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# Discrimination in Bankruptcy Situations.

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**Abstract**

In a bankruptcy situation, not all claimants are equally affected. Some depositors may enter into a situation of *personal bankruptcy* if they lose part of their investments. Events of this kind may lead to a social catastrophe. As a solution to this problem we discriminate among claimants by some claim independent characteristic (wealth, net-income, GDP, etc.) We propose some progressive transfers from richer to poorer claimants with the purpose of distributing losses as evenly as possible. Finally, we characterize our solution by means of the *Lorenz criterion*. Endogenous convex combinations between solutions are also considered.

*Keywords:* Bankruptcy, Discrimination, Compensation, Rules
*JEL classification:* C71, D63, D71.

## 1. Introduction

When an institution goes bankrupt, which bankruptcy solution should be implemented? The formal analysis of situations where agents' deposits exceed the available resources of a bank was originated by O'Neill (1982) under the name of *bankruptcy problems* (see Thomson (2003) for a survey). Clearly, not all depositaries are affected in the same way: some families are dragged into delicate situations of insolvency while others are able to remain financially stable. The final status depends not only on the individual's exposition to the bankruptcy but also on the wealth outside it, income and potential to generate it, number of dependents, etc. To serve a well defined social propose, in addition to the individual claims, it is natural to think that the distribution

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of the bankruptcy resources must depend on a discriminant criteria as well. In this sense there are many theoretical models and contexts where some redistribution and discrimination mechanisms are justified. See for instance Roemer (1986), Young (1988), Moulin and Shenker (1992), Bossert (1995), Pulido et al. (2002, 2007), Luttens (2010) and Moreno-Ternero and Roemer (2012), among others. Moreover, we can find actual instances of discrimination. When the Madoff case happened, the Santander Bank decided to fully reimburse the losses of their private investors, but not those of large companies. The American bankruptcy laws also consider variables other than the claim to establish priority among claimants. The 9/11 Victims Compensation Fund had to estimate how much each victim would have earned in a full lifetime. In addition to income and age, victims were discriminated by number of dependents, consumption habits, among others.

In our context, discrimination attempts to mitigate the social damages induced by a bankruptcy situation. We propose that discrimination among the agents involved in a bankruptcy problem should be based on some variable or individual characteristic (to be sufficiently general) that can be translated into a *monetary value*. Our goal is an equity allocation that reduces the households deficit (i.e., the ones with a negative *monetary value*) after bankruptcy. We assume no third party intervention (government or investors). Consequently, any discrimination in favor of the "in deficit" households must be financed by the "in surplus" ones, through transfers.

Our starting point is a bankruptcy situation with an exogenous allocation (*statu quo*). Given this initial allocation, we add a discriminant criteria and define as *net-receivers* those agents with a negative *personal value* after bankruptcy. Otherwise, the agent is a *net-contributor*.

In the second step, we set a bound for the maximum effort that can be demanded to the *net-contributors* and the maximum benefit that can be provided to the *net-receivers*. These *transfer upper bounds* are the result of a set of axioms that, from our point of view, form the basis of any equity concerned allocation. In short, in the final allocation, no *net-receiver* can become better off than before the bankruptcy or enter into a positive position. Otherwise, we would be moving away from the social objective of the reallocation. On the other hand, we must guarantee that no *net-contributor* ends in a negative position and, in addition, that her value outside of the bankruptcy (e.g., assets) cannot be transferred. Moreover, any reallocation mechanism must satisfy *value order preservation*, i.e., the transfers must not change the existent *value hierarchy*.

In the third step, constrained by the resulting set of possible allocations, we search for the most equitable solution, called the *value discriminant* solution. We show that it not only satisfies some of the most important properties discussed in the bankruptcy literature, but also *Lorenz dominates* any other solution that satisfies the *transfer upper bounds*. Specifically, starting from the poorest *net-receiver* to the richest one, incurred losses from the initial allocation are distributed equally, conditional on *transfer upper bound*.

Finally, we introduce some practical issues. On the one hand, because *"When two rules express opposite points of views on how to solve a bankruptcy problem, it is natural to compromise between them by averaging" (Thomson and Yeh (2006)),* we discuss alternative allocations that balance the opposed interests of the involved parties. In particular, and following Thomson and Yeh (2006, 2008) and Giménez-Gómez and Peris (2011), we propose an endogenous convex combination between the (initial) exogenous (e.g., the *proportional)* and the *value discriminant* solutions. We also discuss homogeneity of flows, inter-temporal adjustment and consumption accountability.

The paper is organized as follows: section 2 proposes the model and divides agents into two groups. section 3 defines the bounds on transfers. sections 4 and 5 provide our solution and characterize it, respectively. Finally, sections 6 and 7 introduce and discuss extensions and practical issues.

## 2. The model

We consider a society with $n$ agents, $i \in \mathbb{N}$, with $\mathbb{N} = \ 1, ..., n$. Each agent has a net-value, $v_i \in \mathbb{R}$, outside the bankrupted institution.[1] This is the monetary translation of some observable characteristic(s) intrinsic to each individual. Besides this value, each agent has net-assets on the bankrupted institution, $s_i \in \mathbb{R}_+$. So each agent is identified by an **initial value**, $V_i \in \mathbb{R}$, given by $V_i = v_i + s_i$.[2]

Note that we rule out neither the existence of agents in financial difficulties before the unexpected bankruptcy, nor the possibility of cross positions

[1] E.g., the depositor's income, wealth, number of children, age, the average amount invested in the bank during the last year (fidelity), the number of years or the risk of the asset under claim, or any other relevant characteristic, as well as combinations of these variables. For discrimination between countries, the GDP can be used.

[2] In our setting this measure aggregates each agent's total position.

between and within institutions, i.e., an agent being both a debtor and creditor at the same time. Relevant agents are net creditors to the bankrupted institution.

After the bankruptcy, only a part $E \in \mathbb{R}_{++}$, of the aggregate claim, $\sum_{i\in\mathbb{N}} s_i$, is available, i.e., $E \leq \sum_{i\in\mathbb{N}} s_i$. Consequently, each **bankruptcy problem** is represented by the pair $(E, s)$, and the set of all problems is represented $\mathcal{B}$ (O'Neill (1982)). A **solution** is a single valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, for each $i \in \mathbb{N}$, $0 \leq \varphi_i(E, s) \leq s_i$ (non-negativity and claim-boundedness), and $\sum_{i\in\mathbb{N}} \varphi_i(E, s) = E$ (efficiency).

