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Document de treball nº - 16- 2010

DEPARTAMENT D’ECONOMIA
Facultat de Ciències Econòmiques i Empresarials

Edita:

Departament d'Economia
http://www.fcee.urv.es/departaments/economia/public_html/index.html
Universitat Rovira i Virgili
Facultat de Ciències Econòmiques i Empresarials
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Dirigir comentaris al Departament d'Economia.

Dipòsit Legal: T - 1999 - 2010

ISSN 1988 - 0812

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Facultat de Ciències Econòmiques i Empresarials

Endogenous Mergers of Complements with Mixed Bundling*

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November 2010

Abstract

This paper studies endogenous mergers of complements with mixed bundling, by allowing both for joint and separate consumption. After merger, partner firms decrease the price of the bundled system. Besides, when markets for individual components are sufficiently important, partner firms raise prices of stand-alone products, exploiting their monopoly power in local markets and making substitute 'mix-and-match' composite products less attractive to consumers. Even though these effects favor the profitability of mergers, merging is not always an equilibrium outcome. The reason is that outsiders respond by cutting their prices to retain their market share, and mergers can be unprofitable when competition is intense. From a welfare analysis, we observe that the number of mergers observed in equilibrium may be either excessive (when markets for individual components are important) or suboptimal (when markets for individual components are less important).

Keywords: complements; merger; mixed bundling; separate consumption

JEL classification: L13; L41; D43

*We thank Alan W. Beggs, Jan K. Brueckner and Jose J. Sempere-Monerris for their helpful comments and suggestions. We are also thankful to all the comments received during the congress *XXXIV Simposio de la Asociación Española de Economía* at Valencia (Spain). The authors acknowledge financial support from the Spanish Ministry of Education and Science and FEDER (SEJ2007-67891 and SEJ2007-66581), the Spanish Ministry of Science and Innovation (ECO2010-19733, ECO2010-17113 and ECO2010-20584), Generalitat de Catalunya (2009SGR900), Generalitat Valenciana (PROMETEO/2009/068) and Ramón Areces Foundation.

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1 Introduction

Consumers are often interested in final goods that are obtained by combining complementary (compatible) products into systems or bundles, which may be substitutes for one another. A merger involving complementary products can have the beneficial effect of reducing a vertical negative externality ('double marginalization') since the price of the bundle falls below the prices the firms would choose if acting independently. However, in oligopoly markets, merged firms may exercise market power through bundling, and this may be a means to foreclose rivals and/or relax price competition. Non-horizontal mergers of this type raise antitrust concerns since the net welfare effect is unclear. This paper addresses the incentives for mergers and its welfare implications in markets with complementary products; the merged firms can engage in bundling and rivalry from competing differentiated systems is considered.

Firms that offer a bundle of complementary products may practice mixed bundling: they set the price for the bundle as well as for the individual components, which may be used to form alternative 'mix-and-match' systems. Although separate consumption is present in this pricing policy, much of the recent literature has assumed that (complementary) components are valuable only when used together. Consumers purchase computer software and hardware components, cold and pain-relief medications, printers and ink cartridges, cellular telephones and SIM cards, ATM services and credit cards, local and connecting flights, train and bus services to get to work, contract services with telephone operators (fixed and mobile lines, Internet, etc.), transportation and hotel services, etc. Apart from bundled, these goods are also available to consumers if sold separately in the market place. Accounting both for joint and separate consumption allows us to study endogenous merger formation in a realistic setting where the issue of competition between bundles can be faithfully examined.

We set up a simple four-firm model, where there are two firms producing a certain product (A_1 and A_2) and two other firms producing a complementary product (B_1 and B_2). Consequently, four different competing systems of complementary products, which are partially substitutable, can be formed. We further assume that firms have monopoly power in the market for individual components. A two-stage game is solved where merging decisions are made before firms compete in prices. Three market structures are analyzed: independent ownership, single integration (one merger is formed) and parallel integration (two mergers are formed).

Our main findings can be summarized as follows. After merger, partners decrease the price of the bundle. Besides, when markets for individual components are sufficiently important, partners raise prices of stand-alone products, exploiting their monopoly power in local markets and making substitute 'mix-and-match' systems less attractive. Even though these effects favor the profitability of mergers, merging is not always an equilibrium outcome. The reason is that outsiders respond by cutting their prices to retain their market share, and mergers can be unprofitable when competition is intense. This explains why independent ownership or just one merger can be observed in equilibrium, which is in contrast with the received literature.

We also find that welfare is highest either under independent ownership (when competition is strong) or under parallel integration (when competition is weak), as in the equilibrium analysis. However, when competition intensity is moderate, social and private interests are not aligned. We may observe either too much integration ('overintegration') or too little integration ('underintegration'), depending on the relative importance of markets for individual components. When markets for individual components are important, price discrimination is used by merging firms to exploit the monopoly power they have in these markets by raising prices. Naturally, this can be socially detrimental ('overintegration'). In contrast, when markets for individual components are less important, the market-power effect is mitigated and the lower system prices set by partner firms turn mergers socially profitable. However, in this case, the lower system price determined by partner firms puts a downward pressure on rivals' prices and mergers become privately unprofitable when competition is intense ('underintegration').

Next section relates our research with the existing literature. Section 3 introduces the model presenting the different market structures. Section 4 provides the subgame perfect equilibrium in merging decisions. The welfare analysis is undertaken in section 5. Some concluding remarks and policy implications close the paper. Proofs are in the Appendix.

2 Related Literature

Our paper is related with three sets of literature, which are examined below: *i*) bundling, *ii*) network-based airline models, and *iii*) endogenous mergers of complements. This section provides the relevant context to understand our contribution.

There exists an extensive literature on bundling. The practice of bundling is said to be either *pure* (i.e., technical tying) when only the bundled product is available, or *mixed* when separate components can also be used to form alternative 'mix-and-match' systems. A multiproduct monopolist that faces heterogeneous consumers will engage in (mixed) bundling because it serves as a price discrimination device (Adams and Yellen, 1976, and McAfee, McMillan, and Whinston, 1989). Strategic motivations arise in oligopoly settings and bundling can soften competition and create entry barriers. A robust conclusion is that bundling is an effective tool to disadvantage rival producers (Whinston, 1990, Choi and Stefanidis, 2001, and Nalebuff, 2004).[1] The analysis of several multiproduct firms that produce differentiated systems is undertaken by Liao and Tauman (2002), who show that an equilibrium exists where firms offer bundle discounts. More recently, Gans and King (2005) discuss when the economic consequences of bundling should be of concern for competition authorities. We contribute to this literature by modeling potential competition between bundles and discerning conditions under which private and social interests are in agreement.

An important feature in the received literature is that the goods in the bundle are valuable when consumed together. Although the idea that separate demands have always been part of the mixed bundling story, it seems to have been lost recently. Nevertheless, the research on airline competition differentiates between two types of passengers on spoke-to-hub routes: *i*) local passengers, who just purchase a direct flight, and *ii*) connecting passengers, who purchase a bundle composed by two (or more) complementary flights (Hendricks *et al.*, 1997 and 1999, Brueckner and Spiller, 1991, and Brueckner, 2001). However, none of these papers focuses on the relative market size of individual components with respect to the markets for systems. In addition, these papers make specific assumptions to capture important characteristics of the airline industry (e.g., economies of traffic density, frequency competition, etc.).

