UNIVERSITAT
ROVIRA I VIRGILI

DEPARTAMENT D'ECONOMIA

(creip)
CENTRE DE RECERCA EN ECONOMIA INDUSTRIAL I ECONOMIA PÚBLICA

WORKING PAPERS

Col·lecció “DOCUMENTS DE TREBALL DEL DEPARTAMENT D’ECONOMIA - CREIP”

Why do we ignore risk in schooling decisions?

Joop Hartog
Luis Diaz-Serrano

Document de treball n.12- 2014

DEPARTAMENT D’ECONOMIA – CREIP
Facultat d’Economia i Empresa

UNIVERSITAT ROVIRA I VIRGILI
DEPARTAMENT D'ECONOMIA

Edita:

Departament d'Economia
www.fcee.urv.es/departaments/economia/public_html/index.html
Universitat Rovira i Virgili
Facultat d'Economia i Empresa
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 759 811
Fax: +34 977 300 661
Email: sde@urv.cat

CREIP
www.urv.cat/creip
Universitat Rovira i Virgili
Departament d'Economia
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 558 936
Email: creip@urv.cat

Adreçar comentaris al Departament d'Economia / CREIP

Dipòsit Legal: T - 834 - 2014

ISSN edició en paper: 1576 - 3382
ISSN edició electrònica: 1988 - 0820

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

Why do we ignore risk in schooling decisions?

Joop Hartog

Universiteit van Amsterdam (j.hartog@uva.nl)

Luis Diaz-Serrano

CREIP - Universitat Rovira i Virgili (luis.diaz@urv.cat)

Abstract

While uncertainty abounds in almost any decision on investment in schooling, it is mostly ignored in research and virtually absent in labour economics tekst books. This paper documents the scope for risk, discusses the tough disentanglement of heterogeneity and risk, surveys the analytical models, laments the absence of a good workhorse model and points out the challenges worth tackling: document ex ante risk that investors face, develop a tractable and malleable analytical model and integrate the option of consumption smoothing in analytical and empirical work. Hedging labour market risk in the stock market can be safely ignored.

This paper builds on Joop Hartog and Luis Diaz Serrano (2014), *Schooling as a risky investment*, Foundations and Trends, NOW publishers, Cambridge, Mass. Details of claims and statements made in this paper can be found there.

We acknowledge the financial support from the Spanish Ministry of Education and Science through Grant ECO2010-20829.

1. Introduction

Schooling decisions are drenched in uncertainty like fish swimming in water, but textbook treatment of human capital theory does not reflect this. With the exception of Yoram Weiss' chapter in the *Handbook of Labor Economics*, introductory and even advanced labour economics textbooks do not pay attention to the obvious uncertainty that surrounds investment in schooling. Abilities, tastes, requirements of the curriculum, graduation, employment, job content, financial rewards are all inherently uncertain when decisions on school careers have to be made. School admission boards have to select students without fully knowing talent and drive, preferences for general over vocational curricula are motivated by supposedly better insurance against the vaguaries of the labour market, but empirical evidence is barely available. Yet labour economics texts only teach the standard static model of investment in schooling under perfect information on present and future.

We shall provide details of the variation of outcomes of schooling decisions, to support our claim that absence of uncertainty cannot justify the neglect of attention for it in standard presentations of the human capital model. We shall then consider models that do acknowledge risk and conclude that the absence of a flexible workhorse model that can be modelled to the needs of variations in the analytical questions is a much more likely explanation: we have no smooth, flexible, easily malleable analytical model to deal with the various dimensions of risk. We will survey the scattered analytical literature that has been developed, consider the empirical evidence related to these models and conclude with a listing of priorities in the research agenda that does acknowledge uncertainties. Both analyticaly and empirically, lots of interesting challenges are waiting.

2. The scope for risk

This paper focusses squarely (and exclusively) on the risk that is associated with investing in education. Following Frank Knight it is common to use the term “risk” to refer to measurable uncertainty and “uncertainty” to unmeasurable uncertainty. In the case of risk, “the distribution of the outcome in a group of instances is known”, while in the case of uncertainty this is not so (Knight, 1921, 233). All the models and theories discussed in this paper refer to measurable uncertainty and we will use the terms risk and uncertainty interchangeably, as confusion can be ruled out.

In empirical work, there is a key distinction between risk and unobserved heterogeneity. In this section, we will discuss empirical evidence to suggest that risk is truly relevant. Indeed, most of what we present here is suggestive, as much of it concerns ex post variance in outcomes (schooling completion rates, earnings variance, unemployment rates etc), and this is not necessarily the same as risk. Following Knight, we speak of risk when the outcome of an action or decision is uncertain at the time of deciding but the probability distribution over possible outcomes is known to the decision maker. There is heterogeneity when there are differences in the probability distribution of the outcome of an action among individuals that are known to the individuals themselves. Much heterogeneity is unobserved to the outsider (like a researcher), and hence, will make observed ex post variation a biased estimate of risk: it combines unobserved heterogeneity and risk. The distinction crucially depends on the information available to the individual: risk is what is uncertain to him (or her). If the individual is no better informed than the outsider and hence, must use ex post variation as his measure if risk, the bias vanishes. Drawing the line between risk and unobserved heterogeneity is a key issue in research and we will discuss it in the next section. In the present section we refer to data on ex post variation in outcomes to demonstrate there is enough variation to assume that risk may play a role, but we also present evidence on student perceptions of variability and risk, to show that risk is a relevant dimension for individual decision making[1].

Ex post variations

Completion rates of schooling are well below hundred percent and vary substantially among countries, suggesting a strong influence of the institutional structure of the schooling system. Averaged over OECD countries, about 2/3 of the students complete secondary education within the standard time, 80 percent graduate within standard time plus two years. Girls do better than boys. Among countries, completion rates within standard time vary between 40 and almost 90 percent. Completion rates in tertiary education are less well documented. On average in OECD countries for which data are available, about 1/3 of the students leave tertiary education without a diploma. Completion rates are uniformly higher in general education than in vocational education. The earnings loss from not graduating is not well documented and there are no simple robust conclusions. Dropping out is not always associated with earnings loss and years spent in school may well have a decent rate of return even if they are not topped off with a diploma. The effect may differ between secondary and tertiary education. The modestly available evidence suggests that perhaps drop-out years spent in high school have no pay-off and drop-out years in college may get a decent return.

Unconditional, crude unemployment rates decline systematically with level of education. On average across the OECD countries, men with only primary education are 4 times more likely

[1] Details substantiating the statements in this section are given in Hartog and Diaz Serrano (2014).

to be unemployed than men with the most advanced tertiary education, women are 3 times more likely to be unemployed. Again, there is large international variation. Among men, the ratio is less than 1:2 in Portugal and more than 1:8 in Slovenia, among women it is less than 1:1 in Turkey and about 1: 12 in Hungary. When controls are added, in duration analyses, some countries deviate from a strict monotonic relationship with level of education. Econometric analyses show that it is the incidence of unemployment that falls with level of education, not so much its duration.

Mismatch between an individual's attained education and education required in the job is well documented. As a rough indication, the sum of under-education and over-education may perhaps be typically take as some 40%. The quality of the match is relevant for earnings. Over-education implies an earnings loss relative to occupying a job precisely demanding attained years of education, under-education brings a premium.

International comparisons of ex post earnings dispersions must be sensitive to the details of definition and measurement of earnings; many different concepts are in use (e.g. hourly or annual, before or after taxes and social security contributions, administrative or self-reported). They are also sensitive to the extent and nature of differentiation in the school system. A sample of countries from the LIS data database (Luxemburg Income Studies) suggests that ex post earnings dispersion is not related to level of education in an international standard pattern. But data from 13 countries in the European Community Household Panel reveals that in all these countries, standard deviations increase with education, while coefficients of variation do not obey a European standard pattern.

Ex ante risk

When searching for measures of risk relevant for individual's decision making, we essentially ask for the quality of information that they possess and we have to distinguish perceptions, heterogeneity and true risk. We may define risk as the dispersion in an individual's probability distribution of the effect of some action. We may also assume that the individual measures risk by positioning him/herself in a perceived, ex post distribution of outcomes. Risk or uncertainty about the returns to education then follows from the perceived distribution of earnings by conditioning on the self-assessed probability to benefit from that education. The conditioning reflects heterogeneity, the perceptions may differ in accuracy, thus allowing the quality of information about outcomes to play a role in analyzing the effects of risk. In this survey we have not paid attention to psychological research on (over-)

confidence and to other systematic factors that may affect the accuracy of information, but no doubt they are relevant (Dominguez and Swank, 2009).

