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Modelling world investment markets using threshold conditional correlation models

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Document de treball n.25 - 2012

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

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Edita:

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Adreçar comentaris al Departament d'Economia / CREIP

Dipòsit Legal: T - 1189 - 2012

ISSN edició en paper: 1576 - 3382
ISSN edició electrònica: 1988 - 0820

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Modelling world investment markets using threshold conditional correlation models

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Abstract

In this paper we propose a parsimonious regime-switching approach to model the correlations between assets, the threshold conditional correlation (TCC) model. This method allows the dynamics of the correlations to change from one state (or regime) to another as a function of observable transition variables. Our model is similar in spirit to Silvennoinen and Teräsvirta (2009) and Pelletier (2006) but with the appealing feature that it does not suffer from the course of dimensionality. In particular, estimation of the parameters of the TCC involves a simple grid search procedure. In addition, it is easy to guarantee a positive definite correlation matrix because the TCC estimator is given by the sample correlation matrix, which is positive definite by construction. The methodology is illustrated by evaluating the behaviour of international equities, govenrment bonds and major exchange rates, first separately and then jointly. We also test and allow for different parts in the correlation matrix to be governed by different transition variables. For this, we estimate a multi-threshold TCC specification. Further, we evaluate the economic performance of the TCC model against a constant conditional correlation (CCC) estimator using a Diebold-Mariano type test. We conclude that threshold correlation modelling gives rise to a significant reduction in portfolio´s variance.

1. Introduction

Understanding and predicting the correlation of returns across different asset classes has key relevance in many financial economics issues such as asset allocation, risk analysis and hedging. In recent years there has emerged a burgeoning literature on multivariate GARCH models with dynamic correlations. Authors have proposed a range of methodologies in capturing the time-varying structure of the correlations. For instance, one of the most frequently used specifications is the Dynamic Conditional Correlation (DCC) model which assumes that the conditional correlation evolves linearly according to a simple GARCH-type structure (Engle, 2002). Estimation of parameters of the DCC model is relatively simple and the model has thus become popular among academics and practitioners.

Recently, two interesting extensions to the DCC model have been proposed in the literature. Colacito, Engle, and Ghysels (2011) develop a MIDAS-DCC approach. The idea is to distinguish between short-run and long-run correlation components. The short-run component varies at high frequencies, while the long-run component does so at low frequencies. Another extension is by Kwan et al. (2010) which allows for different regimes in the short-run DCC dynamics. However, in this model the long-run correlation dynamics are assumed to be constant over time.

We can ask ourselves if a GARCH-type model is appropriate for the correlations because the dynamics of a correlation can be intrinsically different than the behavior of a variance, e.g. a correlation is bounded from below and above while a variance is not. Another way of allowing for time-varying correlations is to define different states of the world or regimes, and to allow for the possibility that the dynamic behaviour of asset correlations depend on the regime that occurs at any given point in time. For instance, Pelletier (2006) proposed a Markov-switching model that allows the correlation depend on the regime that prevails at any given point in time. The Markov-switching model assumes that the regime is governed by an underlying Markov-chain process and one can only assign probabilities to the occurrence of the different correlation regimes. Thus, according to this approach the determinants of correlations cannot be observed.

Silvennoinen and Teräsvirta (2009) developed a different approach to the regime-switching correlation modeling. The proposed smooth transition conditional correlation (STCC) model allows the correlation change smoothly as a function of observable transition variables.[1] Consequently, the regimes that have occurred in the past and present are known with certainty (though they have to be found by statistical techniques, of course). From an estimation point of view, we understand that estimation of the parameters of both the Pelletier and smooth transition correlation models is not an easy task, particularly in high dimensions.

Other authors have estimated the conditional correlations using semi-parametric and non-parametric techniques (Hafner et al., 2006, Aslanidis and Casas, 2010 and Long et al., 2011). Typically, their estimation combines a parametric estimation (e.g., GARCH(1,1)) of the volatility with a subsequent nonparametric estimation of the correlations. The non-parametric methods appear flexible as correlations *a priori* can take any functional form. Nevertheless, so far there is no evidence that they produce economically superior results compared to parametric DCC models (see, for example, Aslanidis and Casas, 2010). Besides, the source of variation is the correlation comes from one or several conditioning variables with the choice of the appropriate conditioning variable(s) being not always clear.

In this paper we propose a parsimonious regime-switching approach for the correlations, the threshold conditional correlation (TCC) model. This method allows the dynamics of the correlations to change from one state or regime to another as a function of observable transition variables. Our model is similar in spirit to Silvennoinen and Teräsvirta (2009) and Pelletier (2006) but with the appealing feature that it does not suffer from the course of dimensionality. In particular, estimation of the parameters of the TCC involves a simple grid search procedure. More importantly, contrary to the previous regime-switching correlations models, in a TCC framework it is easy to guarantee a positive definite correlation matrix. This is because the proposed TCC estimator is the sample correlation matrix, which is positive definite by construction.

With regard to the application, we examine the behaviour of international equities, government bonds and major exchange rates. Our approach can also test and allow for different parts in the correlation matrix to be governed by different transition variables. For this, we estimate a multi-threshold TCC specification. Finally, we evaluate the economic significance of the threshold model against the constant correlation estimator using a Diebold-Mariano type test proposed by Engle and Colacito (2006). The results show that the reduction in portfolio variance obtained by the TCC

[1] See also earlier work by Silvennoinen and Teräsvirta (2005). A special case of this model with time transition was independently introduced by Berben and Jansen (2005).

specification is statistically significant. Therefore, from an asset allocation point of view our method improves significantly on the benchmark constant correlation specification.

The paper is organised as follows. Section 2 presents the threshold conditional correlation model as well as discusses estimation issues and the test of constant correlations. Section 3 presents the results and tests for the economic significance of the threshold model. In Section 4, we test for large TCC models where we allow for different parts in the correlation matrix to be governed by different transition variables. Finally, Section 5 briefly concludes.

2. The threshold conditional correlation model

2.1Model specification

Consider the following *N*-dimensional vector process of asset returns (r_t)

$$r_t = \mu + u_t \qquad t = 1,...,T \tag{1}$$

where μ denotes the vector of mean returns. The conditional covariances of the shocks in (1) are time-varying, such that

$$u_t|\Im_{t-1} \sim N(0, H_t) \tag{2}$$

where $\Im_{t-1}$ is the information set at time t-1 and $N(.)$ denotes the multivariate normal distribution. Each of the univariate error processes has the specification

$$u_{i,t} = h_{i,t}^{1/2}\eta_{i,t} \text{ for } i = 1,..., N$$

where the errors $\eta_{i,t}$ form a sequence of independent random variables with mean zero and variance one, for each of the asset returns $i = 1,..., N$. Each conditional variance $h_{i,t}$ is calculated as the realized variance (RV) of asset prices over a fixed time period or assumed to follow a GARCH(1,1) or any other univariate volatility process.