As mentioned in the introduction, our goal is to mitigate the social damage derived from a bankruptcy situation. With this aim, the claimants are divided into two groups based on their *personal values* after bankruptcy (or *post-values*). These values are composed of their net-values outside the bankrupted institution and the allocations they would receive as a result of an exogenously given solution.[3] In practical terms, the *proportional solution* is a natural candidate. However, more general allocation rules can be considered as well. Formally;

**Definition 1.** ***Post-value***, $\mathbf{V}^\varphi$. *For each* $(E, s) \in \mathcal{B}$ *and each* $i \in N, V_i^\varphi = v_i + \varphi_i(E, s)$.

In spite of the fact that a discriminant criteria has been introduce into the problem, the standard bankruptcy framework remains valid.

Given this distribution and a threshold $\kappa \in \mathbb{R}$, we might have a situation $(i)$ where all agents have a "positive" *post-value* $(V_i^\varphi \geq \kappa)$, a situation $(ii)$ where all agents have a "negative" *post-value* $(V_i^\varphi \leq \kappa)$, or $(iii)$ a combination of both. We focus on the latter scenario, which is the most common and interesting. In addition, to make the exposition more clear and intuitive, we normalize $\kappa = 0$.

For instance, suppose that $v_i$ represents net-income, which can be negative if basic consumptions are greater than income. In this case, some agents

[3]In practical terms, when an individual deposits a given amount on a financial institution agrees on a set of conditions, which implicitly defines an allocation in case of bankruptcy. This is the exogenous or *statu quo* allocation. Since ex-ante, it has received the agreement of all depositaries, it must be the departing point of any other equity concerned allocation.

with a prior balanced budget constraint ($V_i \geq 0$) may become in a negative position after the bankruptcy ($V_i^{\varphi} < 0$), while others are able to retain a positive *post-value* position ($V_i^{\varphi} \geq 0$). Clearly, without the injection of external resources (government or investors), there is no allocation that can fully solve this problems.[4] However, we can think in allocations that reduce the deficit of the agents in negative positions, and consequently provide partial relief to the individuals with a negative *post-value*.

Many authors have attempted to discriminate the final allocation among agents, either imposing "priorities" (Aggarwal (1992)) or giving more "weights" (Lee (1994)) to some claims (see Thomson (2003) for further details). In our context, discrimination is based on the individuals' value. Agents with a negative *post-value* are favored with respect to the others. In other words, we propose implementing transfers from the latter to the former. To do this, we must separate the claimants population into two groups.[5] Before proceeding, we define *final-value* as the value associated with the final allocation which has been readjusted by means of transfers and takes the initial solution as a starting point. In this way a redistribution of the resources is achieved.

**Definition 2.** *The **final-value**, $\mathbf{V}^{\varphi+\mathbf{t}}$. For each $(E, s) \in \mathcal{B}$ and each $i \in N, V_i^{\varphi+t} = v_i + \varphi_i(E, s) + t_i$, where $t_i$ denotes the transfers, such that for each claimant $t_i \in \mathbb{R}$ and $\sum_{i \in N} t_i = 0$.*

A *net-receiver* is an agent who presents a negative *post-value* position after the application of the exogenous bankruptcy solution. In the final allocation she cannot obtain less than this solution, i.e., $x_i \equiv \varphi_i(E, s) + t_i \geq \varphi_i(E, s)$ ($V_i^{\varphi} \leq V_i^{\varphi+t}$). These agents receive a non-negative transfer from the *net-contributors* ($t_i \geq 0$). In contrast, a *net-contributor* presents a *post-value* surplus. In the final allocation, she cannot obtain more than the allocation

[4] For example, Merrill Lynch and Countrywide Financial were acquired by Bank of America, the Bear Stearns was acquired by JPMorgan Chase. IndyMac Bank was converted into a bridge bank by the FDIC. The government of the United Kingdom took a shareholder position in Northern Rock and Bradford & Bingley, and it also played an important role in the cases of RBS, HBOS and Lloyds TSB. In continental Europe, Fortis was supported by the government of the Netherlands, while Dexia was rescued in a joint effort of the governments of Belgium, France and Luxembourg.

[5] The division of the claimants into two groups is not restrictive. Without loss of generality, the reader can think of an arbitrary number of partitions.

proposed by the solution, i.e., $x_i \equiv \varphi_i(E,s) + t_i \leq \varphi_i(E,s)$ ($V_i^{\varphi+t} \leq V_i^{\varphi}$). These agents transfer part of their claims to the pool of *net-receivers* ($t_i \leq 0$). Formally,

**Definition 3.** *For each* $(E,s) \in \mathcal{B}$*, a* ***net-receiver*** *is an agent* $i \in \mathbb{N}$ *with* $V_i^{\varphi} \leq 0$*. A* ***net-contributor*** *is an agent* $i \in \mathbb{N}$ *with* $V_i^{\varphi} > 0$.

Our argument shares similarities with the usual progressive taxation, which is a redistributive principle based on income rather than total value.

Hereafter and for notational convenience, we list agents in increasing order in terms of their *post-values*, i.e., $V_1^{\varphi} \leq V_2^{\varphi} \leq \cdots \leq V_n^{\varphi}$. Consequently, there exists an agent $i = m \in \mathbb{N}$ such that $V_m^{\varphi} \leq 0$ and $V_{m+1}^{\varphi} > 0$, i.e., a total of $m$ *net-receivers* and $n-m$ *net-contributors*.

In this paper, we are looking for a more equitable way of assigning *final-values*, so we will use the well-known *Lorenz (equity) criterion* (Lorenz (1905)), which is considered to be a general equity principle. Many authors agree that it captures the idea that the desirable social goal is to treat everyone as equally as possible (see for instance Dutta and Ray (1989) and Arin (2007)).

Let $A = \left\{x \in \mathbb{R}^N : x \geq 0\right\}$ and for each vector $x \in A$, we denote by $\sigma(x)$ the vector that results from $x$ by permuting the coordinates in such a way that $\sigma_1(x) \leq \sigma_2(x) \leq ... \leq \sigma_n(x)$. Let $x, y \in \mathbb{R}^N$. We say that $x$ Lorenz dominates $y$, $x \succ_L y$, if for each $k = 1, 2, \ldots, n-1$: $x_1 + x_2 + \cdots + x_k \geq y_1 + y_2 + \ldots + y_k$ and $x_1 + x_2 + \ldots + x_n = y_1 + y_2 + \ldots + y_n$. If $x$ Lorenz dominates $y$ and $x \neq y$, then at least one of these $n-1$ inequalities is a strict inequality.