Most studies suggest that prospective students have a fairly good picture of the general structure of earnings, the effect of schooling, of gender, but with substantial variation among students. Variances tend to be smaller for starting wages than for wages for experienced workers. There is also evidence of focused perceptions: accuracy tends to increase when students approach graduation, and to be better for chosen majors than for rejected majors.

Individually expected earnings, whether for the actually chosen education or for rejected alternatives (such as going to work with secondary education only when a tertiary education is pursued) show large variation among individuals. This translates into large variation among individuals in implicitly anticipated rates of return. A simulation based on available estimates of relevant parameters confirms this as substantial risk in the individual rate of return: the distribution has positive skew and a coefficient of variation of about 0.3. Girls tend to expect lower wages than boys, but often so across the board, thus leaving expected rates of return equal to those expected by boys. Evidence has been found that students on average expect higher wages for themselves than for their peers, in particular among male students. There is clear evidence that expectations for own income are anchored to perceptions on contemporaneous market incomes: overestimation of graduates' earnings translates systematically into higher own expected earnings.

One study presents direct evidence on earnings risk by asking students for the highest and the lowest log earnings they expect after graduation, and finds a standard deviation of the difference of 0.3 at a mean of 0.65. The Manski method (Dominitz and Manski, 1996) of asking for the probabilities of surpassing earnings levels at some threshold level above or below the median, applied in several studies, indicates substantial individual risk in the anticipated earnings that are associated with a given type of education. Evidence on systematic variation of level and dispersion of the anticipated earnings distributions with individual ability is ambiguous. Expected mean (or median) earnings are sometimes significantly related to ability, or class rank, sometimes they are not. Individually measured risk does not appear to be systematically related to personal characteristics.

We conclude that there is enough ex post variation in outcomes to suspect an important role for risk and there is direct evidence that individuals' anticipated benefits from education are not degenerated to a single value. But we only have bits and pieces of information, a mosaïque rather than a full account. Ideally, we would like to know how potential students perceive the actual distributions of potential earnings in the alternatives they consider and how actual earnings vary with choices that can be made later (e.g. working hours, or choosing among jobs with different amenities and hence different compensating wage differentials) and with choices that should be made in the schooling ages of life. The potential earnings they expect for themselves may deviate from perceived actual distributions, as individuals may condition the distributions that apply for themselves, on abilities they believe to have or on preferences they will follow during their life course. These perceptions may be right or wrong, but still condition schooling choices. Here we face an important methodological divide. One may attempt to uncover the structure of actual expectations and anticipations of students (and their parents), by just asking them. Or one may impose an econometric structure and estimate the apparently employed information structure from observing the actual choices that have been made. The former route requires more resources than the latter, is more pedestrian and will no doubt lead to confrontation with many inconsistencies: many students will not be aware of their own way of decision making, let alone follow a strictly rational pattern. But it will most likely be descriptively more accurate and certainly much richer in detail and variation among individuals. The econometric method will be intellectually more challenging, be cheaper in data collection, and in a sense more efficient in the exploitation of available data. But it will also border on a more normative specification of rational decision making. In the end, the choice of methodology is an empirical matter. Yet testing one method against the other by comparing predictive performance is not easy, as typically, the direct method collects data in the early stages of school-work careers, and looks forward, while the econometric method tends to collect data in the advanced stage and looks backward to the schooling decisions. In the next section we will pay attention to the econometric method.

3. Imposing an information structure *a priori*

As just noted, there is an important methodological divide among attempts to measure the information available to individuals when deciding on schooling and hence on distinguishing between unobserved heterogeneity and risk. The obvious, direct method has been pioneered by Manski: just interview individuals on their perceived outcomes of alternative schooling choices, and allow for non-degerate distributions. This is a square focus on ex ante risk as stated by interviewees. The other method is to specify an econometric model, including an imposed structure of information, and estimate the parameters of this model on ex post outcomes. Part of such models is often the decomposition of the random disturbance term in wages or earnings into a transitory and a permanent component and the assumption that the transitory component is fully unpredictable and hence reflects risk, while the permanent component is at least partially known and to that extent reflects unobserved heterogeneity. In this section we will first consider empirical evidence on the decomposition of ex post residual variance in permanent and transitory and then consider more extensive econometric modelling.

3.1 Transitory and permanent residual variance

A standard decomposition measures the permanent component as the dispersion of single lifetime shocks between individuals and the transitory component as the dispersion of annual shocks for given individuals. Table 1 gives results of such a decomposition for data from the European Community Household Panel. Estimates cover the period 1994-2001 and only male workers have been considered. In these data, the permanent component strongly dominates the transitory component. Depending on the country and education level, the ratio varies from 1:2 to 1:4. But the results are quite heterogeneous in international comparison. In nine of the thirteen countries, residual variance of the permanent component, and hence overall residual variance, increases when we move from secondary to tertiary education, whereas we observe the opposite in the remaining six countries. An analogous estimate for Germany using the GSOEP covering the period 1984-2007 provides results similar to the ones using the ECHP. While the transitory shocks may be hard to predict, and thus mostly reflect risk, the permanent effect will also hide much heterogeneity related to the differentiation of school types and educational programmes at both secondary and tertiary level. From that perspective it is perhaps not surprising that international variation in the permanent component is larger than in the transitory component: there are marked international differences in the differentiation of school systems.

Table 1. Residual variance: permanent and transitory component

	Secondary		Tertiary	
	Permanent	**Transitory**	**Permanent**	**Transitory**
Germany (ECHP)	0.404	0.164	0.533	0.149
Germany (GSOEP)	0.359	0.300	0.424	0.301
Denmark	0.415	0.272	0.278	0.155
Netherlands	0.467	0.182	0.784	0.185
Belgium	0.377	0.184	0.481	0.179
France	0.542	0.208	0.594	0.210
UK	0.308	0.242	0.259	0.158
Ireland	0.587	0.184	0.697	0.190
Italy	0.340	0.152	0.401	0.178
Greece	0.374	0.168	0.667	0.184
Spain	0.455	0.239	0.339	0.133
Portugal	0.429	0.177	0.697	0.172
Austria	0.462	0.174	0.392	0.174
Finland	0.431	0.178	0.496	0.148

Note: Regressions include controls for age, job tenure, occupation, industry, hours worked, private/public worker, region and year dummies; men only.
Source: Hartog and Diaz Serrano (2014)

The error term in earnings equations can also be decomposed into a persistent component υ_{it} and a transitory component ε_{it} with the persistent component following a first-order autoregressive scheme, thus allowing for innovation shocks:

$$\ln Y_{it} = X_{it}\beta + \nu_{it} + \varepsilon_{it}$$
$$\nu_{it} = \nu_{i,t-1} + \eta_{it}$$

where the transitory component ε_{it} and the innovation η_{it} are independent. With this specification, residuals from d-period log wage differences, $\sigma_i^2(d)$ are equal to

$$\sigma_i^2(d) = d^2\sigma_\eta^2 + 2\sigma_\varepsilon^2$$

This allows to estimate the variances of the innovation and the transitory shock to be estimated from regressions on d and 2. This so-called Caroll-Samwick decomposition

(Carroll and Samwick, 1997) has been applied widely in lifetime consumption-savings models with information on the risk of different levels of education as a by-product.

Saks and Shore (2005) analyse data from the American PSID (Panel Study of Income Dynamics), focusing on heads of households with 16 or more years of education (i.e. a college education), in 8 occupations: teachers, healthcare workers, computer workers, engineers, math/science workers, sales people, managers, and entertainers. This is not exactly a grouping by college major, but there is an obvious link, and career choices by intended occupation are important components of choices of education. Except for entertainers, the variance of the persistent component is larger, often much larger than the variance of the transitory component. According to this result, the biggest risk is to not know your own talent or aptitude for a profession, and not the volatility during the career. Brown, Fang and Gomes (2012) also use the PSID data, with slightly different specification of the Caroll-Samwick decomposition. They only distinguish between high school and university. Again the persistent component dominates the transitory component, now by at least 1:3. However, high school and college graduates have about equal variances; the variances for high school drop-outs are somewhat higher. Gourinchas and Parker (2002) also using the PSID data, estimate a model with age-specific drift. The results are different from those above: now persistent shocks are substantially smaller than transitory shocks, and the variance of the persistent shock is inverted U-shaped in education, while the variance of the transitory shock is U-shaped. All these results taken together confirm that results may vary substantially depending on model specification. Skyt Nielsen and Vissing-Jorgenson (2006) have estimated earnings variances for 50 types of tertiary education in Denmark with the Carroll-Samwick specification. Annual labour income growth rates are age specific and allowed to depend on prior education and parental background. Results are differentiated by deciles of the distribution. The variance of the transitory shock is about three times the variance of the permanent shock. The variation of dispersion across educations is substantial. The variance of transitory shocks at the 90th percentile is 8 times larger than at the 10th percentile, for persistent shocks it is 10 times larger.

