist's choice

changes as the substitutability between the two transportation options increases.

We test the predictions of our theoretical model concerning the relationship between the length of haul and frequency using data on annual frequencies at the airline-route level. Our sample includes about 900 routes that link the ten largest airports in Europe with other European destinations (EU27 + Switzerland and Norway) in the period 2006-2007. The empirical application examines the relationship between airlines' frequency choices and distance controlling for demand shifters at the route level, the intensity of competition and airline attributes. A spline regression that shows the relationship between frequency and distance in our dataset, makes it advisable to differentiate between routes shorter and longer than 500 kilometers (311 miles). Interestingly, the empirical application shows that airlines' frequency increases with distance for routes shorter than 500 kilometers. In contrast, frequency decreases with distance for routes longer than 500 kilometers. Thus, the predictions of the theoretical model are confirmed. As expected, frequency increases with demand (captured by several variables). We also find that airlines compete aggressively in frequency of service, low-cost carriers provide lower quality products, and airport presence strongly influences the number of flights that airlines offer on the routes served.

Previous empirical work has analyzed the determinants of airlines' flight frequency. Borenstein and Netz (1999) and Salvanes *et al.* (2005) find that airlines cluster the departure times of flights when competition increases in their studies for the US and Norway, respectively. Our empirical application is more closely related to the studies by Pai (2007) and Wei and Hansen (2007). Pai (2007) estimates the determinants of flight frequency in the US airline market, observing a decreasing relationship between frequency and distance. From a different perspective, Wei and Hansen (2007) develop an application for three game-theoretic models of airline choices, obtaining that frequency on long-haul routes is less than on short-haul routes. These findings are in line with our results when driving is a dominated option. Thus, although these two previous studies adequately explain the relationship between frequency and distance on long-haul routes, our results suggest that the applicability of their findings to short-haul markets is limited.[1]

To sum up, the main contribution of this paper is to point out that the relationship

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[1]Additionally, unlike the US airline industry, European airline markets have been relatively under-researched (due predominantly to the lack of sufficient data).

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between frequency choices and distance depends crucially on the presence of personal transportation, a finding identified theoretically and tested empirically for the European airline industry. Thus, the distance between endpoints in city-pair markets constitutes a potentially important factor to be considered when analyzing scheduled transportation services.

The plan of the paper is as follows. Section 2 presents the model and the equilibrium and compares the equilibrium outcome with the social optimum. An empirical application to the European airline market is provided in Section 3 and a brief conclusion closes the paper. All the proofs are provided in the Appendix.

2   The model

Our model is based on indirect utilities of heterogeneous travelers choosing between scheduled services and personal transport. We consider an air carrier as the provider of scheduled services, for the purpose of exposition, but other modes of transportation (in particular, the high-speed train) can easily fit into our framework.

The model combines elements of Brueckner's (2004) monopoly scheduling model along with a differentiation of consumers by their value of time similar to the one suggested in Bilotkach (2006). In the model, utility for a consumer *traveling by air* is given by *Consumption − Schedule delay disutility + Value of available time.*

Consumption is $y - p_{air}$ where y is the common level of income and p_{air} is the airline's fare.

Letting H denote the time circumference of the circle, consumer utility then depends on *expected schedule delay* (defined as the difference between the preferred and actual departure times) which equals $H/4f$, where f is number of (evenly spaced) flights operated by the airline. The *Schedule delay disutility* is equal to a disutility parameter $\delta > 0$ times the expected schedule delay expression from above, thus equaling $\delta H/4f = \gamma/f$, where $\gamma \equiv \delta H/4$.

Finally, the available time at the destination is computed as the difference between passengers' total trip time (T) and the actual traveling time which depends on the distance between the origin and the destination (d) and the plane's speed (V), thus equaling $T - d/V$. We assume a large enough T so that $T > d/V$. Thus, taking into account the traveler's specific value of time α, the *Value of available time* at the destination equals $\alpha (T - d/V)$,

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where α is assumed to be uniformly distributed over the range $[0, 1]$. Hence, utility from air travel is $u_{air} = y - p_{air} - \gamma/f + \alpha[T - d/V]$.

However, consumers can also make use of an alternative surface transport mode (i.e., car) obtaining a utility of $Consumption + Value\ of\ available\ time$ since there is no schedule delay in this case. Therefore utility from driving is $u_{car} = y - cd + \alpha[T - d/(\beta V)]$, where cd is the cost of the trip that increases with distance, β captures the airline/car speed differential and $\beta \in (0, 1/2)$, i.e., we assume that traveling by car is at least twice as slow as air travel, and T is large enough so that $T > d/(\beta V)$.[2]

Additionally, we allow for partially-served markets because consumers can also choose not to travel and stay at home, obtaining a utility of $u_o = y$.

Observe that, since air travel is faster than travel by private vehicle, the $u_{air} - u_{car}$ differential is greater the higher the consumer's value of time (α). In other words, both u_{air} and u_{car} increase with α, but u_{air} is steeper than u_{car}. This basically ensures that the higher a consumer's value of time, the more likely she is to fly rather than drive, other things being equal.

Disregarding the trivial cases (either where nobody travels or where everyone uses the same mode of transport), we can state the following: a consumer will undertake air travel when $u_{air} > \max\{u_{car}, u_o\}$. The inequality $u_{air} > u_{car}$ requires $\alpha > \widetilde{\alpha}$ with

$$\widetilde{\alpha} = \frac{(p_{air} - cd + \gamma/f)\beta V}{d(1 - \beta)}; \tag{1}$$

and the inequality $u_{air} > u_o$ holds for $\alpha > \overline{\alpha}$ with

$$\overline{\alpha} = \frac{p_{air} + \gamma/f}{T - d/V}. \tag{2}$$

Finally, a consumer will drive when $u_{car} > \max\{u_{air}, u_o\}$, where $u_{car} > u_{air}$ requires $\alpha < \widetilde{\alpha}$ and $u_{car} > u_o$ requires $\alpha > \widehat{\alpha}$ with

$$\widehat{\alpha} = \frac{cd}{T - d/(\beta V)}. \tag{3}$$

Consumers with a sufficiently high value of time will undertake air travel and consumers with a sufficiently low time value will stay at home. Consequently, we are left with two

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[2]Note that cars are much slower than new high-speed trains, which have been designed to reach a speed above 250 km/h.

possible scenarios depending on whether driving is a dominated alternative or not: the *case with drivers* where $0 < \widehat{\alpha} < \widetilde{\alpha} < 1$ (Scenario 1); and the *situation where there are no drivers* with $0 < \widetilde{\alpha} < \widehat{\alpha} < 1$ (Scenario 2). These two scenarios are represented in Figures 1 and 2 below.

$$-\text{Insert here Figures 1 and 2}-$$

As suggested before, we observe that u_{air} is steeper than u_{car} because $\beta \in (0, 1/2)$ and thus, as α approaches 1, the airline carries more traffic.

The case with drivers (Scenario 1) requires $\widehat{\alpha} < \widetilde{\alpha}$ and, from this inequality we obtain $\left(\frac{p_{air}V}{\beta} + cTV^2 + \frac{\gamma V}{\beta f} \right) d - TV^2 \left(p_{air} + \frac{\gamma}{f} \right) < cd^2V$. Then, it can be shown that this condition requires $d < d^*$, so we conclude that Scenario 1 is only relevant for short distances, meaning that driving is considered a viable alternative by some consumers only for sufficiently short-haul trips. Lemma 1 below summarizes this result (a detailed proof is provided in Appendix A).

Lemma 1 *Given the indirect utilities specified above, there is a single cut-off distance d^* such that Scenario 1 (with drivers) is observed for $d < d^*$ (short-haul city-pair markets); while Scenario 2 (without drivers) emerges for $d > d^*$ (long-haul city-pair markets).*

Looking at our data for the European airline markets examined in the empirical application in Section 3, this cut-off distance d^* is evaluated at around 500 kilometers (311 miles).

For each of the two scenarios, the analysis that follows derives the demand functions, specifies the airline's cost structure, and describes the profit functions. Afterwards, both the equilibrium and the social optimum are analyzed in each scenario.

2.1   Scenario 1: driving is not a dominated alternative

Under Scenario 1, a traveler will fly when $\alpha > \widetilde{\alpha}$. Otherwise, the consumer will use private transportation. Then, using (1), the airline's demand is given by

$$q_{air} = \int_{\widetilde{\alpha}}^{1} d\alpha = 1 - \widetilde{\alpha} = 1 - \frac{(p_{air} - cd + \gamma/f)\beta V}{d(1 - \beta)}. \tag{4}$$

To characterize the equilibrium in fares and frequencies,[3] we need to specify the carrier's cost structure. A *flight's operating cost* is given by $\theta(d) + \tau s$ where s stands for aircraft size (i.e., the number of seats). The parameter τ is the marginal cost per seat of serving the passenger on the ground and in the air. Finally, the function $\theta(d)$ stands for the cost of frequency (or cost per departure) that captures the aircraft fixed cost which includes landing and navigation fees, renting gates, airport maintenance and the cost of fuel. We assume that $\theta(d)$ is continuously differentiable with respect to $d > 0$ and that $\theta'(d) > 0$ because fuel consumption increases with distance.

As in Brueckner (2004), it is assumed that all seats are filled, so that load factor equals 100% and therefore $s = q_{air}/f$, i.e., aircraft size can be determined residually dividing the airline's total traffic on a route by the number of flights. Note that cost per seat, which can be written $\theta(d)/s + \tau$, visibly decreases with s capturing the presence of economies of traffic density (i.e., economies from operating a larger aircraft holding the load factor constant) which are unequivocal in the airline industry.[4] In other words, having a larger traffic density on a certain route reduces the impact on the cost associated with higher frequency.