This equity criterion has been extended to other rules (see for instance Bosmans and Lauwers (2011)). In our context, note that given two solutions $y$ and $z$, $y$ **Lorenz dominates** $z$ if for each $(E,s) \in \mathcal{B}$, $y(E,s) \succeq_L z(E,s)$.

Finally, a minimal requirement of fairness is that the social transfers cannot alter the existent value hierarchy. This requirement is crucial in any bankruptcy context. Otherwise, as a result of the reallocation of existent resources, some agents could end up poorer or richer than those who precede or follow them in the value hierarchy, respectively. Formally,

**Axiom.** ***Value order preservation,*** **ValOrd.** For each $i \in \mathbb{N}$, if $V_{i-1}^{p} \leq V_i^{p}$, then $V_{i-1}^{\varphi+t} \leq V_i^{\varphi+t}$.

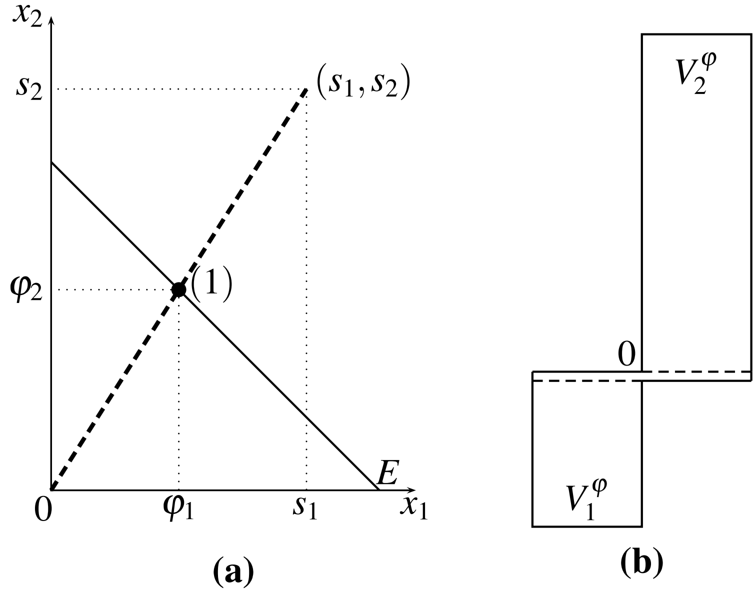


Figure 1: **Initial allocation.** **(a)** The solid line represents all the possible distributions between the agents of the estate, $E$. The broken line shows the paths of awards corresponding to the *statu quo*. Point (1) is the division of $E$ provided by this solution. **(b)** The agents post-value at point (1): agent 1 is a *net-receiver* and agent 2 is a *net-contributor*.

## 3. Bounds on transfers

Until now, we have distinguished *net-receivers* from *net-contributors.* The next step is to rearrange the initial allocation by means of transfers. These transfers cannot be arbitrary and must satisfy a set of minimum requirements. In fact, we consider these as the basis to any "equity" concerned final allocation. "Equity" in our setting is an unconstrained move toward a more equal distribution of *final-values.* Clearly, there are natural and social restriction on how far we can go.

Figure 1 illustrates a situation in which the initial exogenous allocation (**a**) recommends a distribution of resources which is proportional to the claims (*proportional solution*). The agents' *post-value* at point (1), is such that agent 1 is a *net-receiver* and agent 2 is a *net-contributor* (**b**).

We propose "transfers" from agent 2 to agent 1, (**c**) such that the latter preserves at least a non-negative value after the final allocation (**d**). That is, we go from position (1) to (2) (see Figure 2). Unfortunately, as we will see, this objective is not always feasible.

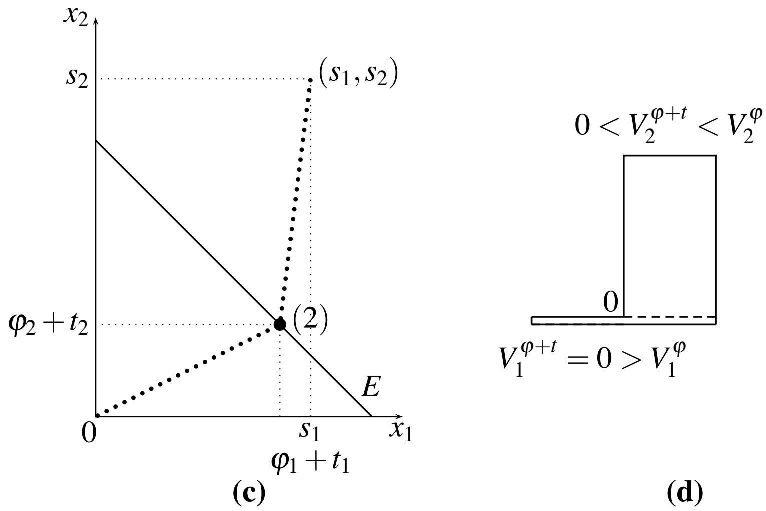


Figure 2: **Transfers. (c)** The point **(2)** represents the division of $E$ reached by transfers $t_1$ and $t_2$. **(d)** The agents' *final-value* at point (2) minimizes social damages — no agent "in deficit".

The following axioms bound the set of acceptable transfers. Our goal is twofold: to define the primitives of an allocation that aims to reduce the deficit of the *net-receivers* (see section 5) and, at the same time, to establish a set of indisputable bounds that apply to every problem of this type. As a result, the set of final allocations is restricted.

Note that the final allocation $\varphi + t$ cannot make a *net-receiver* better off than before the bankruptcy. Formally,

**Axiom.** ***No gains from bankruptcy,*** **NoGain**. For each $i \leq m$, $V_i^{\varphi+t} \leq V_i$.

The previous axiom states that no agent can become better off than before the bankruptcy, but it does not exclude the possibility that the *net-receivers' final-value* could become positive. If this possibility is not restricted, then some *net-contributors* could end up in worse positions than some *net-receivers*. In addition, this would go against the social purpose of the reallocation. Formally,

**Axiom.** ***No surplus net-receiver,*** **NoSurp**. For each $i \leq m$, if $V_i^{\varphi} \leq 0$, then $V_i^{\varphi+t} \leq 0$.

The next requirement implies that a *net-contributor* cannot end up in a worse value position than any *net-receiver* who benefits from her transfers. Thus, an agent with a non-negative *post-value* must end up with a non-negative *final-value.* Formally,

**Axiom.** ***No deficient net-contributor,*** **NoDefi.** For each $i > m$, if $0 \leq V_i^{\varphi}$, then $0 \leq V_i^{\varphi+t}$.

