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An axiomatic characterization of the strong constrained egalitarian solution*

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Abstract

In this paper we axiomatize the strong constrained egalitarian solution (Dutta and Ray, 1991) over the class of weak superadditive games using *constrained egalitarianism, order-consistency*, and *converse order-consistency.*

JEL classification: C71, C78

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1 Introduction

On the space of transferable utility cooperative games, several solution concepts have been motivated by the idea of egalitarianism. One of the best known is the **weak constrained egalitarian solution** (WCES for short), introduced by Dutta and Ray (1989). This solution is defined under the assumption that agents believe in egalitarianism as a social value, but their individual preferences dictate selfish behavior. On the domain of convex games, the WCES has many desirable properties and it has been axiomatized by different authors (see Dutta, 1990; Klijn et al., 2000; Hougaard et al., 2001; Arin et al., 2003). Nevertheless, existence is only guaranteed for the class of convex games. In order to widen the potential class of applications, Dutta and Ray (1991) introduced the **strong constrained egalitarian**

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solution (SCES for short), a parallel concept that exists for a large class of games. However, there is no a proper characterization result for the SCES. Thus, it seems a worthwhile exercise to provide an axiomatic characterization for this solution concept. With this objective in mind, we begin introducing some notation and terminology.

The set of natural numbers $\mathbb{N}$ denotes the universe of potential players. By $N \subset \mathbb{N}$ we denote a finite set of players, in general $N = \{1, \ldots, n\}$. A **transferable utility coalitional game (a game)** is a pair (N, v) where $v : 2^N \longrightarrow \mathbb{R}$ is the characteristic function with $v(\emptyset) = 0$ and 2^N denotes the set of all subsets (coalitions) of N. Here we only consider games with $|N| \geq 2$. Let Γ denote the set of all games. We use $S \subset T$ to indicate strict inclusion, that is $S \subseteq T$ but $S \neq T$. By $|S|$ we denote the cardinality of the coalition $S \subseteq N$.

The set of **feasible payoff vectors** of a game (N, v) is defined by $X^*(N, v) := \{x \in \mathbb{R}^N \,|\, x(N) \leq v(N)\}$. A **solution** on a class of games $\Gamma' \subseteq \Gamma$, is a mapping σ which associates with each game $(N, v) \in \Gamma'$ a subset $\sigma(N, v)$ of $X^*(N, v)$. Notice that $\sigma(N, v)$ is allowed to be empty. The **pre-imputation set** of a game (N, v) is defined by $X(N, v) := \{x \in \mathbb{R}^N \,|\, x(N) = v(N)\}$, and the set of **imputations** by $I(N, v) := \{x \in X(N, v) \,|\, x(i) \geq v(\{i\}),\ \text{for all } i \in N\}$. The core of (N, v) is defined by $C(N, v) = \{x \in X(N, v) \mid x(S) \geq v(S) \text{ for all } S \subseteq N\}$. A game (N, v) is **convex** (Shapley, 1971) if, for every $S, T \subseteq N$, $v(S)+v(T) \leq v(S \cup T)+v(S \cap T)$. We denote by Γ_{vex} the class of all convex games. A game (N, v) is **weakly superadditive** if for all partition, $\{S_1, \ldots, S_m\}$ of N, $v(S_1) + \cdots + v(S_m) \leq v(N)$. We denote by Γ_{ws} the class of all weak superadditive games. Notice that $\Gamma_{vex} \subseteq \Gamma_{ws}$.