Rather than modelling the off-diagonal elements of H_t directly, we use the following decomposition

$$H_t = D_t R_t D_t \tag{3}$$

allows the focus to be placed on the conditional correlation matrix R_t, where $D_t = diag(h_{1t}^{1/2},..., h_{Nt}^{1/2})$ is the diagonal standard deviation matrix. We model the conditional correlations as follows

$$R_t = R_1 I[s_{t-1} < \gamma] + R_2(1 - I[s_{t-1} < \gamma]) \quad (4)$$

where R_1 and R_2 are (constant) positive definite correlation matrices, and $I[s_{t-1} < \gamma]$ is an indicator function with s_{t-1} being the transition variable and γ the threshold value.

2.2 Estimation procedure

The parameters of interest are mean vector μ, the variance matrix D_t parameters (denoted by ϕ) , the correlation matrices R_1 and R_2 (denoted by ψ) and the threshold γ. Estimation of the TCC model is carried out by maximum likelihood (ML) under the assumption of normality. Given a value of γ, the Gaussian log-likelihood is given by

$$\begin{aligned}\ln L(\theta/\gamma) &= -\frac{1}{2}\sum_t (N\ln(2\pi) + \ln(|H_t|) + (r_t - \mu)' H_t^{-1}(r_t - \mu)) \\ &= -\frac{1}{2}\sum_t (N\ln(2\pi) + 2\ln|D_t| + \ln(|R_t|) + (r_t - \mu)' D_t^{-1} R_t^{-1} D_t^{-1}(r_t - \mu)) \\ &= -\frac{1}{2}\sum_t (N\ln(2\pi) + 2\ln|D_t| + \ln(|R_t|) + \eta'_t R_t^{-1}\eta_t) \quad (5)\end{aligned}$$

where $\theta \equiv (\phi', \psi')'$ and $\eta_t = D_t^{-1}(r_t - \mu)$. We follow the literature and divide the estimation procedure of θ into two separate estimations: the mean and volatility estimation first and then the correlation estimation (see, Engle and Sheppard, 2001). In particular, conditional on the mean and volatility estimates, the conditional log-likelihood is

$$\ln L(\psi/\hat{\phi}, \gamma) = -\frac{1}{2}\sum_t (N\ln(2\pi) + 2\ln|\hat{D}_t| + \ln(|R_t|) + \hat{\eta}'_t R_t^{-1}\hat{\eta}'_t)$$

Note that the only portion of the log-likelihood that will influence the parameter selection is $\ln(|R_t|) + \eta'_t R_t^{-1}\eta_t$, thus, excluding the constant terms we simply maximize

$$\ln L(\psi/\hat{\phi}, \gamma) = -\frac{1}{2}\sum_t (\ln(|R_t|) + \hat{\eta}'_t R_t^{-1}\hat{\eta}_t) \quad (6)$$

Notice that is computationally convenient to first concentrate out (R_1, R_2). That is, holding γ fixed the sample correlation matrices compute the ML correlation estimator

$$\hat{R}_1(\gamma) = \frac{1}{\sum_t I[s_{t-1} < \gamma]} \sum_t \hat{\eta}_t \hat{\eta}'_t I[s_{t-1} < \gamma]$$
$$\hat{R}_2(\gamma) = \frac{1}{\sum_t I[s_{t-1} \geq \gamma]} \sum_t \hat{\eta}_t \hat{\eta}'_t I[s_{t-1} \geq \gamma] \qquad (7)$$

where $(\hat{R}_1(\gamma), \hat{R}_2(\gamma))$ are the sample correlations matrices in each regime, which are positive definite by construction.

Notice that given the information matrix between the parameters ϕ and ψ is not block diagonal, we use the derivatives of the full likelihood function in Eq. (5) to estimate the asymptotic covariance matrix of the parameters.

Finally, estimation of γ is given by

$$\hat{\gamma} = \arg\max_{\gamma \in [\gamma_L, \gamma_U]} \ln L(\hat{R}_1(\gamma), \hat{R}_2(\gamma) / \hat{\phi}) \qquad (8)$$

It is undesirable for a threshold to be selected which sorts too few observations into one or the other regime. This possibility can be excluded by restricting $P[s_{t-1} \leq \gamma_L] = P[s_{t-1} > \gamma_U] = \pi$ such that a minimal percentage of the observations (say, $100\pi = 20\%$) lie in each regime. Estimation of γ is performed by a grid search procedure. The final correlation estimates are $(\hat{R}_1(\hat{\gamma}), \hat{R}_2(\hat{\gamma}))$.

2.3 Threshold effect test

Before considering the TCC model it is important to determine whether the change in correlation is statistically significant. The relevant null hypothesis of no threshold effect (or constancy) is $H_0 : R_1 = R_2$. To that purpose and for exposition purposes, we redefine Eq. (5) as follows

$$R_t = R + \Lambda I[s_{t-1} < \gamma] \qquad (9)$$

where $R = R_2$ and $\Lambda = R_1 - R_2$. With this notation, the null of constancy is tested as $H_0^{\Lambda} : \Lambda = 0$. We perform an extension of the Lagrange Multiplier test developed by Tse (2000). More specifically, for a given γ, we denote $s(\gamma)$ as the score vector, $s(\gamma) = \partial \ln L(\theta / \gamma) / \partial \theta$. Then, the partial derivatives with respect to λ_{ij} (with $\Lambda = \{\lambda_{ij}\}$) parameters are

$$\frac{\partial \ln L(\theta / \gamma)}{\partial \lambda_{ij}} = I[s_{t-1} < \gamma] \left(\varepsilon_{i,t} \varepsilon_{j,t} - (\rho + \lambda)^{ij} \right) \qquad (9)$$

where $\varepsilon_t = (\varepsilon_{1,t},...,\varepsilon_{N,t})' = R_t^{-1}\eta_t$ and $[R+\Lambda]^{-1} = \{(\rho+\lambda)^{ij}\}$.[2] Further, we denote $S(\gamma)$ as the $T \times N$ matrix the rows of which are the partial derivatives $\partial \ln L_t(\theta/\gamma)/\partial\theta'$, for $t = 1,...,T$. Thus, the LM statistic for testing the hypothesis H_0^{Λ}, denoted as $LMC(\gamma)$, is calculated as follows

$$LMC(\gamma) = \hat{s}(\gamma)'(\hat{S}(\gamma)'\hat{S}(\gamma))^{-1}\hat{s}(\gamma) \tag{10}$$

where the hats denote evaluation at $\hat{\theta}$.

We propose two different statistics: the $Sup\,LMC$ and the $Ave\,LMC$ (see Andrews and Ploberger, 1994, and Hansen, 1996). The $Sup\,LMC$ is simply the maximum of the individual LMC statistics

$$\sup LCM = \max_{\gamma_L \leq \gamma \leq \gamma_U} \left(LCM(\gamma)\right) \tag{11a}$$

The $Ave\,LMC$ is the simple average of the individual LMC statistics

$$AveLCM = \frac{1}{k}\sum_{\gamma=\gamma_L}^{\gamma_U} LCM(\gamma) \tag{11b}$$

Note that as the function $LCM(\gamma)$ is non-differentiable in γ, we perform a grid-LM evaluation over $[\gamma_L, \gamma_U]$.