Furthermore, we need to guarantee that the *net-contributors* do not suffer value losses to assets outside of the bankruptcy. In other words, nobody will face worse consequences than loosing all her bankruptcy assets because only the bankruptcy-available resources can be subject to transfers. Formally,

**Axiom. *No loss outside the bankruptcy,* NoLoss.** For each $i > m$, $V_i^{\varphi+t} \geq v_i$.

Note that we need to impose $VOP$ because, in general, it is not guaranteed by the axioms *NoGain*, *NoSurp*, $NoDefi$ and $NoLoss$. The following result and its subsequent proof explain why this is the case.

**Proposition 1.** *Axioms NoGain, NoSurp,* $NoDefi$ *and* $NoLoss$*, do not imply* $ValOrd$*.*

*Proof.*- The proof of this theorem is straightforwardly obtained by efficiency: that is, by the total distribution of the available resources. Consider the following three-agent problem, $v = (-16, 4, -64)$, $s = (25, 1, 74)$, then $V = (9, 5, 10)$. Now, $E = 20 \leq \sum_{i=1,2,3} s_i$. Without loss of generality, let the exogenous allocation $\varphi$ be the *proportional* solution; then $p = (5, 0.2, 14.8)$ and $V^p = (-11, 4.2, 10.8)$. The upper bounds on transfers defined by *NoGain*, $NoDefi$, *NoSurp* and $NoLoss$ are given by $r = (-11, 0.2, 10.8)$. Thus, agent 1 (the *net-receiver*) can receive as much 11, and agent 2 and 3 (the *net-contributors*) pay as much as 0.2 and 10.8, respectively. Note that we are not imposing $VOP$. Then, by *efficiency*, the transfers are $t = (+11, -0.2, -10.8)$, thus $V^t = V^p + t = (0, 4, 0)$, contradicting $ValOrd$. An example with the same implications can be constructed for any exogenous allocation and vector of transfers pair.

**q.e.d.**

Therefore, besides *NoGain*, $NoDefi$, *NoSurp* and $NoLoss$, we impose $ValOrd$ to avoid changes in the existing value hierarchy. Obviously, these axioms restrict the set of feasible allocations. Next, we will derive these bounds.

Regarding the *net-contributors* side, i.e., for each $i > m$, Definition 3 and $NoLoss$ imply that $0 \leq \varphi_i(E, s) + t_i \leq \varphi_i(E, s)$. Moreover, $NoDefi$ imposes $-v_i \leq \varphi_i(E, s) + t_i$.[6] In addition, $ValOrd$ requires that $\varphi_{i-1}(E, s) + t_{i-1} +$

[6]Recall that $v_i$ can take negative values, and at the same time, agent $i$ can be a net-contributor. In this case, individual $i$ must hold a sufficiently strong position in the bankrupted institution.

$(v_{i-1} - v_i) \leq \varphi_i(E,s) + t_i$. The aggregation of these conditions leads to

$$0 \leq -t_i \leq \min\left\{\varphi_i(E,s), V_i^{\varphi}, -t_{i-1} + \left(V_i^{\varphi} - V_{i-1}^{\varphi}\right)\right\}.$$

Because the transfer of agent $i$ depends on the transfer made by the previous agent in the value hierarchy, to compute the *net-contributors* transfers, we must begin from the agent $m+1$ and move up until $n$. Note that $ValOrd$ between the agents $m$ and $m+1$ is guaranteed by $NoDefi$ and $NoSurp$. Consequently, for $i = m+1$ the transfer received by agent $i = m$ is irrelevant, and thus we have $0 \leq -t_{m+1} \leq \min\left\{\varphi_{m+1}(E,s), V_{m+1}^{\varphi}\right\}$.

With respect to the *net-receivers* side, i.e., for each $i \leq m$, Definition 3 and $NoGain$ imply that $\varphi_i(E,s) \leq \varphi_i(E,s) + t_i \leq s_i$. Moreover, $NoSurp$ imposes $\varphi_i(E,s)+t_i \leq -v_i$. In addition, $ValOrd$ requires that $\varphi_i(E,s)+t_i \leq \varphi_{i+1}(E,s) + t_{i+1} + (v_{i+1} - v_i)$. The aggregation of these conditions can be written as

$$0 \leq t_i \leq \min\left\{V_i - V_i^{\varphi}, -V_i^{\varphi}, t_{i+1} + \left(V_{i+1}^{\varphi} - V_i^{\varphi}\right)\right\}.$$

Because the transfer of agent $i$ depends on the transfer made by the next agent in the value hierarchy, for the *net-receivers* we must start from the agent $i = m$ and move down until $i = 1$. Similarly, for $i = m$ the transfer of the agent $i = m+1$ is irrelevant, and we have $0 \leq t_i \leq \min\left\{V_i - V_i^{\varphi}, -V_i^{\varphi}\right\}$.

Formally, the *transfers upper bound* for a given *net-contributor* and a given *net-receiver*, denoted by $b_i$ and $b_i$, respectively, will be the maximum possible transfer.

**Definition 4.** ***Transfers upper bounds.*** *For each $(E,s) \in \mathcal{B}$ and each $i \in N$, $|t_i| \leq b_i$, where the bounds $b_i$ are defined by:*
*$\forall i \leq m$,*

$b_m = \min\left\{\varphi_m(E,s), -V_m^{\varphi}\right\}$,

$b_i = \min\left\{s_i - \varphi_i(E,s), -V_i^{\varphi}, b_{i+1} - \left(V_i^{\varphi} - V_{i+1}^{\varphi}\right)\right\}$;

*$\forall i \geq m+1$,*

$b_{m+1} = \min\left\{\varphi_{m+1}(E,s), V_{m+1}^{\varphi}\right\}$,

$b_i = \min\left\{\varphi_i(E,s), V_i^{\varphi}, b_{i-1} + \left(V_i^{\varphi} - V_{i-1}^{\varphi}\right)\right\}$.

**Example 1.** *Let* $v = (-8, -5, -2, y)$ *and* $s = (8, 4, 10, 8)$ *where* $y \in (0, \infty)$, [7] *then* $V = (0, -1, 8, y+8)$. *Suppose that* $E = 15$, *i.e., half of the total claims, and that the exogenous allocation is the proportional, i.e.,* $\varphi = p$, *then* $V^p = (-4, -3, 3, y+4)$. *Starting from the poorest net-contributor, the bound of* $i = 3$ *is restricted by* $NoDefi$ *and equals to* $b_3^+ = 3$. *The bound of* $i = 4$ *is restricted by* $NoLoss$ *and equals to* $b_4^+ = 4$. *To compute the net-receivers bounds we start from* $i = 2$, *which is restricted by* $NoGain$ *and equals to* $b_2^- = 2$. *The bound of* $i = 1$ *is restricted by* $ValOrd$ *and equals to* $b_1^- = 3$. *Then,* $b = (3, 2, 3, 4)$.