Some previous research has studied endogenous mergers. Kamien and Zang (1990) first formulated a two-stage game where the merger decision is endogenized prior to quantity competition. With perfect complements and price competition, Gaudet and Salant (1992) generalize their model to establish that some socially desirable mergers may fail to occur. Economides and Salop (1992) introduce competition across systems composed of compatible complementary products, and provide an extensive analysis of the effects on prices of alternative (exogenously given) market structures. Beggs (1994) studies endogenous merger formation between two groups of firms where products within a group are complementary but are substitutes across groups. However, he does not assume full compatibility among components and, the merged firm engages in pure bundling (or technical tying), thus not making the individual components available separately. Choi (2008), assuming that the merged firm engages in mixed bundling, builds on Economides and Salop (1992) to find cases where an exogenous merger, which is always privately profitable, is socially detrimental. We further delve into the idea developed by Anderson *et al.* (2010), where a merger of complements turns out profitable under some conditions (on the shape of the demand function) in the presence of competition.[2] Our model complements and extends these contributions. As in these papers, we consider complementarities that arise from the demand side and cost synergies are assumed away. In addition, we permit consumers to derive utility both from separate purchases of individual components, as well as from their joint purchase. As shall be seen, the results depend on the degree of substitutability among systems and the relative importance of markets for individual components.

3 A model of endogenous mergers with mixed bundling

Suppose that there are two differentiated brands of each of two components, A (A_1 and A_2) and B (B_1 and B_2), each offered by an independent firm, so that there are four firms. Consumers combine A and B in fixed proportions on a one-to-one basis to form a final composite product (or system). Under full compatibility, there are four ways to form a system: A_1B_1, A_1B_2, A_2B_1 and A_2B_2. Denote by p_i the price of brand A_i and by q_j the price of brand B_j, where $i, j = 1, 2$. The system A_iB_j is available at total price $s_{ij} = p_i + q_j$. The four systems are equally substitutable. The demand for system A_1B_1, denoted by D^{11}, decreases with s_{11} and increases with the prices of the substitute systems, s_{12}, s_{21} and s_{22}. Thus, brand A_i is sold as part of systems A_iB_1 and A_iB_2 and, similarly, B_j is sold as part of composite products A_1B_j and A_2B_j. We assume linear and symmetric demand functions as follows[3]

$$
\begin{aligned}
D^{11}(s_{11}, s_{12}, s_{21}, s_{22}) &= a - bs_{11} + c\left(s_{12} + s_{21} + s_{22}\right), \\
D^{12}(s_{12}, s_{11}, s_{21}, s_{22}) &= a - bs_{12} + c\left(s_{11} + s_{22} + s_{21}\right), \\
D^{21}(s_{21}, s_{22}, s_{11}, s_{12}) &= a - bs_{21} + c\left(s_{22} + s_{11} + s_{12}\right), \\
D^{22}(s_{22}, s_{21}, s_{12}, s_{11}) &= a - bs_{22} + c\left(s_{21} + s_{12} + s_{11}\right),
\end{aligned}
\tag{1}
$$

where $a, b, c > 0$. Further, assume that $b > 3c$ to ensure that composite products are gross substitutes. The demand system in (1) has been employed by Economides and Salop (1992) and Choi (2008). In case the components were not compatible, we would be left with two composite products, i.e., $D^{11}(s_{11}, s_{22}) = a - bs_{11} + cs_{22}$ and $D^{22}(s_{22}, s_{11}) = a - bs_{22} + cs_{11}$, which is the demand system in Beggs (1994) who considers pure bundling (or technical tying). We follow the approach in Choi (2008) by introducing the possibility of mixed bundling. Thus if, say firms producing A_1 and B_1 merge, then the merged entity can offer three prices: one for the bundled product A_1B_1, and one for each of the individual components A_1 and B_1. Let us also assume that marginal costs of production are zero. Then, parameter a represents the basic level of demand for each composite product if prices were zero; parameter b represents the own-price effect; and parameter c reflects a cross-price effect. The higher c is, the more substitutable the composite products are and thus the stronger competition is.

As argued in the Introduction, there are consumers who derive utility from individual consumption of each of the four brands.[4] The demand functions are given by[5]

$$\bar{D}^{A_1} = \lambda a - bp_1,\ \bar{D}^{A_2} = \lambda a - bp_2,\ \bar{D}^{B_1} = \lambda a - bq_1,\ \bar{D}^{B_2} = \lambda a - bq_2, \tag{2}$$

where $\lambda \in (0, 1)$. Thus, λ measures the relative importance of markets for individual components with respect to the markets for systems (i.e., the degree of demand asymmetry), and both the demands for joint and for separate consumption are more similar the closer is λ to 1. In other words, λ captures the maximum willingness to pay for the individual component relative to the composite product, so that λ can also be seen as a measure of the degree of complementarity between systems and individual brands.[6] Since we focus on the role played by λ, for the sake of the exposition, we assume components A_1 and A_2 to be independent (and the same for components B_1 and B_2) and that the parameter reflecting the own-price elasticity of demand of individual components (b) is the same as the one for systems.[7]

Let us normalize $a = b = 1$ because the level of the demand intercept (a) has no effect on the relative prices, and the parameters b and c only affect the results through the ratio of b/c. With the assumption of the gross substitutability of composite goods, the normalization of $b = 1$ implies that $c \in (0, 1/3)$. Hence, we are left with two parameters: c and λ.

We are interested in solving the following two-stage game. In the first stage, firms A_1 and A_2 simultaneously and respectively propose to firms B_1 and B_2 whether or not to merge. If offered a merger, firm B_i then simply accepts or rejects (if not offered a merger, then no merger is formed), with $i = 1, 2$. Since firms are symmetric, no merger offer will be declined and no firm A_i will fail to offer a merger that firm B_i would accept. Thus, we may have three different scenarios: *independent ownership*, *single integration* and *parallel integration*. In stage two, given the inherited outcome from the first stage, firms set prices. More specifically, under independent ownership (the pre-merger situation), the firms engage in pure component pricing. The other two scenarios imply that the merged firm can practice mixed bundling since it can offer a price for the bundled product, as well as prices for their individual components.

♦ *Independent ownership (I)*

The profit functions of the four firms are given by

$$\begin{aligned} \pi^{A_1} &= p_1(D^{11} + D^{12}) + p_1\bar{D}^{A_1},\ \pi^{A_2} = p_2(D^{21} + D^{22}) + p_2\bar{D}^{A_2}, \\ \pi^{B_1} &= q_1(D^{11} + D^{21}) + q_1\bar{D}^{B_1},\ \pi^{B_2} = q_2(D^{12} + D^{22}) + q_2\bar{D}^{B_2}. \end{aligned} \tag{3}$$

Solving the system of first-order conditions $\partial\pi^{A_1}/\partial p_1 = 0$, $\partial\pi^{A_2}/\partial p_2 = 0$, $\partial\pi^{B_1}/\partial q_1 = 0$ and $\partial\pi^{B_2}/\partial q_2 = 0$, we get the following equilibrium prices

$$p_1^I = p_2^I = q_1^I = q_2^I = \frac{2 + \lambda}{8 - 14c}, \tag{4}$$

where superscript I stands for independent ownership. Thus, the total price for each composite product is $s_{ij} = \frac{2+\lambda}{4-7c}$, for $i, j = 1, 2$. These equilibrium prices yield $(D^{ij})^I = \frac{2-\lambda+c(3\lambda-1)}{4-7c}$ for composite products, and $(\bar{D}^x)^I = \lambda - \frac{2+\lambda}{8-14c}$ with $x = A_1, A_2, B_1, B_2$, for individual brands. It is easy to observe that we need to impose a lower bound on λ to ensure positive demands for individual brands. This is well discussed below and in the Appendix. Finally, the equilibrium profits to any of the four firms can then be written as

$$\pi^I = \frac{(3 - 2c)(2 + \lambda)^2}{4(4 - 7c)^2}. \tag{5}$$

♦ *Single integration (S)*

Suppose now that firms A_1 and B_1 merge and jointly offer the bundle A_1B_1. The merged entity engages in mixed bundling so that it chooses the price for the bundled product (s_{11}), as well as the prices of the individual brands (p_1 and q_1). Its profit is given by

$$\pi^{A_1-B_1} = s_{11}(D^{11}) + p_1(D^{12}) + q_1(D^{21}) + p_1\bar{D}^{A_1} + q_1\bar{D}^{B_1}. \tag{6}$$