Given that asymptotic critical values of the sampling distribution of the above statistics cannot be tabulated since in general the distribution depends upon moments of the sample, a model-based bootstrap is proposed and performed in the following computer algorithm format:

Algorithm (*Model-based Bootstrap procedure*)

1. $l = 1$

2. Generate $\left\{\varepsilon_t^{(l)}\right\}_{t=1}^{T}$ resampling from $\left\{\hat{\varepsilon}_t\right\}_{t=1}^{T}$, with $\hat{\varepsilon}_t = \hat{R}_t^{-1/2}\hat{\eta}_t \sim iid(0, I_N)$ (estimated standardized errors under the alternative hypothesis, threshold conditional correlation).

3. Generate $\left\{r_t^{(l)}\right\}_{t=1}^{T}$ from $r_t^{(l)} = \hat{\mu} + \hat{D}_t\hat{R}^{1/2}\varepsilon_t^{(l)}$ (model under the null hypothesis, constant conditional correlation).

4. Compute $\sup LCM^{(l)}$ and $AveLCM^{(l)}$ for the bootstrap sample series $\left\{r_t^{(l)}\right\}_{t=1}^{T}$.

[2] For details regarding the other derivatives, we refer to Tse (2000).

5. $l = l + 1$. Go to step 2 while $l \le B$.

6. Estimate the *p*-value, from the bootstrap approximation

$$p_{\sup} = \frac{1}{B}\sum_{l=1}^{B} I[\sup LCM^{(l)} \ge \sup LCM]$$

$$p_{ave} = \frac{1}{B}\sum_{l=1}^{B} I[AveLCM^{(l)} \ge AveLCM]$$

Note that in step 2, we resample $\{\hat{\varepsilon}_t\}_{t=1}^{T}$ all the vector $\hat{\varepsilon}_t$, so we do not do an *individual resampling*. We also treat the realized volatility and threshold variable series as given, holding their values fixed in repeated bootstrap samples.

2.4 Finite-sample properties of threshold effect test

In this section, we study the finite-sample properties of the threshold effect test by Monte Carlo simulations. We present two different experiments. In particular, we simulate a bivariate volatility model with normal errors using as transition variables the realized volatility or the first principal component of the series. The parameter values for the DGPs are as follows

$$r_{1,t} = h_{1,t}^{1/2}\eta_{1,t}, \quad h_{1,t} = 0.4rv_{1,t}, \; rv_{1,t} = 1 + (0.15\eta_{1,t-1}^{2} + 0.8)rv_{1,t-1}$$

$$r_{2,t} = h_{2,t}^{1/2}\eta_{2,t}, \; h_{2,t} = 0.2rv_{2,t}, \; rv_{2,t} = 1 + (0.2\eta_{2,t-1}^{2} + 0.7)rv_{2,t-1}$$

$$\begin{pmatrix}\eta_{1,t}\\ \eta_{2,t}\end{pmatrix} \sim N\left[\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right], \qquad \rho_{sc1} = 0.8, 0.3, -0.3, -0.8$$

For the correlation parameter, we tried four different values. The sample size is *T*=500, the number of Monte Carlo replications is *M*=300, and the number of bootstrap replications is *B*=300. Values of the statistic in (10) are calculated using the analytical expression for the first derivatives of the maximum likelihood (ML) estimation.

In Table 1 we report the rejection frequencies at the significance levels of 1%, 5% and 10%. As seen, the actual size of the test is close to the nominal size. Thus, we conclude the test does not generally seem to suffer from any size distortion.

3. Empirical results

3.1 Data and preliminary statistics

We estimate the correlations of a portfolio containing equities, bonds and exchange rates. In particular, we include the stock market indices of Australia (ASX 200), Britain (FTSE 100), France (CAC 40), Germany (DAX 30), Hong Kong (HANG SENG), Japan (NIKKEI 225) and the US (S&P 500). We also consider the 10-year constant maturity bonds of Britain (UK 10Y), France (FR 10Y), Germany (GER 10Y), Italy (IT 10Y), Japan (JP 10Y) and the US (US 10Y). As for the exchanges rates, we use bilateral exchange rates (vs. US $) of Australian dollar (AUD), British pound (GBP), the Euro (EUR), Japanese yen (JPY), South African rand (ZAR) and Swiss franc (CHF). All data is obtained from DataStream. The sample period starts on June 1992 and ends on March 2011, which yields 980 weekly observations. This long time span features several episodes of financial market turbulence (e.g., Asian crisis, LTCM, Russian crisis, terrorist attacks in September 2001 and recent financial crisis) as well as tranquil periods.

Descriptive statistics of the data are presented in Table 2. During this period, bonds provide lower standard deviations, but also surprisingly, higher returns compared to equities. Further, the standardized returns are less skewed and less fat-tailed than the raw returns. Tables 3-5 reports the sample (unconditional) correlations of returns across markets, using the whole sample period. As seen, the correlation is very high for the stock pair DAX 30 vs. CAC 40, between the three Eurozone bond indices for the exchange rate CHF vs. EUR. On the other hand, the three Japanese assets to show low correlation with the other markets, suggesting that the Japanese markets are comparatively disconnected from global market developments.

3.2 Modelling cycle

As mentioned before, we divide the estimation procedure into two separate estimations: the mean and variance estimation first and then the correlation estimation. In particular, for the conditional variances we calculate the realized volatility using daily returns. We also tested whether the realized variances sufficiently capture the dominant volatility dynamics in the data. For this, we applied the Ljung-Box statistic for testing autocorrelation up to $m = \sqrt{T}$ lags in the squared standardized returns. The results (last column of Table 2) showed that the null hypothesis of no serial correlation is rejected only in the case of the Euro exchange rate with a *p*-value of 0.035). Hence, we concluded that the realized variances capture the volatilities dynamics in the data quite adequately.[3]

As for the correlations, we first consider the three financial markets/blocks separately (equities, bond and exchange rates). For each market, we estimate bivariate as well as higher-dimensional TCC models. The bivariate results give us insights regarding how “homogeneous” are the blocks in terms of the chosen transition variables for the conditional correlations. We make use of the following candidate transition variables (1-week lag):

- Realized variances of each block return series, $rv_{i,t}$, using daily frequency returns.
- Principal components of realized variances in the block.
- Each standardized return series, $r_{i,t}/\sqrt{rv_{i,t}}$, separately.
- Absolute value of block standardized return series separately.
- Principal components of standardized return series in the block.
- Absolute value of principal components of standardized return series in the block.
- CBOE (Chicago Board of Options Exchange) VIX volatility index. The VIX is considered a leading measure of the market´s near term volatility. It is a measure of market expectations of future volatility of S&P 500 implied by the options trading on this index. In practice, we calculated the realized mean of the VIX index over the week using daily data.

In total, we employ between 43 and 50 candidate transition variables for each block/market.

3.2 Pairwise and block analyses

The pairwise estimation results of three markets are shown in Tables 6-11. As mentioned, the paiwise results would give us insights regarding how “homogeneous” are the blocks with regards to the chosen transition variables. We use the following evaluation criteria: For each bivariate model we first calculate the log-likelood value

[3] Nevertheless, we also estimated a standard GARCH(1,1) process and our results remain qualitatively the same.

using each candidate transition variable. Then, for the transition variable with the achieved maximun log-likelood value we assign position 1, for the second best position 2 and so on, and report the mean position (denoted by Mean Position). Then, for each bivariate model we also calculate the difference between the log-likelood obtained by the candidate variable and the log-likelood achieved by the *best* candidate for this pair and summed the difference over all pairs (denoted by Loss Function). The lower the Loss Function of the *best* transition variable compared to the other candidates, the more "homogemeous" would be the block in terms of the transition variable. In Tables 6-8, we report the results for the seven best transition variables as well as for the worst candidate for the three markets, respectively.