Therefore, the airline's total cost from operating on a route is $f[\theta(d) + \tau s]$ or equivalently

$$c = \theta(d) f + \tau q_{air}. \tag{5}$$

Thus the airline's profit is $\pi_{air} = p_{air} q_{air} - c$, which can be rewritten using (5) as

$$\pi_{air} = (p_{air} - \tau) q_{air} - \theta(d) f, \tag{6}$$

indicating that average variable costs are independent of the number of flights.

After plugging (4) into (6) and maximizing, the first-order condition for the fare is $\frac{\partial \pi_{air}}{\partial p_{air}} = 1 - \frac{\beta V}{d(1-\beta)} (2p_{air} - cd + \gamma/f - \tau) = 0$ and, from this condition, it is easy to obtain the following expression

$$p_{air} = \frac{1}{2} \left(cd + \tau - \gamma/f + \frac{d(1-\beta)}{\beta V} \right), \tag{7}$$

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[3] On the other hand, a traveler will prefer to drive instead of stay at home (i.e., not travel at all) for $\alpha > \widehat{\alpha}$; and thus, $q_{car} = \int_{\widehat{\alpha}}^{\widetilde{\alpha}} d\alpha = \widetilde{\alpha} - \widehat{\alpha}$ and $q_o = \int_0^{\widehat{\alpha}} d\alpha = \widehat{\alpha}$. The demand for the three possible consumer options is determined by the choices made by the airline. However, the focus of the paper is the airline's frequency choice.

[4] Empirical studies confirming presence of economies of traffic density in the airline industry include Caves *et al.* (1984), Brueckner and Spiller (1994) and Berry *et al.* (1996).

so that fares increase with variable costs and with distance, and they decrease with schedule delay and with the speed of personal transportation (βV).

On the other hand, the first-order condition for frequency is given by $\frac{\partial \pi_{air}}{\partial f} = \frac{(p_{air}-\tau)\gamma\beta V}{d(1-\beta)f^2} - \theta(d) = 0$ or equivalently by

$$f = \left(\frac{(p_{air} - \tau)\gamma\beta V}{\theta(d)\, d\, (1-\beta)} \right)^{1/2}, \tag{8}$$

indicating that frequency increases with passengers' disutility of delay, carrier's margin $(p_{air} - \tau)$ and the speed of personal transportation, and decreases with the cost of frequency and distance.

The second-order conditions $\partial^2\pi_{air}/\partial p_{air}^2, \partial^2\pi_{air}/\partial f^2 < 0$ are satisfied by inspection and the remaining positivity condition on the Hessian determinant is discussed below. By combining the two first-order conditions, we obtain the following equilibrium condition

$$\underbrace{\frac{2\theta(d)d(1-\beta)}{\gamma\beta V}f^3}_{Cf^*} = \underbrace{\left(cd - \tau + \frac{d(1-\beta)}{\beta V} \right) f - \gamma}_{Lf^*}. \tag{9}$$

The equilibrium frequency is shown graphically in Figure 3, as in Brueckner (2004) and Brueckner and Flores-Fillol (2007), where we observe that the f solution occurs at an intersection between a cubic expression (Cf^*) and a linear expression (Lf^*) whose vertical intercept is negative. The slope of Lf^* must be positive for the solution to be positive and thus we assume that τ is small enough for this to be the case. We observe that there are two possible positive solutions, but only the second one satisfies the second-order condition.[5]

$$-\text{Insert here Figure 3}-$$

Looking at (9) together with Figure 3, we can carry out a comparative statics analysis for all the parameters in the model. Although some effects do not seem trivial from inspection of (9), the lemma below ascertains the overall effect by analyzing the sign of the total differential of the equilibrium frequency with respect to each parameter.

Lemma 2 *In Scenario 1, the equilibrium frequency falls with an increase in the marginal cost per seat (τ). Frequency also decreases with the speed of both plane (V) and car (β) if*

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[5] Positivity of the Hessian determinant requires $p_{air} - \tau > \frac{\gamma}{4f}$. Observe that for the second intersection to be relevant, the slope of Cf^* must exceed the slope of Lf^*, i.e., $\frac{6\theta(d)d(1-\beta)}{\gamma\beta V}f^2 > cd - \tau + \frac{d(1-\beta)}{\beta V}$. Using the first-order conditions for p_{air} and f, this expression reduces to $p_{air} - \tau > \frac{\gamma}{4f}$, which is exactly the condition required by the positivity of the Hessian determinant.

$\tau > cd$. *However, it rises with the cost of driving (c) and the disutility of delay (γ) for small values of β.*

Looking at the effect of distance on the equilibrium flight frequency, although the effect is in general indeterminate, we observe that $\frac{df^*}{dd} > 0$ can hold at least for sufficiently low distances.

As expected, when the cost of driving (c) rises, the equilibrium frequency also increases because the car becomes a worse option and more passengers choose air travel. There is also a positive relationship between γ and f^* when β is small since carriers increase frequency as passengers' disutility of delay increases. When the marginal cost per seat (τ) increases, frequency falls since air travel becomes a less competitive option.

Note that β is the airline/car speed differential and that when β increases the car's speed increases relative to that of the airplane, and driving becomes a more attractive option for travel (for $\tau > cd$, so that the cost per seat is higher than the cost of driving). Thus, more passengers prefer personal transportation and flight frequency falls. Finally, the effect of plane's speed (V) seems to be somewhat counterintuitive since frequency falls when V rises (again for $\tau > cd$). Yet, higher speed of the aircraft means that the traveler reaches her final destination faster, compensating for the disutility of schedule delay.

Looking at the effect of distance on equilibrium frequency, our result suggests that the airline may choose to increase the frequency of service as distance increases. This result appears counterintuitive at first. Indeed, with longer distance driving becomes a less attractive substitute for flying, as the ratio of remaining time for the driver to remaining time for the flyer $\frac{T - d/(\beta V)}{T - d/V}$ shrinks with distance. Then, as distance increases, the airline will have less incentive to increase the quality of its product, and can simply charge the customers more as demand grows by itself. In fact, the first-order condition for f (see (8)) tells us exactly that: with higher distance, other things being equal, our carrier can afford to reduce the frequency (and save money). Thus, distance has a *negative direct effect* on frequency.

The answer to this puzzle lies in the first-order condition for p_{air} which shows a *positive indirect effect* of distance on frequency through fares (see (7)) since $p_{air}(\underset{+}{d})$ and $f(\underset{+}{p_{air}})$. The lemma above suggests that the *indirect effect* may prevail.

Air travel is perceived as a better option than driving as distance increases because it is faster and reduces travel time and, as a consequence, our monopolist carrier faces a higher demand as distance increases. Since there is a positive relationship between p_{air} and f, when facing a higher demand (as distance increases) our monopolist can boost profits by increasing flight frequency. Thus, in addition to the marginal cost associated with higher frequency,

the airline can obtain the marginal benefit in terms of the higher price it will be able to charge (in addition to the price increase due to longer distance already present in (7)). What Lemma 2 says, then, is that the airline will increase its service quality (frequency) when the marginal benefit from doing so outweighs the marginal cost; moreover, the set of parameter values where this will happen will be non-empty.

2.2 Scenario 2: driving is a dominated alternative

The case without drivers is not nearly as exciting: we observe the empirically confirmed decreasing relationship between frequency and distance.

Under Scenario 2, a traveler will fly when $\alpha > \overline{\alpha}$ and stay home otherwise. From (2), demand for air travel is given by

$$q_{air} = \int_{\overline{\alpha}}^{1} d\alpha = 1 - \overline{\alpha} = 1 - \frac{p_{air} + \gamma/f}{T - d/V},$$ (10)

and costs and profits are as in Scenario 1 (see (5) and (6)). After plugging (10) into the profit function and maximizing, the first-order condition for the fare is $\frac{\partial \pi_{air}}{\partial p_{air}} = 1 - \frac{1}{T-d/V}\left(2p_{air} + \gamma/f - \tau\right) = 0$ and, from this condition, we obtain

$$p_{air} = \frac{1}{2}\left(T - \gamma/f - d/V + \tau\right),$$ (11)

which shows that fares rise with passengers' total time, variable costs and aircraft's speed, and fall with schedule delay and distance. Comparing (7) and (11), we observe that p_{air} increases with distance in Scenario 1, but the sign of this effect changes in Scenario 2. This is explained by the different kinds of competition existing in the two scenarios. On the one hand, when competing against driving, flying becomes more attractive as distance increases and thus the airline can increase fares. On the other hand, when competing against staying at home, flying becomes less attractive for longer distances and the airline tries to compensate this negative effect by lowering fares.

The first-order condition for frequency is given by $\frac{\partial \pi_{air}}{\partial f} = \frac{(p_{air}-\tau)\gamma}{(T-d/V)f^2} - \theta(d) = 0$ or equivalently by

$$f = \left(\frac{(p_{air} - \tau)\gamma V}{\theta(d)(TV - d)}\right)^{1/2},$$ (12)

showing that frequency increases with passengers' disutility of delay, carrier's margin $(p_{air}-\tau)$ and airline's speed, whereas it decreases with passengers' total time and the cost of frequency. Differently from Scenario 1, the effect of distance on f is unclear.

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By combining the two first-order conditions, we obtain the following equilibrium condition

$$\underbrace{\frac{2\theta(d)(TV-d)}{\gamma}f^3}_{Cf^*} = \underbrace{(TV-d-\tau V)\,f-\gamma V.}_{Lf^*} \tag{13}$$

As in Scenario 1, the f solution occurs at an intersection of Cf^* and Lf^* whose vertical intercept is negative. It is sufficient to assume a large T to have a positive sloping Lf^* so that there are positive values for f^*. As in Scenario 1, there are two possible positive solutions, but only the second one satisfies the second-order condition.[6]

The comparative statics effects are summarized in the lemma below.