Let $\mathbb{R}^N$ stand for the space of real-valued vectors indexed by N, $x = (x_i)_{i \in N}$, and for all $S \subseteq N$, $x(S) = \sum_{i \in S} x_i$, with the convention $x(\emptyset) = 0$. For each $x \in \mathbb{R}^N$ and $T \subseteq N$, x_T denotes the restriction of x to T: $x_T = (x_i)_{i \in T} \in \mathbb{R}^T$. Given two vectors $x, y \in \mathbb{R}^N$, $x \geq y$ if $x_i \geq y_i$ for all $i \in N$. We say that $x > y$ if $x \geq y$ and for some $j \in N$, $x_j > y_j$. Moreover, $x \gg y$ if $x_i > y_i$ for all $i \in N$. For any $x \in \mathbb{R}^N$, denote by $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ the vector obtained from x by rearranging its coordinates in a non-decreasing order, that is, $\hat{x}_1 \leq \hat{x}_2 \leq \ldots \leq \hat{x}_n$. For any two vectors $y, x \in \mathbb{R}^N$ with $y(N) = x(N)$, we say that y **Lorenz dominates** x, denoted by $y \succ_L x$, if $\sum_{j=1}^{k} \hat{y}_j \geq \sum_{j=1}^{k} \hat{x}_j$, for all $k \in \{1, \ldots, |N|\}$, with at least one strict inequality. Given a coalition $\emptyset \neq S \subseteq N$ and a set $A \subseteq \mathbb{R}^S$, EA denotes the set of allocations that are Lorenz undominated within A. That is, $EA := \{x \in A \mid \text{there is no } y \in A \ \text{ such that } \ y \succ_L x\}$. Given a game (N, v), the **strong Lorenz core** (Dutta and Ray, 1989) is defined in a recursive way as follows: the strong Lorenz core of a singleton coalition is $L^*(\{i\}, v) = \{v(\{i\})\}$. Now suppose that the strong Lorenz core for all coalitions of cardinality k or less have been defined, where $1 < k < |N|$. The strong Lorenz core of a coalition $S \subset \mathbb{N}$ of size $(k + 1)$ is defined by $L^*(S, v) = \{x \in \mathbb{R}^S \mid x(S) = v(S),\ \text{and there is no } T \subset S \text{and } y \in EL^*(T, v) \text{ such that } y \gg x_T\}$. The **weak Lorenz core** is defined similarly, but replacing $\gg$ by $>$. The **strong constrained egalitarian**

solution (Dutta and Ray, 1991) selects the Lorenz-undominated vectors within the strong Lorenz core. The **weak constrained egalitarian** solution (Dutta and Ray, 1989), denoted by **DR**, selects the Lorenz-undominated vectors within the weak Lorenz core. For $(N, v) \in \Gamma$, $|DR(N, v)| \leq 1$.

2 An axiomatic characterization of the strong constrained egalitarian solution

In this section, we provide an axiomatization of the SCES based on the **consistency** principle and its converse, together with **constrained egalitarianism** (Dutta, 1990), a prescriptive property which fix the solution for two person games.

Before introducing consistency we need to define the concept of a reduced game. The terminology is taken from Thomson (2006a).

Definition 1. Let $(N, v) \in \Gamma$, $y \in \mathbb{R}^N$ and $\emptyset \neq T \subset N$. The **max reduced game** (Davis and Maschler, 1965) **relative to** T **at** y is the game $\left(T, r^T_{y,\,DM}(v)\right)$ defined by

$$r^T_{y,\,DM}(v)(S) := \begin{cases} 0 & \text{if } S = \emptyset, \\ \max\limits_{Q \subseteq N \setminus T} \{v(S \cup Q) - y(Q)\} & \text{if } S \subset T, \\ v(N) - y(N \setminus T) & \text{if } S = T. \end{cases}$$

For an interpretation of the max reduced game see Peleg (1986).

Roughly speaking, consistency says that there is no difference in what the players of the reduced game will get in both the original game and in the reduced game. The dual of consistency is named **converse consistency**. This property states that if an efficient payoff vector is accepted for every pair of players, then it is accepted for the set of all players.[1] Let us now introduce formally these properties.

Let $(\{i, j\}, v)$ be a 2-person weak-superadditive game. Without loss of generality, assume $v(\{i\}) \leq v(\{j\})$. The **constrained egalitarian solution** of the game $(\{i, j\}, v)$, denoted by $CE(\{i, j\}, v)$, is defined as follows: $CE_j(\{i, j\}, v) := \max\left\{\frac{v(\{i,j\})}{2}, v(\{j\})\right\}$, and $CE_i(\{i, j\}, v) := v(\{i, j\}) - CE_j(\{i, j\}, v)$.

A solution σ on Γ_{ws} satisfies

- **constrained egalitarianism** if it coincides with the constrained egalitarian solution for all 2-person weak-superadditive games.