The equity returns analysis shows that the *best* transition variable is the VIX volatility index (Table 6). It achieves a Mean Position of 1.95 and has a Loss Function of 30.75. In second place comes the realized variance of the FTSE 100 with the realized variance of S&P500 following next as the third best transition variable. In terms of loss, the difference between the VIX and the other candidates can be large implying that Ccan be considered as the homogeneous transition for the different bivariate models. On the other hand, the results for bonds show that the *best* transition variable is the absolute value of the 5th principal component of series (Table 7). Interestingly, this principal component is basically the difference between the German/French and the Italian bonds:

$$Abs(Pc_5)_{t-1} = -0.48(GER10Y)_{t-1} - 0.376(FR10Y)_{t-1} + 0.779(IT10Y)_{t-1} + ...$$

Thus, this spread may reflect the premium over the Italian bonds. Further, the VIX volatility index comes in third place with a Loss Function of 307 and a Mean Position of 13.8. As for the exchange rate results in Table 8, once again the VIX provides the best-fit transition variable, though it achieves a Loss Function of 110 and a Mean Position of 15.19. Interestingly, the difference now between the VIX and other candidates such as the realized variance of the South African rand and Australian dollar is small. This implies that different candidate variables contain similar information making the choice of the most suitable transition variable not being clear as in the case of equities or bonds.

Turning to the blocks, the results for three markets are reported in Tables 9-11. In particular, for each block we estimated high-dimensional models by using each candidate transition variable and in the tables we reported the correlation matrices obtained by using the best-fit candidate. As seen, for all three cases the results are consistent with the bivariate analysis. That is, for equities the best fit is obtained using the VIX volatility index. More specifically, the TCC model gives a threshold estimate of $\hat{\gamma} = 21.33$, and one may thus speak about high and low volatility regimes (Tables 9a-b). During the former, i.e., when the volatility index exceeds the estimated threshold, which constitutes 38.2% of the sample, the correlations are higher than during calm periods. That is, the uncertainty of the investors shows as an increase in the correlations. This result is in line with the other studies in the literature, for example, Ang and Bekaert (2002), Baele (2005), Longin and Solnik (2001), Ramchand and Susmel (1998), among others. For bonds, the TCC results in Tables 10a-b show that the lower the premium over the Italian bonds the stronger is the correlation between international bond markets, a result which is expected. As for the exchange rates (Tables 11a-b), the model gives a threshold estimate of $\hat{\gamma} = 16.14$, which is about in the middle of the VIX distribution. We find that in the high-volatility regime the correlations between the exchanges rates become weaker (in absolute value).

The next step of our analysis is to test the hypothesis of constant correlations against threshold-type correlations with the application of the sup*LCM* and Ave*LCM* tests given by Eq. (11a-b). This is done for the three blocks seperately using the *best* transition variables from the aforementioned analysis. The *p*-values were calculated using the model-based bootstrap experiment with *B*=300 replications (described in Section 2.4). The results, reported in Table 12, show that all six *p*-values are effectively zero. Thus, we conclude that there is strong support for the threshold specifications for those three transition variables.

3.3 Economic significance

In this section, we investigate the implications of time-varying correlations on asset allocation. So far, the literature has centred on evaluating the statistical performance of correlation models rather than their economic significance. In contrast, we focus on the latter. Specifically, we examine the performance of the TCC specification against the constant conditional correlation (CCC) model (benchmark) using the Diebold-Mariano type test proposed by Engle and Colacito (2006).

Suppose that we have two different time series of the covariance matrices $\{H_t^j\}_{j=1}^2$ and a set of hypothesized vectors of expected returns $\{\mu^k\}_{k=1}^K$ (divided by the required excess return, μ_0), where $j = 1,2$ corresponds to the benchmark and alternative (TCC) models. For each period we calculate a set of optimal portfolio weights, $w_t^{j,k}$ based on a covariance matrix and on an expected return. The portfolio´s return is given by

$$\pi_t^{j,k} = (w_t^{j,k})'(r_t - \bar{r})$$

where $w_t^{j,k} = \frac{(H_t^j)^{-1}(\mu^k)}{(\mu^k)'(H_t^j)^{-1}(\mu^k)}$ and $\bar{r}$ denotes (sample) mean returns. The Diebold-Mariano type test statistic is calculated as the difference between realized portfolio variance obtained from the CCC and the TCC models

$$u_t^k = (\pi_t^{1,k})^2 - (\pi_t^{2,k})^2$$

Under the null hypothesis, the expected value of u_t^k is zero for all k implying that the threshold model does not reduce portfolio' s variance. To improve the sampling properties of the test, we also perform the weighted version of the test where we divide u_t^k by its standard deviation

$$v_t^k = u_t^k \left[2\left(\mu^{k'}(H_t^1)^{-1}\mu^k\right)\left(\mu^{k'}(H_t^2)^{-1}\mu^k\right)\right]^{1/2}$$

We implement a GMM procedure to estimate jointly

$$\begin{aligned} u_t^1 &= \beta + \varepsilon_{u,t}^1 \\ u_t^2 &= \beta + \varepsilon_{u,t}^2 \\ &\ldots \\ u_t^K &= \beta + \varepsilon_{u,t}^K \end{aligned} \qquad (12)$$

or the weighted version of the test

$$\begin{aligned} v_t^1 &= \beta + \varepsilon_{v,t}^1 \\ v_t^2 &= \beta + \varepsilon_{v,t}^2 \\ &\ldots \\ v_t^K &= \beta + \varepsilon_{v,t}^K \end{aligned} \qquad (13)$$

The null hypothesis of equal variances is $H_0: \beta = 0$. In a multivariate setting, one problem in running Eq. (12)-(13) is the choice of the appropriate vector of expected returns, which may lead to an unbearable number of possible combinations. We follow Engle and Colacito (2006) and focus on hedging portfolios. More specifically, we select vectors of expected returns for which one entry is equal to one, while everything else is zero. That is, one asset is hedged against all other assets.

Table 13 shows the GMM estimates of coefficients and *t*-statistics for the test. As seen, the threshold correlation modelling gives rise to a significant reduction in portfolio´s variance. This holds for all three markets. As expected, the weighted version of the test implies stronger rejections of the null of equal variance. Thus, we conclude that from an asset allocation point of view, the threshold model for correlations improves significantly on the benchmark constant correlation specification.

4. Testing for large TCC models

One important issue is whether the previous three blocks can be unified in a single time-varying correlation framework. In particular, given the different transition variables for equities and for bonds in our application, can we estimate a single TCC specification for these two financial markets (blocks)? More importantly, can we allow for different parts in the correlation matrix to be governed by different transition variables? If so, how can we guarantee that the resulting correlation matrix is positive definite?