Lemma 3 *In Scenario 2, equilibrium flight frequency falls with an increase in the marginal cost per seat (τ). However, frequency rises with the disutility of delay (γ), the plane's speed (V) and passengers' total time (T).*
Finally, looking at the effect of distance, we observe that $\frac{df^*}{dd} < 0$, i.e., equilibrium flight frequency decreases with distance.

As under Scenario 1, we observe $f^*(\underset{+}{\gamma})$ and $f^*(\underset{-}{\tau})$. When passengers' total time (T) rises, more passengers are willing to undertake air travel since the utility of flying increases and, as a consequence, the equilibrium frequency increases. Finally, when the plane's speed increases (V), we observe the same effect as with T, i.e., the valuation of air travel increases and thus the equilibrium frequency rises.

Looking at the effect of distance, from the first-order conditions we observe that the *direct effect on frequency is unclear* (see (12)) and there is a *negative indirect effect* through fares (see (11)) since $p_{air}(\underset{-}{d})$ and $f(\underset{+}{p_{air}})$. The above lemma states that the indirect effect outweighs the direct one.

Combining the results from Lemmas 1-3, we deduce that the equilibrium frequency can increase with distance when driving is not a dominated option (i.e., Scenario 1), whereas it decreases with distance when driving is disregarded by consumers as a relevant mode of transport (i.e., Scenario 2), as stated in the proposition that follows.

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[6]The second-order conditions $\partial^2\pi_{air}/\partial p_{air}^2, \partial^2\pi_{air}/\partial f^2 < 0$ are satisfied by inspection and the remaining positivity condition on the Hessian determinant is the same as in Scenario 1, i.e., $p_{air} - \tau > \frac{\gamma}{4f}$. Again as in Scenario 1, for the second intersection to be relevant, the slope of Cf^* must exceed the slope of Lf^*; and this condition exactly reduces to $p_{air} - \tau > \frac{\gamma}{4f}$.

Proposition 1 *From Lemmas 1-3 we conclude that equilibrium frequency*
i) may increase with distance for $d < d^*$;
ii) decreases with distance for $d > d^*$.

Hence, when driving is a dominated alternative (i.e., $d > d^*$), flight frequency decreases with distance, confirming the results in Wei and Hansen (2006) and Pai (2007). More interestingly, when driving is not a dominated alternative (i.e., $d < d^*$), air travel is perceived as a better option than driving as distance increases because it is faster and reduces travel time. Then airlines increase service quality with distance by offering higher frequency at a higher fare.

As we suggested above, our result is due to a trade-off between two forces. On the one hand, increasing frequency always implies an extra cost for the provider of scheduled transportation services in terms of higher fixed costs and lower benefits from density economies which, in the case of airlines, arise from using bigger aircraft at high load factors. On the other hand, an increase in distance may boost the demand for high-speed scheduled transportation services on short-haul routes where the use of private vehicles is a relevant option for travelers. This is because an increase in distance makes the high-speed transportation mode more competitive and so providers of scheduled transportation services are able to increase frequency and charge higher fares. Thus, the relationship between frequency choices and distance depends crucially on the presence of personal transportation.

2.3 The social optimum

Having analyzed the monopoly airline's choice, our attention now shifts to welfare analysis in which a social planner decides flight frequency and traffic so as to maximize social surplus, which is computed as the sum of total utility and airline profit. We need to differentiate between the two scenarios.

Scenario 1: driving is not a dominated alternative
Total utility for passengers undertaking air travel is $u_{air} = \int_{\alpha^*}^{1} [y - p_{air} - \gamma/f + \alpha (T - d/V)] \, d\alpha$, where α^* denotes the air-travel/driving margin. Therefore, carrying out the integration we obtain

$$u_{air} = (y - p_{air} - \gamma/f)(1 - \alpha^*) + \frac{T - d/V}{2} \left(1 - \alpha^{*2}\right). \tag{14}$$

Total utility for driving passengers is $u_{car} = \int_{\alpha^{**}}^{\alpha^*} [y - cd + \alpha (T - d/(\beta V))] \, d\alpha$, where

α^{**} denotes the driving/staying margin. Integrating across drivers we obtain

$$u_{car} = (y - cd)(\alpha^* - \alpha^{**}) + \frac{T - d/(\beta V)}{2}(\alpha^{*2} - \alpha^{**2}). \tag{15}$$

Finally, total utility for "stayers" is

$$u_o = \int_0^{\alpha^{**}} y d\alpha = y\alpha^{**}. \tag{16}$$

From (6), airline's total profit equals $\pi_{air} = (p_{air} - \tau) \int_{\alpha^*}^1 d\alpha - \theta(d)f$ and after integrating across flyers it becomes

$$\pi_{air} = (p_{air} - \tau)(1 - \alpha^*) - \theta(d)f. \tag{17}$$

The total welfare function is computed by adding utilities and profits, i.e., $W = u_{air} + u_{car} + u_o + \pi_{air}.$[7] This expression, after substituting (14), (15), (16) and (17), becomes

$$W = y - (\gamma/f + \tau)(1 - \alpha^*) - \theta(d)f + \frac{T - d/V}{2}(1 - \alpha^{*2}) + \frac{T - d/(\beta V)}{2}(\alpha^{*2} - \alpha^{**2}) - cd(\alpha^* - \alpha^{**}). \tag{18}$$

The planner chooses α^* and α^{**}, which determine the optimal air and road traffic, along with flight frequencies to maximize (18). Observe that airfares do not appear in the expression because they are transfers between airlines and air travelers.

From the first-order condition for frequency we obtain

$$f^2 = \frac{\gamma(1 - \alpha^*)}{\theta(d)}, \tag{19}$$

which indicates that the optimal frequency increases with the disutility of delay and with the proportion of air travelers, whereas it decreases with the cost of frequency.

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[7]This analysis implicitly assumes that the "gasoline market" is perfectly competitive. If the "gasoline market" is either imperfectly competitive or regulated so that the price is kept above cost, then (at least part of) the cost drivers incur is somebody's profit. In the case in which all the cost of driving represents the profit of "gasoline companies" (i.e., $\pi_{car} = cd \int_{\alpha^{**}}^{\alpha^*} d\alpha$), the welfare function that the social planner is to maximize has to incorporate this new element, so that it becomes $W = u_{air} + u_{car} + u_o + \pi_{air} + \pi_{car}$. In fact, taking into account a positive profit for "gasoline firms" makes driving a better travel option from the viewpoint of the social planner. In this framework, the socially desired number of flyers and "stayers" decreases, the number of drivers increases, and equilibrium frequency can be both suboptimal or excessive depending on the value of cd. More information on this particular case is available from the authors upon request.

The first-order condition for choice of α^* yields

$$\alpha^* = \frac{(\tau - cd + \gamma/f)\beta V}{d(1-\beta)} \equiv \widetilde{\alpha}_{so}, \tag{20}$$

and, by comparing (1) with (20) it is easy to check that $\widetilde{\alpha} > \widetilde{\alpha}_{so}$ since $p_{air} > \tau$ (because otherwise the airline would have negative profits). Therefore, air traffic is suboptimal and there are too many drivers in equilibrium.

The first-order condition for choice of α^{**} yields

$$\alpha^{**} = \frac{cd}{T - d/(\beta V)} \equiv \widehat{\alpha}_{so}, \tag{21}$$

so that $\widehat{\alpha} = \widehat{\alpha}_{so}$ and thus the amount of "stayers" is socially optimal.

$$-\text{Insert here Figure 4}-$$

From (19) and (20), we obtain the following expression

$$\underbrace{\frac{\theta(d)d(1-\beta)}{\gamma\beta V}f^3}_{Cf^{SO}} = \underbrace{\left(cd - \tau + \frac{d(1-\beta)}{\beta V}\right)f - \gamma}_{Lf^* = Lf^{SO}}. \tag{22}$$

The social optimum and equilibrium are easily compared because the RHS is identical and the only difference in the LHS is the absence of the 2 factor multiplying the expression. As a result, the socially optimal flight frequency is higher than the equilibrium frequency, as show in the figure below

$$-\text{Insert here Figure 5}-$$

The results are summarized in the lemma below.

Lemma 4 *Under Scenario 1, both the equilibrium flight frequency and air traffic are suboptimal (i.e., $f^* < f^{SO}$ and $\widetilde{\alpha}_{so} < \widetilde{\alpha}$).*

Therefore, in the eyes of the social planner, more drivers should undertake air travel. To achieve this, the airline should increase flight frequency. This result is consistent with those in Brueckner (2004), Brueckner and Flores-Fillol (2007) and Flores-Fillol (2009). As pointed out in Flores-Fillol (2009), the underprovision of frequency is the natural result in monopolistic situations and even under competition when carriers operate point-to-point

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networks.[8]

Scenario 2: driving is a dominated alternative

Proceeding in a similar way as in Scenario 1, we have $u_{air} = (y - p_{air} - \gamma/f)(1 - \alpha^*) + \frac{T - d/V}{2}(1 - \alpha^{*2})$, $u_o = y\alpha^*$ and $\pi_{air} = (p_{air} - \tau)(1 - \alpha^*) - \theta(d)f$ where α^* now denotes the air-travel/driving margin. Computing $W = u_{air} + u_o + \pi_{air}$, we obtain

$$W = y - (\gamma/f + \tau)(1 - \alpha^*) - \theta(d)f + \frac{T - d/V}{2}\left(1 - \alpha^{*2}\right). \qquad (23)$$

From the planner's first-order conditions for f and α^*, we obtain $f^2 = \frac{\gamma(1 - \alpha^*)}{\theta(d)}$ which is the same condition as in Scenario 1 (see (19)); and

$$\alpha^* = \frac{\tau + \gamma/f}{T - d/V} \equiv \overline{\alpha}_{so}, \qquad (24)$$

and, by comparing (2) with (24) it is easy to check that $\overline{\alpha}_{so} < \overline{\alpha}$ since $p_{air} > \tau$. Therefore, air traffic is again suboptimal. Finally, from (19) and (24), we obtain the following expression

$$\underbrace{\frac{\theta(d)(TV - d)}{\gamma}f^3}_{Cf^{SO}} = \underbrace{(TV - d - V\tau)f - \gamma V}_{Lf^* = Lf^{SO}}. \qquad (25)$$

As in Scenario 1, the social optimum and equilibrium are easily compared because the RHS is identical and the only difference in the LHS is the absence of the 2 factor multiplying the expression. As a result, there is also an underprovision of flight frequency in absence of drivers as summarized in the lemma below.