[1] See Thomson (2006a) for a survey on consistency and its converse

A solution σ on Γ' satisfies

- **max consistency** if for each $(N,v) \in \Gamma'$, each $\emptyset \neq T \subset N$, and each $y \in \sigma(N,v)$, then $\left(T, r^T_{y,\,DM}(v)\right) \in \Gamma'$ and $y_T \in \sigma\left(T, r^T_{y,DM}(v)\right)$.
- **converse max consistency** if for each $(N,v) \in \Gamma'$, $y \in X(N,v)$, and for each $T = \{i,j\} \subseteq N$, $\left(T, r^T_{y,\,DM}(v)\right) \in \Gamma'$ and $y_T \in \sigma\left(T, r^T_{y,DM}(v)\right)$, then $y \in \sigma(N,v)$.

These properties have been widely studied and appear in axiomatizations of several solution concepts. For instance, they appear among other properties in Peleg's (1986) characterization of the core. On the domain of convex games, Dutta (1990) characterizes the WCES by means of constrained egalitarianism and max consistency. The next example shows that the SCES is not *max consistent*.

Example 1. Dutta and Ray (1991)
Let $(N,v) \in \Gamma$ *where* $N = \{1,2,3\}$, $v(S) = v(N) = 1$ *if* $|S| = 2$, *and* $v(S) = 0$ *if* $|S| = 1$. *Here* $EL^*(N,v) = \{(0.5, 0.5, 0), (0.5, 0, 0.5), (0, 0.5, 0.5)\}$ *(Dutta and Ray, 1991). Consider the max reduced game* $\left(\{1,2\}, r^{\{1,2\}}_{x,\,DM}(v)\right)$, *where* $x = (0.5, 0.5, 0)$. *As the reader can easy check,* $I\left(\{1,2\}, r^{\{1,2\}}_{x,\,DM}(v)\right) = \emptyset$, *thus* $EL^*\left(\{1,2\}, r^{\{1,2\}}_{x,\,DM}(v)\right) = \emptyset$.

Hence, to characterize the SCES by means of consistency we introduced a different notion of a reduced game. To do this, we need additional definitions.

An ordering $\theta = (i_1, i_2, \ldots, i_n)$ of N where $|N| = n$, is a bijection from $\{1, 2, \ldots, n\}$ to N. We denote by Θ_N the set of all orderings of N.

Definition 2. Given $(N,v) \in \Gamma$ and $\theta = (i_1, \ldots, i_n) \in \Theta_N$, let $x^\theta \in \mathbb{R}^N$ be defined as follows:

$$x^\theta_{i_k} := \max_{S \in P_{i_k}} \left\{ \frac{v(S)}{|S|} \right\}, \text{ for } k = 1, \ldots, n, \tag{1}$$

where $P_{i_1} := \{S \subseteq N \mid i_1 \in S\}$ and $P_{i_k} := \{S \subseteq N \mid i_1, \ldots, i_{k-1} \notin S,\ i_k \in S\}$, for $k = 2, \ldots, n$.

Observe that in the construction of x^θ underlines the principle of equal division. As Selten (1972) showed by a great number of experimental games, this principle is a strong distributive norm which influences the behavior of players. Thus, we can consider x^θ as a vector of "natural" claims that players can require in a sequential way.

Given $(N,v) \in \Gamma$ and $\theta \in \Theta_N$, let

$$\Delta^{\theta}(v) := \{y \in X(N,v) \,\text{such that}\, y \geq x^{\theta}\}, \text{ and}$$

$$\Theta_N^{\preceq}(v) := \{\theta^{'} \in \Theta_N \mid \nexists\, \theta \in \Theta_N \,\text{such that}\, \theta \succ_L \theta^{'}\},$$

where $\theta \succ_L \theta^{'}$ means that there is $x \in \Delta^{\theta}(v)$ such that $x \succ_L y$, for all $y \in \Delta^{\theta^{'}}(v)$.

Definition 3. Let $(N,v) \in \Gamma$, $\emptyset \neq T \subset N$, $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N^{\preceq}(v)$ and $y \in \mathbb{R}^N$. The **order-reduced game on** T **at** y is the game $\left(T, r_{y,\theta}^T(v)\right)$, where

$$r_{y,\theta}^T(v)(S) := \begin{cases} 0 & \text{if } S = \emptyset \\ \max\limits_{Q \subseteq N \setminus T} \left\{x^{\theta}(S \cup Q) - y(Q)\right\} & \text{if } S \subset T, \\ v(N) - y(N \setminus T) & \text{if } S = T. \end{cases}$$

In the definition of the order-reduced game, we suppose that the worth of a non empty coalition $S \subset T$ is revaluated under the assumption that the members of S can choose the best partners in $N \setminus T$ in order to maximize the "natural" requirements given by x^{θ}, provided that it pays them their components of y.This is as in the Davis and Maschler (1965) definition. Assuming that all the members of N agree that the members of $N \setminus T$ will get $y_{N \setminus T}$, the members of T may get $v(N) - y(N \setminus T)$.

