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Capital accumulation and income inequality

Núria Barrubés

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Capital accumulation and income inequality

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Abstract

In this paper, I study an alternative mechanism to the one proposed by Sato et al. (2008). Like Sato et al.' (2008), I propose a theoretical model where the individuals' innate potential and the technological level determine the type of technology that can be accessed by each agent. Furthermore, the model generates a new prediction both at the individual and at the aggregate level. Depending on the individuals' characteristics and the economic environment, the agents accumulate different capital levels. This distribution among the agents ends up determining their economic outcomes in terms of income and life quality, with respect to leisure and the different socioeconomic orders.

Keywords income inequality · capital accumulation · technological level · technological change · socioeconomic orders

JEL Classification D31, D33, O11, O12, O33

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1 Introduction

Let us consider an economy divided into two social groups and two types of technology. Each individual *i* has a finite life. In the first period, each agent has an initial income amount and decides on his investment in education. This investment depends on the individuals' innate potential p_i and the technological level of the economy A. Both determinant factors end up determining the type of technology that can be accessed by each individual *i* in the next period.

Sato et al. (2008) analyze the context explained above by using an overlapping-generations model in order to explain the key development patterns: the skilled agent fraction, the fertility rate, and the income inequality. In particular, Sato et al. (2008) construct a growth model which considers two different types of technology: modern, which can be accessed by the skilled agents, and traditional, which can be accessed by the unskilled ones. The productivity in modern technologies is considered higher than that in traditional technologies (see Harley, 1996). Sato et al. (2008) also consider the complementarity between human capital and technological progress. In particular, they assume that the current productivity levels are related to the past educational level (see Buiter and Kletzer, 1993; Galor and Tsiddon, 1997; Mountford, 1997). Thus, Sato et al. (2008) describe an economy that is initially in a poverty trap or Malthusian state (see Galor, 2004). Their mechanism raises the skilled wage premium duo to the skilled-biased technological change and enables the economy to escape from this scenario by reaching a better equilibrium in terms of agents' well-being. Concerning the income inequality, Sato et al. (2008) confirm the Kuznets Curve phenomenon.

In this paper, I replicate Sato et al.' (2008) model by using an alternative mechanism which generates a different pattern of income inequality. In contrast to Sato et al. (2008), the proposed analysis presents a new approach about how agents can generate positive economic outcomes in terms of income and life quality, with respect to leisure. However, these events that are privately beneficial for each agent, end up determining the several socioeconomic orders at the aggregate level. These scenarios not necessarily lead to a situation of equality. In sum, young individuals *i* work and receive formal education in order to become rentiers and enjoy the benefits provided by their accumulated capital once they are elder. This capital promotes technological change and fosters a new type of technology that can be accessed by the rentiers. Thus, the individuals *i* obtain more income and improve their life quality or well-being. On the contrary, the workers' investment is not high enough in order to access the capital that allows using a new type of technology. Therefore, the income level and the quality of life of this social group almost unchange. In the initial period, innate potential, technological level and the gross rate of return of the economy are exogenous and constitute the environment, while income, consumption and savings or investment in formal education in the initial period are all endogenous. Otherwise, in the last period, labor input, leisure time, acquired potential, capital level, income and consumption are all endogenously determined.

The rest of this paper is organized as follows. Section 2 presents and discusses the model. Section 3 establishes the equilibrium. Section 4 concludes the paper.

2 The model

Setup

The economy is populated by two social groups: rentiers and workers. Both types of individuals have a finite life. They live for two periods and produce one unit of an only one kind of good in the last one. This economy extends over an infinite horizon being the first period t=0. A cohort born in time t is called generation t. In the first and the second period of life, the agents are called young and old individuals, respectively.

Following Bertinelli and Black (2004), it is assumed that each agent is member of a single-worker firm when old. I also consider two types of technology: new and traditional, which can be accessed by rentiers and workers, in the order given. In addition, I assume that the productivity in new technologies is higher than that in traditional technologies as more recent evidence, for instance Laitner (2000), Galor and Mountford (2003) and Sato, Tabata and Yamamoto (2008) suggest. For the sake of simplicity, the productivity in traditional technologies is considered constant (i.e. $A_t = A$). Thus, considering the past statements, this study only focuses on the evolution of economic outcomes in new technologies.