## 4. The value discriminant solution

Because a *net-contributor* cannot transfer more than $b_i$ and a *net-receiver* cannot receive more than $b_i$, Definition 4 restricts the set of final allocations. However, it does not give us a unique solution. There are several solutions in this new set (the exogenous solution for instance). Let $B^- \equiv \sum_{i=1}^{m} b_i$ and $B^+ \equiv \sum_{i=m+1}^{n} b_i$ denote the total amount of resources needed by the *net-receivers* and the total amount of available resources from *net-contributors*, respectively. Therefore, associated to each bankruptcy problem there is a **sub-problem** represented by the pair $(B^*, V^\varphi)$, where now $B^* = \min\{B^-, B^+\}$ and, by efficiency, $\sum_{i \in \mathbb{N}} \varphi_i(B^*, V^\varphi) = B^*$. We consider that if there were enough resources to satisfy the *net-receivers'* needs, they would be fulfilled. Consequently, our proposal has a strong concern for "equity".

The *value discriminant solution*, where $d$ denotes the transfers component, works as the *cel* rule but takes into account the above value ordering and imposes that no one contributes ($d \leq 0$) or receives ($d \geq 0$) more that her bound. The *constrained equal losses solution* (discussed by Maimonides (Aumann and Maschler (1985))) chooses the awards vector at which all agents incur the same losses, subject to no one receiving a negative amount. Formally,

**The constrained equal losses solution, cel.** For each $(E, s) \in \mathcal{B}$ and each $i \in N$, $cel_i(E, s) \equiv \max\{0, s_i - \mu\}$, where $\mu$ is chosen so that $\sum_{i \in \mathbb{N}} \max\{0, s_i - \mu\} = E$.

[7] We do not fix individual $i = 4$ value other than $y \in (0, \infty)$. Such does not place any restriction and will be useful later in Example 2, which is a continuation of this one.

Specifically, our solution has two cases:

1st. From the *net-contributors'* (*net-receivers'*) side, the richest (poorest) agents pay (receive) their full bound if this is lower than the value distance with the next richest (poorest) agent, and while the available resources have not been exhausted, i.e., $a_i = b_i$, if $V_i^\varphi - V_{i-1}^\varphi \geq b_i$ and $\sum_{j\geq i} b_j \leq B^*$ ($a_i = b_i$, if $V_{i+1}^\varphi - V_i^\varphi \geq b_i$ and $\sum_{j\leq i} b_j \leq B^*$).

2nd. From the *net-contributors'* (*net-receivers'*) side, the poorest (richest) agents pay (receive) when the estate is high enough relative to distances and bounds. In other words, we look for the poorest (richest) agent such that her bound is lower than the value distance, say $j$. If the aggregate bound of the richer (poorer) agents ($i \geq j$) is enough to satisfy (capture) all the resources, then the poorest (richest) agents do not pay (receive) anything, i.e., $\overline{s_i} = 0$, if $V_i^\varphi - V_{i-1}^\varphi \geq b_i$ and $\sum_{i\geq j} b_i \geq B^*$ ($\overline{s_i} = 0$, if $V_{i+1}^\varphi - V_i^\varphi \geq b_i$ and $\sum_{i\leq j} b_i \geq B^*$).

**Definition 5.** ***The discriminant transfers,*** **d**. *For each* $(E, s) \in \mathcal{B}$ *and each* $i \in N$*,*

$$d_i(B^*, V^\varphi) = \begin{cases} a_i + cel_i\left(B^* - \sum_{i\leq m} a_i, \overline{s}\right), & \forall i \leq m \\ \\ -a_i - cel_i\left(B^* - \sum_{i>m} a_i, \overline{s}\right), & \forall i \geq m+1 \end{cases}$$

$a_i = b_i$, *for each* $i = \{1, \ldots, k\}$ *when* $V_k^\varphi = \min\left\{V_i^\varphi : V_{i+1}^\varphi - V_i^\varphi \geq b_i\right\}$ *and* $\sum_{i\leq k} b_i \leq B^*$.

*Otherwise,* $a_i = 0$.

$\overline{s_i} = 0$, *for each* $i = \{j, \ldots, m\}$ *when* $V_j^\varphi = \min\left\{V_i^\varphi : V_{i+1}^\varphi - V_i^\varphi \geq b_i\right\}$ *and* $\sum_{i\leq j} b_i \geq B^*$.

$\overline{s_-} = b_i$, *otherwise.*

$a_i = b_i$, *for each* $i = \{k, \ldots, n\}$ *when* $V_k^\varphi = \min\left\{V_i^\varphi : V_i^\varphi - V_{i-1}^\varphi \geq b_i\right\}$ *and* $\sum_{i\geq k} b_i \leq B^*$.

*Otherwise,* $a_i = 0$.

$\overline{s_i} = 0$, *for each* $i = \{m+1, \ldots, j\}$ *when* $V_j^\varphi = \min\left\{V_i^\varphi : V_i^\varphi - V_{i-1}^\varphi \geq b_i\right\}$ *and* $\sum_{i\geq j} b_i \geq B^*$.

$\overline{s_i} = b_i$, *otherwise.*

Graphically (Figure 3) the **discriminant transfers** solves the problem $(B^*, V^\varphi)$ as the *cel* solution, but taking into the account each agent's transfer upper bound. Particularly, this figure represents the problem where there are two *net-contributors* (agents $i = 3, 4$). In the case **(a)** the bound of the richest contributor is higher than the distance between agents' *post-values*, so our solution works as the *cel* solution (2nd case discussed above). In the case **(b)** the bound of the richest contributor is lower that the distance between agents' *post-values*. Then, this agent will pay her total bound, and the rest is paid by the other agent (1st case discussed above).

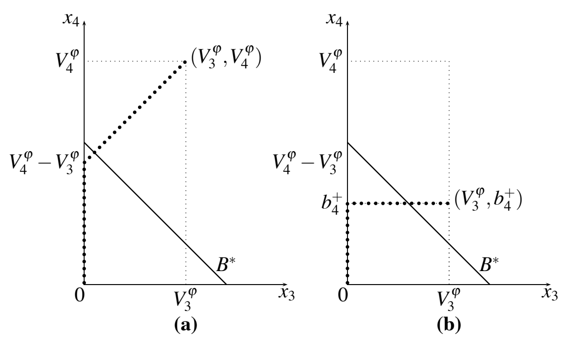


Figure 3: **The discriminant transfers from the *net-contributors'* side.** The figure represents the problem when there are two *net-contributors* (agents $i = 3, 4$).