A solution σ on Γ_{ws} satisfies

- **order-consistency** if for each $(N,v) \in \Gamma_{ws}$ and each $y \in \sigma(N,v)$, there is $\theta \in \Theta_N^{\preceq}(v)$ such that, for all $\emptyset \neq T \subset N$, $\left(T, r_{y,\theta}^T(v)\right) \in \Gamma_{ws}$ and $y_T \in \sigma\left(T, r_{y,\theta}^T(v)\right)$.
- **converse order-consistency** if for each $(N,v) \in \Gamma_{ws}$ and $y \in X(N,v)$, each $\theta \in \Theta_N^{\preceq}(v)$ and for each $T = \{i,j\} \subseteq N$, $\left(T, r_{y,\theta}^T(v)\right) \in \Gamma_{ws}$ and $y_T \in \sigma\left(T, r_{y,\theta}^T(v)\right)$, then $y \in \sigma(N,v)$.

Now we characterize the SCES.

Theorem 2.1. *On the domain of weak superadditive games, the SCES is the only solution satisfying constrained egalitarianism, order-consistency, and converse order-consistency.*

Proof: Constrained egalitarianism follows directly from the fact that for 2-person games both the strong and the weak constrained egalitarian solution coincide. To prove the remaining axioms, we need a geometrical decomposition of the strong Lorenz core. Let $(N,v) \in \Gamma$, we claim that

$$L^*(N,v) = \bigcup_{\theta \in \Theta_N} \Delta^{\theta}(v). \tag{2}$$

Indeed, let $y \in L^*(N,v)$ and $S_1 \in \arg\max_{S\subseteq N}\left\{\frac{v(S)}{|S|}\right\}$. Since $y \in L^*(N,v)$, there is $i_1 \in S_1$ such that $y_{i_1} \geq \frac{v(S_1)}{|S_1|}$. Now let $S_2 \in \arg\max_{S\subseteq N\setminus\{i_1\}}\left\{\frac{v(S)}{|S|}\right\}$ and $i_2 \in S_2$ such that $y_{i_2} \geq \frac{v(S_2)}{|S_2|}$. Following this process step by step, we can generate an order $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N$. Let $x^\theta \in \mathbb{R}^N$ as given in Definition 2. Since $y \geq x^\theta$ and $y(N) = v(N)$, we have that $y \in \Delta^\theta(v)$. To show the reverse inclusion, let $\theta \in \Theta_N$ and $y \in \Delta^\theta(v)$. Let $S \subset N$ be a non-empty coalition and $i_k \in S$ the first player in S with respect to θ. Then, $y_{i_k} \geq x^\theta_{i_k} \geq \frac{v(S)}{|S|}$ and so $y \in L^*(N,v)$.

Next we prove *order-consistency*. Let $(N,v) \in \Gamma_{ws}$ and $y \in EL^*(N,v)$. From expression (2) we know that there is $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N$ such that $y \in \Delta^\theta(v)$. Suppose that $\theta \notin \Theta_N^{\preceq}(v)$. Then, there are $\theta' \in \Theta_N$ and $y' \in \Delta^{\theta'}(v)$ such that $y' \succ_L y$, a contradiction because, from (2), $\Delta^{\theta'}(v) \subseteq L^*(N,v)$. Hence, $\theta \in \Theta_N^{\preceq}(v)$. Let $\emptyset \neq T \subset N$, and consider the order-reduced game $\left(T, r^T_{y,\theta}(v)\right)$. Now define the game (N, v_θ) as follows:

$$v_\theta(R) := \begin{cases} x^\theta(R) & \text{if } R \neq N, \\ \\ v(N) & \text{if } R = N. \end{cases} \tag{3}$$

Since $x^\theta(N) \leq v(N)$, we have $(N, v_\theta) \in \Gamma_{vex}$. Consider the max reduced game $\left(T, r^T_{y,DM}(v_\theta)\right)$. Notice that

$$\left(T, r^T_{y,\theta}(v)\right) = \left(T, r^T_{y,DM}(v_\theta)\right). \tag{4}$$