The agents do not work in their first period of life but they have an initial endowment of income q. They invest in formal education when young in order to be more productive when old. Note that they endogenously choose the amount of income allocated to this expenditure. Otherwise, individuals are distributed in a continuum of agents with innate potential parameter p (see Razin et al. (2002)) considered a random value within the range $[p_{min}, p_{max}]$. The density function is defined by f (p). I consider that the innate potential can be improved through formal education.

If the investment in education is enough, the agents also obtain high enough capital to become rentiers when old. Through the technological change, they can access to a new type of technology $B_{i,t+1}$ in period $t+1$. The property of technological change is specified later.

The production function of the rentiers in modern technologies can be posed in terms of the following equation:

$$q_{i,t+1}^{M} = B_{i,t+1} p_{i,t+1} l_{i,t+1} , \tag{1}$$

Where $B_{i,t+1} \equiv AR\alpha e_{i,t}$ and α>1

I assume that α transforms the investment in formal education in capital at period $t+1$. A is the productivity of the traditional technology in period $t+1$, R represents the gross rate of return of the economy and $e_{i,t}$ is the individual i's investment amount in formal education. In turn, $B_{i,t+1}$ is the labor productivity of the new technology, $p_{i,t+1}$ captures the individual' acquired potential which depends on the investment in formal education and $l_{i,t+1}$ is the labor input. I also consider that the only variable that is not constant is the investment amount in formal education. Thus, the higher the value of $e_{i,t}$, the higher the level of productivity of the new technology $B_{i,t+1}$. This implies that formal education allows improving the individuals' economic outcomes. Note that the total income is equivalent to the agent' output $q^M_{i,t+1}$. Consequently, it is treated as a numeraire.

However, if young agents' investment in formal education is not high enough to obtain enough capital, they become workers and only can access the traditional technology A in $t+1$. Their production function is defined as follows:

$$q^T_{i,t+1} = Ap_{i,t+1}l_{i,t+1}, \tag{2}$$

Where A represents the less intensive technological level of the economy in period $t+1$, $p_{i,t+1}$ captures the individual' acquired potential which depends on the investment in formal education and $l_{i,t+1}$ is the labor input. Again, the output $q^T_{i,t+1}$ of each individual is equivalent to the income.

In turn, individuals receive utility from their own consumption and from their leisure time when old. Therefore, the utility function of the agent i in generation t can be outlined in terms of the following expected lifetime utility function:

$$V_{i,t+1} = \beta \ln(c_{i,t+1}) + \gamma ln(z_{i,t+1}) \tag{3}$$

Where $c_{i,t+1}$ and $z_{i,t+1}$ are the consumption and the leisure time in the adulthood. Additionally, β and γ are the propensities for consumption and for leisure time when old.

In period t, the agents distribute the income q between consumption and savings which are equivalent to the investment in formal education. This investment depends on the individual' innate potential $p_{i,t}$ and the technological level A. The gross rate of return of the economy transforms young individuals' savings into capital in the next period.

Following Arrow (1962), if the investment mentioned above is high enough, the accumulated capital facilitates the access to a more productive type of technology through the technological change. Both the individual' acquired potential $p_{i,t+1}$ and the new technology $B_{i,t+1}$ end up determining the leisure time in *t+1*, which reduces the time destined to work for the rentiers. Thus, the budget constraints of each member of this group are as follows:

$$c_{i,t+1}^{r} = B_{i,t+1}\, p_{i,t+1}^{r} l_{i,t+1}^{r} \;, \tag{4}$$

$$l_{i,t+1}^{r} + z_{i,t+1}^{r} = 1, \tag{5}$$

Where $c_{i,t+1}^{r}$, , $l_{i,t+1}^{r}$, $z_{i,t+1}^{r}$ and p_{i}^{r} are the adulthood consumption, the time destined to work in the adulthood, the leisure time when old and the acquired potential, respectively.

By maximizing Eq. (3), subject to Eqs. (4) and (5), we have that:

$$z_{i,t+1}^{r*} = \frac{\gamma}{\beta} W_{i,t+1}^{r} \;, \tag{6}$$

$$c_{i,t+1}^{r*} = W_{i,t+1}^{r} \;, \tag{7}$$

Where: $W_{i,t+1}^{r} \equiv B_{i,t+1}\, p_{i,t+1} l_{i,t+1}$ is the potentially income of the rentiers.

