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Product differentiation, competitive pressure and the effects of innovation on the success and survivability of firms in oligopoly markets

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Abstract

In a recent paper Tishler and Milstein (2009) find that increased competition may increase aggregate R&D spending while market output decreases. Therefore, they obtain the surprising result that R&D spending is excessive when competition becomes intense. Their result is based on the standard linear demand function for differentiated products introduced by Bowley (1924) where decreased product differentiation is interpreted as more competitive pressure. In this paper I show that at an aggregate level this interpretation is problematic because equilibrium effects are dominated by a demand reduction effect. A slight modification of the standard demand function eliminates this effect. For the Tishler and Milstein (2009) setting it is shown that then increased competition increases both R&D spending and aggregate market output. Therefore, at least for consumers, more intense competition increases welfare.

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1 Introduction

The linear demand system for differentiated products introduced by Bowley (1924) and extended by Dixit (1979) and Singh and Vives (1984) is a standard tool to analyze market performance in oligopolistic markets. The degree of product substitutability, a basic parameter of the model, thereby is interpreted as an indicator of competitive pressure. While this is certainly true for a single firm one must take care to interpret the parameter as an indicator of competitive market pressure at an *aggregate level*. To make this point clear consider a simple version of the model in which two firms compete à la Cournot each of them producing one differentiated product. The inverse demand function of firm i is:

$$p_i = a_i + bq_i + dq_j,\ i, j = 1, 2, i \neq j, \tag{1}$$

where $a_i > 0$, $b < 0$, $d \leq 0$ and $b-d \leq 0$. For $d = b$ the products of both firms are homogeneous. For $d = 0$ each firm is a monopolist in its market. Therefore, a lower value of d or a higher value of d/b can be interpreted as more intensive market competition for firm i. Furthermore, assume that firms' cost functions are linear:

$$C_i = c_i q_i,\ i = 1, 2, \tag{2}$$

where the marginal cost $c_i > 0$.

If firm i chooses q_i to maximize profits, $\pi_i = (a_i + bq_i + dq_j - c_i)\, q_i$, given the output produced by its rival, q_j, the Nash-Cournot equilibrium outcome is:

$$q_i^* = \frac{-2b\,(a_i - c_i) + d\,(a_j - c_j)}{4b^2 - d^2},\ i = 1, 2. \tag{3}$$

Total equilibrium output is: $q^* = -b\,(a_i + a_j - c_i - c_j)\,/(2 + d/b)$. So, we get the result that total output is decreasing in the degree of product differentiation, i.e. with more intensive competition. What is problematic with this interpretation becomes clear if we consider the aggregate demand function:

$$q = \frac{a_i + a_j - p_i - p_j}{-b - d}. \tag{4}$$

From equation (4) we see that even if equilibrium prices were the same for different intensities of competition, aggregate demand already decreases when competition becomes more intensive. Therefore, we have two effects when d is interpreted as an indicator of competitive pressure at an *aggregate level*. First, products become closer substitutes which increases competitive pressure for firms and should lead to lower prices and higher quantities in equilibrium. Second, total demand decreases which means less equilibrium output. With the inverse demand function in (1) this second effect dominates the first one. However, this second effect has nothing to do with competitive pressure. Therefore, if we want to interpret changes in d as changes in the intensity of competition at an aggregate level we should not use the demand function in (1).

Instead, such an interpretation would be more adequate if the inverse demand function of firm i were

$$p_i = a_i + (2b - d)\, q_i + dq_j,\ i, j = 1, 2, i \neq j. \tag{5}$$

Now, a decrease of d shifts the influence of firm i's quantity setting on price i to firm j. With this change with decreasing d products become closer substitutes while aggregate demand remains constant:

$$q = \frac{a_i + a_j - p_i - p_j}{-2b}. \tag{6}$$

Now, changes in the competitiveness of markets, interpreted as changes in the degree of product differentiation, can affect aggregate demand only through changes in equilibrium prices. The Nash-Cournot equilibrium outcome with this demand function is:

$$q_i^* = \frac{-2\,(2b-d)\,(a_i - c_i) + d\,(a_j - c_j)}{(4b-d)\,(4b-3d)},\ i = 1, 2 \tag{7}$$

and total demand is $q^* = -b\,(a_i + a_j - c_i - c_j)\,/(4 - d/b)$. So, as expected, total equilibrium output increases with more intensive market competition.