Clearly $C(N, v_\theta) = \Delta^\theta(v)$, and so $y = DR(N, v_\theta)$ (Dutta-Ray, 1989). Since the WCES is *max consistent* on the domain of convex games (Dutta, 1990), $\left(T, r^T_{y,DM}(v_\theta)\right) \in \Gamma_{ex} \subseteq \Gamma_{ws}$ and $y_T = DR\left(T, r^T_{y,DM}(v_\theta)\right)$. Observe that $C\left(T, r^T_{y,DM}(v_\theta)\right) = L^*\left(T, r^T_{y,DM}(v_\theta)\right)$. From Dutta-Ray (1989) we know that $y_T \in C\left(T, r^T_{y,DM}(v_\theta)\right)$. Thus, from (4) we conclude that $\left(T, r^T_{y,\theta}(v)\right) \in \Gamma_{ws}$ and $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$.

Next we prove *converse order-consistency*. Let $y \in X(N,v)$ and $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N^{\preceq}(v)$ such that, for all $T = \{i_k, i_l\} \subseteq N$, $\left(T, r^T_{y,\theta}(v)\right) \in \Gamma_{ws}$ and $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$. Thus, for all $i \in N$, $y_i \geq r^T_{y,\theta}(v)(\{i\}) \geq x^\theta_i$. Since $y(N) = v(N)$, we have that $y \in \Delta^\theta(v)$. Consider the game (N, v_θ) as defined in (3). Let $T = \{i, j\} \subseteq N$. As we have seen before $\left(T, r^T_{y,\theta}(v)\right) = \left(T, r^T_{y,DM}(v_\theta)\right)$. For 2-person games, the SCES and WCES coincide, so $y_T = DR\left(T, r^T_{y,DM}(v_\theta)\right)$. Since the WCES is *converse max consistent* on the domain of convex games (Dutta, 1990), $y = DR(N, v_\theta)$. Moreover, $y \in C(N, v_\theta) = \Delta^\theta(v) \subseteq L^*(N,v)$ (expression (2)). Suppose that $y \notin EL^*(N,v)$. Then, there is $y' \in L^*(N,v)$ such that $y' \succ_L y$. From (2) there is $\theta' \in \Theta_N$ such that $y' \in \Delta^{\theta'}(v)$. Since $y' \succ_L y \succ_L x$, for all $x \in \Delta^\theta(v)$, we get $\theta' \succ_L \theta$, a contradiction. Thus, $y \in EL^*(N,v)$.

Finally, to show uniqueness, suppose there is a another solution σ on Γ_{ws} satisfying the above three axioms. For 2-person games, constrained egalitarianism implies $\sigma = EL^*$. Let $(N, v) \in \Gamma_{ws}$ with $|N| \geq 3$. First note that *constrained egalitarianism* and *order-consistency* imply *efficiency*. Let $y \in \sigma(N, v)$. By *order-consistency* there is $\theta \in \Theta_N^{\prec}(v)$ such that, for each $T \subset N$, $|T| = 2$, $y_T \in \sigma\left(T, r^T_{y,\theta}(v)\right)$. By *constrained egalitarianism*, $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$. Now applying *converse order-consistency* we get $y \in EL^*(N, v)$. Following a symmetric argument we obtain the reverse inclusion, $EL^*(N, v) \subseteq \sigma(N, v)$. [2] $\square$

The following examples show that in Theorem 2.1 the axioms are independent:

- Let $\sigma^1(N, v)$ be defined as follows: $\sigma^1(N, v) := \emptyset$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^1 satisfies *order-consistency*, *converse order-consistency*, but not *constrained egalitarianism*.
- Let $\sigma^2(N, v)$ be defined as follows: $\sigma^2(N, v) := CE(N, v)$ if $|N| = 2$ and $\sigma^2(N, v) := X(N, v)$ if $|N| > 2$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^2 satisfies *constrained egalitarianism*, *converse order-consistency*, but not *order-consistency*.
- Let $\sigma^3(N, v)$ be defined as follows: $\sigma^3(N, v) := CE(N, v)$ if $|N| = 2$, and $\sigma^3(N, v) := \emptyset$ if $|N| > 2$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^3 satisfies *constrained egalitarianism*, *order-consistency*, *but not converse order-consistency*.

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[2] Notice that this is a standard application of the Elevator Lemma (for more details see Thomson (2006b)).

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