Suppose that we have two different groups of assets such as equities and bonds. Their conditional correlation matrix is given by

$$Var\begin{pmatrix} \eta_t^s \\ \eta_t^b \end{pmatrix} = \begin{pmatrix} R_t^s & R_t^{sb} \\ R_t^{bs} & R_t^b \end{pmatrix} = R_t$$

where η_t^s is a $n_1 \times 1$ vector of standardized equity return shocks and η_t^b is a $n_2 \times 1$ vector of standardized bond return shocks. The upper block of the diagonal of R_t denoted by R_t^s is the correlation matrix of the equities, while the lower block of the diagonal of R_t denoted by R_t^b is the correlation matrix of the bonds. Then, the off-diagonal blocks R_t^{sb} (and $R_t^{bs} = (R_t^{sb})'$) are the cross-correlations between equities and bonds.[4]

Based on our previous results, consider a TCC model with two threshold variables

$$R_t = R_1 I[s_{t-1}^s \le \gamma_s \wedge s_{t-1}^b \le \gamma_b] + R_2 I[s_{t-1}^s \le \gamma_s \wedge s_{t-1}^b > \gamma_b] + R_3 I[s_{t-1}^s > \gamma_s \wedge s_{t-1}^b \le \gamma_b] + R_4 I[s_{t-1}^s > \gamma_s \wedge s_{t-1}^b > \gamma_b]$$

[4] Note that this part of the correlation matrix is neither a correlation matrix with ones on the diagonal nor necessarily a square matrix.

(14)

where $R_i = \begin{bmatrix} R_i^s & R_i^{sb} \\ R_i^{bs} & R_i^b \end{bmatrix}$, i =1,…,4. It is of interest to test whether s_{t-1}^b implies a statistically significant change in the correlation matrix of the equities and/or s_{t-1}^s produces a statistically significant change in the correlation matrix of the bonds. Regarding the cross-correlations between equities and bonds, it is intuitive to allow for both variables to act as transition variables. In particular, the relevant null hypothesis is given by[5]

$$H_0 : R_1^s = R_2^s \wedge R_3^s = R_4^s \wedge R_1^b = R_3^b \wedge R_2^b = R_4^b \qquad (15)$$

In this case, under H_0 the correlations of equities are governed only by s_{t-1}^s while the correlations of bonds are driven only by s_{t-1}^b. Notice that given that s_{t-1}^s is the threshold of equities (e.g., VIX_{t-1} from the previous analysis) and s_{t-1}^b is the threshold of bonds (e.g., $Abs(Pc_5)_{t-1}$ from the previous analysis) under the null hypothesis both threshold parameters are identified. Thus, under the null there are no nuisance parameters. That is, the model is a TCC both under the null and the alternative. This implies that the threshold estimates are superconsistent and thus a classical Wald-type test can be applied taking γ_s and γ_b as known.

As seen, under the alternative hypothesis, the model is a TCC model with two thresholds (double-threshold TCC) for all correlations pairs. Thus, if we reject the null we estimate a four-regime TCC model. In this case, estimation can be carried out by extending the estimation procedure of the two-regime TCC described in section 2.2.

Interestingly, under the null hypothesis the model is a restricted four-regime TCC model that allows for different parts in the correlation matrix to be governed by different transition variables. Thus, if we do not reject the null we should estimate this model by guaranteeing that the resulting correlation matrix is positive definite. To do so we proceed as follows. Consider the following transformation of the standardized shocks

[5] Notice that the null can be decomposed in two null hypotheses: $H_0 : R_1^s = R_2^s \wedge R_3^s = R_4^s$ and $H_0 : R_1^b = R_3^b \wedge R_2^b = R_4^b$. Under the former null the correlations of equities are governed only by s_{t-1}^s while under the latter, the correlations bonds are driven only by s_{t-1}^b.

$$\eta_t^* = \begin{pmatrix} \eta_t^{s*} \\ \eta_t^{b*} \end{pmatrix} = \begin{pmatrix} R_t^{s^{-1/2}} \eta_t^s \\ R_t^{b^{-1/2}} \eta_t^b \end{pmatrix}$$

The variance of the vector η_t^* is given by

$$Var(\eta_t^*) = \begin{pmatrix} I_{n_1} & R_t^{s^{-1/2}} R_t^{sb} R_t^{b^{-1/2}} \\ R_t^{b^{-1/2}} R_t^{bs} R_t^{s^{-1/2}} & I_{n_2} \end{pmatrix}$$

with

$$R_t = \begin{pmatrix} R_t^{s^{1/2}} & 0 \\ 0 & R_t^{b^{1/2}} \end{pmatrix} \begin{pmatrix} I_{n_1} & R_t^{s^{-1/2}} R_t^{sb} R_t^{b^{-1/2}} \\ R_t^{b^{-1/2}} R_t^{bs} R_t^{s^{-1/2}} & I_{n_2} \end{pmatrix} \begin{pmatrix} R_t^{s^{1/2}} & 0 \\ 0 & R_t^{b^{1/2}} \end{pmatrix} \qquad (16)$$

Notice that by construction Eq. (16) guarantees that R_t is positive definite if $Var(\eta_t^*)$ is positive definite.

To estimate Eq. (16), we use two-regime TCC models for equities and for the bonds, separately (as in the previous section) and obtain consistent estimates for R_t^s and R_t^b. Then, we calculate $\hat{\eta}_t^* = \left(\hat{R}_t^{s^{-1/2}} \hat{\eta}_t^s,\ \hat{R}_t^{b^{-1/2}} \hat{\eta}_t^b \right)'$ and use again a four-regime TCC specification to estimate the variance matrix $Var(\eta_t^*)$. Finally, we recover the estimate of R_t^{sb} from the estimate of $R_t^{s^{-1/2}} R_t^{sb} R_t^{b^{-1/2}}$.

Table 14 shows that the null hypothesis in Eq. (15) is strongly rejected. This applies to both the joint and block-wise tests. Thus, we proceed by estimating a four-regime (double-threshold) TCC specification. The results are reported in the Appendix. To investigate the economic implication of such a large TCC specification we perform the Engle and Colacito (2006) test described in Section 3.3. Under the null hypothesis the unrestricted four-regime TCC model in Eq. (14) reduces the portfolio's variance compared to the restricted four-regime TCC in Eq. (16). Table 15 shows the GMM estimates of coefficients and *t*-statistics for the test. As seen, both versions of the test are highly significant. Thus, we conclude that from an economic point of view, the unrestricted four-regime TCC improves significantly over the restricted four-regime TCC model.

Regarding the blocks of equities and exchange rates, our previous analysis shows that these two financial markets have a common transition variable (VIX index). Therefore, the issue here is to test for an additional regime in a two-regime TCC specification for these two blocks jointly. In particular, under the null hypothesis a two-regime TCC specification is adequate whereas the alternative supports a three-regime TCC specification. The correlation model under the alternative is given by

$$R_t = R_1 I[s_{t-1} < \gamma_1] + R_2 I[\gamma_1 \leq s_{t-1} < \gamma_2] + R_3 I[s_{t-1} \geq \gamma_2]$$

or what is the same

$$R_t = R_1 I[s_{t-1} < \gamma_1] + R_2 I[\gamma_1 \leq s_{t-1}] + \Lambda I[s_{t-1} \geq \gamma_2] \quad (17)$$

with $H_0 \colon \Lambda = R_3 - R_2$. The null of two-regime TCC is tested as $H_0 \colon \Lambda = 0$. This is a special case of the unrestricted four-regime TCC as the transition variables are the same. In order to ensure identification we require $\gamma_1 < \gamma_2$ and hence that the two correlation transitions occur at different values of the transition variable.