2 Innovation and competitive pressure measured by product differentiation

To see the implications of different inverse demand specifications on a firm's incentives to invest in R&D consider the two-stage model of Tishler and Milstein (2009). In stage 1 a firm chooses an R&D investment λ_i to maximize its expected profits. R&D investment either is demand enhancing or cost reducing or both. Specifically:

$$a_i = a_{i0} + \theta\lambda_i \text{ and} \tag{8}$$
$$c_i = c_{i0} - \rho\lambda_i \tag{9}$$

where $a_{i0} > 0$, $\theta \geq 0$, $c_{i0} > 0$ and $\rho \geq 0$. The profits of firm i are

$$\Pi_i = \pi_i - \eta E(\lambda_i)^2 \tag{10}$$

where the expected value $E(\lambda_i)$ and the variance $V(\lambda_i)$ of the R&D investment are bounded and the upper and lower limits are known constants. In stage 2 firms compete á la Counot where firm i's inverse demand function is given by (5) and production costs are given by (2).

This model is identical to that of Tishler and Milstein (2009) except that the inverse demand function (1) is substituted by (5). With this new demand function a firm's stage 2 Nash-Cournot equilibrium output is given by:

$$q_1^* = \frac{-2(2b-d)[\delta M+\varphi\lambda_1]+d[(1-\delta)M+\varphi\lambda_2]}{4(2b-d)^2-d^2} \tag{11}$$

$$q_2^* = \frac{-2(2b-d)[(1-\delta)M+\varphi\lambda_2]+d[\delta M+\varphi\lambda_1]}{4(2b-d)^2-d^2} \tag{12}$$

where $\varphi=\theta+\rho$ is the overall R&D parameter, $M=a_{i0}-c_{i0}+a_{j0}-c_{j0}$ and $\delta=(a_{i0}-c_{i0})/M$ is a measure of firm 1's relative market strength. In stage 1, firm 1 chooses expected R&D investment to maximize expected profits, which are given by:

$$\begin{aligned} E\Pi_1 &= E(\pi_1)-\eta E(\lambda_1)^2 \\ &= \frac{-(2b-d)}{\left[4(2b-d)^2-d^2\right]^2}\left[\begin{array}{c} M^2(d-4b\delta+d\delta)^2 \\ -4M\varphi(2b-d)(d-4b\delta+d\delta)E(\lambda_1) \\ +2d\varphi M(d-4b\delta+d\delta)E(\lambda_2) \\ +4\varphi^2(2b-d)^2\left[V(\lambda_1)+E(\lambda_1)^2\right] \\ -4d\varphi^2(2b-d)[Cov(\lambda_1\lambda_2)+E(\lambda_1)E(\lambda_2)] \\ +d^2\varphi^2\left[V(\lambda_2)+E(\lambda_2)^2\right] \end{array}\right] \\ &\quad -\eta E(\lambda_1)^2 \end{aligned} \tag{13}$$

The maximization of (13) with respect to $E(\lambda_1)$ and of a similar expression for firm 2 with respect to $E(\lambda_2)$ yields the Nash equilibrium in expected R&D outcomes:

$$E^*(\lambda_1) = 2(2b-d)^2\varphi\left[\frac{\left(\frac{1}{2}-\delta\right)}{A_1}-\frac{\frac{1}{2}}{A_2}\right]M \tag{14}$$

$$E^*(\lambda_2) = 2(2b-d)^2\varphi\left[\frac{\left(\delta-\frac{1}{2}\right)}{A_1}-\frac{\frac{1}{2}}{A_2}\right]M \tag{15}$$

where

$$A_1 = \eta(4b-3d)^2(4b-d)+2(2b-d)^2\varphi^2 \tag{16}$$

$$A_2 = \eta(4b-3d)(4b-d)^2+2(2b-d)^2\varphi^2 \tag{17}$$

The following assumption guarantees a unique and stable Nash equilibrium with positive expected R&D outcomes:[1]

Assumption 1.