Lemma 5 *Under Scenario 2, both the equilibrium flight frequency and air traffic are suboptimal (i.e., $f^* < f^{SO}$ and $\overline{\alpha}_{so} < \overline{\alpha}$).*

Hence, the number of "stayers" is no longer efficient in Scenario 2, since some of them should undertake air travel to achieve the social optimum. To carry the extra passengers needed to obtain efficiency, our carrier should increase frequency, as in Scenario 1.

To sum up, the social optimum analysis performed under the two scenarios considered suggests that higher flight frequency should be provided by the carrier so that more passengers make use of air travel.

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[8] Flores-Fillol (2009) claims that the apparent overprovison of flight frequency in the current unregulated context, requires us to consider airline competition where carriers operate in hub-and-spoke networks and markets are partially served.

3   Empirical application

This section offers an empirical test of the model's predictions, using the data on frequency choices by airlines in the deregulated EU market. Specifically, we use airline-route-level frequencies on European routes over the period from May 2006 until April 2007.[9] Our sample includes routes from the ten largest airports in Europe to all European destinations (EU27 + Switzerland and Norway) with direct flights. Data of airline frequencies have been provided by Official Airlines Guide (OAG Data market analysis publication). We exclude observations for airlines that offer fewer than 52 frequencies per year on a particular route: operations with less than one flight per week should not be considered as scheduled.

Our theoretical model predicts the possibility of increasing frequency with length of haul for routes on which potential customers may consider driving as a viable alternative to flying. Figure 6 shows the spline that estimates the relationship between distance and frequency in our dataset without imposing any restriction or shape in the functional form of this relationship.

−Insert here Figure 6−

Results of the spline suggest that the frequency-distance relationship changes at a haul length of about 500 kilometers (311 miles). Specifically, this figure suggests a higher frequency on longer routes for distances less than 500 kilometers; the direction of the relationship is reversed for routes longer than 500 kilometers (note that, for the sake of clarity, we depict the spline only for distances less than 1, 000 kilometers). However, since Figure 6 presents a rough picture, we must control for several route features and airline specific factors to come to a definite conclusion.

An obvious but important feature of the theoretical model is that there is a positive relationship between flight frequency and demand for air services. The empirical application is based on examining determinants of airlines' frequency choices. Thus, to be consistent with our theoretical model, we have to include *distance* together with the usual *regional variables (population, income, etc.)* as explanatory variables to capture relevant air demand shifters at the route level.

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[9]We define routes as air services between two different airports, so that one city-pair market (e.g., London-Milan) may include several routes.

Lemmas 2 and 3 in the theoretical model state that frequency decreases with the airline/car speed differential (β), while it increases with the disutility of delay (γ). We also observe that distance is a relevant measure of the possible advantage of airlines with respect to cars, but it is not the only one: availability and quality of surface road network will also play a major role. In our empirical specifications, we use an indicator for the absence of direct roadways between the endpoints (i.e., cases in which at least one of the cities is located on an island) as a crude measure of car/plane substitutability beyond simple distance. Thus, distance and island endpoints explain, to a large extent, aircraft speed advantage in comparison with other transportation modes. In the same way, the disutility of delay should be more important on short-haul routes in which personal transportation is a relevant option (i.e., driving is not a dominated alternative).

Additionally, the theoretical model also predicts a negative relationship between frequency and the marginal cost per seat (τ); and in the empirical application we include some *airline attributes (i.e., airport presence, exploitation of connecting traffic)* that may influence the marginal cost per seat of carriers or, alternatively, their marginal revenue given the marginal cost per seat (Borenstein, 1989; Berry *et al.*, 1996).

Beyond the theoretical model, the *intensity of airline competition* is also taken into account as an explanatory variable since, other things being equal, the specific demand for an airline is influenced both by service quality of other airlines operating on the same route and of surface transportation modes. Note that our theoretical model considers a monopoly provider of scheduled transportation services, while our empirical analysis uses a sample of routes with different market structures. However, a large proportion of the routes in our sample have a high level of concentration. In fact, 54% of routes are monopoly routes and the concentration index (HHI) is higher than 0.80 in an additional 6% of routes, so the monopoly seems to be the best-fitting market structure for about 60% of the routes included in our sample.

Eventually, the following equation will be estimated:

$$Frequency_{ijk} = \alpha + \beta_1 Distance_{ij} + \beta_2 Population_{ij} + \beta_3 GDPC_{ij} + \beta_4 D_{ij}^{capital} + \beta_5 D_j^{island} +$$
$$\beta_6 Tourism_j + \beta_7 HHI_{ij} + \beta_8 D_k^{LCC} + \beta_9 D_k^{interhub} + \beta_{10} Airport\text{-}presence_k + \varepsilon_{ijk},$$

(26)

where the dependent variable is annual frequencies of airline k from airport i to airport j

($Frequency_{ijk}$). We consider the following variables as exogenous explanatory variables of airlines' frequency choices:[10]

1. $Population_{ij}$: Weighted average of population at the origin and destination regions of the route (NUTS 2 level). Airline frequencies should be higher on routes that link more populated regions due to higher demand for air travel.

2. $GDPC_{ij}$: Weighted average of Gross Domestic Product per capita at the origin and destination regions of the route (NUTS 2 level). Weights are based on population. Airline frequencies should be higher on routes that link richer regions due to higher demand for air travel.

3. $D_{ij}^{capital}$: Dummy variable that takes value 1 when one of the route endpoints is located in the political capital of the corresponding country. Employees of public administrations may require air services to carry out their professional duties.

4. D_{j}^{island}: Dummy variable that takes value 1 when at least one of the endpoints is located on an island. On islands, alternative transport modes (ships) are poorer substitutes to airline services than surface transport modes (cars, trains) on the mainland. Therefore, demand for air services should be higher when one of the endpoints of the route is an island.

5. $Tourism_j$: Percentage of employment in hotels and restaurants in the destination region of the route (NUTS 2 level). Airline frequencies should be higher on routes where the destination has significant tourist activity due to the higher demand for air services.

6. HHI_{ij}: Herfindahl-Hirschman index in terms of airline frequencies at the route level. Since airlines compete in fares and frequencies, they should increase frequencies when competition from other carriers is tougher.

7. D_{k}^{LCC}: Dummy variable that takes value 1 for low-cost carriers. We define as low-cost carriers those airlines that do not use a business fare class on any route and have only economy class cabins. At least in Europe, the route network of low-cost carriers is based on point-to-point services. Therefore, their frequencies should be lower than those of other airlines that may be operating hub-and-spoke networks.

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[10]Data for population, GDP and tourism specialization at the NUTS 2 level (the statistical unit used by Eurostat) have been provided by Cambridge Econometrics (European Regional Database publication).

8. $D_k^{interhub}$: Dummy variable that takes value 1 for hubbing airlines that use the origin airport as the spoke of its hub (e.g., Lufthansa's flights to Frankfurt). These airlines should have a high number of frequencies to feed their hubs.

9. $Airport\text{-}presence_k$: Given the importance of this element, we use two variables to define the presence of the airline at the origin airport. First, the total number of destinations served by the airline from the origin airport ($Destination_k$). Second, the market share of the airline in terms of total departures from the origin airport ($Airportshare_k$). These variables are highly correlated so they should be estimated separately to avoid multicollinearity. Our baseline specification includes $Destination_k$ as explanatory variable, but the results are very similar regardless of the indicator of airport presence used. Note that the frequency that an airline sets at the route level should be high when its presence in the origin airport is strong. Airport dominance allows an airline to have control of the slots and facilities at the terminal building (gates, check-in counters, etc.) that are needed to offer high frequencies. This has been well documented in the literature since the seminal work of Borenstein (1989). In addition to this, a high amount of operations in a large airport by an airline usually indicates the use of that airport to exploit connecting traffic (in our sample, the only exception is Easyjet at Gatwick).

Our empirical strategy involves estimating equation (26) for routes with less and more than 500 kilometers. This will allow us to examine the relationship between frequencies and distances depending on whether driving is a dominated alternative or not. One may be tempted to suggest a regression over the entire sample with a dummy variable indicating routes shorter than 500 kilometers. However, our hypothesis does not really tell us what the sign of this variable should be. Rather, it just says that the slope of the frequency-distance relationship is positive up to 500 kilometers and negative afterwards, a conclusion that does not say how average frequencies in the two ranges compare.[11]

Note that the dummy variable for islands may distort the results with regard to our main hypothesis. Indeed, driving may not be a viable option even on short-haul routes when one of the endpoints is an island. Furthermore, the variable for islands may also distort results

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[11]In any case, it is interesting to mention results of estimating equation (26) for the entire sample including the dummy variable for routes shorter than 500 kilometers. This dummy variable takes a positive value and is clearly significant from a statistical point of view. Thus, airline frequency seems to be higher for short-haul routes. The results of this estimation are available from the authors upon request.

of the variable for tourism since tourist activity on islands tends to be significant. Hence, we estimate equation (26) for all routes within the corresponding distance range (less or more than 500 kilometers), and then compute the estimation excluding routes that have an island as an endpoint (origin and/or destination) as well. In this latter estimation, we exclude flights from London-Heathrow and London-Gatwick to the continent. We also exclude flights that have any island as destination.