The profits of the firms that remain separate (A_2 and B_2) are as under independent ownership. Making the corresponding changes in (1), we can solve the system formed by $\partial\pi^{A_1-B_1}/\partial s_{11} = 0$, $\partial\pi^{A_1-B_1}/\partial p_1 = 0$, $\partial\pi^{A_1-B_1}/\partial q_1 = 0$, $\partial\pi^{A_2}/\partial p_2 = 0$ and $\partial\pi^{B_2}/\partial q_2 = 0$. The Nash equilibrium in prices results in[8]

$$\begin{gathered} p_1^S = q_1^S = K\left[5 + 6\lambda + c(5 - 4\lambda)\right], \; p_2^S = q_2^S = K\left[7 + 3\lambda + c(2 + \lambda) - 3c^2\right], \\ s_{11}^S = \frac{3K}{2}\left[9 + 4c(1 + 3\lambda) - c^2(1 + 4\lambda)\right], \end{gathered} \tag{7}$$

with $K = \frac{1}{27-36c-31c^2+12c^3}$, where superscript S stands for single integration.[9] Finally, equilibrium profits of the merged firm and outside firms are given by

$$\pi^S = s_{11}^S(D^{11})^S + 2p_1^S\left[(D^{12})^S + (\bar{D}^{A_1})^S\right], \; (\pi^{A_2})^S = (\pi^{B_2})^S = (3 - 2c)\left(p_2^S\right)^2, \tag{8}$$

since $p_1^S = q_1^S$, $p_2^S = q_2^S$, $(D^{12})^S = (D^{21})^S$ and $(\bar{D}^{A_1})^S = (\bar{D}^{B_1})^S$.

♦ *Parallel integration (P)*

We next consider a double merger scenario where both firms A_1 and B_1, and firms A_2 and B_2 merge. Both merged entities engage in mixed bundling so as to maximize profits given by

$$\begin{aligned} \pi^{A_1-B_1} &= s_{11}(D^{11}) + p_1(D^{12}) + q_1(D^{21}) + p_1\bar{D}^{A_1} + q_1\bar{D}^{B_1}, \\ \pi^{A_2-B_2} &= s_{22}(D^{22}) + p_2(D^{21}) + q_2(D^{12}) + p_2\bar{D}^{A_2} + q_2\bar{D}^{B_2}. \end{aligned} \tag{9}$$

Given the symmetry of this scenario, it follows that the equilibrium prices for bundled products and individual brands turn out to be

$$p_1^P = p_2^P = q_1^P = q_2^P = Z\left[2(1+\lambda) + c(2-\lambda)\right], \; s_{11}^P = s_{22}^P = Z\left[5 + 3c(1+2\lambda)\right], \tag{10}$$

with $Z = \frac{1}{10-11c-15c^2}$, where superscript P indicates parallel integration.[10] Finally, the equilibrium profits for any pair of merged entities

$$\pi^P = s_{11}^P (D^{11})^P + 2p_1^P (D^{12})^P + 2p_1^P (\bar{D}^{A_1})^P, \tag{11}$$

with $(D^{11})^P = (D^{22})^P$, $(D^{12})^P = (D^{21})^P$ and $(\bar{D}^{A_1})^P = (\bar{D}^{B_1})^P = (\bar{D}^{A_2})^P = (\bar{D}^{B_2})^P$.

♦ *Effects of mergers of complements on prices*

Prior to analyzing the equilibrium in merger formation and the effects on social welfare in the space (c, λ), we must impose a lower bound on λ to ensure positive demands for individual products. Lemma 1 below identifies this condition.

Lemma 1 *For all $c \in (0, 1/3)$, there exists a function $\underline{\lambda}(c) < 1$ such that $\lambda > \underline{\lambda}(c)$ ensures positive demands for individual consumption in all the scenarios under consideration.*

As a consequence, the following assumption is needed to guarantee comparable results.

Assumption 1 *We restrict attention to values of c and λ such that $\lambda > \underline{\lambda}(c)$. The relevant region in the space (c, λ) is depicted in Fig. 1.*

−Insert here Fig. 1−

The next proposition summarizes the effects on prices whenever two complementary firms merge, considering both a move from I to S and a move from S to P.

Proposition 1 *For any (c, λ) in the relevant region, when two firms merge*

i) the price of the post-merger bundle is lower than the sum of the pre-merger prices,

ii) the price of outsiders' system and individual components decreases for $\lambda > \lambda_1^*(c)$ in the move from I to S, and for $\lambda > \lambda_3^*(c)$ in the move from S to P,

iii) the price of partners' individual components increases for $\lambda > \lambda_2^*(c)$ in the move from I to S, and for $\lambda > \lambda_4^*(c)$ in the move from S to P.

Fig. 2 below depicts the effect on prices when moving from I to S, and Fig. 3 captures the move from S to P.

–Insert here Figs. 2 and 3–

Interestingly, the same pattern is repeated for both changes in market structure. Proposition 1(i) states that a complementary merger internalizes the externality that arises when firms set prices independently thus ignoring the effects on their individual markups. This is a well-known result in the literature and explains why the price of the bundle is reduced below the pre-merger situation. For instance, Brueckner (2003) shows that the presence of codesharing and antitrust immunity on an international interline itinerary (that has similar pricing effects as a merger) reduces the fare by 17%-30%.

Proposition 1(ii) claims that the lower the price set by one merged entity, the lower the price the outsiders will set when λ is high enough, as shown in Figs. 2(a) and 3(a). Under pure bundling (as in Beggs (1994)), the decrease in outsiders' prices occurs due to strategic complementarity in prices. The incentive to reduce prices is reinforced under mixed bundling (as in Choi (2008)), and outsiders strategically respond to rivals by cutting their prices to retain their market share. We observe that the need to set lower prices is weaker the lower is the demand for individual components (i.e., the lower is λ) because outsiders can take advantage of the lower prices set by partners when pricing their portion of 'mix-and-match' composite products (since the demand only depends on the overall price).

Finally, Proposition 1(iii) asserts that the merged firms increase the price of their stand-alone components for λ high, as shown in Figs. 2(b) and 3(b). When markets for individual components are sufficiently important, partners raise prices of stand-alone products, exploiting their monopoly power in local markets and making substitute 'mix-and-match' systems less attractive to consumers. However, when markets for individual components are less important, partners prefer to decrease prices and make 'mix-and-match' systems more competitive.

Thus, Proposition 1 states conditions under which the results in Beggs (1994) and Choi (2008) would hold. We can conclude that their results are robust to the introduction of separate consumption as long as the market for individual components is sufficiently important relative to the market for systems. These findings constitute the cornerstone of the results that follow.

4 Equilibrium analysis

In the light of the equilibrium results obtained under the three different considered scenarios, attention shifts now to the first stage of the game where merger decisions are made. Given the symmetry of the model, it suffices to examine the best response for firms producing A_1 and B_1. Let us then define the best-reply functions $\Psi_1(c, \lambda) = \frac{\pi^S}{2} - \pi^I$ and $\Psi_2(c, \lambda) = \frac{\pi^P}{2} - (\pi^{A_2})^S$. Hence, $\Psi_1(c, \lambda) > 0$ defines when they will merge, given that the rivals do not; and $\Psi_2(c, \lambda) > 0$ defines when they will merge given that the rivals also merge. The joint analysis of these best-reply functions leads to the following equilibrium result.

If neither pair of firms finds it profitable to merge, we will have a market structure with independent ownership (I equilibrium). Parallel integration results when it is profitable for both pairs of firms to merge (P equilibrium), and single integration entails just one merger in equilibrium (S equilibrium). Fig. 4 below displays the equilibrium in merging decisions.

–Insert here Fig. 4–

Note that $\widetilde{\lambda}(c)$ and $\widetilde{\widetilde{\lambda}}(c)$ are functions obtained from solving $\Psi_1 = 0$ and $\Psi_2 = 0$, respectively (see the details in the Appendix). The proposition below specifies this result.