**Definition 6.** *For each $(E, s) \in \mathcal{B}$, the* ***discriminant solution*** *proposes the allocation $\varphi(E, s) + d(B^*, V^\varphi)$.*

Note that, on the one hand, for each $(E, s)$ and due to the exogenous (*statu quo*), the *discriminant transfers* allocate $B^*$ in terms of $V^\varphi$ and its associated *transfers upper bounds*. On the other hand, the *net-contributors* (*net-receivers*) with a higher (lower) *post-value* tend to pay (receive) more than those ones with a lower (higher) value.

The *final-value* associated with the *discriminant solution*, is the sum of the value associated to the applied solution with the resulting transfers $d$ (positive or negative, depending on the agent's type).

**Definition 7.** ***The final discriminant value,*** $\mathbf{V}^{\varphi+\mathbf{d}}$**.** *For each $(E, s) \in \mathcal{B}$ and each $i \in N$,*

$$V_i^{\varphi+d} = v_i + \varphi_i(E, s) + d_i(B^*, V^\varphi).$$

This solution reproduces the target allocation that we previously introduced. It departs from the exogenous solution, without penalizing (benefiting) the *net-contributors* (*net-receivers*) population more than their bound (until the exhaustion). In our view, this is a crucial point because, in a bankruptcy, everybody losses something. Consequently, we limit the effort (behind the claim) demanded on the *net-contributors,* while maintaining the objective of equity.

Bankruptcy problems induce social issues that destabilize the functioning of society as a whole (in particular among the agents in deficit). On the other hand, departures from the initial solution cause discontentment among *net-contributors*. The proposed allocation searches for a balance between these opposed interests.

The most intuitive way to understand how transfers $d_i(B^*, V^\varphi)$ are computed is through a hydraulic representation (Figure 4).[8] Agents are listed in increasing order of their *post-values* ($V_i^\varphi$). This figure shows a four agent problem where two agents are *net-contributors* and the other two *net-receivers*. Each agent's *post-value* is displayed by a vessel with the poorest agent on the left and the richest one on the right. Note that the *net-receivers* go from the center (the normalized zero point) to the left, and the *net-contributors* go in the opposite direction. The horizontal solid line represents each agent's value and the dotted line is each agent's value plus or minus the respective bound. The *net-contributors'* vessels are initially full (grey color), so the goal is to draw out as much water (resources) as possible. In case **(a)** first the tallest glass is poured out until satisfy the difference with the second one. Then, both vessels are emptied (white color) at the same time to the bound. And so on, until either all the the needs are met or all the *net-contributors*' have participated. In case **(c)** first the tallest glass is poured out until the bound. Then, the second one is emptied until the bound. From the *net-receivers'* ($i \leq m$) side, the vessels are initially empty, so the goal is to fill them as much as possible. Now, the first left vessel is filled to the distance with the second one. Then, the process continues until either all the available resources have been exhausted or all needs satisfied. Example 2 gives a numerical case.

**Example 2.** *In continuation of example 1, we have* $B^- = 5$ *and* $B^+ = 7$*, then* $B^* = 5$*. If* $y = 1$*, we have a vector of transfers* $d(B^*, V^\varphi) = (3, 2, -1.5, -3.5)$*, a vector of final allocations* $p(E, s) + d(B^*, V^\varphi) =$

[8] See Kaminski (2000) for a deep learning about hydraulic rationing.

$(7, 4, 3.5, 0.5)$, *and the vector of final-values* $V^{p+d} = (-1, -1, 1.5, 1.5)$. *As shown in Figure 3, the agent* $i = 4$ *transfers bound can be larger than the distance between agents' post-values (cases* ***(a)*** *and* ***(c)****), or smaller (****(b)*** *and* ***(d)****). In particular, for the latter case, if* $y = 3.5$, *we have a vector of transfers* $d(B^*, V^\varphi) = (3, 2, -1, -4)$, *a vector of final allocations* $p(E, s) + d(B^*, V^\varphi) = (7, 4, 4, 0)$, *and the vector of final-values* $V^{p+d} = (-1, -1, 2, 3.5)$.