The results from the US and Denmark suggest that suggest that decomposition results strongly depend on the extent of detail in the classification of education. With crude educational classifications, the permanent component often dominates the transitory component. In the refined distinction in Denmark, with 50 categories, the transitory components generally surpass the persistent component, suggesting that in the cruder aggregates much variance reflects heterogeneity of educational programs and disciplines. Students may well be aware of such heterogeneity. Variance in narrowly defined categories may perhaps very well be equated to risk, while the variance in broader categories may only be risk if the individual does not know what his later specialisation will be.

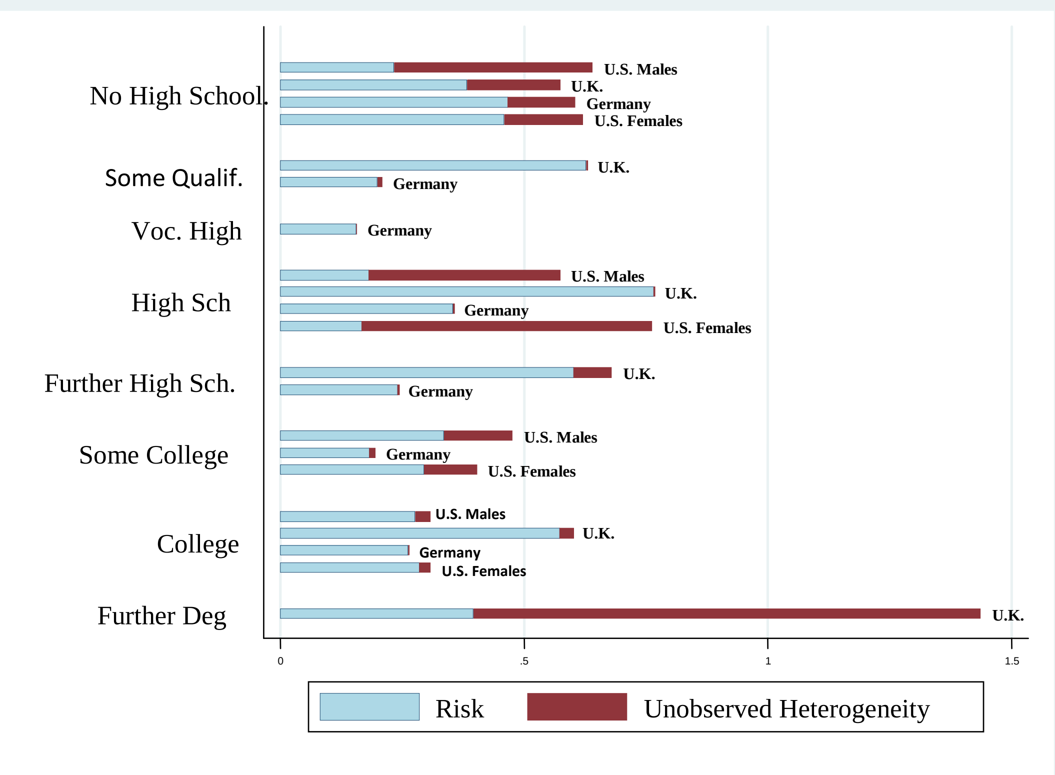
3.2. Imposing an information structure

Stacey Chen (2008) imposes an information structure based on the estimated correlation coefficient between a latent variable that rules schooling choices and a permanent component in residual wages, assuming that individuals know this correlation and know how this shifts the distribution of permanent shocks. Chen wants to find out if higher education leads to lower earnings risk and builds a model of self-selection, but does not allow for risk to affect these choices. She models individuals as endowed with a factor v that rules the choice of education: a single parameter reflecting the tastes and abilities, known to the individual, unobserved by the outsider. Educational choice is modeled as an ordered probit on this taste factor. Potential wage after completing an education has three components: rewards for individual characteristics, a permanent individual fixed effect and an annual transitory shock. Each component is education specific. The rewards for individual characteristics are known to the individual, the transitory shock is fully relegated to uncertainty. The fixed effect (permanent shock) is partly known: only to the extent that it correlates with the schooling taste factor. The remaining part, the extent of imperfect correlation, is an element of the uncertainty faced by an individual.

Chen's estimates for the US are based on the NLSY 1979-2000. Mazza, Van Ophem and Hartog (2013) have replicated Chen's estimates and have also estimated the replication model on the women in the American NLSY sample and on data for the UK and Germany, all with the same instruments for schooling: unemployment and GDP growth at the time of schooling decisions[2]. The UK data are taken from 18 waves of the British Household Panel, the German data are based on 10 waves of the SOEP panel. As shown by the results in Figure 1, neither in the US, nor in the UK or Germany is there a monotonic relationship of risk to level of education. The key conclusion is that risk strongly dominates unobserved heterogeneity, in particular in Germany (there are exceptions in the UK).

[2] Chen's instrument for schooling in the US, college tuition in the respondent's county of residence, is only available to researchers in the US (and not available in the datasets for the other countries).

Figure 1. Risk and unobserved heterogeneity.



Source: Mazza, Van Ophem and Hartog (2013)

Heckman has extensively studied the question of information available at the time of schooling decisions and summarised results in several papers. Cunha et al. (2005) argue that there must be a set of factors affecting future wages which are known by individuals, and hence used as input in their schooling choices. If the factors are not observed by the researcher, then risk defined as the variance of earnings will be overestimated, since these factors will be estimated as part of the risk when in reality it is unobserved heterogeneity. Cunha et al. (2005) build their model on this assumption to separate uncertainty from heterogeneity in the estimation of the variance of earnings by schooling level. The approach resorts to very sophisticated econometric modeling, but essentially entails replacing a simple set of fixed effects in earnings functions and the schooling choice equation by a system of independent latent factors, each constructed from a mixture of independent normal variables. If a factor is significant in both the earning equations and the schooling choice function, that factor must have been in the individual's information set when choosing an education. If the factor is significant in the earnings function but not in the choice equation, then apparently that factor was not in the information set. Earnings distributions for schooling alternatives can be constructed, for chosen and not chosen alternatives, conditional on assumptions about the information set (which factors are known ex ante, which are not?).

The most relevant conclusion is that the variance of lifetime earnings facing an individual decreases significantly if individual heterogeneity is separated from uncertainty. Cunha et al. consider information scenarios differing in the number of factors included in the model. When only one factor is considered, the variance, compared to having no information on any factor, barely falls, by about 1% for college individuals, while for high school the reduction is even smaller (0.27%). The consideration of a second factor reduces the variance by approximately one third for both college and high school graduates. Finally, adding the third factor is unsubstantial for high school graduates, but reduces the variance of college graduates by a bit more than two thirds. According to this result, only one third of the variance of earnings is risk, while the remaining two thirds is just individual unobserved heterogeneity.

The evidence on separating risk from heterogeneity is far from conclusive. Reducing unconditional earnings variance to residual variance within an education category after taking out the effect of observables usually brings a modest reduction in variance, as reflected in low coefficients of determination. This suggests that most observed variance can be interpreted as risk. In a decomposition in panel data of transitory and persistent shocks, the transitory shocks are routinely relegated to risk. In many cases, the persistent shock has larger variance than the transitory shock, but in the refined distinction in Denmark, with 50 categories, the transitory components generally surpass the persistent component, suggesting that in crude aggregates much variance reflects heterogeneity of educational programs and disciplines. Students may well be aware of such heterogeneity. The results by Chen also indicate that most observed variance is just risk[3].

Heckman's claim, however, differs. Cunha and Heckman (2007, p.892) conclude from their survey of several contributions by Heckman and co-authors: "For a variety of market environments and assumptions about preferences, a robust empirical regularity is that over 50% of the *ex post* variance in the returns to schooling are foreseeable at the time students make their college choices". This conclusion is in line with Chen's results for US men. Our

[3] Low, Meghir and Pistaferri (2010) build a model of lifecycle consumption and saving for the US, where labour market risk is decomposed in wage risk and employment risk (with a search model). Permanent risk and transitory risk in the wage differ little and are about equal for high and low educated. The worker-firm match specific component in the wage has substantially larger variance, but is also about equal for high and low educated. Low educated have much higher risk of job destruction, however.

own results with Chen's model (Mazza, Van Ophem and Hartog, 2013) are more in line with the picture emerging from directly asking individuals about their wage expectations. The available literature from several countries suggests that (potential) students' expectations on their future earnings distributions are indeed simply anchored to observed wages for graduates already in the labour market and that deviations between their own expectations and the observed means are not systematically related to their own (perceived) qualities (see Hartog and Diaz Serrano 2013, for details).