Under the null hypothesis γ_2 is not identified, so we use a supremum-type test. As in the case of no threshold effect test (described in Section 2.3) we use the Lagrange Multiplier statistic developed by Tse (2000). For that we need to calculate the derivatives of the log-likelihood with respect to all the parameter (that appear under the alternative) evaluated in the estimated parameters under the null.

Table 16 shows that the null hypothesis of an adequate two-regime TCC model for stock and exchange rates is not rejected. Thus, we do not proceed with estimation of a three-regime TCC specification. All in all, for these two groups of assets (equities and exchange rates) we conclude that a two-regime TCC specification captures adequately the correlation dynamics.

In principle, one could follow the same steps and extend the above analysis to estimate an even larger TCC model (one-threshold or multi-threshold) for all three financial markets. However, this is beyond the scope of this paper and is left to the interested reader.

5. Conclusions

In this paper we propose a (single and multi-threshold) threshold conditional correlation (TCC) model. The appealing feature of this correlation model is that it does not suffer from the course of dimensionality. In particular, estimation of the parameters of the TCC involves a simple grid search procedure. In addition, it is easy to guarantee a positive definite correlation matrix because the TCC estimator is given by the sample correlation matrix, which is positive definite by construction. The methodology is illustrated by evaluating the behaviour of international equities, govenrment bonds and major exchange rates, first separately and then jointly. We also test and allow for different parts in the correlation matrix to be governed by different transition variables. For this, we estimate a multi-threshold TCC specification. Further, from an economic point of view, we conclude that TCC model gives rise to a significant reduction in portfolio´s variance.

Table 1: Finite-sample properties of threshold effect test

Transition variable		Realized volatility			Principal components		
		1%	5%	10%	1%	5%	10%
$\rho = 0.8$	sup*LCM*	0.026	0.053	0.10	0.016	0.08	0.136
	Ave*LCM*	0.023	0.053	0.103	0.026	0.093	0.13
$\rho = 0.3$	sup*LCM*	0.01	0.046	0.10	0.013	0.07	0.12
	Ave*LCM*	0.006	0.05	0.096	0.016	0.066	0.126
$\rho = -0.3$	sup*LCM*	0.01	0.056	0.086	0.026	0.066	0.13
	Ave*LCM*	0.013	0.06	0.106	0.02	0.056	0.136
$\rho = -0.8$	sup*LCM*	0	0.056	0.116	0.01	0.056	0.106
	Ave*LCM*	0.013	0.043	0.096	0.01	0.073	0.116

Notes: Actual size of the threshold effect test (rejection frequencies). The sample size is *T*=500, the number of Monte Carlo replications is *M*=300, and the number of bootstrap replications is *B*=300.

Table 2: Summary statistics

Abbr.	Mean	St.dev	Skewness	Kurtosis	Standardized Skewness	Standardized Kurtosis	LB- Squared Standardized
Australia Equities - ASX 200	0.103	2.067	-0.581	5.713	-0.156	2.035	0.856
UK Equities - FTSE 100	0.079	2.336	-0.569	5.873	-0.097	2.104	0.275
Hong Kong Equities – HANG SENG	0.139	3.694	-0.565	6.917	-0.123	1.855	0.189
France Equities – CAC 40	0.070	2.848	-0.346	5.266	-0.130	2.100	0.766
US Equities - S&P 500	0.116	2.416	-1.139	14.98	-0.148	2.114	0.550
Germany Equities - DAX 30	0.140	3.018	-0.575	6.256	-0.150	2.104	0.157
Japan Equities - NIKKEI 225	-0.055	3.043	-0.285	5.567	-0.039	2.084	0.214
Germany bonds - GER 10Y	0.139	1.583	0.372	5.009	-0.081	2.138	0.409
France bonds - FR 10Y	0.147	1.568	0.288	4.927	-0.095	2.141	0.703
Italy bonds - IT 10Y	0.151	1.813	-0.237	7.050	-0.100	2.058	0.129
Japan bonds - JP 10Y	0.128	1.666	0.639	7.201	-0.033	2.144	0.268
UK bonds - UK 10Y	0.130	1.532	-0.486	5.875	-0.104	2.126	0.574
US bonds – US 10Y	0.116	1.017	0.031	4.633	-0.096	2.028	0.499
British pound – GBP	-0.013	1.358	-0.946	9.034	-0.094	2.102	0.454
Australian dollar – AUD	0.027	1.647	-0.842	7.349	-0.020	2.071	0.200
Swiss franc – CHF	-0.046	1.533	-0.591	6.935	0.051	2.123	0.088
Euro – EUR	-0.005	1.410	-0.074	5.980	0.060	2.049	0.035**
Japanese yen - JPY	-0.043	1.492	-0.921	9.798	-0.008	2.123	0.103
South African rand – ZAR	0.090	2.101	0.834	11.91	0.007	2.084	0.724

Notes: The standardized skewness and kurtosis are the skewness and kurtosis of the returns standardized by the realized standard deviation. The Ljung-Box statistic (*p*-value) tests autocorrelation up to 31 lags in the squares of standardized returns, distributed as chi-2 with 31 degrees of freedom. Source is DataStream.

Table 3: Unconditional correlations between equity returns

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225
ASX 200	1						
FTSE 100	0.588	1					
HANG SENG	0.539	0.539	1				
CAC 40	0.556	0.829	0.507	1			
S&P 500	0.547	0.694	0.474	0.696	1		
DAX 30	0.555	0.771	0.527	0.851	0.706	1	
NIKKEI 225	0.522	0.438	0.430	0.451	0.438	0.452	1

Table 4: Unconditional correlations between bond returns

	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
GER 10Y	1					
FR 10Y	0.958	1				
IT 10Y	0.774	0.818	1			
JP 10Y	0.415	0.379	0.241	1		
UK 10Y	0.680	0.679	0.618	0.236	1	
US 10Y	0.391	0.387	0.308	0.248	0.401	1

Table 5: Unconditional correlations between exchange rates

	GBP	AUD	CHF	EUR	JPY	ZAR
GBP	1					
AUD	0.398	1				
CHF	-0.606	-0.346	1			
EUR	-0.684	-0.452	0.895	1		
JPY	-0.174	-0.099	0.387	0.319	1	
ZAR	-0.284	-0.424	0.268	0.345	0.027	1

Table 6: Equities pairwise analysis

Transition Variable	Mean Position	Loss Function
VIX	1.95	-30.75
Realized variance of FTSE 100	3.14	-122.71
Realized variance of S&P 500	3.33	-131.58
Realized variance of DAX 30	8.04	-199.10
Absolute value of the 7th principal component of series	9.85	-240.74
Realized variance of CAC 40	9.09	-243.92
Realized variance of ASX200	14.04	-259.50
…		
Absolute value of CAC 40	41.23	-387.23