Proposition 2 *For any (c, λ) in the relevant region, the equilibrium in merging decisions yields*

i) a unique P equilibrium for c sufficiently low,

ii) a unique S equilibrium for c intermediate and λ low,

iii) a unique I equilibrium for c sufficiently large and λ intermediate,

iv) a multiple equilibrium P and I for c intermediate and λ high,

in a way made clear by Fig. 4.

The above proposition highlights that merging is not always an equilibrium strategy, and that mergers of complements are unprofitable when markets for individual components are sufficiently important and competition is intense.[11]

Looking at the effect of competition intensity (c), there is a positive effect and a negative effect at work when firms merge. The positive effect comes from partner firms jointly deciding on the price of the system; this permits to internalize the negative externality arising from the 'double marginalization' existing under independent pricing of complementary components. Thus, if there were no competition from a substitute composite good, then mergers would always turn out profitable – as in Cournot (1838). However, there is a negative cross-price effect coming from the presence of competing systems because a merger typically puts a downward pressure on rivals' prices. This effect is more important as competition becomes stronger. Therefore, the negative effect offsets the positive effect when competition is more intense, turning mergers unprofitable. Beggs (1994) uses the example of shops in a mall selling complementary products that may not be interested in forming a hyperstore when competition is intense. Although firms would internalize a 'double marginalization' externality allowing them to reduce prices, other competing malls would do the same making profits go down.

The effect of the size of the demand for individual components (λ) is not straightforward. However, we observe that incentives towards merger formation are typically high when markets for individual components are not important (i.e., for low values of λ). Note that P is the only equilibrium that would be obtained in Choi (2008), where separate consumption is not possible.[12] As independent components become more relevant and λ increases, outsiders become more competitive if a merger is formed since their prices decrease. In addition, partners' prices increase, discouraging the demands for the 'mix-and-match' systems, which affects the merged firms through a lower demand for independent components (see Figs. 2 and 3).

Fig. 4 also identifies the equilibrium in integration decisions obtained in Beggs (1994) and Choi (2008) in the (c,λ) space. By Assumption 1 above and looking at the degree of substitutability across systems, we find that forming a merger is a dominant strategy when systems are sufficiently poor substitutes (i.e., competition is weak), whereas independent ownership arises when system competition is strong. We observe that the equilibrium configuration obtained in Beggs (1994) can be recovered when $\lambda \in (\lambda_3, \lambda_4)$. On the other hand, Fig. 4 also displays the equilibrium that would follow in the setting considered by Choi (2008) for $\lambda \in (\lambda_1, \lambda_2)$, where both mergers take place in equilibrium. Furthermore, in contrast with these previous papers, the possibility of a single merger arises in equilibrium, a result that seems to be sensible in the light of the observed behavior in certain industries. For instance, looking at the airline industry after its deregulation, among the major European carriers, only Air France and KLM merged on 2004. Such asymmetric equilibrium has been obtained without alluding to firms' asymmetries or demand shocks and without considering internal 'governance' costs associated with merger, and it occurs for intermediate values of λ. Therefore, our setting offers a more parsimonious transition between the two extreme market structures.

5 Welfare analysis

Choi (2008) constitutes the first attempt to provide a welfare analysis of complementary mergers, studying the move from independent ownership to single integration. This section complements his analysis in a setting with separate consumption and extends it to the case with parallel integration. In addition, the comparison between the socially-optimal results and the equilibrium results allows us to identify potential market failures.

Choi (2008) concludes that, as c approaches 0, competition across systems vanishes and the 'vertical' positive externality (between complements) stemming from the elimination of the 'double marginalization' problem turns mergers welfare enhancing. However, as c increases, a merger becomes socially harmful due to the 'horizontal' negative externality coming from the increase of partners' individual component prices, which yields an increase in the price of the 'mix-and-match' systems. Our analysis with individual consumption complements this analysis, confirming that mergers of complements may be socially detrimental.

We denote by W^I, W^S and W^P the social welfare under each market structure, which is computed as the sum of firm profits and utilities from system and individual components consumption. Let us define $\Omega_1 = W^S - W^I$, $\Omega_2 = W^P - W^S$ and $\Omega_3 = W^P - W^I$. From solving $\Omega_1 = 0$, $\Omega_2 = 0$ and $\Omega_3 = 0$, we can find the functions $\overline{\lambda}(c)$, $\overline{\overline{\lambda}}(c)$ and $\overline{\overline{\overline{\lambda}}}(c)$, respectively, which determine the different regions plotted in Fig. 5 below (details in the Appendix).

−Insert here Fig. 5−

The function $\overline{\overline{\overline{\lambda}}}(c)$ delimits the two different regions appearing in Fig. 5. Proposition 3 below summarizes the effect of complementary mergers on social welfare.

Proposition 3 *For any (c, λ) in the relevant region, mergers are welfare reducing for high values of c together with high values of λ. Mergers are welfare enhancing for low values of c.*

On the one hand, we observe that the 'vertical' positive externality prevails when in weak competition environments (i.e., c low), regardless of the relative importance of markets for individual components. On the other hand, the 'horizontal' negative externality is more marked and overcomes the previous positive effect when competition is sufficiently intense and the markets for systems and for components are rather symmetric (i.e., λ is high). This result is related to the price behavior commented before since, as shown in Figs. 2 and 3, when two firms merge, the price of their individual components increases only for high values of λ, thus making the 'horizontal' negative externality more important.

Although a priori the result in the proposition above seems to be similar to the one obtained in equilibrium, setting the welfare results against firms' equilibrium choices (by comparing Figs. 4 and 5) uncovers a market failure. This is shown in Fig. 6 below

−Insert here Fig. 6−

In the figure above, we can distinguish three regions where the best outcome from the social viewpoint differs from firms' equilibrium.[13] A conflict between public and private interests may arise when competition intensity is moderate (intermediate values of c). We observe that the number of mergers observed in equilibrium may be either excessive ('overintegration') or suboptimal ('underintegration'), depending on the relative importance of markets for individual components. In the upper conflict region, there is 'overintegration' since the socially preferred structure is I but the equilibrium leads to P. However, in the other two conflict regions, we observe 'underintegration' because P is socially preferred and the equilibrium may be either I or S. This result is summarized in the corollary below.

Corollary 1 *There is 'overintegration' when the demand for individual components is high. There is 'underintegration' when the demand for individual components is low.*

There are two opposing forces driving this result. On the one hand, market power increases with mergers ('horizontal' negative externality) and partners can price-discriminate and set different prices to systems and individual components. On the other hand, complementary mergers lead to lower bundle prices because they internalize the externality that arises when firms set prices independently ('vertical' positive externality), as shown in Proposition 1(i). When markets for individual components are important, price discrimination is used by partner firms to exploit the monopoly power they have in these markets by raising prices. Naturally, this can be socially detrimental. In contrast, when markets for individual components are less important, the market-power effect is mitigated and the lower bundle prices set by partner firms turn mergers socially profitable. However, in this case, the lower system price determined by partner firms puts a downward pressure on rivals' prices and, when competition is intense, mergers become privately unprofitable.

The aforementioned effects are clearly observed in the airline industry. When airlines price-cooperate, either by means of alliances (with antitrust immunity) or mergers,[14] they separate passengers into connecting and local. Fares charged to connecting passengers are determined by the competitive characteristics of the interline markets. However, fares charged to local passengers often increase and the overall effect could yield a worse social outcome, depending on the relative size of local and connecting markets (as well as on demand elasticities).

6 Conclusions and policy implications

We have developed a model of complementary mergers with mixed bundling, allowing for separate consumption, to identify conditions under which mergers are profitable for their members, and we have studied the possible discrepancies between private and social interests.

Since the foregoing analysis has assumed away the presence of any efficiency gains, one could expect a negative effect of mergers on consumers and society at large, since partners can price-discriminate and market power increases with mergers ('horizontal' negative externality). Yet a merger can rationalize production when there are complementary products involved ('vertical' positive externality). In this framework, we have analyzed when socially detrimental mergers may occur in equilibrium and when some socially beneficial mergers may fail to occur.