4. Modelling demand for education

With a perfect capital market and no uncertainty, investment and consumption decisions are separable, as long as alternative ways of using time have no direct effect on utility (i.e. if labour supply is fixed). The human capital investment decision can then be analysed in isolation from maximising lifetime earnings, as is indeed common practice. Even under certainty, separability breaks down if labour supply is endogenous. Separability breaks down under uncertainty that cannot be hedged, smoothed or perfectly insured.

4.1 Maintaining separability

The simplest way to introduce uncertainty in the investment decision is to make wages stochastic in the basic Becker –Mincer model, thus maintaining separability. So, suppose, an individual faces potential earnings, depending on realised schooling *s*, in a simple multiplicative stochastic specification.

$$Y_{st} = \theta_{st} Y_s ,$$

where Y_{st} is earnings at age *t* for given schooling length *s*, Y_s is a non-stochastic shift parameter and θ_{st} is a stochastic variable. For a start, the model is simplified to $\theta_{st} = \theta_s$ and

$$E\left(\theta_s\right) = 1;$$
$$E\left\{\theta_s - E\left(\theta_s\right)\right\}^2 = \sigma_s^2.$$

θ_s is a stochastic shock around Y_s, with a single lifetime realisation,[4] but with variance dependent on schooling length *s*. This simple specification is similar in spirit to Levhari and Weiss's two period model (see below), with a wage unknown when deciding on schooling, but with a single lifetime realisation (one wage rate for the entire post-school period). We will assume that individuals maximise the lifetime utility of post-school earnings, *W*. Earnings risk cannot be insured and consumption smoothing is ignored: annual consumption is equal to annual earnings. A year in school always has zero utility: students survive on transfers, from parents or government and the utility from that consumption is exactly countered by the disutility from school attendance.

[4] A generalisation of the model, with uncorrelated annual earnings shocks, yields essentially the same conclusions. See our IZA Discussion Paper for details, Hartog and Diaz-Serrano (2002).

$$
\begin{aligned}
W &= E\int_s^{\infty} U\left\{\theta_s Y_s\right\} e^{-\rho t}\, dt \\
&= \frac{1}{\rho} e^{-\rho s}\, E\left[U\left(\theta_s Y_s\right)\right].
\end{aligned}
$$

If we replace $E\left[U\left(\theta_s Y_s\right)\right]$ by its certainty equivalent Y_{cs}, the first-order condition for a maximum is simply

$$\frac{\partial Y_{CE}}{\partial s}\frac{1}{Y_{CE}} = \rho$$

Optimal schooling length is defined by equality of discount rate and rate of return in income adjusted for risk. For further analysis, equate $U(Y_{CE}) = E\{U(\theta_s Y_s)\}$, apply a second-order Taylor series expansion around Y_s to the right-hand side and a first-order expansion to the left-hand side to get the standard risk compensation rule

$$Y_{CE} = (1 - \frac{1}{2}\alpha_s \sigma_s^2) Y_s$$

where α_s is relative risk aversion:

$$\alpha_s = -\frac{U^{''}(Y_s)}{U^{'}(Y_s)} Y_s$$

Differentiating Y_{CE} to *s* and substituting in the first-order condition then yields the optimum condition

$$\mu_s = \frac{1}{2}\frac{Y_s}{Y_{CE}}(\sigma_s^2 \frac{\partial \alpha_s}{\partial s} + \alpha_s \frac{\partial \sigma_s^2}{\partial s}) + \rho$$

where

$$\mu_s = \frac{\partial Y_s}{\partial s}\frac{1}{Y_s}$$

Hence, with constant relative risk aversion and risk constant across schooling, we have the standard condition under certainty of rate of return equal to discount rate. If these conditions do not hold, the rate of return in expected earnings should surpass the discount rate by the changes in risk and risk aversion with schooling. With the rate of return declining in schooling level and the compensation term positive, individuals would choose lower levels of schooling. At constant relative risk aversion, declining risk at higher schooling levels will stimulate schooling, increasing risk will discourage it. In the particular case that the effect of declining relative risk aversion exactly counters the effect of increasing risk, uncertainty would have no effect on the optimal level of schooling. We can easily summarise the result in neat standard predictions: ceteris paribus, lower level of schooling for higher earnings risk, steeper slope of risk across schooling levels, stronger risk aversion and steeper slope of risk aversion across schooling levels (and conversely for the sometimes more realistic reverse cases).

4.2 Giving up separability

The model developed by Levhari and Weiss (1974) is often referred to as the standard model for human capital decisions under uncertainty. The model has two periods, a schooling and a working period, time is allocated in the first period between school and work, in the second period working time is fixed. Leisure has no utility. There are no borrowing constraints, first period consumption can exceed first period earnings. The second period wage is uncertain and affected by investment in schooling. Uncertainty enters as second period earnings depend on time invested in human capital in the first period and a parameter reflecting the state of nature, with a known probability distribution and revelation of the outcome at the beginning of period 2.

Utility only depends on consumption, C_1 in period 1, C_2 in period 2. The objective function is to maximise expected utility $E\{U(C_1, C_2)\}$. In period 1, time is allocated between hours worked H_w and hours spent investing in human capital (schooling), H_s, both expressed as a proportion of time endowment; the constraint implies that we may write $(1-H_s)$ for H_w. The first period wage rate is given as W_1, initial assets are given as A. Second period earnings are a function of the time invested in period 1:

$$W_2 = h(H_s, x)$$

where x is the random variable representing the state of the world that is revealed at the beginning of period 2. By assumption $h_x = \partial h / \partial x > 0$: a higher value of x refers to a better state of the world in period 2, with a higher wage rate. Uncertainty covers both input risk and output risk in Levhari and Weiss' sense (not knowing the quality of your inputs and not knowing what your inputs will yield): the state of the world, as revealed at the beginning of period 2, may inform you ex post about your ability to benefit from education, or about the condition of the labour market.

Maximizing $E\{U(C_1,C_2)\}$ with the instruments H_s and C_1 requires as first-order conditions

$$\left\{\frac{\delta U}{\delta C_1}-(1+r)\left[\frac{\delta U}{\delta C_2}\right]\right\}=0 \tag{1}$$

$$E\left\{\frac{\delta U}{\delta C_2}\left[\frac{\delta h}{\delta H_s}-(1+r)w_1\right]\right\}=0 \tag{2}$$

An immediate effect of uncertainty is the breakdown of separability of consumption and human capital investment decisions. If there would be no uncertainty, conditions (1) and (2) would reduce to the standard conditions under certainty: inter-temporal marginal rate of substitution of consumption equal to the rate of interest, investment in human capital should proceed until marginal benefit in period 2 discounted at (1+r) equals marginal opportunity cost (assuming non-satiation), and both conditions independent. With uncertainty, we cannot eliminate the marginal utility of consumption from condition (2). The contribution of a marginal unit of investment is now weighted by the marginal utility of consumption in each state of the world. Optimal investment in human capital is inevitably related with optimal allocation of consumption, and conversely, of course.

Rewrite (2), with simplified symbols U_2 and h_s, as

$$\frac{EU_2h_s}{w_1EU_2}=1+r$$

and subtract expected return on human capital from both sides to get

$$\frac{EU_2h_s}{w_1EU_2}-\frac{Eh_s}{w_1}=(1+r)-\frac{Eh_s}{w_1}$$

Then, the sign of the difference between marginal return on non-human and human capital depends on the sign of the left-hand side, which can be rewritten as

$$\frac{E(U_2)E(h_s)+\text{cov}(U_2,h_s)}{w_1EU_2}-\frac{Eh_s}{w_1}=\frac{\text{cov}(U_2,h_s)}{w_1EU_2}$$

Thus, the expected marginal return to investment in human capital is greater than the return to non-human capital investment if $\text{cov}(U_2,h_s)<0.$. With diminishing marginal returns, this implies that under uncertainty, investment in human capital is lower than without uncertainty, as in the latter case returns to human and non-human capital would be equal. In other words, with uncertainty, there would be underinvestment in human capital.