Table 1 includes some characteristics of origin airports included in our dataset (ten largest European gateways in terms of total passenger traffic). Our sample includes a total of 887 routes. Some differences across airports should be noted. Amsterdam and Barcelona serve the most European destinations, and London-Heathrow and Paris-Orly the fewest. Note that the three largest airports in terms of total traffic (London-Heathrow, Paris-CDG and Frankfurt) do not necessarily have the highest amount of frequencies to European destinations. Capacity constraints may explain the fact that low-cost carriers do not operate in London-Heathrow, but have a significant share in airports like Barcelona, Amsterdam, Paris-Orly and especially London-Gatwick. For the rest of airports, the role of low-cost carriers is modest. The position of the dominant airline seems to be especially strong in the case of Air France (Paris-CDG, Paris-Orly) and Lufthansa (Frankfurt, Munich) although the dominant airline also controls about half of total operations in the rest of airports. In all the cases, the average number of route competitors is lower than 3 and it is lower than 2 in London airports, Paris-Orly and Amsterdam. Average route distance is about 1,000 kilometers, a figure that varies depending on the geographical location of each airport (except in London-Gatwick which is very high).

Table 2 shows some preliminary evidence of airline choices concerning frequencies and aircraft size as a function of route distance. For all airlines in our sample, frequencies are substantially higher on short-haul routes than on long-haul ones. Differences in average frequencies between short-haul and long-haul routes are statistically significant when considering all airlines, hubbing airlines at the origin airport, and low-cost carriers. In contrast, planes are bigger on long-haul routes although differences from a statistical point of view are modest. Low-cost airlines are the exception, as they typically use the same type of aircraft on most of their routes. The analysis must take into account other variables that influence airline choices, but it seems that airlines are required to offer high frequencies on short-haul routes. In contrast, they may exploit density economies on long-haul routes from the use of

bigger planes at high load factors.[12]

Table 3 provides descriptive statistics for the continuous and discrete variables used in the empirical analysis. All variables have enough variability to capture relevant differences across the routes of the airlines in our sample. Table 4 shows results from estimates of equation (26) for routes of less than 500 kilometers, while Table 5 presents the results for routes of more than 500 kilometers.

We estimate our equation using both Ordinary Least Squares (OLS) and Zero Truncated Poisson (ZTP) techniques. The latter technique allows us to exploit the form of the dependent variable which takes positive integer values. However, since the number of counts is high, we do not expect substantial differences in our results when using either OLS or ZTP. The overall explanatory power of the models estimated is reasonably good. Results are similar when using OLS or ZTP techniques, and the variation inflation factors (VIF) show that multicollinearity is absent in our regressions. Different specifications are estimated with the ZTP technique. Specification (2) is the baseline, specification (3) uses $Airportshare_k$ as indicator of airport presence and specification (4) excludes routes with islands as endpoints. Finally, specification (5) uses the same sample of routes as in specification (4), but it also includes as explanatory variable a dummy variable that captures the presence of high-speed train services, as explained below.

As regards flight frequency and distance, we find a positive relationship between airline frequencies and distance for routes shorter than 500 kilometers. This relationship is statistically significant at the 5 percent level when including routes with islands as endpoints, and is statistically significant at the 1 percent level when excluding routes with islands as endpoints. In terms of elasticities, a 10 percent increase in route distance implies an increase of about 4 percent in airline frequency. Interestingly, 4 percent of the average frequency for routes with less than 500 kilometers amounts to 59 flights, meaning that a 10 percent increase in distance adds about a flight per week, other things being equal.

For routes longer than 500 kilometers, we find a negative relationship between airline frequencies and distance. This relationship is statistically significant at the 1 percent level. Focusing on elasticities, a 10 percent increase in route distance implies a decrease of about 5

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[12]Note also that aircraft costs correspond to three stages: takeoff, inflight time and landing. With regard to the size of the aircraft scale diseconomies arise in takeoff and landing, while scale economies arise at the cruise speed. This explains why aircraft that minimize costs are smaller on short-haul than on long-haul routes.

percent in airline frequency. Note that 5 percent of the average frequency for routes longer than 500 kilometers is only 38 extra flights per year. So, while the sensitivity of frequency to distance appears similar (though the direction of the effect is different) for shorter and longer routes, in absolute value the effect is bigger for short-haul markets.

Thus, we provide empirical evidence in support of the hypothesis stated in our theoretical framework. On the one hand, airline frequency decreases with distance on long-haul routes. From our empirical analysis, this seems to be the case on routes longer than 500 kilometers. On the other hand, on short-haul routes (routes shorter than 500 kilometers), airline frequency increases with route distance.

Looking at demand shifters at the route level, population has a positive influence on airline frequency, as expected, although its statistical significance is modest in some of the specifications estimated. The role of the city as a political capital has a political influence on airline frequencies but is only statistically significant on short-haul routes that include a higher proportion of domestic links.

The dummy variable for islands as endpoints has a positive influence on airline frequencies (as expected) but it is not statistically significant. Moreover, tourism specialization at the destination has a positive influence on airline frequencies for short-haul routes when routes that have an island as endpoint are excluded. Interestingly, Gross Domestic Product per capita has a statistically significant positive influence on airline frequencies on long-haul routes but is not a relevant factor on short-haul routes. The variable for intensity of competition is statistically significant at the 1 percent level in all the specifications. Airline frequencies decrease with the level of route concentration. As expected, airlines compete strongly in frequencies, so the monopolization of a route allows an airline to operate fewer flights.

Concerning airline attributes, the two different variables used to capture presence at the origin airport have a positive sign and are statistically significant at the 1 percent level in all the specifications. The results for the rest of variables do not change substantially when using one or the other measure of airport presence, so that we present results for specifications (1), (2), (4), and (5) for short-haul routes when using the variable $Destination_k$ as the indicator of airport presence (recall that specification (3) uses $Airportshare_k$ as indicator of airport presence).

The dummy variable for airlines that use the origin airport as spoke is also clearly signif-

icant in all specifications. Finally, the dummy variable for low-cost carriers is negative and is statistically significant in most of the specifications.

The results of variables for intensity of competition and airline attributes are consistent with those obtained by Carlsson (2004). Indeed, this study analyzes the effect of market structure on flight frequency for a sample of European city-pair markets, finding that former flag carriers provide more flights than other airlines; and that market concentration has a negative influence on flight frequency. In this vein, Schipper *et al.* (2002) find that bilateral airline liberalization in Europe led to a higher frequency in city-pair markets for the period 1988-1992.

Note that some air routes in our sample are also affected by competition from high-speed trains. This is particularly the case for routes that have Paris as origin airport. The interaction between air services and high-speed trains is beyond the scope of this paper. However, this interaction is fully consistent with the empirical predictions of our theoretical model. High-speed trains barely influence airline behavior on long-haul routes but, on shorter routes, airlines are required to provide higher quality products to prevent travelers from using other transport modes (i.e., cars and trains on certain routes).

Specification (5) in Table 4 includes as an explanatory factor a dummy variable (denoted by D_{ij}^{HST}) that takes value 1 for routes on which airlines compete with high-speed train services.[13] Following the definition used by the International Union of Railways (Union Internationale des Chemins de Fer - UIC), we consider high-speed train lines to be those lines with trains able to reach a speed above 250 km/h. This dummy variable takes a positive value but is not statistically significant. This result does not mean that airlines and high-speed trains do not compete: it is explained by the fact that airlines are required to offer high frequencies on short-haul routes to be competitive against surface transportation modes (either cars or trains). In addition to this, it is worth noting that airlines react to new high-speed services by adjusting aircraft size and maintaining flight frequency. For example, the reaction of Iberia to the new high-speed train service on the route Madrid-Barcelona (the densest route in our sample) was to reduce aircraft size but maintain high frequencies. Moreover, an accurate analysis of competition between airplanes and trains should take into account the situation before and after the start of high-speed train services since these new

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[13]Note that we do not estimate the specification (5), which includes the variable for high-speed train services, when considering routes longer than 500 kilometers. In our dataset, a very few number of long-haul routes have available high-speed train services.

services typically cause a reduction in the number of airlines offering services on the route.

To sum up, we have shown the same result both theoretically and empirically: airline frequency choices are dependent upon competition from private vehicles. Airlines always incur extra costs when adding flights on a route since economies of traffic density require the use of big aircraft at high load factors and each additional flight is associated with additional fixed costs. However, the demand for air services rises with distance on routes where airlines compete with personal transportation since the latter is a slower transportation mode and, in this case, airlines can increase frequency and charge higher fares as they become more competitive.

In this regard, our empirical analysis shows that airlines increase frequency as distance increases on routes shorter than 500 kilometers. Nevertheless, for routes longer than 500 kilometers, on which driving is a dominated alternative, airlines decrease frequency as distance increases. In the latter scenario, the demand for air services may not increase when the origin and destination airports are more remote because some potential travelers may prefer to stay at home, and airlines prefer to save costs by exploiting density economies.

4   Concluding remarks

The main contribution of this paper is to underscore that presence of the personal transportation option crucially affects frequency choice by a provider of scheduled transportation services. We have shown this to be true, using distance as a proxy for the substitutability between higher-speed scheduled services and private vehicles; our findings are identified theoretically and tested empirically for the European airline industry. Analysts and policy-makers should consider this factor when analyzing investment in transportation infrastructures and regulation of scheduled services.

A large proportion of air traffic involves short-haul routes where airlines must provide high flight frequency to compete with cars. Our social optimum analysis shows that there is an underprovision of flight frequency supposing that airports are not congested, implying an overuse of personal transportation that may create problems such as pollution, noise and road congestion.