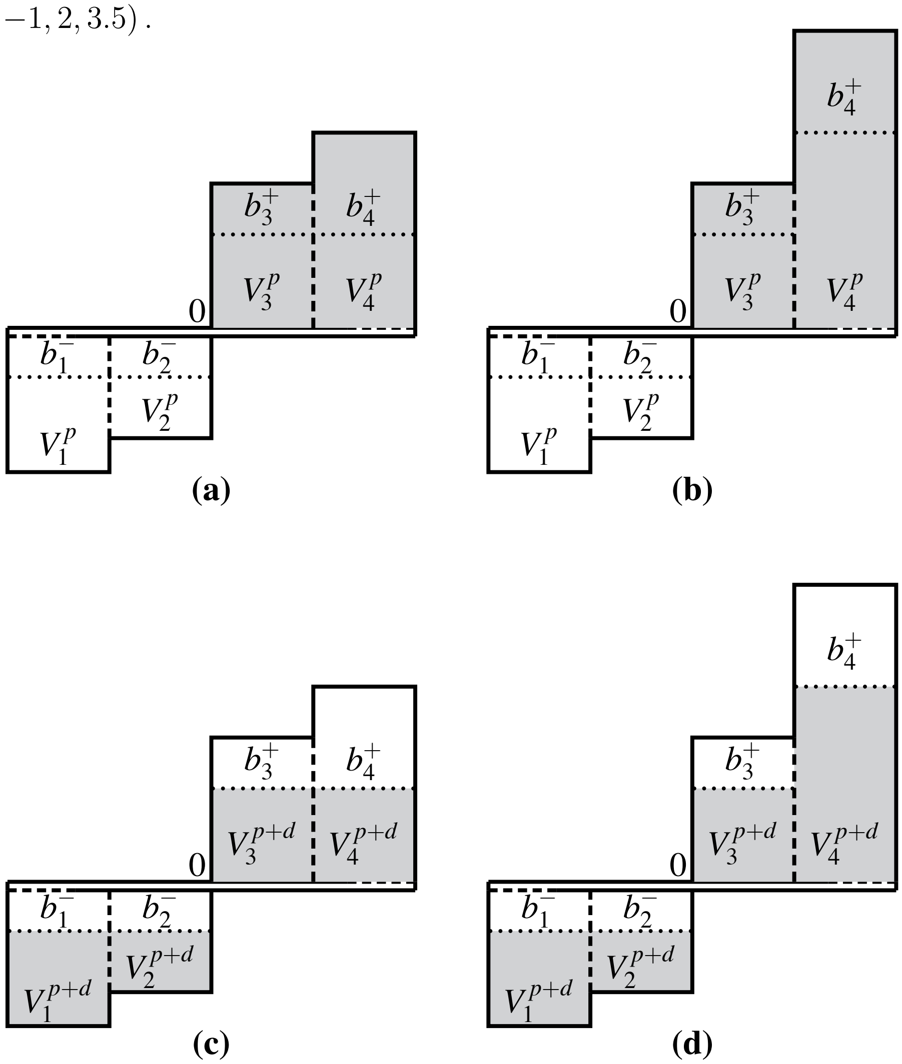


Figure 4: **The hydraulic representation of the discriminant solution.**

## 5. Axiomatic characterization

| | $d$ |
|---|---|
| Anonymity | Yes |
| Equal treatment of equals | Yes |
| Continuity | Yes |
| Homogeneity | Yes |
| Order preservation (Aumann and Maschler (1985)) | No |
| value order preservation | Yes |
| resources monotonicity (Curiel et al. (1987), Young (1987)) | Yes |
| Claims monotonicity | Yes |
| Value Progressivity in bounds | Yes |
| Lorenz criterion in $\mathcal{R}$ | Yes |

Table 1: **General axioms.** Formal definitions and interpretations can be found in Thomson (2003).

In order to characterize our solution we will use the well-known *Lorenz (equity) criterion*, which is considered a general equity principle.[9] In bankruptcy problems, it is well known that the *Constrained Equal Awards* solution, $CEA$, (Maimonides 12th century, see Thomson (2003)) Lorenz dominates any other solution. Such a result does not generalize to our setting. The addition of a monetary discriminant variable to the problem changes this result in a crucial away. The proposed *discriminant solution,* Lorenz dominates any other solution in terms of the agents' *final-value,* which is the relevant measure of "equity" in our setting.

For instance, consider $V = (0, 3, 5)$, $s = (2, 3, 4)$ and let $E = 3$. Then, $CEA(E, s) = (1, 1, 1)$ and $V^{CEA} = (-1, 1, 2)$. The *discriminant* solution recommends, $d = (1, 0, -1)$ and $V^{\varphi+d} = (0, 1, 1)$. Then, $V_1^d \geq V_1^{CEA}$, $V_1^d + V_2^d \geq V_1^{CEA} + V_2^{CEA}$, so $V^{\varphi+d} \succeq_L V^{CEA}$.

**Theorem 1.** *Let $\varphi$ be a solution and consider the family of transfers $t_i$ satisfying the transfers upper bounds in Definition 4. Then, the discriminant solution Lorenz dominates any other redistribution solution.*

[9] See Hougaard and Osterdal (2005), KasaGima and Velez (2010) and Thomson (forthcoming).

*Proof.*- It is sufficient to show by contradiction that $d$ Lorenz dominates every solution that satisfies the *transfers upper bounds* in Definition 4. Let $\varphi \in \mathcal{R}$ and $(E, s) \in \mathcal{B}$ be such that $y = V^{\varphi+d}$ does not Lorenz dominate $z = V^{\varphi}$ in *value* terms. Note $V^{\varphi+d} = v + \varphi(E, s) + d$ and $V^{\varphi} = v + \varphi(E, s) + t$, where $d$ and $t$ denote the transfers recommended by the *discriminant solution* and any other one, respectively. We focus on the *net-receivers* side. Let $k \in N$ be the smallest number such that $\sigma_1(y) + \sigma_2(y) + ...\sigma_k(y) < \sigma_1(z) + \sigma_2(z) + ...\sigma_k(z)$.

Hence, $y_k < z_k \leq b_k$. **(a)** If $B^- \leq B^+$, then $B^* = B^-$ and each agent should receive her $b_i$, by efficiency. By definition, $y_k = v_k + \varphi(E, s) + b_k < v_k + \varphi(E, s) + t_k = z_k$. Then, $t > b$ contradicting $\mathcal{R}$. **(b)** If $B^- \geq B^+$, then $B^* = B^+$. Because $y_k < z_k$, by efficiency for some agent $k^* > k$, $y_{k^*} > z_{k^*} > 0$. In this case, by the definition of the *discriminant* solution, **(b.1)** $d_{k^*} = b_{k^*}$, or **(b.2)** $V^{\varphi}_{k^*-1} - V^{\varphi}_{k^*} < d_{k^*} \leq b_{k^*}$. On the one hand, (b.1) implies that for each $j < k^*, d_j = b_j$. If $y_k < z_k$, then $z$ contradicts $\mathcal{R}$, as in case (a). On the other hand, (b.2) implies that for each $j < k^*, V^{\varphi}_{j-1} - V^{\varphi}_{j} < y_j \leq b_{k^*}$. If $d_j < t_j$, then $V^z_j > V^z_{j+1}$, because the definition of the *discriminant* solution states that $V^y_j = V^y_{j+1}$. So the allocation $z$ contradicts $ValOrd$. Following a similar reasoning, it would be straightforward to explain the same for the *net-contributors*.

**q.e.d.**

This result shows that our proposal is the most equitable redistribution solution in the *Lorenz* sense that fulfills a reasonable and intuitive set of axioms ($ValOrd$, $NoGain$, $NoDefi$, $NoSurp$ and $NoLoss$). Moreover, this is true for any choosing of proposal $\varphi$.

Finally, Table 5 shows some relevant properties and their relationship with our solution. The proofs can be made available by the authors upon request. We would like to distinguish the fact that $d$ satisfies progressiveness in value with respect to the bounds. Thus, richer (the poorer) *net-contributors* (*net-receivers*) tend to pay (receive) relatively more than poorer (richer) ones, with respect to their bound. Formally,

**Definition 8.** ***Value progressiveness in bounds,* VPB.** *For each* $i \in \mathbb{N}$, *if* $|V_i| \leq |V_j|$, *then* $\frac{|t_i|}{b_i} \leq \frac{|t_j|}{b_j}$.

## 6. Extensions: lobbies and convex combinations

In this section, we assume that the final allocation is the result of a grim of arguments between the interested groups. The population of *net-contributors* will attempt to block any departure from the *initial exogenous solution*, i.e., a solution having no *value discriminant* transfers. On the other hand, the *net-receivers* population will argue in favor of discriminant transfers, i.e., the *value discriminant* solution. Taking these opposing interest into account and following Thomson and Yeh (2006, 2008) and Giménez-Gómez and Peris (2011), we propose a convex combination between the allocations $\varphi$ and $\varphi+d$ meaning that we are including all the possible distributions between points (1) and (2) in Figures 1 and 2. Formally,

$$^{\alpha}\varphi_i = \alpha \left(\varphi_i(E,s) + d_i(B^*, V^{\varphi})\right) + (1-\alpha)\varphi_i(E,s). \tag{1}$$

When $\alpha = 0$ (respectively, $\alpha = 1$) the initial (respectively, *value discriminant*) solution is implemented. Otherwise, we move away from this initial solution.