Kodde (1985, Chapter 7) shows, using a second-order Taylor expansion around E(x), that the covariance term can be written as

$$\text{cov}(U_2,h_s)=U_{22}h_xh_{sx}\sigma^2-\left[U_{222}h_x^2h_{sxx}+U_{22}h_{xx}h_{sxx}\right]\sigma^4/4$$

If h_{sxx} = 0 , as Levhari and Weiss implicitly appear to have assumed, the sign of h_{sx} determines the sign of the covariance. H_{sx} is the sensitivity of the marginal return of schooling to the stochastic shock in the wage rate. With an additive specification of the returns, $w_2 = g(H_s) + x$, the derivative is zero, the covariance is zero, expected return from human capital equals return to non-human capital (in equilibrium) and risk has no effect on human capital investment. If the marginal return to schooling increases in good states ($h_{sx}>0$), the covariance is negative (from $U_{22}<0$), expected return from schooling surpasses the rate of interest and investment responds negatively to (increasing) risk σ^2 (from decreasing marginal returns to schooling). Conversely, if the marginal return to schooling decreases in good states ($h_{sx}<0$), investment in schooling is increasing in risk σ^2 . With a multiplicative specification for risk, $w_2 = xg(H_s)$, $h_{sx}>0$ and investment reacts negatively to increasing risk.

Still assuming $h_{sxx} = 0$, a little rewriting shows

$$(1+r)-\frac{Eh_s}{w_1}=\frac{\text{cov}(U_2,h_s)}{w_1EU_2}=\frac{U_{22}h_xh_{sx}\sigma^2}{w_1EU_2}=w_1\frac{U_{22}}{EU_2}h_xh_{sx}\frac{\sigma^2}{w_1^2} \qquad (3.18)$$

w_1U_{22}/EU_2 is a measure of relative risk aversion, evaluated at the first period wage and the expected value of U_2. The larger the degree of risk aversion, the greater the gap between returns to non-human capital and expected returns to human capital, and the greater the effect of increasing risk.

The pure human capital model, under conditions of certainty and with a perfect capital market, predicts the following comparative static results. An increase in initial assets (or family wealth) has no effect on investment or returns, because of the separability of consumption and investment. An increase in the market rate of interest, in the direct cost of education (tuition, books, etc), in the opportunity cost (the unskilled wages) and a decrease in scholarships and grants *a fonds perdu*, all reduce investment in human capital[5] and increase the marginal rate of return to human capital (assuming declining marginal returns to investment). An increase in the marginal productivity schedule increases investment and returns.

The Levhari-Weiss model, under the assumption of increasing risk (returns up in better states of the world) generates different predictions. More initial assets (family wealth) now increases investment in human capital (because higher wealth reduces risk aversion). An increase in the market rate of interest leads unambiguously to a lower investment for a net borrower (as the substitution effect is re-inforced by a negative income effect). The prediction for a net saver is ambiguous (as the income effect now counters the substitution effect), but we can take net borrowing as the more realistic case. Kodde (1985, p. 136) additionally derives that under an intertemporal additive separable utility function and decreasing absolute risk aversion, an increase in the direct cost of education or a decrease in schooling grants reduces the level of investment (the negative substitution effect is reinforced by moving away from the risky investment if real income is reduced). For the case of multiplicative risk, $h(H_s, x) = a + xh(H_s)$ Levhari and Weiss show that for any multiplicative shift in the distribution of x, investment in human capital will decrease. Thus, only under this restriction can it be said that increased risk reduces investment.

The Levhari and Weiss model shows that even when schooling can be financed by borrowing in the capital market, consumption and investment decisions cannot be separated, as the level of investment will affect the distribution of marginal utility of consumption. It requires zero covariance between marginal productivity of investment in schooling and marginal utility of consumption to restore the separation. Thus, with an additive separable production function of human capital, and marginal productivity not affected by uncertainty, uncertainty will have no effect on the level of investment in human capital.

[5] Scholarships have a different impact than (family) wealth because they are conditional on schooling and thus affect opportunity cost.

In terms of predicted effects, there are two key conclusions. The first is underinvestment in human capital under uncertainty. Under uncertainty, optimal investment will leave the expected marginal benefits from investment in human capital higher than the marginal cost (the rate of interest), and thus, from decreasing marginal investment benefits, human capital investment under uncertainty will be lower than without uncertainty. This conclusion only holds if good states of the world generate higher marginal returns to schooling. If x is ability, the condition entails higher marginal returns for higher ability individuals. This will show up as a positive correlation between marginal and average returns to schooling (higher ability individuals have higher average returns)[6]. Levhari and Weiss point out that increasing risk can be tested: the required positive correlation between marginal and average return to human capital holds if the variance of earnings increases with the level of investment in human capital (o.c., p. 954). The authors cite some research that indeed suggests a positive relationship, but as noted in section 2, this is not a robust and well-established result. Lower investment under uncertainty than without uncertainty cannot be tested, as we lack observations for a world without uncertainty. Marginal rate of return to human capital investment above the cost of borrowing, however, is a testable prediction[7].

The second key conclusion is a negative effect of increasing risk on human capital investment. This prediction, however, could only be derived for a multiplicative specification of the effect of risk on the benefits from investment. It is, in principle, a testable prediction.

Arguably the richest specification of a human capital investment model has been given by Williams (1979). Williams builds a model of stochastic dynamic programming of the lifetime consumption-investment problem allowing for human and non-human capital, as a substantial generalisation of Levhari and Weiss. The model considers marketable skills (human capital) and allocation of time to education, leisure and work. At moments of decision, the present wage is known and future wages are uncertain. Individuals can invest their wealth in a single riskless asset at safe return, in N risky marketable assets or spend it on consumption. Human capital depreciates at an unknown, stochastic rate. Human capital is produced in a multiplicative stochastic specification of initial stock and time spent investing. At the beginning of any period, the individual knows his human capital, his non-human wealth and his present wage rate. The model is solved analytically by maximising

[6] Snow and Warren (1990) show that with endogenous labour supply this only holds when human capital investment is inferior (and there is decreasing risk aversion). When human capital investment is normal, the effect of introducing risk is ambiguous.

[7] Rates of return estimated by the Mincer equation seem to support this prediction, but note that Mincer equations usually assume a constant rate of return and do not allow for falling returns. Heckman, Lochner and Todd (2008) show substantial variation of returns across schooling years. With 6 classes of education, a U shaped pattern seems most common, with only 3 classes, rates of return increase without exception.

expected discounted lifetime utility, where utility depends on consumption, leisure, human capital stock and bequest.

Human capital investment reacts positively to the expected rate of return corrected for the rate of substitution between human and non-human wealth. More non-human wealth will reduce its marginal indirect utility and increase investment in human capital. Investment in human capital reacts positively to the covariance between depreciation and school productivity. If high depreciation comes along with high school productivity, individuals invest more. With negative covariance they invest less, as the two components act like each other's insurance. The effect of increased risk, as a greater variance in school productivity, is to reduce investment in human capital, unless the covariance between depreciation and productivity is highly negative and risk aversion is very strong. The optimal portfolio of risky assets generally may respond to labour market conditions. But if the covariance between marketable assets and wage adjustment is zero, the portfolio does not respond to the wage risk. If human and non-human wealth are substitutes, investment in an asset responds negatively to covariance between wage and asset: there is less investment in an asset that co-varies positively with wage adjustment, and more in an asset that co-varies negatively with wage adjustment. In a specification of the model where time allocated to education also improves information on ability, the added uncertainty from not knowing true ability reduces investment, but at the same time investment is increased to generate information about ability; hence, unambiguous predictions cannot be made.

The Levhari and Weiss model predicts lower investment in human capital when the risk in the returns increases. Williams' base model predicts that human capital investment does not respond to increased risk in future wages, as this will be (mostly) absorbed in portfolio adjustment (o.c., p 531). However, it does respond to increased risk in school productivity and to the covariance between depreciation and productivity of schooling. And it responds negatively to uncertainty about individual ability. Thus, Levhari and Weiss' identical treatment of input risk and output risk no longer holds.

Williams (1979) is by far the most elaborate model of human capital investment under uncertainty, with a number of specific testable predictions. It also gives a detailed treatment of interactions between investment in human and non-human capital, with analytical results. Later models that integrate human capital investment with the consumption-savings-financial decisions are mostly simulation models or models for applied econometric work.

4.3 Separability through the backdoor?

Potentially, there are two ways to restore separability: hedging in the stock market and consumption smoothing. Palacios-Huerta (2003) has specifically analysed the effect of combining human and non-human capital investment portfolios, by considering integrated efficient mean-variance frontiers: portfolios that have minimum variance for given rates of expected return, for data on the US during the period 1964-1996. Return to human capital is measured as the year-to-year wage growth for *(s-1)* years of schooling multiplied by the wage ratio for *s* years of schooling over *(s-1)* years of schooling, in the given year. Thus, it combines the one-period wage growth from owning *(s-1)* units of skill in year *(t+1)* and the skill premium from owning one more unit of skill in year *(t+1)*. Returns from financial investment are taken from the US securities index. Returns per unit of risk (Sharpe ratio's) are typically substantially higher for human capital investment than for financial investment. Focus of the analysis is on "spanning": does the frontier improve, i.e. provide lower variance for given return, if new assets are added? Typically, if human capital investment is added to the frontier for financial investment, the frontier does not improve for the lower educated (below college), but does improve for some of the higher educated groups. If financial portfolios are added to human capital investment, there is no gain for the higher educated (college or more for men, high school or more for women) and gain for the lower educated.