For antitrust authorities, non-horizontal mergers are less likely to significantly impede competition than horizontal mergers.[15] However, they may raise antitrust concerns if creating or strengthening a dominant position. In a controversial decision, the European Commission blocked the proposed GE/Honeywell merger (Nalebuff, 2002) and the primary concern was on 'conglomerate effects' of bringing complements together.[16] The authorities feared that mixed bundling would restrict competition in the markets for jet aircraft engines and avionics; that rivals could not match the bundle offer could lead to foreclosure in the component markets. All in all, a more faithful assessment of the effects on competition requires the consideration of the markets for individual products and rivals' counter-strategies. These are indeed incorporated in the EU non-horizontal merger guidelines (2008, section V on conglomerate mergers).[17]

Our setting may prove useful as it precisely combines endogenous integration of complements (rivals' response), along with the markets for the bundle and for separate components. We have found that, in line with most of the received literature, a non-horizontal merger pushes rivals' profits down. However, this is not in itself deemed a problem. Rather, the Commission's main focus is on the likely harm to consumers. Clearly, consumers that purchase the bundle will face a lower price than before the merger; but those who 'mix-and-match', could be worse off. Consumer surplus indeed diminishes when the cross-price effects are important. Given the symmetry assumptions of our model, compatibility is preferred by firms and foreclosure issues do not arise. Still, mixed bundling by integrated producers of complementary products can adversely affect consumer and social welfare. The divergence between private and social interests might be resolved by imposing some constraints on firms' bundling behavior, e.g., that the bundle discount be fixed to a maximum (behavioral remedies).

In June 2007, the Commission prohibited the hostile takeover by Ryanair of Aer Lingus. As is known (and as advanced in the literature review), complementarities play an important role when airlines that join an alliance (or merge) are granted antitrust immunity. At the time of the decision, Ryanair and Aer Lingus competed directly on 35 routes to and from Ireland. On 22 of these routes, customers would have faced a monopoly after the merger (i.e., routes where networks of the two airlines overlap). This aspect gave the merger a horizontal nature and a price regression analysis helped to confirm these effects. Only recently, the US Department of Justice has cleared the merger between United and Continental Airlines arguing that complementarities prevail. Our analysis emphasizes the role played by the cross-price elasticity of demand across systems (c) and the relative importance of the markets for individual components (λ). Thus, regulators might use econometric demand specifications to elicit values for own-price and cross-price elasticities. These values, coupled with relative measures of the market for composite and stand-alone products, can be employed to calibrate the model and obtain an assessment of the (expected) variations in prices. For example, when composite products are sufficiently good substitutes (high values of estimated c) and both the composite and individual markets are similar (λ rather high), the price of the bundle is close to the sum of the pre-merger prices; mixed bundling is not marked and firms have no incentive to merge, yet remark that independent ownership is the socially-preferred setting. Interests are also aligned when c is rather low ending up with parallel integration. It is for intermediate values of the estimated c that the authorities need to discourage (high λ) or to encourage (low λ) integration processes, to have private interests conform to social ones.

Future work should introduce more complex network structures and cost considerations in the analysis that would allow for more precise assessments by looking into efficiency justifications for mergers of the type herein analyzed. The consideration of asymmetries would make the model suitable to investigate any likely exclusionary behavior, the main theory of harm specified in the legislation regarding non-horizontal mergers.

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Notes

[1]The case of bundling with complements has been studied by Matutes and Regibeau (1992), Economides and Salop (1992), Denicolo (2000), and Church and Gandal (2000), among others.

[2]A merger with complements typically results in lower profits for firms outside the merger. Boyer (1992) shows that this can be so even in a horizontal merger between producers of substitute products.

[3]This demand structure for differentiated products follows from solving the optimization problem of a representative consumer with a quasi-linear utility function. See Choi (2008) for the details.

[4]The consideration of both separate and joint consumption with complementary products in a duopolistic vertical differentiation model is taken up in Gabszewicz *et al.* (2001).

[5]They follow from the optimization problem of a representative consumer with a quadratic quasi-linear utility function of the type $U(\bar{D}^{A_1}, \bar{D}^{A_2}, \bar{D}^{B_1}, \bar{D}^{B_2}) = \sum_x \left(\frac{\lambda a}{b} \bar{D}^x - \frac{1}{2b} \left(\bar{D}^x\right)^2 \right) + m$, where $x = A_1, A_2, B_1, B_2$ and m is the amount of numeraire good.

[6]An alternative demand specification would be $\bar{D}^{A_1} = \lambda(a - bp_1)$, so that changes in λ would affect both the intercept and the slope of the demand curve. However, under this modeling specification, we would lose the interpretation of λ as the relative market size of individual components with respect to the markets for systems, which is useful from an antitrust perspective.

[7]The case of substitutability between components is discussed in Footnote 11.

[8]Non-arbitrage conditions need to hold: *i*) the price of the bundle has to be greater than the price of the individual components ($s_{11} > p_1, q_1$), and *ii*) the price of the bundle has to be smaller than the sum of prices of individual components ($s_{11} < p_1 + q_1$). The equilibrium prices in (7) and (10) fulfill these conditions.

[9]The equilibrium quantities are $(D^{11})^S = \frac{K}{2}\left[27 + 4c(3\lambda - 2) - c^2(23 - 4\lambda)\right]$, $(D^{12})^S = (D^{21})^S = \frac{K}{2}\left[6(5 - 3\lambda) + c(36\lambda - 7) + 2c^2(17\lambda - 11) + 3c^3(1 - 4\lambda)\right]$, $(D^{22})^S = \frac{K}{2}\left[2(13 - 6\lambda) + c(32\lambda - 5) + 2c^2(12\lambda - 5) + 3c^3(3 - 4\lambda)\right]$, $(\bar{D}^{A_1})^S = (\bar{D}^{B_1})^S = \lambda - K\left[(5 + 6\lambda) + c(5 - 4\lambda)\right]$, $(\bar{D}^{A_2})^S = (\bar{D}^{B_2})^S = \lambda - K[(7 + 3\lambda) + c(2 + \lambda) - 3c^2]$. As in the I scenario, a lower bound on λ is needed to ensure positive demands (details in the Appendix).

[10]The equilibrium quantities are $(D^{11})^P = (D^{22})^P = Z\left[5 - c(1 - 2\lambda) - 2c^2(2 - \lambda)\right]$, $(D^{12})^P = (D^{21})^P = Z\left[6 - 4\lambda + 5c^2(2\lambda - 1) + c(6\lambda - 1)\right]$, $(\bar{D}^x)^P = \lambda - Z\left[2(1 + \lambda) + c(2 - \lambda)\right]$ with $x = A_1, A_2, B_1, B_2$. As in the previous scenarios, a lower bound on λ is needed to ensure positive demands (details in the Appendix).

[11]The analysis can be extended to consider substitutability between individual components. This setting would sit well with some examples in the airline industry. For instance, passengers may travel from an

airport, say Orange County (SNA), to another, say London-Heathrow (LHR), while stopping at a hub, say Chicago O'Hare (ORD); suppose that each segment of the interline trip is served by two differentiated carriers (e.g., American and United on the route SNA-ORD, and British Airways and British Midland on the route ORD-LHR). We assume that demand functions for individual components A_1 and A_2 are given by $\bar{D}^{A_1} = \lambda a - bp_1 + cp_2$, $\bar{D}^{A_2} = \lambda a - bp_2 + cp_1$, and the corresponding functions for B_1 and B_2. Thus, local markets are duopolies characterized by the same competition intensity as the market for systems (c). The results remain qualitatively unchanged under this specification (computations available from the authors on request).