Investing in road infrastructures may place strong pressure on public budgets. In fact, the US Department of Transportation predicts a required expenditure of $225 billion (which

represents over 1.5% of the current US GDP) annually for the next 50 years to upgrade the existing road network. In this vein, a major goal in the transportation policy of the European Commission is to alleviate road congestion by promoting the use of scheduled transportation services.[14] Since road and airport infrastructures are communicating vessels, we suggest that policy makers could take into account capacity at national airports as an instrument available to reduce road congestion. Similarly, high-speed train lines providing a high frequency of service may also be useful in alleviating road congestion.

While our empirical application relates to the airline industry (a competitive industry for which relevant data are readily available), our analysis has policy implications for any transport market with private transportation and scheduled services like inter-city and intra-city surface transportation. Furthermore, the logic of the model goes beyond the transportation sector since a similar setup could be used to analyze the behavior of a firm in situations where better alternatives in certain dimensions are either present or absent. When these alternatives are present, the firm may be required to find other ways to improve its position in the market even when this implies a higher cost; when they are absent, the main concern of the firm will be related to the customers using its services.

A natural extension of our theoretical model would be to introduce competition across scheduled carriers to capture the interaction among airlines. Additionally, the emerging intermodal competition in Europe between airlines and high-speed trains should be examined thoroughly. Although the dummy variable capturing the competition from high-speed trains is not statistically significant in our empirical application, this interaction needs further analysis since airlines typically react to new high-speed services by adjusting aircraft size and maintaining flight frequency.

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[14]See the Report of the National Surface Transportation Policy and Revenue Study Commission (2007), and the White Paper "European Transport Policy for 2010: Time to Decide" of the European Commission (2001).

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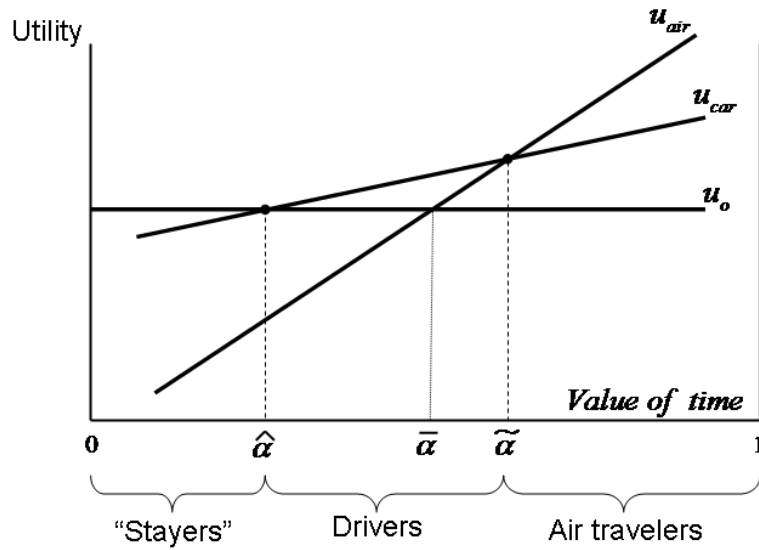
Figures and Tables

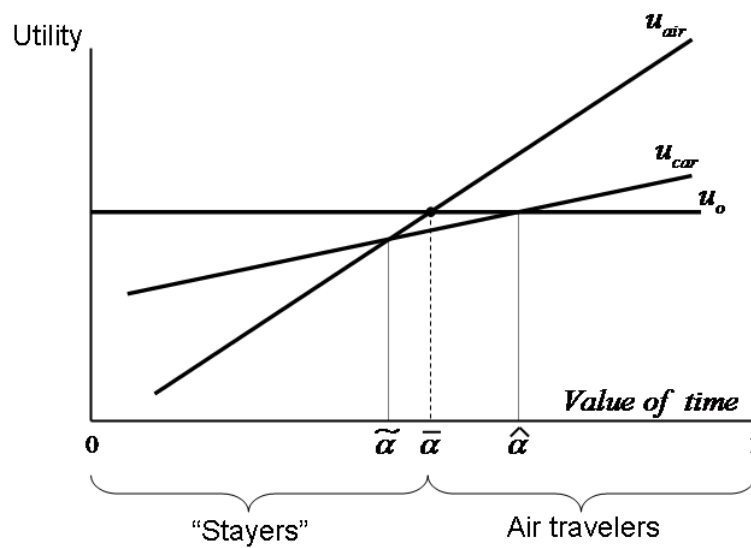Figure 1: Utilities in Scenario 1
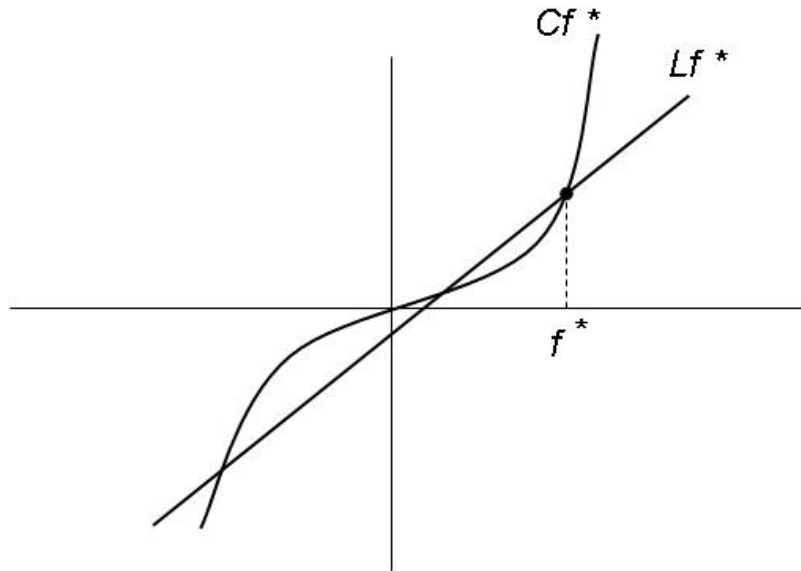

Figure 2: Utilities in Scenario 2
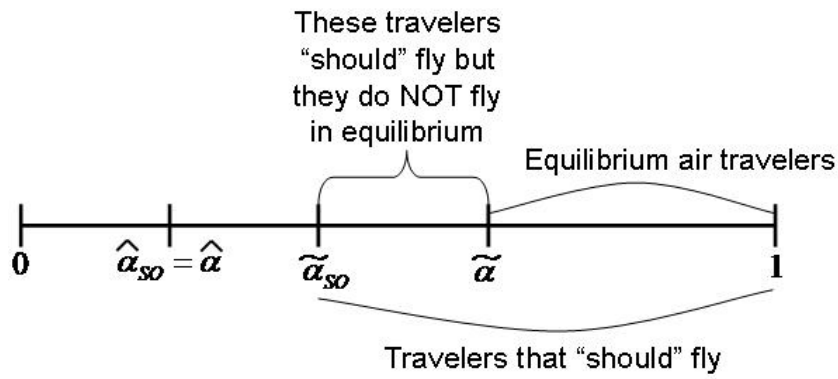
Figure 3: The f^* solution
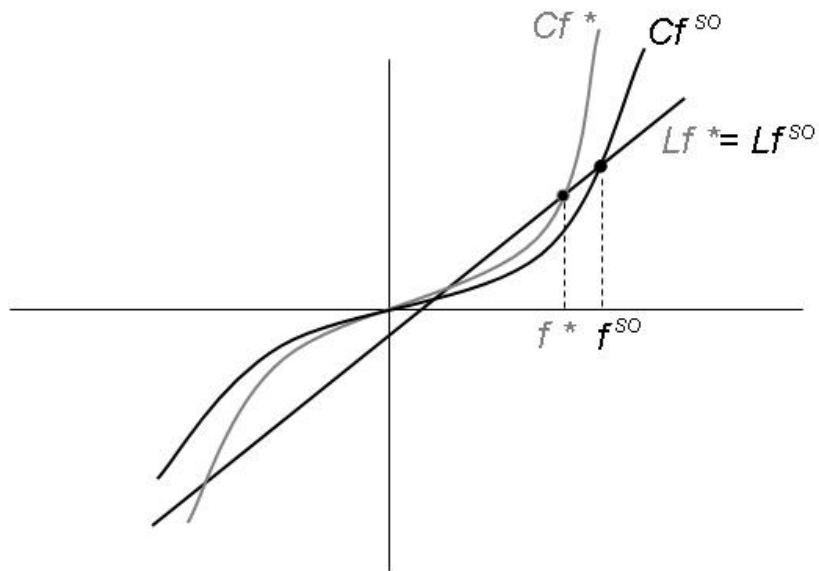

Figure 4: Suboptimal air traffic

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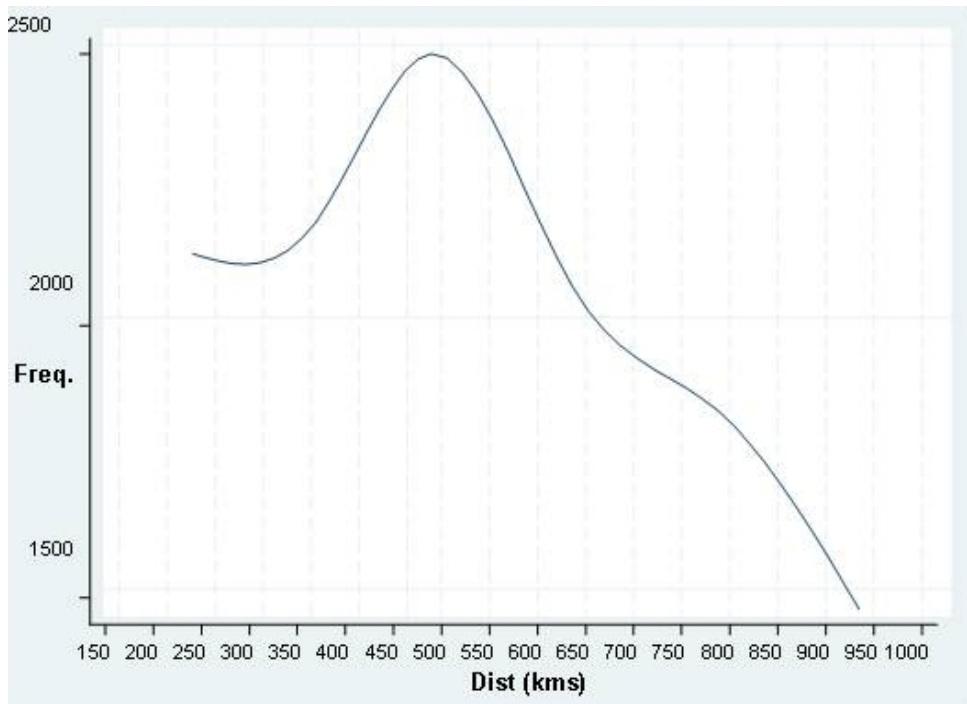Figure 5: Underprovision of flight frequency