Hedging has also been studied from the perspective of optimum portfolio literature, where the assumption of a given financial wealth has been replaced by the assumption of risky labour income. An example is Davis and Willen (2000). To Davis and Willen, the significance of risky labour income is the breakdown of another separability: the classical two-fund separation theorem of financial investment. According to this theorem, the optimal investment portfolio is a linear combination of a risk free asset and a portfolio of risky assets. The risky assets portfolio composition is identical for all investors and is found as the portfolio with the optimum return/risk ratio given the availability of the safe return on the risk free asset. The individually optimal mix of safe and risky assets is determined by the investor's risk attitude but all mixtures have the same return/risk ratio. With risky labour income, the composition of the risky portfolio is no longer identical for all individuals as they have human capital endowments with different risk properties. In particular, their human capital will have different covariance with financial assets. The approach can be inverted to consider how labour market risk can be hedged in the stock market. This is only feasible if the covariance between labour income and asset returns is non-zero.

Davis and Willen (2000b) reports that correlations between labour income shocks and asset returns increase with education. They use earnings data by occupation, rather than education, from the US CPS files 1967-1994, to assess earnings volatility. The 10 occupations they consider, however, clearly represent different types and levels of education. For none of the occupations was there significant correlation with aggregated market returns (returns on the value-weighted market portfolio). Neither was there any correlation with six other broadbased equity indexes. Thus, for these data and this model, just hedging on a market portfolio brings no benefit. Fama and Schwert (1977) also find that capital market returns and labour market returns are poorly correlated. Davis and Willen calculate optimal investment in a combination of three portfolios: the market portfolio, the SMB portfolio and the HML portfolio, based on the covariances of the occupational level shocks and the returns on these portfolios. The SMB portfolio is an equity portfolio that pays the difference in returns of a portfolio of firms with small market value and one with large market value firms. The HML portfolio pays a return equal to the difference in return to a portfolio of firms with a high book-to-market value and one with firms with low book-to-market value. These two Fama-Fench portfolios, according to Davis and Willen, are known to catch much variability in the market. The results are completely unrealistic. For example, a 40 year old truck driver should hold a portfolio of 550 000 dollars, including a short position in one fund of 141 000 dollars. On top of that, the capital market is not as perfect as their exercise assumes. "If investors must borrow at an interest rate that approximates the expected return on risky assets (rather than the risk-free interest rate) the optimal risky asset position is approximately zero when asset returns and labor income are uncorrelated." (o.c., p 21). Hence, the conclusion is devastatingly simple: workers must stand on their own feet when confronted with financial risk in the labour market. They cannot hedge their way out through the stock market.

The other way to soften or even eliminate the effect of earnings risk is self-insurance through consumption smoothing: if consumption volatility is the real problem, but can be isolated from earnings volatility, earnings risk itself is no longer an issue. Or at least a less pressing issue. Brown, Fang and Gomes (2012) construct an empirical model for the US to estimate the gains in lifetime utility from education, considering stochastic labour income from work, unemployment benefits and retirement income, under the assumption that individuals cannot borrow against future income. Utility only derives from consumption, with a Constant Relative Risk Aversion utility function per period, weighted over future periods by age-specific mortality rates and discounting at an exogenously fixed and constant discount rate. Each period's consumption is constrained by that period's income and wealth, where wealth is build up from savings earning a riskless return of 2 %. Labour income for an individual has a stochastic component following the Carroll-Samwick specification. The earnings function is estimated on American data from the PSID, a panel initiated in 1968, with an average of 8 observations per individual (the sample uses records from 7050 individuals). The equations are estimated separately for three levels of education: no high school, high school graduate, college graduate. Individuals with incomplete college education or with any amount of post-college education are excluded. Unemployment is modeled as the probability of incurring a spell of unemployment during the year and benefits are specified to represent the institutional specifications. After estimating the income process, lifecycle consumption patterns are determined from maximizing expected lifetime utility, solving the model backwards from the final period (age 100). The optimisation exercise mimics the experience of the cohort born in 1945, working for 44 years after age 22 and hence retiring at age 66. Focus is on after-tax income, with tax rates measured as actually experienced during the cohort's lifetime.

The results focus on the benefits from having the possibility of consumption smoothing. Just adding up gross lifetime income brings high school graduates a gain of 30% over drop-outs and college graduates a gain of 67% over high school graduates. Taxes are progressive and mostly reduce the college premium, to 54%, while barely affecting the high school graduation premium. The probability of experiencing a spell of unemployment differs dramatically by education, in a ratio of 3:2:1 from no high school to high school to college, but has negligible effect on schooling income premiums. This, by itself, is quite noteworthy. When education is portrayed as an investment with risky returns, most people spontaneously refer to unemployment, rather than to poorly predictable wages. Clearly, the disutility of unemployment is not primarily associated with income loss (unless people are simply inadequately informed). Presumably, the disutility of unemployment is related to a strong sense of being constrained and socially excluded. If so, the benefits of a college education are higher than just the gain in lifetime income. Not surprisingly, adding Social Security retirement income reduces benefits most for college graduates, as their replacement rate is substantially lower than for lower income groups. Allowing for consumption smoothing through saving, removing the constraint to consume all income in the period in which it is received, reduces the benefits from education: measured in consumption the benefit is smaller than measured in income. The model is used to calculate the annually constant level of consumption that would yield the same level of utility as the utility derived from consumption equal to expected income in each period. The additional degrees of freedom reduce the benefits of a high school diploma by 7 points, the benefits of a college degree by 10 points. Considering only labour income, the transitory component in the standard deviation is 1/3 to ¼ of the permanent component. Differences in income risk among schooling levels are remarkably small, with only the transitory labour income risk for high school drop-outs standing out. This of course implies a modest effect of risk on returns to education.

It turns out that income risk only affects schooling benefits to a meaningful degree if risk aversion is sufficiently high. The impact of risk is assessed from considering Certainty Equivalent consumption: the annually constant level of consumption that would yield a utility level equal to expected utility that would result from the stochastic profiles of income and the associated optimal levels of savings and consumption. With CRRA = 1, the consumption gains from completing high school (measured by Certainty Equivalents) are

20%, with CRRA = 4, the gains are 38%. In those case, the gains from college over high school are 42 and 33%, respectively.

Huggett, Ventura and Yaron (2011) analyse the growing inequality in the US, with a lifecycle stochastic dynamic programming model in which an individual maximizes lifetime consumption utility. The utility function for a given period has constant relative risk aversion (CRRA = 2). Individuals are endowed with initial financial wealth and a stock of human capital (including a given, exogenous level of schooling at age 23), and they can use their resources (including their ability) to produce additional human capital after leaving school. Borrowing against future earnings is ruled out, but optimal consumption smoothing is included, with saving at fixed market rate of interest. Parameters of human capital are estimated from US earnings data. In particular, human capital depreciation and shocks are estimated from earnings profiles for older workers (above age 55), when investment has ceased and the human capital stock no longer grows. The variance of annual shocks to individual human capital is estimated at 0.111, meaning that a one standard deviation shock changes wages by 11 %. Lifetime inequality is measured in utility (the equivalent variation of consumption), earnings and wealth. The key conclusion is that for lifetime inequality, initial conditions (financial and human capital, learning ability) are most important, being responsible for some 60 to 65 %. Initial human capital has by far the biggest impact, which is not surprising as it includes schooling completed at age 23. In this setting, risk, as the dispersion of the shocks to human capital has modest effect. Risk has a direct effect on inequality but also an indirect effect, as increasing risk reduces the human capital investment of risk averse individuals. Changing the dispersion of the shocks from one standard deviation below to one standard deviation above its base value changes the contribution of initial conditions to lifetime inequality from 0.656 and 0.695 for lifetime earnings and utility respectively, to 0.570 and 0.620.

5. Testing the effect of risk on demand for education

The empirical literature on demand for education under uncertainty is depressingly small. There is some evidence that individuals sort themselves into more or less risky positions in line with their risk attitude, some evidence of the effect of risk on attained levels of education and some studies deal with risk in relation to university education. But robust, repeatedly confirmed results are lacking.

