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José Manuel Giménez Gómez
Josep E. Peris

Document de treball n.5 - 2012

DEPARTAMENT D'ECONOMIA – CREIP
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ROVIRA I VIRGILI
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Edita:

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Universitat Rovira i Virgili
Facultat d'Economia i Empresa
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 759 811
Fax: +34 977 300 661
Email: sde@urv.cat

CREIP
www.urv.cat/creip
Universitat Rovira i Virgili
Departament d'Economia
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 558 936
Email: creip@urv.cat

Adreçar comentaris al Departament d'Economia / CREIP

Dipòsit Legal: T - 403 - 2012

ISSN edició en paper: 1576 - 3382
ISSN edició electrònica: 1988 - 0820

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Solidarity and uniform rules in bankruptcy problems.

José M. Giménez-Gómez[a], **Josep E. Peris**[b]

[a]*Universitat Rovira i Virgili, Dep. d'Economia, CREIP and GRODE, Av.Universitat 1, 43204 Reus, Spain.*
(e-mail: josemanuel.Gimenez@urv.cat)
[b]*Universitat d'Alacant, Dep. de Mètodes Quantitatius i Teoria Econòmica, 03080 Alacant, Spain.*
(e-mail: peris@ua.es)

Abstract

The idea of ensuring a guarantee (a minimum amount of the resources) to each agent has recently acquired great relevance, in both social and political terms. Furthermore, the notion of Solidarity has been treated frequently in redistribution problems to establish that any increment of the resources should be equally distributed taking into account some *relevant* characteristics. In this paper, we combine these two general concepts, guarantee and solidarity, to characterize the uniform rules in bankruptcy problems (*Constrained Equal Awards* and *Constrained Equal Losses* rules).

Keywords: Constrained Equal Awards, Constrained Equal Losses, Lower bounds, Bankruptcy problems, Solidarity
JEL classification: C71, D63, D71.

1. Introduction.

The concern of ensuring minimum individual rights has been figured prominently in a large number of contexts. Specifically, a classical issue that has captured most of the attention in the social policy literature and the political agenda during the last two decades, is the *Universal Basic Income*. This proposal involves the payment of a universal cash benefit to all citizens by the Administration, to which they would be entitled by the simple fact of being a full member of a community politics, regardless of income, employment history, availability to work, or the composition of her family (see for instance Noguera (2010)). Another context where the idea of a guarantee appears is the establishment of a minimum wage in the labour market or, more currently, the U.S. Senate's debate of ensuring universal minimum health coverage. In bankruptcy problems, we can also find these guarantees. In fact, the solution for the *"Contested Garment Problem"* proposed in the Babylonic Talmud suggests that each agent should receive at least some part of the available amount (Aumann and Maschler (1985)). Moreover, this idea has underlaid the theoretical analysis of bankruptcy problems from its beginning (it appears in the formal definition of a bankruptcy rule, requiring that no agent receives less than zero) to present day (Giménez-Gómez and Marco-Gil (2008)). In this paper we consider the idea defined in Moulin (2002) that establishes a guarantee to each agent, which only depends on her claim and on the estate.

On the other hand, Solidarity is a extensively used property in redistribution problems, since it expresses how variations in an economy should affect their members. The first idea of solidarity requires all agents to be affected in the same direction when a change in the estate occurs. But this is a mild condition satisfied by any bankruptcy rule. The (more restrictive) idea of group solidarity requires that changes in the estate are equally distributed among agents. But there is not bankruptcy rule satisfying this condition. We define here a Group Solidarity axiom, which is an intermediate compromise between the above mentioned notions, referred to the selected guarantee.

Therefore, in this paper, (i) we combine the lower bound defined in Moulin (2002) with a solidarity axiom; (ii) we use them to characterize the uniform gains rule (*Constrained Equal Awards*); (iii) similarly, we find the conditions that characterize the uniform losses rule (*Constrained Equal Losses*).

The article is organized as follows. In the next section, we present the model and introduce the *fair* lower bound. Section 3 contains the axiomatic characterization of the uniform rules. The proof of our main result is given in an Appendix.