There are two main arguments in favor of a large value of $\alpha$ and consequently to an allocation which is more concerned with "equity".

(*i*) $B^+ >> B^-$ : In aggregate terms, the *net-contributors* population is in good financial conditions with respect to the *net-receivers* needs. Consequently, the relative effort required from the former is low.

(*ii*) $B^- >> B^+$ : In aggregate terms, the *net-receivers* population has great needs. Consequently, there is a serious social problem and solution concerned with equity becomes easier to defend, even if a large effort is required from the *net-contributors*.

Points (*i*) and (*ii*) strengthen the *net-receivers* claims for more "equity" (the reverse supports the opposite). Let ratios $B^+/(B^+ + B^-)$ and $B^-/(B^+ + B^-)$ in connection with $\max\{B^+, B^-\}$ capture these statements. Consequently, we propose,

$$\alpha = \min\left\{\beta \frac{\max\{B^+, B^-\}}{B^+ + B^-}, 1\right\}. \tag{2}$$

The higher the value of $\beta \in (0, \infty)$, the more policy makers are concerned with issues equity. This bias is important for the determination of the convex allocation. A value of $\beta = 1$ denotes an independent arbitrator. For $\beta \downarrow 0$ (respectively, $\beta \uparrow \infty$) we have $\alpha \downarrow 0$ (respectively, $\alpha \uparrow 1$) and the *exogenous solution* (respectively, *value discriminant solution*) is implemented. The ratio inside $\min\{.\}$ takes values in the interval $[1/2, 1]$.[10] Moreover,

[10] In fact, for $\beta \geq (B^+ + B^-)/\max\{B^+, B^-\}$ we have $\alpha = 1$.

note that $\alpha$ weakly increases with the difference between $B^+$ and $B^-$. When $\max\{B^+, B^-\} = B^+$ the argument $(i)$ is the one to consider, otherwise the argument $(ii)$ is the relevant one. In addition, and in line with the previous discussion, when either $B^+$ or $B^-$ goes to $\infty$, the ratio in (2) converges to one. The lowest value of this ratio equals to $1/2$ and obtains when $B^+ = B^-$.

## 7. Final remarks

Discrimination among claimants, tax payers, or social beneficiaries is a must. The difficulty lies in the implementation. We conclude with comments on the EU funds transfer policy, which is already in place, and on some practical issues associated with the application of our model in the cases where individuals' total wealth is the discriminant variable.

### *7.1. The EU structural and cohesion funds*

An actual case where policy makers apply discrimination for distributing resources occurs in the EU. A total of 81.9 per cent of the EU's structural funds resources are allocated to its poorest countries. Countries are divided into two groups, depending on their GDP. Specifically, those EU members with a GDP below 75 per cent of the average EU GDP are considered poor. Note that this allocation of resources pursues the aim of increasing cohesion among members states, so that the GDP of EU convergence's: a goal captured by the solution proposed in this paper.

### *7.2. Wealth as a discriminant variable: comments*

We have suggested a wide range of potential discriminatory variable. For practical proposes the net-wealth outside the bankrupted institution is a natural candidate. In this case $v_i = y_i - c_i + a_i$, where $y_i \in \mathbb{R}_+$ represents income derived from labor activities, employment insurance, asset returns, or any other kind of monetary inflow, $c_i \in \mathbb{R}_+$ represents consumption needs or any other kind of monetary outflow, and $a_i \in \mathbb{R}_+$ represent capital stocks, real estate, or any other kind of wealth stock outside the bankruptcy. $y_i$ and $c_i$ are monetary flows and $a_i$ is a stock that can be seen as the aggregate of multiple past flows.[11]

[11] To discriminate among victims The 9/11 Victims Compensation Fund has implemented a rather complex formula, which included variables like income, age, number of dependents, consumption habits among others.

Next, we take a close look at practical issues.

*Homogeneity of Flows and Adjustment Times* - Agents typically have short- or medium-term difficulties in increase their flows of income. This is particularly relevant for unexpected bankruptcies, after which some individuals are forced to reduce their consumption and/or increase their income. Consequently, we must allow a sufficient period of time for these changes to be feasible. Similarly, to establish some homogeneity between flows and stocks, we should increase the weight of monetary flows.

*Income and Wealth Manipulation* - Agent income and wealth stocks can be subject to counterfeit and/or misreporting, for example, due to an underground economy or the existence of savings accounts in Switzerland or in tax havens lacking of transparency. Clearly, the likelihood of some of these events is specific to the sophistication of recordkeeping in each country. In spite of these unavoidable issues, for developed countries, we can assume the existence of sufficiently accurate information.[12]

*Consumption Related Problems* - However, we cannot make such assumptions with respect to consumption needs. The most the agent can hold is a common knowledge distribution over consumption. Because the final allocation is biased in favor of the agents with weaker value positions, we would naturally expect agents to over report their consumption needs.[13] Moreover, not all expenditures present an equal relevance in the budget. For example, the monthly mortgage repayment by an agent with a tide budget does not have the same relative importance as the acquisition of an additional million dollar automobile by an extremely wealthy agent. The problem is that we cannot make the rich agent accountable for the poor agent mortgage nor censure her apparently excessive consumption habits. It would be ideal to restrict *net-receivers* without constraining the *net-contributors'* consumption. Creating a balanced proposal that respect these trade-offs is not an easy task. Finally, there is a large degree of heterogeneity between consumption and wealth. We can observe agents living above their means (low wealth and high consumption) and others with low consumption with respect to their wealth. We therefore need to make sure that agents with relatively low (respectively, high) consumption are not penalized (respectively, benefited) by

[12]This information can be requested directly at the institutions that hold these data or to each agent to present proof of it.

[13]This comment opens the question of truth-telling mechanisms in the context of bankruptcy problems.

that fact.

Following this brief discussion and because personal consumption is private information, we must define an acceptable/standard level of consumption $\bar{c}$ that applies to all claimants (average or median consumption can be considered as well).[14] This consumption level can be adjusted to consider expenses such as rent and mortgage payments, costs related with long-term health treatments, positive discrimination per dependent, etc.

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