Table 7: Bonds pairwise analysis

Transition Variable	Mean Position	Loss Function
Absolute value of the 5th principal component of series	2.13	-68.99
Absolute value of the 6th principal component of series	3.66	-102.62
VIX	13.88	-307.45
3rd principal component of the realized variance of series	8.73	-387.80
5th principal component of the absolute value of series	8.73	-420.13
6th principal component of series	6.20	-424.55
6th principal component of the absolute value of series	7.80	-469.72
…		
Absolute value of JP 10Y	31.93	-630.29

Table 8: Exchange rates pairwise analysis

Transition Variable	Mean Position	Loss Function
VIX	15.19	-110
Realized variance of ZAR	18.09	-132.19
Realized variance of AUD	13.85	-134.38
Realized variance of GBP	11.80	-141.82
1st principal component of the realized variance of series	21.80	-148.06
2nd absolute value of principal component of series	21.57	-149.51
2nd principal component of series	22.04	-153.44
…		
2nd principal component of absolute value of series	31.33	-192.10

Table 9a: Single-transition TCC correlations for equities
Regime1: $VIX_{t-1} \leq 21.33$ (low volatility)

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225
ASX 200	1						
FTSE 100	0.410	1					
HANG SENG	0.362	0.397	1				
CAC 40	0.356	0.692	0.368	1			
S&P 500	0.347	0.514	0.361	0.500	1		
DAX 30	0.413	0.645	0.401	0.710	0.516	1	
NIKKEI 225	0.365	0.315	0.263	0.313	0.320	0.292	1

Table 9b: Single-transition TCC correlations for equities
Regime 2: $VIX_{t-1} > 21.33$ (high volatility)

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225
ASX 200	1						
FTSE 100	0.582	1					
HANG SENG	0.624	0.598	1				
CAC 40	0.603	0.860	0.598	1			
S&P 500	0.621	0.717	0.590	0.775	1		
DAX 30	0.611	0.773	0.622	0.870	0.732	1	
NIKKEI 225	0.573	0.474	0.578	0.516	0.458	0.503	1

Table 10a: Single-transition TCC correlations for bonds
Regime 1: $Abs(Pc_5)_{t-1} \leq 0.201$ (low Italian bond premium)

	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
GER 10Y	1					
FR 10Y	0.970	1				
IT 10Y	0.939	0.947	1			
JP 10Y	0.407	0.385	0.363	1		
UK 10Y	0.689	0.697	0.666	0.323	1	
US 10Y	0.417	0.397	0.366	0.301	0.465	1

Table 10b: Single-transition TCC correlations for bonds
Regime 2: $Abs(Pc_5)_{t-1} > 0.201$ (high Italian bond premium)

	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
GER 10Y	1					
FR 10Y	0.892	1				
IT 10Y	0.622	0.672	1			
JP 10Y	0.366	0.326	0.132	1		
UK 10Y	0.570	0.567	0.492	0.164	1	
US 10Y	0.189	0.181	0.176	0.108	0.298	1

Table 11a: Single-transition TCC correlations for exchange rates
Regime 1: $VIX_{t-1} \leq 16.14$ (low volatility)

	GBP	AUD	CHF	EUR	JPY	ZAR
GBP	1					
AUD	0.302	1				
CHF	-0.706	-0.293	1			
EUR	-0.719	-0.282	0.901	1		
JPY	-0.331	-0.116	0.498	0.475	1	
ZAR	-0.365	-0.177	0.340	0.413	0.127	1

Table 11b: Single-transition TCC correlations for exchange rates
Regime 2: $VIX_{t-1} > 16.14$ (high volatility)

	GBP	AUD	CHF	EUR	JPY	ZAR
GBP	1					
AUD	0.288	1				
CHF	-0.535	-0.310	1			
EUR	-0.618	-0.403	0.880	1		
JPY	-0.114	-0.155	0.287	0.244	1	
ZAR	-0.280	-0.377	0.319	0.367	0.036	1

Table 12: Constancy tests

	sup*LCM*	Ave*LCM*
Equities	202.1	129.8
	(0.000)	(0.000)
Bonds	280.5	176.6
	(0.000)	(0.000)
Exchange rates	105.3	66.7
	(0.000)	(0.000)

Notes: sup*LCM* and Ave*LCM* statistics, see Eq. (11a-b). Bootstrapped *p*-values in parentheses, where the number of bootstrap replications is *B*=300. The tests are performed using the following transition variables: VIX_{t-1} for equities and exchange rates, $Abs(Pc_5)_{t-1}$ for bonds.

Table 13: Asset allocation tests 1

	Diebold-Mariano test (unweighted)	Diebold-Mariano test (weighted)
Equities	-0.014	-0.019
	(-2.229)	(-5.104)
Bonds	-0.004	-0.017
	(-3.200)	(-3.952)
Exchange rates	-0.006	-0.029
	(-2.736)	(-4.872)

Notes: GMM estimates of coefficients and *t*-statistics for testing that the TCC produces a smaller variance than the constant correlation model (CCC). Negative values are evidence in favour of the TCC.

Table 14: Testing for large TCC models (equities and bonds)

	Wald statistic (*p*-value)
H_0: $R_1^s = R_2^s \wedge R_3^s = R_4^s$	105.8 (0.000)
H_0: $R_1^b = R_3^b \wedge R_2^b = R_4^b$	117.6 (0.000)
H_0: $R_1^s = R_2^s \wedge R_3^s = R_4^s \wedge R_1^b = R_3^b \wedge R_2^b = R_4^b$	257.9 (0.000)

Notes: Tests restricted four-regime TCC vs. unrestricted four-regime TCC. The first null hypothesis tests for the significance of $Abs(Pc_5)_{t-1}$ as a threshold variable (additional to VIX_{t-1}) for the correlation matrix of the equities. The number of restrictions for the equities is 42, so the Wald statistic follows a χ^2_{42}. The second null tests for the significance of VIX_{t-1} as a threshold variable (additional to $Abs(Pc_5)_{t-1}$) for the correlation matrix of the bonds. The number of restrictions for the bonds is 30, so the Wald statistic follows a χ^2_{30}. The third null hypothesis tests jointly for the aforementioned two hypotheses.

Table 15: Asset allocation tests 2 (equities and bonds)

Diebold-Mariano test (unweighted)	Diebold-Mariano test (weighted)
-0.011 (-8.890)	-0.045 (-8.438)

Notes: GMM estimates of coefficients and *t*-statistics for testing that the unrestricted four-regime TCC in Eq. (14) produces a smaller variance than the restricted four-regime TCC in Eq. (16). Negative values are evidence in favour of the unrestricted four-regime model.

Table 16: Testing for large TCC models (equities and exchange rates)

sup*LCM*	Ave*LCM*
0.699	**0.995**
(0.150) ?	**(0.125) ?**

Notes: Tests of two-regime TCC vs. three-regime TCC models. sup*LCM* and Ave*LCM* statistics. Bootstrapped *p*-values in parentheses, where the number of bootstrap replications is *B*=300. The tests are performed using VIX_{t-1} as transition variable.