2. Preliminaries.

Throughout the paper we will consider a set of agents $N = \{1, 2, ..., n\}$. Each agent is identified by her *claim*, c_i, $i \in N$, on the *estate* E. A *bankruptcy problem* appears whenever the estate is not enough to satisfy all the claims; that is, $C = \sum_{i=1}^{n} c_i > E$. Without loss of generality, we will order the agents according to their claims: $c_1 \leq c_2 \leq \cdots \leq c_n$. The pair (E, c) represents the bankruptcy problem, and we will denote by $\mathcal{B}$ the set of all bankruptcy problems. A *bankruptcy rule* (*solution*) is a single valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, $0 \leq \varphi_i(E, c) \leq c_i \quad \forall i \in N$ (non-negativity and claim-boundedness), and $\sum_{i=1}^{n} \varphi_i(E, c) = E$ (efficiency).

Many solution concepts have been defined in the literature about bankruptcy problems (see for instance Thomson (2003)). Two of the most important solution concepts are the uniform ones (Maimonides, 12th century Aumann and Maschler (1985))). Specifically, the *Constrained Equal Awards* rule, which recommends equalizing awards across agents subject to no-one receiving more than her claim; and the *Constrained Equal Losses* rule, which is obtained by focusing on the losses claimants incur (what they do not receive), and choosing the awards vector at which these losses are equal subject to no-one receiving negative amount[1].

Definition 1. *The* ***Constrained Equal Awards*** *rule, CEA: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E, c) \equiv \min \{c_i, \mu\}$, where μ is chosen so that $\sum_{i \in N} \min \{c_i, \mu\} = E$.*

Definition 2. *The* ***Constrained Equal Losses*** *rule, CEL: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEL_i(E, c) \equiv \max \{0, c_i - \mu\}$, where μ is chosen so that $\sum_{i \in N} \max \{0, c_i - \mu\} = E$.*

Among all the guarantees defined in the literature we use the lower and upper bounds defined in Moulin (2002), that we call *fair bounds*.

Definition 3. *(Moulin (2002))* ***Fair Lower Bound****, f^l: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $f_i^l(E, c) = \min \left\{c_i, \frac{E}{n}\right\}$.*

Note that, contrary to other lower bounds, this bound does not depend on the size of other agent's claims, and it guarantees a strictly positive amount of the resources to each agent, independently of her claims size.

[1] In Moulin (2002) these rules are called *Uniform Gains* and *Uniform Losses*, respectively.

3. Main results.

This section provides a characterization of uniform rules (*Constrained Equal Awards* and *Constrained Equal Losses*) by means of two properties, defined below: respect of fair lower bound and group solidarity for equal changes in fair bound.

Solidarity is a well known principle in the literature about redistribution (see Fleurbaey and Maniquet (2007)). The main idea is that a change in the estate affects all agents in the same direction. In our context (bankruptcy problems) this condition is known as *Resource Monotonicity*, a condition that is fulfilled by all defined rules. This condition has been strengthened in *Group Solidarity* (see Bossert (1995)), that in our context can be written as:

$\varphi_i(E, c) - \varphi_i(E', c) = \varphi_j(E, c) - \varphi_j(E', c)$, for all $i, j \in N$.

When applied to the particular case with $E' = 0$, Group Solidarity implies $\varphi_i(E, c) = \frac{E}{n}$ for all i, a condition that is not possible for bankruptcy rules when there exists some claim $c_i < \frac{E}{n}$. When looking for a weaker condition (intermediate between Solidarity and Group Solidarity) we use the next property[2]. It requires that if the estate increases, then this increment should be shared equally among agents who experiment an equal change in the *fair lower bound*.

Definition 4. ***Group solidarity for equal changes in fair lower bound,*** *$GSFL$***:** *for each $(E, c) \in \mathcal{B}$ and each $i, j \in N$ such that $C \geq E > E'$, if $f_i^l(E, c) - f_i^l(E', c) = f_j^l(E, c) - f_j^l(E', c)$, then $\varphi_i(E, c) - \varphi_i(E', c) = \varphi_j(E, c) - \varphi_j(E', c)$.*

The next property establishes that each agent should receive at least the fair lower bound.

Definition 5. ***Respect of fair lower bound,*** *RFL***:** *for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $\varphi_i(E, c) \geq f_i^l(E, c)$.*

In Moulin's words, this condition "says that agent i is guaranteed a fair share of the resources unless she demands no more than the fair share, in which case her demand is meet in full".