Let $\eta > -[2(2b-d)^2\varphi^2]/[(4b-d)(4b-3d)^2]$ and $\frac{1}{2}[1-A_1/A_2] < \delta < \frac{1}{2}[1+A_1/A_2]$.

[1] The proof is similar to that in Tischler and Milstein (2009).

We get the following result:

Proposition 1.

Suppose assumption 1 holds. Then,

i. The industry's aggregate level of optimal R&D outcome is increasing in d/b.

ii. The industry's aggregate level of optimal expected output is increasing in d/b.

These results are opposed to those in Tishler and Milstein (2009). The authors find that for values above $d/b = 2/3$ the industry's aggregate level of optimal expected R&D outcome is increasing in d/b while the industry's aggregate level of optimal expected output is decreasing. Therefore, the authors conclude that R&D expenditure might be too high when market competition is substantial. The results in Proposition 1 suggest that this is not true if the inverse demand function (1) is substituted by (5). Then, more competition always increases the aggregate level of R&D expenditure and the aggregate level of output. Therefore, at least for consumers, more competition increases welfare. As mentioned before, the difference in the results is due to the fact that with the inverse demand function used by Tishler and Milstein (2009) an increase in product differentiation not only increases competitive pressure for a single firm but also reduces aggregate demand even if firms do not change their price setting behavior. This second effect changes completely the results. Therefore, one should take care to interpret an increase in d/b as more intensive competition at an aggregate level in the standard inverse demand model.

Appendix A

Proof of Proposition 1.

i. Notice that using conditions (14) and (15), the industry's aggregate level of optimal expected R&D outcome is

$$E^*(\lambda_1) + E^*(\lambda_2) = -\frac{2\left(2b-d\right)^2 \varphi M}{A_2}.$$

Differentiating this expression with respect to d/b yields:

$$\frac{\partial\left[E^*(\lambda_1) + E^*(\lambda_2)\right]}{\partial\left(d/b\right)} = -2b\varphi M \frac{\eta b^4 \left(4-d/b\right)\left(2-d/b\right)\left(3\left(d/b\right)^2 - 6\left(d/b\right) + 8\right)}{\left(A_2\right)^2} > 0.$$

ii. Substituting (14) and (15) into (11) and (12), the expected outputs of firm 1 and 2 are:

$$\begin{aligned} E\left(q_1^*\right) &= \left(4\left(2b-d\right)^2 - d^2\right)\eta\left[\frac{\left(\frac{1}{2}-\delta\right)}{A_1} - \frac{\frac{1}{2}}{A_2}\right]M \\ E\left(q_2^*\right) &= \left(4\left(2b-d\right)^2 - d^2\right)\eta\left[\frac{\left(\delta-\frac{1}{2}\right)}{A_1} - \frac{\frac{1}{2}}{A_2}\right]M. \end{aligned}$$

Then, the industry's aggregate level of optimal expected output is

$$E\left(q_1^*\right) + E\left(q_2^*\right) = -\frac{\left(4\left(2b-d\right)^2 - d^2\right)\eta M}{A_2}.$$

Differentiating this expression with respect to d/b yields:

$$\frac{\partial\left[E^*(q_1) + E^*(q_2)\right]}{\partial\left(d/b\right)} = -\eta M b \frac{\eta\left(4b-d\right)^2\left(3d-4b\right)^2 - 8db\left(2b-d\right)\varphi^2}{\left(A_2\right)^2} > 0$$

under assumption 1.

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