Figure 6: Spline of total frequency with respect to distance

Table 1: Data characteristics of origin airports

Airport	Total frequency	# destin.	Dominant airline	Share dominant airline[1]	Share LCC[1]	Av. distance[2]	Av. # competitors[2]
Madrid	186,464	94	Iberia	53.34%	6.33%	1,133.06	2.88
Paris-CDG	176,520	99	Air France-KLM	59.97%	7.02%	886.81	2.08
Munich	167,101	98	Lufthansa	64.75%	18.67%	928.23	2.69
Frankfurt	163,655	97	Lufthansa	67.57%	5.25%	1,026.93	2.13
Amsterdam	152,449	106	Air France-KLM	53.04%	15.45%	917.53	1.97
London-LHR	148,805	57	British Airways	45.83%	0.16%	1,002.52	1.88
Barcelona	143,063	102	Iberia	40.26%	17.49%	1,083.66	2.47
Rome-FCO	127,178	80	Alitalia	46.63%	8.50%	897.74	2.64
Paris-Orly	95,624	72	Air France-KLM	65.91%	14.43%	782.40	1.40
London-LGW[3]	74,232	82	British Airways	44.93%	41.12%	1,365.72	1.93

Notes: 1. Share in terms of departures at the origin airport to European destinations (destinations within EU27 + Switzerland and Norway); 2. Data at the route level; 3. In London-LGW, Easyjet has a share of 28.21%.

Table 2: T-test for mean differences in frequency and aircraft size choices of airlines

Routes	Average frequency			Average seats per flight		
	All airlines	Hubbing airlines	LCC	All airlines	Hubbing airlines	LCC
< 500 kms (1)	1,477.95	1,962.04	656.47	116.93	100.63	160.58
	(1,491.03)	(1,648.43)	(615.99)	(53.28)	(37.35)	(45.53)
# obs.	365	176	57	365	175	57
> 500 kms (2)	769.64	1,127.79	380.52	152.25	165.47	166.61
	(760.38)	(924.27)	(353.31)	(470.31)	(834.22)	(50.36)
# obs.	1,163	367	343	1,163	367	343
T.statistic mean diff. (1)-(2)	11.98***	8.08***	4.81***	−1.43*	−1.20	−0.84

Note: Data at the route level. Hubbing airlines at origin airport.

Table 3: Descriptive statistics (# obs.=1,528)

	Continuous variables			
	Mean	Std. deviation	Minimum	Maximum
Frequency (annual airline flights at the route)	938.83	1,030.14	52	13,441
Distance (kms)	1,014.51	637.17	137	3,335
Population (000 inhabitants)	5,556.27	2,470.76	2,161	11,134
GDPC - pps (index: EU25=100)	133.32	24.07	80	228
HHI (concentration index)	0.65	0.26	0.007	1
Airport share (share airline at origin airport)	0.22	0.25	0.0003	0.67
Destination (# destinations airline at origin airport)	25.39	25.43	1	70
Tourism (% employment in hotels and restaurants)	0.06	0.03	0.01	0.20
	Discrete variables			
	Total obs.	# obs. with value 1	# obs. with value 0	
$D^{capital}$	1,528	477	1,051	
D^{island}	1,528	483	1,045	
D^{LCC}	1,528	400	1,128	
$D^{Interhub}$	1,528	156	1,372	

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Table 4: $Frequency_{ijk}$ equation estimates (routes with less than 500 kms)

	Specif. (1): OLS	Specif. (2): ZTP	Specif. (3): ZTP	Specif. (4): ZTP	Specif. (5): ZTP
$Distance_{ij}$	1.62	0.0010	0.0009	0.0014	0.0014
	$(0.68)^{**}$	$(0.0004)^{**}$	$(0.0004)^{**}$	$(0.0005)^{***}$	$(0.0005)^{***}$
$Population_{ij}$	0.048	0.000027	0.000022	0.000033	0.000028
	$(0.02)^{*}$	$(0.000014)^{*}$	$(0.000014)^{+}$	$(0.000017)^{**}$	$(0.000018)^{+}$
$GDPC_{ij}$	-1.00	-0.00033	-0.002	-0.00097	-0.0010
	(4.02)	(0.0018)	(0.0018)	(0.0020)	(0.002)
$D^{capital}_{ij}$	453.09	0.25	0.29	0.29	0.28
	$(238.52)^{*}$	$(0.12)^{**}$	$(0.11)^{**}$	$(0.15)^{*}$	$(0.15)^{*}$
D^{island}_{j}	169.14	0.09	0.07	–	–
	$(119.57)^{+}$	(0.08)	(0.08)		
$Tourism_{j}$	$-1,107.31$	-0.65	0.64	4.77	4.79
	$(2,214)$	(1.53)	(1.49)	$(2.71)^{*}$	$(2.70)^{*}$
HHI_{ij}	$-1,165.64$	-0.87	-1.01	-0.66	-0.67
	$(239.76)^{***}$	$(0.16)^{***}$	$(0.15)^{***}$	$(0.21)^{***}$	$(0.20)^{***}$
D^{LCC}_{k}	-196.49	-0.33	-0.15	-0.37	-0.37
	(154.15)	$(0.14)^{**}$	(0.14)	$(0.19)^{*}$	$(0.19)^{*}$
$D^{interhub}_{k}$	2,167.59	1.01	1.10	1.05	1.05
	$(700.814)^{***}$	$(0.15)^{***}$	$(0.15)^{***}$	$(0.17)^{***}$	$(0.17)^{***}$
$Destination_{k}$	25.27	0.016	–	0.017	0.017
	$(3.85)^{***}$	$(0.0018)^{***}$		$(0.002)^{***}$	$(0.002)^{***}$
$Airportshare_{k}$	–	–	1.97	–	–
			$(0.20)^{***}$		
D^{HST}	–	–	–	–	0.12
					(0.13)
$Intercept$	570.37	6.66	6.93	6.07	6.08
	(501.37)	$(0.35)^{***}$	$(0.34)^{***}$	$(0.44)^{***}$	$(0.43)^{***}$
N	365	365	365	276	276
R^2	0.35	0.45	0.48	0.44	0.44
VIF	1.33	–	–	–	–
$L-pseudolikelihood$	–	$-114,248.54$	$-106,445.22$	$-92,196.67$	$-92,009.10$
$Test\ F$ (joint sign.)	17.53^{***}	199.65^{***}	313.93^{***}	199.65^{***}	213.93^{***}

Notes: Standard errors in parenthesis (robust to heteroscedasticity). Significance at 1% (***), 5% (**), 10% (*), 15% (+).

Elasticities (evaluated at sample mean) of frequencies with regard to distance (OLS estimation): 0.41.

In the ZTP estimation, R^2 refers to pseudo R^2.

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Table 5: $Frequency_{ijk}$ **equation estimates (routes with more than 500 kms)**

	Specif. (1): OLS	Specif. (2): ZTP	Specif. (3): ZTP	Specif. (4): ZTP
$Distance_{ij}$	−0.30	−0.0006	−0.0006	−0.0006
	(0.03)***	(0.00004)***	(0.00004)***	(0.00005)***
$Population_{ij}$	0.017	0.000022	0.000022	0.000017
	(0.009)*	(0.000010)**	(0.000010)**	(0.000012)+
$GDPC_{ij}$	6.41	0.0070	0.006	0.008
	(1.00)***	(0.0012)***	(0.001)***	(0.0014)***
$D_{ij}^{capital}$	−58.16	−0.016	0.000026	−0.017
	(46.85)	(0.055)	(0.055)	(0.068)
D_j^{island}	60.63	0.035	0.025	–
	(42.00)	(0.054)	(0.053)	
$Tourism_j$	−855.55	−1.28	−0.49	0.75
	(589.06)+	(0.90)	(0.91)	(1.39)
HHI_{ij}	−228.46	−0.36	−0.42	−0.26
	(61.79)***	(0.09)***	(0.08)***	(0.10)***
D_k^{LCC}	−149.10	−0.34	−0.21	−0.42
	(32.13)***	(0.06)***	(0.06)***	(0.08)***
$D_k^{interhub}$	661.67	0.76	0.83	0.78
	(62.52)***	(0.06)***	(0.07)***	(0.07)***
$Destination_k$	11.13	0.012	–	0.013
	(0.73)***	(0.0009)***		(0.001)***
$Airportshare_k$	–	–	1.58	–
			(0.11)***	
$Intercept$	113.94	6.12	6.18	5.87
	(123.74)	(0.17)***	(0.17)***	(0.18)***
N	1,163	1,163	1,163	769
R^2	0.38	0.49	0.52	0.49
VIF	1.44			
$L-pseudolikelihood$	–	−181,964.61	−168,593.81	−121,616.29
$Test\ F$ (joint sign.)	68.36***	1,075.64***	892.95***	829.31***

Notes: Standard errors in parenthesis (robust to heteroscedasticity). Significance at 1% (***), 5% (**), 10% (*), 15% (+).
Elasticities (evaluated at sample mean) of frequencies with regard to distance (OLS estimation): Specification (5): −0.48.
In the ZTP estimation, R^2 refers to pseudo R^2.