[2] A similar idea, in the context of redistribution problems, can be found in Luttens (2010).

Theorem 1. *The CEA rule is the only bankruptcy rule satisfying RFL and $GSFL$.*

Proof. *See Appendix.*

Finally, if we apply our reasoning to an upper bound (see Moulin (2002)), we obtain a characterization of the CEL rule. Let L denote the total losses agents incur, $L = C - R$.

Definition 6. *(Moulin (2002))* ***Fair upper bound,*** *f^u: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $f_i^u(E, c) = \max\left\{0, c_i - \frac{L}{n}\right\}$.*

Definition 7. ***Group solidarity for equal changes in fair upper bound,*** *$GSFU$**:** for each $(E, c) \in \mathcal{B}$ and each $i, j \in N$ such that $C \geq E > E'$, if $f_i^u(E, c) - f_i^u(E', c) = f_j^u(E, c) - f_j^u(E', c)$, then $\varphi_i(E, c) - \varphi_i(E', c) = \varphi_j(E, c) - \varphi_j(E', c)$.*

Definition 8. ***Respect of fair upper bound,*** *RFU**:** for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $\varphi_i(E, c) \leq f_i^u(E, c)$.*

Theorem 2. *The CEL rule is the only bankruptcy rule satisfying RFU and $GSFU$.*

ACKNOWLEDGEMENTS:

Personal acknowledgements will be included.
The usual caveat applies. Financial support from Universitat Rovira i Virgili, Banco Santander and Generalitat de Catalunya under project 2011LINE-06 and the Barcelona GSE is gratefully acknowledged.

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Appendix: Proof of Theorem 1

For each $(E, c) \in \mathcal{B}$ and each $i \in N$,

1. if $E \leq nc_1$, by the definition of the fair lower bound, $f_i^l(E, c) = \frac{E}{n}$. Then, by $GSFL$ and efficiency, $\varphi_i(E, c) = \frac{E}{n}$, which coincides with the CEA rule.
2. If $nc_1 \leq E \leq c_1 + (n-1)c_2$, by definition of the fair lower bound, $f_1^l(E, c) = c_1$, and $f_i^l(E, c) = \frac{E}{n}$, $\forall i \geq 2$. By RFL and claim-boundedness, $\varphi_1(E, c) = c_1$. By $GSFL$ and efficiency, $\varphi_i(E, c) = \frac{E-c_1}{n-1}$, $\forall i \geq 2$, which coincides with the CEA rule.
3. If $c_1 + (n-1)c_2 \leq E \leq nc_2$, by definition of the fair lower bound, $f_1^l(E, c) = c_1$, and $f_i^l(E, c) = \frac{E}{n}$, $\forall i \geq 2$. By RFL, $GSFL$, claim-boundedness and efficiency, $\varphi_1(E, c) = c_1$, $\varphi_2(E, c) = c_2$ and $\varphi_i(E, c) = \frac{E-c_1-c_2}{n-1}$, $\forall i \geq 3$, which coincides with the CEA rule.
4. If $nc_2 \leq E \leq c_1 + c_2 + (n-2)c_3$, by definition of the fair lower bound, $f_1^l(E, c) = c_1, f_2^l = c_2$, and $f_i^l(E, c) = \frac{E}{n}$, $\forall i \geq 3$. By RFL, $GSFL$, claim-boundedness and efficiency, $\varphi_1(E, c) = c_1$, $\varphi_2(E, c) = c_2$ and $\varphi_i(E, c) = \frac{E-c_1-c_2}{n-1}$, $\forall i \geq 3$, which coincides with the CEA rule.
5. If $c_1 + c_2 + (n-2)c_3 \leq E \leq nc_3$, by definition of the fair lower bound, $f_1^l(E, c) = c_1, f_2^l = c_2$, and $f_i^l(E, c) = \frac{E}{n}$, $\forall i \geq 3$. By RFL, $GSFL$, claim-boundedness and efficiency, $\varphi_1(E, c) = c_1$, $\varphi_2(E, c) = c_2$,

$\varphi_3(E, c) = c_3$ and $\varphi_i(E, c) = \frac{E - c_1 - c_2}{n-1}$, $\forall i \geq 4$, which coincides with the CEA rule.

6. It is straightforwardly the replication of this reasoning retrieving in each case the CEA rule.

q.e.d.