Furthermore, this modeling would allow us to examine a horizontal merger A_1-A_2. It has been proven that a merger between firms producing components of the same type is always profitable to partners and that it leads to higher profits than a merger between complementary components when systems are not too differentiated (Pardo-Garcia, 2010). Nevertheless it must be noted that a horizontal merger of this type would be challenged by antitrust authorities (see the last section of the paper for policy implications).

[12] Although merger formation is not studied in Choi (2008), the equilibrium can be computed in his setup.

[13] In addition, other conflicts could arise in the multiple-equilibria region (i.e., between $\widetilde{\lambda}(c)$ and $\widetilde{\widetilde{\lambda}}(c)$), but these cases are not analyzed since there is not a clear equilibrium prediction.

[14] Brueckner (2001), Brueckner and Whalen (2000) and Whalen (2007) analyze the effects of alliances on airfares and their pro and anticompetitive effects.

[15] Neven and de la Mano (2009) study some economic analyses used for merger control in recent EU cases.

[16] A strand of antitrust law in the EU and the US finds that mixed bundling can be anticompetitive, absent any merger. See, e.g., *i*) *Bristish Airways v Commission*, where discounts to travel agents who used a particular airline were provided, *ii*) the *LePage's Inc v 3M* case on bundling rebates in the Scotch-brand tape, and *iii*) the renowned *US v Microsoft* on bundling Internet Explorer with Windows OS.

[17] Concentrations in the EU are evaluated on the basis of Regulation 4064/89, amended by Council Regulation 139/2004. In the US, the ruling is under the 1992 Horizontal Merger Guidelines, later amended in 1997. Price discrimination in the EU is ruled under Art. 82 of the Treaty, while in the US it is judged according to Section 2 of the Clayton Act amended by the Robinson-Patman Act (1936).

Figures

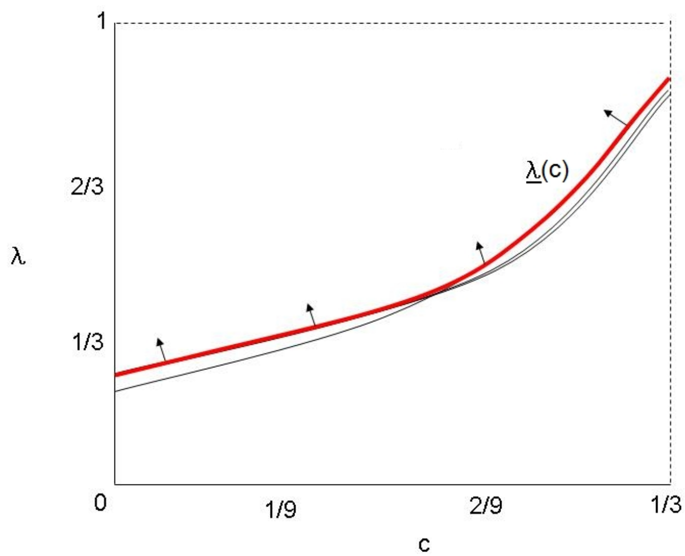
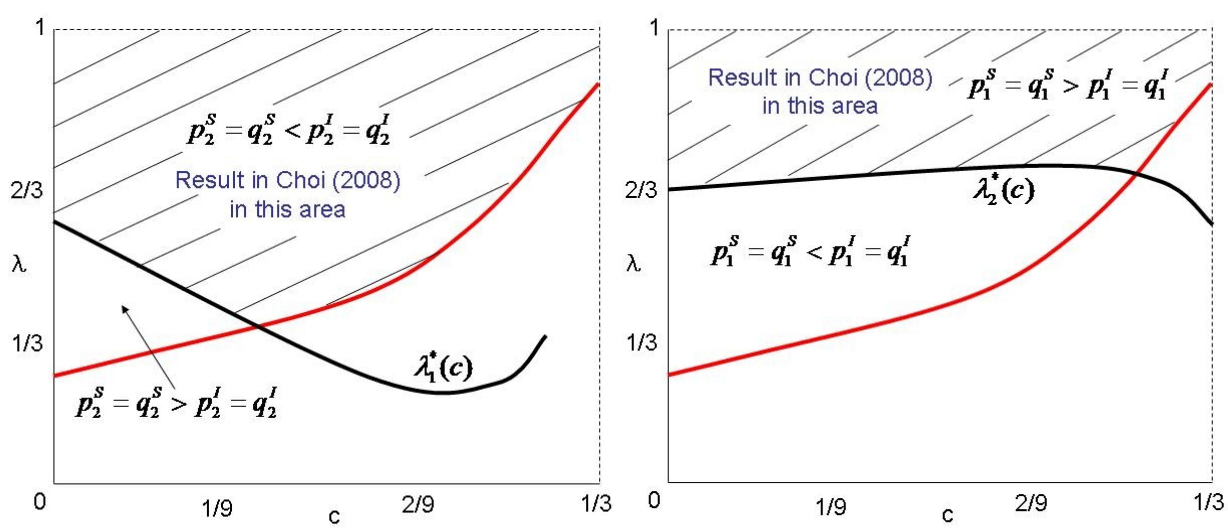