There are two results on sorting. Bonin, Dohmen, Falk, Huffman and Sunde (2007) relate individual risk attitude to financial risk in the individual's occupation, to test the hypothesis that more risk averse individuals will seek out occupations with smaller financial risk. As occupation and education are strongly linked, we can take these result as also relevant for educational choice. Using some 4000 observations on men from the 2004 wave of the German Socioeconomic panel (SOEP), Bonin et al. regress individual wages on experience, tenure, schooling, region, sector of employment and dummies for 50 2-digit occupations. Risk is measured as the standard deviation of the residuals from the regression within an occupation. Risk in the individual's occupation is then regressed on personal characteristics (experience, tenure, schooling, marital status, body height), region, sector of employment and monthly income, and on risk attitude. Risk attitude is measured by asking individuals to rate their willingness to take risks in general on a 0-10 scale. In the private sector, risk attitude is significantly related to risk, in the public sector the effect is not significant. In the private sector, a 10 point increase in willingness to take risk is associated with 0.01 increase in occupational risk. Risk attitude 0 means "completely unwilling to take risks", 10 means "completely willing to take risks". A 0.01 increase in risk represents 2.5 percent of mean risk across occupations, or 1.5 percent of the range.

Saks and Shore (2005) explain college major choice in the US from characteristics of the institution attended, SAT scores and parental background. Eight majors are linked to nine occupations. Each occupation is characterized by total labour risk, the sum of permanent and transitory risk estimated from panel data. A simple graph is used to show a clear positive association between greater parental wealth (and presumably lower risk aversion) and the likelihood to choose a major with higher earnings risk. Similar results are available for Turkey.

There are a few applied structural dynamic programming models of educational choices[8]. The Belzil-Hansen model (2002) assumes maximization of expected lifetime utility from income, where period income is evaluated with a CRRA utility function. When in school, students receive financial support from parents, after school earnings derive from wage and employment which are both explicitly modeled. Individual heterogeneity is acknowledged in different intercepts for equations determining parental transfers, wage and employment probability. The model also allows for school interruptions. The model is estimated on an American panel, the NLSY 1979, restricting the sample to white males, aged 20 or less at the start of the panel, and interviewed every year from 1979 till 1990. The estimated discount rate is 0.09 and the coefficient of relative risk aversion is 0.93 (with a standard deviation of only 0.04). Given the estimated parameters of variance equations, wage risk increases up to 9 years of education and then decreases, hours risk decreases continuously; taken together,

[8] Two other studies applying stochastic dynamic programming may seem relevant. Keane and Wolpin (1997) allow for uncertain future wages, future valuation of effort and of leisure, but individuals aim to maximise present value of net rewards under risk neutrality. Hence, risk and risk attitude play no role in the decisions. The model in Eckstein and Wolpin (1999) is similar.

earnings risk strongly declines with education. Predicted level of schooling appears very sensitive to the degree of risk aversion: an increase from 0.93 to 3.00, in a simulation exercise, increases schooling from 12.45 years to 18.50 years. This happens because schooling operates as an insurance: risk strongly falls with schooling level. The estimated schooling elasticity of earnings risk is positive but small, at 0.07. It is positive, as with increasing risk of earnings, the alternative of staying in school rather than go to work becomes more attractive: schooling is only evaluated as receiving safe parental transfers. This is no doubt an insufficient representation of the response to increasing risk in the benefits from schooling. It reflects mostly that the costs of attending school are not adequately accounted for.

Belzil and Leonardi (2007) model schooling attainment as grade progression through four schooling levels g in Italy, primary, junior high, high school and college or more. The core equation is a hazard function for stopping at grade g, where the hazard rate is specified as a mixture of five normal random variables Belzil and Leonardo allow for endogeneity of risk aversion as a function of family wealth, background risk and permanent individual-specific risk aversion. Basic information on risk aversion is taken from the stated reservation price for a specified lottery ticket. The model is estimated on data from the Bank of Italy Survey of Income and Wealth 1995. The key result is a positive effect of risk aversion on leaving before senior high school graduation and a negative effect on leaving before college graduation. This implies that senior high school is seen as a risky investment, while college is seen as an insurance. The effect of risk aversion is quite small, certainly in comparison with the effect of family background. Increasing risk aversion from the bottom to the top of the distribution barely affects the termination rates, while raising parental education levels from low (less than high school) to high (high school and higher) increases the probabilities of leaving with a high school diploma or a college education fourfold.

In the small literature on demand for university education, two papers consider the effect of probability of success. Montmarquette, Cannings, and Mahseredjian (2002) relate the choice among four aggregate college major groups by US students to expected earnings and probability of success. Students have been asked for their perceived probability to graduate with their chosen major, and these probabilities are also used to calculate expected earnings. The probability of success is a significant determinant for the observed choice of major. Rochat and De Meulemeester (2001) estimate the effect of probability of success in the first year of university on students' choice among 7 disciplines in Belgium. Results, in their small sample, are mixed. Probability of success has a significant positive effect on choice of discipline in the full sample. Among students from families with the highest incomes, predicted success is not significant, but among students with a scholarship (171 students, typically from low-income families), success rates have a significant positive effect.

Kodde (1985) intends to test the Levhari-Weiss model on intentions to go on to university among Dutch high school students in their last year. The sample is restricted to children from high income families, not eligible for grants and scholarships, to obtain homogeneity in financial conditions. Students have been asked for their forgone earnings (work after high school graduation), and for their expected earnings after completing tertiary education, as well as for the highest and the lowest earnings it might generate. The average gap, in logs, is 0.65, with a standard deviation of 0.32, is used as the measure of perceived earnings risk. Perceived risk has a positive effect, violating the Levhari and Weiss prediction. Taken separately, the extreme earnings also have significant effect: the expected highest earnings have positive effect (coefficient 1.81), the expected lowest earnings have negative effect (coefficient -2.20). The unexpectedly positive effect of earnings risk is thus based on the unexpected negative effect of lowest earnings. One can speculate about possible causes such as a preference for low paying cultural studies among the rich, but Kodde gives no further information.

Hartog and Diaz-Serrano (2007) estimate earnings functions within regions in Spain, separately for graduates from secondary and from university education and then derive regional measures of returns to university education and the risk gradient (the ratio of residual earnings variance for university graduates' relative to secondary school graduates). The regional measures of return and risk are used as explanatory variables in a probit for college attendance of youth. The earnings variance ratio (risk) has a significant negative effect. The trade-off between risk and returns (the marginal returns required to maintain constant probability of going to university when risk increases, where both returns and risk are measured in relative terms) is about -0.2: if the risk ratio increases by 10 percentage points (e.g. from 1.2 to 1.3), compensation requires an increase in the returns ratio by 2 percentage points. The negative effect of risk is significantly dampened if household expenditures on lottery tickets, a measure of heterogeneity in risk attitudes, are higher.

Hartog, Ding and Liao (2012) apply the Dominitz-Manski method to elicit students' anticipated earnings distributions among Chinese bachelor students to test the effect on choosing for continued education or going to work after graduation. Risk attitudes were also measured and included in the testing procedure. Earnings risk, measured as tail probabilities in anticipated earnings distributions, never has significant effect on intention to continue for a Master degree in any of the specifications of the earnings variables (levels, logs, ratios, starting, midlife or lifetime).

Support on the effect of anticipated unemployment is presented by Kodde (1985). In a logit estimate on the sample described above, that also includes family background, high school grades in mathematics and languages and anticipated earnings, the stated probabilities have statistically significant effect on intention to enter university. The elasticity of perceived employment probability after high school is -0.064, the elasticity of perceived employment probability after university is 0.132. This also implies that if both probabilities increase, demand for university education increases, as indeed predicted in Kodde's analytical extension of the Levhari-Weiss model.

6. A challenging "to do" list

In the daily life of students, school administrators and policy makers, uncertainty about the outcome of decisions on education abounds. In economics of education, the literature on uncertainty is something like inherited family porcelain: proudly exposed but never used. It is hard to think of a good reason to leave it there just to gather more dust.

There is convincing evidence of a substantial scope for risk to be relevant. A large variation in ex post outcomes that cannot be taken as obviously foreseeable when decisions have to be made. Clear evidence ex ante of uncertainty among agents about the outcomes of alternative actions. However, we have no reliable, robust information on available information sets and we cannot comfortably distinguish ex post variance and ex post risk. In fact, we are still methodologically divided on how to proceed: straightforward along the ex ante interview route shown by Manski, or with high powered econometrics to tap ex post information as applied by Heckman.