A   Appendix: Proofs

Proof of Lemma 1.

The case with drivers (Scenario 1) requires $0 < \widehat{\alpha} < \widetilde{\alpha} < 1$. From the inequality $\widehat{\alpha} < \widetilde{\alpha}$ and after some computations we get:

$$\underbrace{\left(\frac{p_{air}V}{\beta} + cTV^2 + \frac{\gamma V}{\beta f}\right) d - TV^2 \left(p_{air} + \frac{\gamma}{f}\right)}_{Ld} < \underbrace{cd^2V}_{Qd}, \tag{A1}$$

that is represented in the Figure $A1$ below

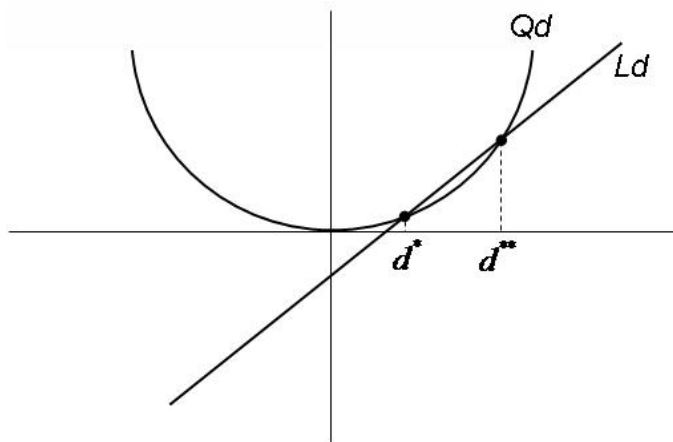

Figure $A1$

Thus we conclude that $\widehat{\alpha} < \widetilde{\alpha}$ requires either $d < d^*$ or $d > d^{**}$, where d^* and d^{**} are the two roots solving the equation $Ld = Qd$. Although d^* and d^{**} can be computed, it is easier to proceed in the following way. In the first possible region (i.e., $d < d^*$), the slope of Qd is smaller than the one of Ld and thus $d < \frac{\frac{1}{\beta}\left(p_{air}+\frac{\gamma}{f}\right)+cTV}{2c}$ and the opposite happens in the second possible region (i.e., $d > d^{**}$). From $\widetilde{\alpha} > 0$ we know that $d < \frac{p_{air}+\frac{\gamma}{f}}{c}$ and this condition is incompatible with $d > \frac{\frac{1}{\beta}\left(p_{air}+\frac{\gamma}{f}\right)+cTV}{2c}$ (and thus with $d > d^{**}$) either for $\beta < 1/2$ (that is assumed in the model) or for a large enough T. Therefore $\widehat{\alpha} < \widetilde{\alpha}$ requires $d < d^*$ and thus Scenario 1 (with drivers) is only relevant for short distances. ∎

Proof of Lemma 2.

From (9), let us define $\Omega \equiv Cf^* - Lf^* = 0$, that is

$$\Omega = \frac{2\theta(d)d(1-\beta)}{\gamma\beta V}f^3 - \left(cd - \tau + \frac{d(1-\beta)}{\beta V}\right)f + \gamma = 0. \tag{A2}$$

The total differential of the equilibrium frequency with respect to a parameter x is $\frac{df^*}{dx} = -\frac{\partial\Omega/\partial x}{\partial\Omega/\partial f}$. Notice that $\partial\Omega/\partial f = slope\,(Cf^*) - slope\,(Lf^*)$ and thus $\partial\Omega/\partial f > 0$ because at the equilibrium frequency the slope of Cf^* exceeds the slope of Lf^*. Thus we just need to explore the sign of $\partial\Omega/\partial x$.

- $\partial\Omega/\partial\gamma = -\frac{2\theta(d)d(1-\beta)}{\gamma^2\beta V}f^3 + 1$ which has an ambiguous sign and is negative for small values of β (more precisely, for $\beta < \frac{2\theta(d)df^3}{V\gamma^2 + 2\theta(d)df^3}$). Thus, when this condition holds, we observe $\frac{df^*}{d\gamma} > 0$.

- $\partial\Omega/\partial c = -df < 0$. Then $\frac{df^*}{dc} > 0$. We observe that $c \uparrow$ increases the slope of Lf^* in Figure 3 so that $f^* \uparrow$.

- $\partial\Omega/\partial\tau = f > 0$. Then $\frac{df^*}{d\tau} < 0$. We observe that $\tau \uparrow$ decreases the slope of Lf^* in Figure 3 so that $f^* \downarrow$.

- $\partial\Omega/\partial V = \frac{1}{V}\left(-\frac{2\theta(d)d(1-\beta)}{\gamma\beta V}f^3 + \frac{d(1-\beta)}{\beta V}f\right)$ and, using (A2), this expression can be rewritten as $\partial\Omega/\partial V = \gamma + (\tau - cd)f/V$. We observe that $\tau > cd$ is a sufficient condition to have $\partial\Omega/\partial V > 0$ and thus $\frac{df^*}{dV} < 0$.

- $\partial\Omega/\partial\beta = -\frac{2\theta(d)}{\gamma}f^2 + 1$ that has the same sign as $\frac{1}{V}\left(-\frac{2\theta(d)d(1-\beta)}{\gamma\beta V}f^3 + \frac{d(1-\beta)}{\beta V}f\right)$ which is the value of $\partial\Omega/\partial V$. Therefore the same sufficient condition ensures $\partial\Omega/\partial\beta > 0$ and thus $\frac{df^*}{d\beta} < 0$.

- $\partial\Omega/\partial d = \frac{2\theta'(d)d(1-\beta)}{\gamma\beta V}f^3 + \frac{2\theta(d)(1-\beta)}{\gamma\beta V}f^3 - \left(c + \frac{1-\beta}{\beta V}\right)f$ and, using (A2), this expression can be rewritten as $\partial\Omega/\partial d = \frac{2\theta'(d)d(1-\beta)}{\gamma\beta V}f^3 - \frac{\gamma+\tau f}{d}$. Notice that when the cost of frequency is independent of distance, i.e., $\theta'(d) = 0$, $\partial\Omega/\partial d < 0$ and $\frac{df^*}{dd} > 0$. Yet, when $\theta'(d) > 0$, the result seems uncertain. Notice that $\theta'(d)$ is a continuous function starting from $d = 0$ because $\theta(d)$ is continuously differentiable with respect to $d > 0$. Therefore, at least for low values of d we will observe $\partial\Omega/\partial d < 0$ and $\frac{df^*}{dd} > 0$. ∎

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Proof of Lemma 3.

From (13), let us define $\Omega \equiv Cf^* - Lf^* = 0$, that is

$$\Omega = \frac{2\theta(d)(TV-d)}{\gamma}f^3 - (TV - d - \tau V)f + \gamma V = 0 \qquad (A3)$$

As before, the total differential of the equilibrium frequency with respect to a parameter x is $\frac{df^*}{dx} = -\frac{\partial\Omega/\partial x}{\partial\Omega/\partial f}$. Notice that $\partial\Omega/\partial f = slope\,(Cf^*) - slope\,(Lf^*)$ and thus $\partial\Omega/\partial f > 0$ because at the equilibrium frequency the slope of Cf^* exceeds the slope of Lf^*. Thus we just need to explore the sign of $\partial\Omega/\partial x$.

- $\partial\Omega/\partial\gamma = -\frac{2\theta(d)(TV-d)}{\gamma^2}f^3 + V < 0$ assuming a large T. Then $\frac{df^*}{d\gamma} > 0$.

- $\partial\Omega/\partial\tau = Vf > 0$. Then $\frac{df^*}{d\tau} < 0$. We observe that $\tau \uparrow$ decreases the slope of Lf^* in Figure 3 so that $f^* \downarrow$.

- $\partial\Omega/\partial V = \frac{2\theta(d)T}{\gamma}f^3 - Tf + \tau f + \gamma$ and, using (A3), this expression can be rewritten as $\partial\Omega/\partial V = \frac{2\theta(d)}{\gamma}f^2 - 1$, so that $\partial\Omega/\partial V < 0$ requires $f^2 < \frac{\gamma}{2\theta(d)}$. Then using (12) this inequality becomes $p_{air} < \frac{1}{2}\left(T - d/V\right) + \tau$, and finally using (11) we obtain $-\gamma/f < \tau$ which is always true. Therefore, $\partial\Omega/\partial V < 0$ and thus $\frac{df^*}{dV} > 0$.

- $\partial\Omega/\partial T = \frac{2\theta(d)V}{\gamma}f^3 - Vf$, so that $\partial\Omega/\partial T < 0$ requires $f^2 < \frac{\gamma}{2\theta(d)}$; and we just showed this inequality always holds. Therefore $\partial\Omega/\partial T < 0$ and thus $\frac{df^*}{dT} > 0$.

- $\partial\Omega/\partial d = \frac{2\theta'(d)(TV-d)}{\gamma}f^3 + f - \frac{2\theta(d)}{\gamma}f^3$ and, plugging (A3) into the derivative we obtain $\partial\Omega/\partial d = \frac{2\theta'(d)(TV-d)}{\gamma}f^3 + \frac{\gamma V + \tau V f}{TV-d}$ that is positive because $T > d/V$.

  Therefore, $\frac{df^*}{dd} < 0$ and thus frequency decreases with distance. ∎