Table A1: Data Overview

Symbol	Name
ASX 200	S&P/ASX 200 - PRICE INDEX (Australia)
FTSE 100	FTSE 100 - PRICE INDEX (United Kingdom)
HANG SENG	HANG SENG - PRICE INDEX (Hong Kong)
CAC 40	CAC 40 - PRICE INDEX (France)
S&P 500	S&P 500 COMPOSITE - PRICE INDEX (United States)
DAX 30	DAX 30 PERFORMANCE - PRICE INDEX (Germany)
NIKKEI 225	NIKKEI 225 STOCK AVERAGE – PRICE INDEX (Japan)
GER 10Y	BD BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$) (Germany)
FR 10Y	FR BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$)(France)
IT 10Y	IT BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$) (Italy)
JP 10Y	JP BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$) (Japan)
UK 10Y	UK BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$) (United Kingdom)
US 10Y	US BENCHMARK 10 YEAR DS GOVT. INDEX - TOT RETURN IND (~U$) (United States)
GDP	GBP TO USD (BOE) - EXCHANGE RATE (British pound vs. $)
AUD	AUD TO USD (BOE) - EXCHANGE RATE (Australian dollar vs. $)
CHF	USD TO CHF (BOE) - EXCHANGE RATE (Swiss franc vs. $)
EUR	USD TO EUR (BOE) - EXCHANGE RATE (Euro vs. $)
JPY	USD TO JPY (BOE) - EXCHANGE RATE (Yen vs. $)
ZAR	USD TO ZAR (BOE) - EXCHANGE RATE (South African rand vs. $)

Table A2: Double-transition TCC correlations for equities and bonds
Regime 1: $VIX_{t-1} \leq 19.9480 \wedge Abs(Pc_5)_{t-1} \leq 0.2054$

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
ASX 200	1												
FTSE 100	0.491	1											
HANG SENG	0.355	0.400	1										
CAC 40	0.455	0.783	0.452	1									
S&P 500	0.445	0.583	0.410	0.617	1								
DAX 30	0.415	0.709	0.470	0.817	0.596	1							
NIKKEI 225	0.430	0.456	0.318	0.426	0.462	0.408	1						
GER 10Y	0.003	-0.126	0.038	-0.119	-0.016	-0.189	-0.183	1					
FR 10Y	-0.007	-0.110	0.053	-0.087	0.007	-0.177	-0.183	0.978	1				
IT 10Y	0.004	-0.102	0.050	-0.088	0.005	-0.171	-0.174	0.989	0.988	1			
JP 10Y	-0.022	-0.233	0.032	-0.199	-0.052	-0.225	-0.267	0.559	0.544	0.524	1		
UK 10Y	-0.045	-0.151	0.112	-0.118	0.003	-0.173	-0.134	0.770	0.769	0.760	0.437	1	
US 10Y	-0.051	-0.173	-0.018	-0.160	-0.074	-0.208	-0.104	0.371	0.379	0.364	0.356	0.478	1

Table A2: Double-transition TCC correlations for equities and bonds
Regime 2: $VIX_{t-1} \leq 19.9480 \wedge Abs(Pc_5)_{t-1} > 0.2054$

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
ASX 200	1												
FTSE 100	0.346	1											
HANG SENG	0.347	0.345	1										
CAC 40	0.264	0.595	0.249	1									
S&P 500	0.259	0.426	0.308	0.378	1								
DAX 30	0.417	0.583	0.297	0.582	0.423	1							
NIKKEI 225	0.313	0.203	0.167	0.212	0.187	0.170	1						
GER 10Y	0.096	-0.045	0.085	-0.175	0.095	-0.046	-0.040	1					
FR 10Y	0.028	-0.020	0.065	-0.031	0.079	-0.048	-0.097	0.844	1				
IT 10Y	0.178	0.085	0.223	0.116	0.158	0.142	0.084	0.418	0.490	1			
JP 10Y	0.015	-0.083	0.029	-0.252	-0.045	-0.139	-0.254	0.377	0.327	0.071	1		
UK 10Y	0.168	0.088	0.159	0.051	0.201	0.068	0.044	0.514	0.526	0.417	0.193	1	
US 10Y	0.090	0.123	0.113	0.132	0.448	0.203	0.048	0.248	0.240	0.254	0.082	0.333	1

Table A3: Double-transition TCC correlations for equities and bonds
Regime 3: $VIX_{t-1} > 19.9480 \wedge Abs(Pc_5)_{t-1} \leq 0.2054$

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
ASX 200	1												
FTSE 100	0.500	1											
HANG SENG	0.556	0.596	1										
CAC 40	0.551	0.837	0.573	1									
S&P 500	0.576	0.648	0.539	0.755	1								
DAX 30	0.574	0.753	0.588	0.864	0.725	1							
NIKKEI 225	0.517	0.453	0.571	0.486	0.445	0.480	1						
GER 10Y	-0.037	-0.131	-0.039	-0.121	-0.055	-0.099	-0.046	1					
FR 10Y	-0.010	-0.108	-0.018	-0.092	-0.012	-0.077	-0.040	0.977	1				
IT 10Y	0.004	-0.085	-0.009	-0.075	-0.003	-0.051	-0.018	0.985	0.983	1			
JP 10Y	-0.220	-0.258	-0.158	-0.254	-0.196	-0.238	-0.225	0.388	0.367	0.313	1		
UK 10Y	-0.015	-0.135	-0.048	-0.055	-0.007	-0.068	-0.109	0.710	0.717	0.692	0.255	1	
US 10Y	-0.201	-0.256	-0.274	-0.236	-0.237	-0.279	-0.222	0.395	0.378	0.358	0.318	0.429	1

Table A4: Double-transition TCC correlations for equities and bonds
Regime 4: $VIX_{t-1} > 19.9480 \wedge Abs(Pc_5)_{t-1} > 0.2054$

	ASX 200	FTSE 100	HANG SENG	CAC 40	S&P 500	DAX 30	NIKKEI 225	GER 10Y	FR 10Y	IT 10Y	JP 10Y	UK 10Y	US 10Y
ASX 200	1												
FTSE 100	0.635	1											
HANG SENG	0.673	0.613	1										
CAC 40	0.590	0.860	0.627	1									
S&P 500	0.599	0.776	0.602	0.737	1								
DAX 30	0.562	0.749	0.631	0.860	0.676	1							
NIKKEI 225	0.551	0.395	0.499	0.457	0.379	0.423	1						
GER 10Y	-0.015	-0.049	0.058	-0.182	-0.083	-0.221	-0.017	1					
FR 10Y	0.029	0.007	0.114	-0.116	-0.009	-0.172	0.066	0.933	1				
IT 10Y	0.113	0.117	0.164	-0.045	0.022	-0.113	0.140	0.811	0.843	1			
JP 10Y	-0.040	-0.044	0.017	-0.051	-0.034	-0.138	-0.184	0.160	0.120	0.131	1		
UK 10Y	0.000	-0.039	0.187	0.034	-0.019	-0.009	0.002	0.473	0.458	0.410	0.116	1	
US 10Y	-0.131	-0.119	-0.096	-0.237	-0.124	-0.192	-0.219	0.247	0.192	0.082	0.074	0.345	1

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