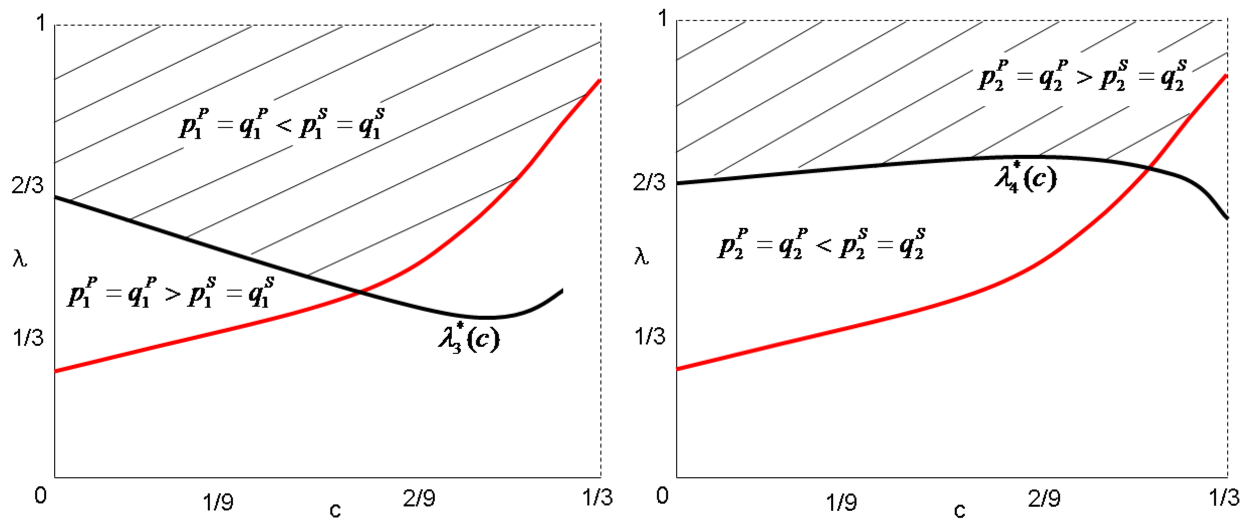
Figure 1: The relevant region



a) Effect on outsiders

b) Effect on partners

Figure 2: Effect on prices of the move from I to S



a) Effect on outsiders *b*) Effect on partners

Figure 3: Effect on prices of the move from S to P

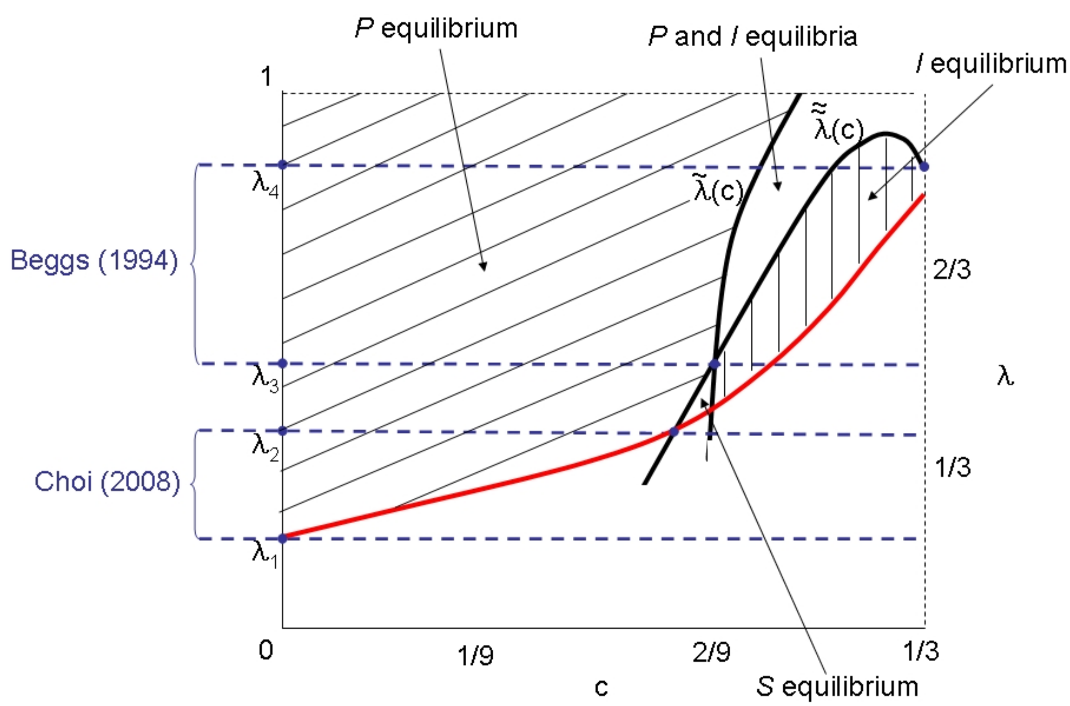


Figure 4: Equilibrium analysis

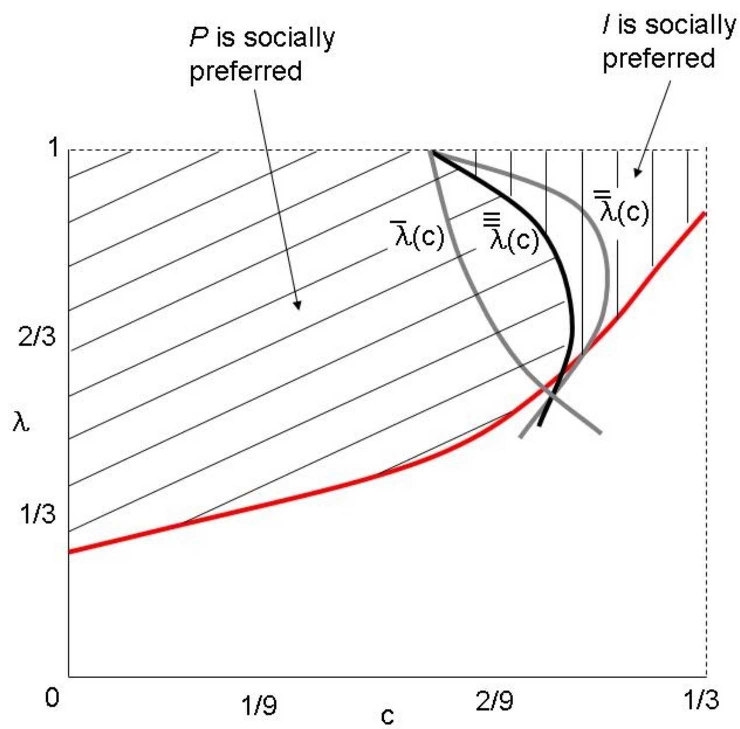


Figure 5: Welfare analysis

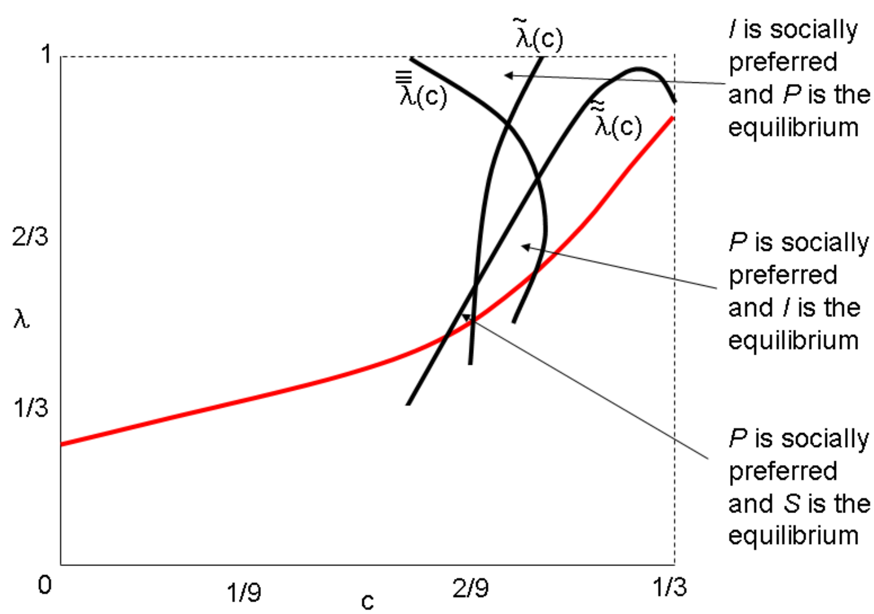


Figure 6: Conflict between private and public interests

A Appendix: Proofs

Proof of Lemma 1.

In our simple model, prices and demand for systems are always positive. Only the demands for individual components (that are given by (2)) could be negative. To ensure that these demands remain positive, under each scenario (i.e., I, S and P), we require λ to be higher than a certain lower bound $\underline{\lambda}_i(c) < 1$ that depends on the degree of substitutability across systems (c). Defining $\underline{\lambda}$ as the upper envelope of all the previous conditions, i.e., $\underline{\lambda}(c) = \max\{\underline{\lambda}_i(c)\} < 1$ for $i = 1, 2, 3, 4$, we just have to require $\lambda > \underline{\lambda}(c)$ to remain within the relevant region.

From $(\bar{D}^x)^I > 0$ with $x = A_1, A_2, B_1, B_2$ we get $\lambda > \frac{2}{7(1-2c)}$; from $(\bar{D}^{A_1})^S = (\bar{D}^{B_1})^S > 0$ we get $\lambda > \frac{5(1+c)}{21-c[32-c(12c-31)]}$; from $(\bar{D}^{A_2})^S = (\bar{D}^{B_2})^S > 0$ we get $\lambda > \frac{7+c(2-3c)}{24-c[37-c(12c-31)]}$; and from $(\bar{D}^x)^P > 0$ with $x = A_1, A_2, B_1, B_2$, we get $\lambda > \frac{2(1+c)}{8-5c(2+3c)}$. It is easy to observe that the latter condition is dominated by the previous ones and thus

$$\underline{\lambda}(c) = \max\left\{\frac{2}{7(1-2c)}, \frac{5(1+c)}{21-c[32-c(12c-31)]}, \frac{7+c(2-3c)}{24-c[37-c(12c-31)]}\right\}, \tag{A1}$$

which is depicted in Fig. 1. ■

The proofs that follow compare prices, profits and welfare under the different scenarios.

Proof of Proposition 1.

In the move from I to S, partner firms A_1 and B_1 merge, and thus A_2 and B_2 are the outsiders. Let us define $\Gamma_1 = s_{11}^S - (p_1^I + q_1^I)$, $\Gamma_2 = p_2^S - p_2^I$ and $\Gamma_3 = p_1^S - p_1^I$.