The explanation for ignoring the risk in human capital decisions is probably the absence of a tractable and malleable workhorse analytical model. The seminal model, no doubt, is Levhari and Weiss (1974). It's routinely cited when uncertainty is mentioned and in October 2013 it had accumulated 405 citations in Google Scholar. Yet, as far as we can see, it's only truly applied in the literature on optimum taxation. The model does not function as a skeleton that is fleshed out in many directions and hence to unite analyses of related problems in a common frame, in the way for example the job search model unites matching, job search, effects of labour market policies, etc. Presumably, the right handles are simply missing and the compact treatment of input and output risk cannot conveniently be unfolded to add the details of specific problems. The main contender, in a sense an abundant extension of Levhari and Weiss, is Williams (1979). Runner-up in the contest of the early starters, but with only 76 citations far behind the front-runner. William's model is actually quite a rare accomplishment, as it provides the analytical solution to a complex stochastic dynamic programming model. But the mathematics is complicated and I wonder how many readers have worked through the 3 page appendix to check the derivation of the optimum conditions. The legacy of Williams' dynamic programming approach (which was in fact also applied by

Ben-Porath in his classic on lifecycle human capital investment) is applied work: estimated, calibrated and simulated dynamic programming models. Interesting and useful, but integrating theory and estimation, and thus not directly suitable to build a base of general analytical results (although results with wide if not general applicability might eventually be found). The dynamic programming models can be used to mimic many details of the problem and its context, and to portray a world that is easily recognized, but they are not easy to handle. The numerical applications are computationally involved and cannot serve as a convenient hand tool.

A key issue in the proper analysis of human capital risk is separability. The basic workhorse model of human capital investment assumes separability, where lifetime earnings maximization and lifetime consumption profiles are treated as independent. This assumption is not warranted with endogenous labour and neither warranted with risk that cannot be eliminated. A fairly extensive literature has indicated that wages provide compensation for earnings risk (Hartog, 2011). But the elasticities are small and full risk shifting to employers cannot be taken for granted. Williams' analytical model shows the scope for compensating labour market by a proper stock market portfolio, but Palacios-Huerta's model of combined efficient investment frontier shows mixed results and models that actually calculate required stock market portfolios find completely unrealistic required portfolios. The stock market as an insurance to human capital risk does not seem a credible option. More credible is the option to self-insure through consumption smoothing. A priori it seems quite plausible that consumption smoothing, even if only through saving when borrowing against future labour income is ruled out, can make a dent and that focus on lifetime earnings only, is too restrictive. The question then becomes to what extent consumption smoothing helps. A simulation exercise has shown that the impact depends on the degree of risk aversion. For a risk neutral individual, availability of consumption smoothing means nothing, but the value increases with increasing levels of risk aversion. We can state for sure that risk aversion is quite heterogeneous among individuals, but actual values of risk aversion are much less known. The small effects of earnings risk on wages reported in Hartog (2011) either corresponds to low level of risk aversion or to weak bargaining power of labour. But to the extent that consumption smoothing is relevant, wages should be related to the risk that remains after allowing for consumption smoothing, rather than to pure earnings risk. That suggests that the magnitude of risk compensation has been underestimated, as risk has been overestimated.

In conclusion, a challenging "to do" list suggests itself.[9]

1. Properly identify the *ex ante* risk associated with schooling decisions.
2. Develop a tractable, malleable, and extendible workhorse model of human capital investment under uncertainty
3. Allow for consumption smoothing in dealing with risk and reconsider the estimates of risk compensation in wages

[9] There is also some literature on the effect of risk taking on wage growth, built on the assumption that human capital is a risky investment and hence that risk attitude must affect wage growth. See Budria et al. (2013).

References

Belzil, C. and J. Hansen (2002), Earnings dispersion, risk aversion and education, CEPR Discussion Paper No. 3600, published under the same title in *Research in Labor Economics*, 23 (2004), 335-358.

Belzil, C. and M. Leonardi (2007), Can risk aversion explain schooling attainments? Evidence from Italy, *Labour Economics*, 14(6), 957-970,

Bonin, H., T. Dohmen, A. Falk, D. Huffman and U. Sunde (2007), Cross-sectional earnings risk and occupational sorting: The role of risk attitudes, *Labour Economics*, 14(6), 926-937,

Brown, J., C. Fang and F. Gomes (2102), Risk and returns to education, Cambridge, Mass: NBER Working Paper 18300

Budria, S., L. Diaz Serrano, A. Ferrer i Carbonell and J. Hartog (2013), Risk Attitude and Wage Growth, Replicating Shaw (1996), *Empirical Economics*, 44 (2), 981-1004

Carroll, C. and A. Samwick (1997), The nature of precautionary wealth, *Journal of Monetary Economics*, 40(1), 41-71

Chen S.H. (2008), "Estimating the Variance of Wages in the Presence of Selection and Unobserved Heterogeneity", *The Review of Economics and Statistics*, 90(2): 275-289.

Cunha, J. Heckman and S. Navarro, (2005), Separating uncertainty from heterogeneity in life cycle earnings, *Oxford Economic Papers*, 57, 191-261

Cunha, F. and J. Heckman (2007), Identifying and estimating the distributions of ex post and ex ante
returns to schooling, *Labour Economics, Special issue: Education and risk*, 14 (6), 870-893

Davis, S. and P. Willen (2000), The Implications of Uncertain Labor Income for Portfolio Choice, in O. Mitchell, Z. Bodie, B. Hammond, and S. Zeldes (eds), *Financial Innovations for Retirement Income*, Pension Research Council, Philadelphia, PA, published on internet as Risky labor income and portfolio choice

Davis, S. and P. Willen (2000b). Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice, Boston, Mass: NBER Working Paper 7905

Dominguez-Martinez, Silvia and Otto H. Swank (2009). A Simple Model of Self- Assessment. *Economic Journal*, 119 (July), 1225-1241

Dominitz, J. and C. Manski (1996), Eliciting student expectations of the return to schooling, *Journal of Human Resources,* 31, 1-26.

Eckstein, Z. and K. Wolpin (1999), Why youths drop out of high school: the impact of preferences, opportunities and abilities, *Econometrica,* 67 (6), 1295-1339

Fama, E. and G. Schwert (1977), Human capital and capital market equilibrium, *Journal of Financial Economics,* 4(1), 95-125

Gourinchas and Parker (2002), *Econometrica* 70(1), 2002, 47-89

Hartog, J. (2011), A Risk Augmented Mincer earnings equation? Taking stock. *Research in Labor Economics*, 33, 129-173

Hartog, J. and L. Diaz-Serrano (2007), Earnings risk and demand for higher education, *Journal of Applied Economics*, X (1), 1-28

Keane, M. and K. Wolpin (1997), The Career Decisions of Young Men, *Journal of Political Economy*, 105(3), 473-522

Kodde, D. (1985), Microeconomic analysis of demand for education, PhD thesis Erasmus University Rotterdam

Levhari, D. and Y. Weiss (1974), The effect of risk on the investment in human capital. *American Economic Review*, 64 (6), 950-963

Low, H., C. Meghir and L. Pistaferri (2010), Wage risk and employment risk over the life cycle, *American Economic Review*, 100 (4), 1432-1467

Mazza, J., H. van Ophem and J. Hartog (2013), Unobserved heterogeneity and risk in wage variance: does more schooling reduce earnings risk?, *Labour Economics*, 24 (), 323-338

Mazza, J. (2012), On the uncertain nature of human capital investments, PhD dissertation University of Amsterdam

Montmarquette, C., K. Cannings and S. Mahseredjian (2002), How do young people choose college majors? *Economics of Education Review*, 21 (6), 543-556

Palacios-Huerta, I.(2003), An empirical analysis of the risk properties of human capital returns, *American Economic Review* 93 (3), 948-964

Rochat. D. and J-L. de Meulemeester (2001) Rational choice under unequal constraints: the example of Belgian higher education, *Economics of Education Review* 20, (1) 15–26

Saks and Shore: "Risk and Career Choice", (with Raven E. Saks), *Advances in Economic Analysis & Policy*, 2005, 5(1), Article 7.

Skyt Nielsen, H. and A. Vissing-Jorgenson (2006), The impact of labour market risk on education choices: estimates and implied risk aversion, Working paper Aarhus/Northwestern.

Snow, A. and R. Warren (1990), Human Capital Investment and Labor Supply under Uncertainty, *International Economic Review*, 31(1), 195-206

Weiss, Y. (1987), The determination of life cycle earnings: A survey, in: O. Ashenfelter and R. Layard (ed.), *Handbook of Labor Economics*, edition 1, volume 1, chapter 11, pages 603-640, Amsterdam: Elsevier.

Williams, J. (1979), Uncertainty and the accumulation of human capital over the lifecycle, *The Journal of Business*, 52 (4), 521- 548