$\Gamma_1 < 0$ requires $\lambda > \lambda_a(c) \equiv c\frac{3c+c(28-27c)}{2c[c(119-30c)-108]+54}$. It can be checked that $\lambda_a < \underline{\lambda}(c)$ for any (c, λ) in the relevant region. Therefore $\lambda > \lambda_a$ is always observed and thus $s_{11}^S < p_1^I + q_1^I$.

$\Gamma_2 < 0$ requires $\lambda > \lambda_1^*(c) \equiv 2\frac{1+c[c(5+9c)-5]}{3-c[2+c(17-12c)]}$. Both $\lambda > \lambda_1^*(c)$ and $\lambda < \lambda_1^*(c)$ are possible, and thus both $p_2^S < p_2^I$ and $p_2^S > p_2^I$ can be observed, as shown in Fig. 2(a).

$\Gamma_3 < 0$ requires $\lambda > \lambda_2^*(c) \equiv 2\frac{7-c(3+2c)(7-6c)}{21-c[80-3c(29-4c)]}$. Both $\lambda > \lambda_2^*(c)$ and $\lambda < \lambda_2^*(c)$ are possible, and thus both $p_1^S > p_1^I$ and $p_1^S < p_1^I$ can be observed, as shown in Fig. 2(b).

In the move from S to P, partner firms A_2 and B_2 merge, and thus A_1 and B_1 are the outsiders. Let us define $\Gamma_4 = s_{22}^P - (p_2^S + q_2^S)$, $\Gamma_5 = s_{11}^P - s_{11}^S$, $\Gamma_6 = p_1^P - p_1^S$ and $\Gamma_7 = p_2^P - p_2^S$.

$\Gamma_4 < 0$ requires $\lambda > \lambda_b(c) \equiv \frac{5-3c\{5+c[17-c(13+18c)]\}}{4\{c[52-c(26+3c(13-6c))]-15\}}$. It can be checked that $\lambda_b < \underline{\lambda}(c)$ for any (c, λ) in the relevant region. Therefore $\lambda > \lambda_b$ is always observed and thus $s_{22}^P < p_2^S + q_2^S$.

$\Gamma_5 < 0$ requires $\lambda > \lambda_c(c) \equiv \frac{c[41+27c(3+c)]-21}{12\{3-c[7+3c(1-c)]\}}$. It can be checked that $\lambda_c < \underline{\lambda}(c)$ for any (c, λ) in the relevant region. Therefore $\lambda > \lambda_c$ is always observed and thus $s_{11}^P < s_{11}^S$.

$\Gamma_6 < 0$ requires $\lambda > \lambda_3^*(c) \equiv \frac{4-c[17-c(13+24c)]}{(1+c)[6-c(19-12c)]}$. Both $\lambda > \lambda_3^*(c)$ and $\lambda < \lambda_3^*(c)$ are possible, and thus both $p_1^P < p_1^S$ and $p_1^P > p_1^S$ can be observed, as shown in Fig. 3(a).

$\Gamma_7 < 0$ requires $\lambda > \lambda_4^*(c) \equiv \frac{16-c(1-c)[39+c(62+21c)]}{24-2c\{38-c[15+c(35-6c)]\}}$. Both $\lambda > \lambda_4^*(c)$ and $\lambda < \lambda_4^*(c)$ are possible, and thus both $p_2^P > p_2^S$ and $p_2^P < p_2^S$ can be observed, as shown in Fig. 3(b). ■

Proof of Proposition 2.

From $\Psi_1 = \frac{\pi^S}{2} - \pi^I$, we get $\frac{H}{(7c-4)^2(27-36c-31c^2+12c^3)^2}$, with

$$H = 568 - 2216c - 1367c^2 + 11320c^3 + 3886c^4 - 18784c^5 - 9039c^6 + 2304c^7$$
$$+\lambda(-2136 + 12368c - 30096c^2 + 36104c^3 + 480c^4 - 36312c^5 + 6984c^6 + 2304c^7)$$
$$+\lambda^2(4842 - 34572c + 85180c^2 - 70408c^3 - 23926c^4 + 54356c^5 - 15600c^6 + 576c^7).$$

From $\Psi_2 = \frac{\pi^P}{2} - (\pi^{A_2})^S$, we get $\frac{K}{(15c^2+11c-10)^2(27-36c-31c^2+12c^3)^2}$, with

$$K = 489 - 2018c - 2343c^2 + 11548c^3 + 16925c^4 - 16590c^5 - 41596c^6 - 18272c^7 + 4818c^8 + 3492c^9$$
$$+\lambda(-1872 + 9012c - 18868c^2 + 13280c^3 + 45864c^4 - 52884c^5 - 54796c^6 + 17504c^7 + 9672c^8 - 4032c^9)$$
$$+\lambda^2(6264 - 36720c + 52074c^2 - 49810c^3 - 126588c^4 - 17324c^5 + 88890c^6 - 422c^7 - 16368c^8 + 3618c^9).$$

The sign of Ψ_1 and Ψ_2 depends on the sign of H and K, respectively. Solving $H = 0$ and $K = 0$ for λ, we obtain two roots in each case (the precise values of these roots are available from the authors upon request). Restricting attention to the relevant region, only the positive root is effective in each case (since the negative root is below $\underline{\lambda}(c)$ for any (c, λ) in the relevant region). We denote these positive roots by $\widetilde{\lambda}(c)$ and $\widetilde{\widetilde{\lambda}}(c)$ respectively (see Fig. 4). ■

Proof of Proposition 3.

From $\Omega_1 = W^S - W^I$, we get $\frac{L}{8(4-7c)^2\{27+c[c(12c-31)]-36\}^2}$, with

$L = 9984 - 58104c + 81401c^2 + 69680c^3 - 134762c^4 - 45440c^5 + 21297c^6 - 7848c^7$

$+\lambda(-144 - 19200c + 59568c^2 + 9112c^3 - 80616c^4 - 31672c^5 + 3048c^6 + 9792c^7)$

$+\lambda^2(3996 - 38592c + 109272c^2 - 86520c^3 - 43348c^4 + 51112c^5 + 2064c^6 - 4608c^7)$.

From $\Omega_2 = W^P - W^S$, we get $\frac{M}{8[c(11+15c)-10]^2\{27+c[c(12c-31)-36]\}^2}$, with

$M = -409404 + 990132c + 1492507c^2 - 3303610c^3 - 2901651c^4 + 3168368c^5 + 2700453c^6$

$-352434c^7 - 271953c^8 + 100008c^9 + \lambda(2354112 - 10867776c + 5964720c^2 + 32106152c^3$

$-27919912c^4 - 41211920c^5 + 28154752c^6 + 26353896c^7 - 7102152c^8 - 4131648c^9 + 1036800c^{10})$

$+\lambda^2(-1862784 + 9279072c - 6757536c^2 - 26915112c^3 + 28997808c^4 + 35155152c^5 - 30400032c^6$

$-24845832c^7 + 7465104c^8 + 3946752c^9 - 1036800c^{10})$.

From $\Omega_3 = W^P - W^I$, we get $\frac{N}{2(4-7c)^2[c(11+15c)-10]^2}$, with

$N = -1904 + 5472c + 2462c^2 - 12282c^3 - 918c^4 + 5442c^5$

$+\lambda(12912 - 71056c + 101708c^2 + 59344c^3 - 188852c^4 + 7248c^5 + 88200c^6)$

$+\lambda^2(-10084 + 58172c - 90269c^2 - 41090c^3 + 171867c^4 - 20952c^5 - 88200c^6)$.

The sign of Ω_1, Ω_2 and Ω_3 depends on the sign of L, M and N, respectively. Solving $L = 0$, $M = 0$ and $N = 0$ for λ, we obtain two roots in each case (the precise values of these roots are available from the authors upon request). Restricting attention to the roots that are effective in the relevant region, we are left with $\overline{\lambda}(c)$, $\overline{\overline{\lambda}}(c)$ and $\overline{\overline{\overline{\lambda}}}(c)$, which are depicted in Fig. 5. ■

Proof of Corollary 1. Straightforward. ■