The indirect utility function is:

$$v_{i,t+1}^{r}\left(W_{i,t+1}^{r}\right) = \ln([\, 1^{\beta}\, [\, \gamma/\beta\,]^{\gamma})\;. \tag{8}$$

Otherwise, the budget constraints of the workers can be denoted in the following way:

$$c_{i,t+1}^{w} = A p_{i,t+1}^{w} l_{i,t+1}^{w}, \tag{9}$$

$$l_{i,t+1}^{w} + z_{i,t+1}^{w} = 1, \tag{10}$$

By maximizing Eq.(3), subject to Eqs. (9) and (10), we have that:

$$z_{i,t+1}^{w*} = \frac{\gamma}{\beta} W_{i,t+1}^{w} , \tag{11}$$

$$c_{i,t+1}^{w*} = W_{i,t+1}^{w} , \tag{12}$$

Where $W_{i,t+1}^{w} \equiv A p_{i,t+1}^{w} l_{i,t+1}^{w}$ is the potentially income of the workers.

The indirect utility function is:

$$v_{i,t+1}^{w}\left(W_{i,t+1}^{w}\right) = \ln([\, 1^{\beta}\, [\, \gamma/\beta\,]^{\gamma}\,) , \tag{13}$$

Assumption 1

$$B_{i,0}\, p_{min}\, l_{min} > A\, p_{max} .$$

I assume that the potentially income of a rentier in the initial period is larger than that of a worker even in the case where a worker has the highest innate potential and spends all the time to work.

The previous hypothesis implies the opportunity cost of the working time. This cost is defined by Lemma 1.

Lemma 1

$$z_{i,t+1}^{r} > z_{i,t+1}^{w} .$$

(Proof of Lemma 1: see Appendix A)

Making use of Eqs. (8) and (13), I obtain the following condition:

$$v_{i,t+1}^{r} - v_{i,t+1}^{w} \geq 0 \leftrightarrow (k_{i,t+1} y_{t+1})^{\beta\gamma} \geq (y_{t+1})^{\beta\gamma} \;, \tag{14}$$

Where $y_{t+1} \equiv Ap_{i,t+1} l_{i,t+1}$.

(Proof of condition (14): see Appendix B)

Assumption 2

Concerning Assumption 1, the less productive rentier (p_{min} , l_{min}) obtains more income than any worker for being a rentier. This assumption implies that all the individuals want to become rentiers. However, not everyone may belong to this social group because there is a barrier to entry when condition (14) is not met:

$$(k_{min}\, y_0)^{\beta\gamma} < (y_0)^{\beta\gamma}$$

Where $y_0 = Ap_{max}$.

Assumption 2 implies that the individual with the lowest capital level becomes worker in the initial period.

Additionally, there exist a unique value of k^* which satisfies the following condition with equality:

$$(k^* y_0)^{\beta\gamma} = (y_0)^{\beta\gamma}$$

For simplicity, I also assume that the threshold's value is $k^* = 1$. Therefore, we can interpret this value as a normalized variable. That is, the necessary condition for becoming a rentier is to have capital.

Let me illustrate what is the individual threshold for each rentier:

$$k_{i,t+1}^{*} = R \frac{y_{t+1}}{Ap_{i,t}\, l_{i,t+1}}$$

Lemma 2 An individual whose capital level $k_{i,t+1}$ is higher or equal than $R \frac{y_{t+1}}{Ap_{i,t} l_{i,t+1}}$ can foster technological change and produce using the new technology $B_{i,t+1}$. Consequently, the individual *i* becomes rentier. Conversely, an individual with capital levels $k_{i,t+1}$ lower than $R\frac{y_{t+1}}{Ap_{i},t l_{i,t+1}}$ cannot foster technological change and becomes worker.

(Proof of Lemma 2: see Appendix C)

By having presented the previous threshold, I am able to show the relationship between capital and leisure time. Since $l_{i,t+1} = (1 - z_{i,t+1})$ and $k^{*}_{i,t+1} = R\frac{y_{t+1}}{Ap_{i,t} l_{i,t+1}}$. By substituting the first equation into the second one, we obtain $k^{*}_{i,t+1} = R\frac{y_{t+1}}{Ap(1-z_{i,t+1})}$. I also consider that $p_{i,t} = p$ because this is an exogenous and constant parameter for each agent in period t. Hence, it may simply verified that the higher the leisure time, the higher the capital level.

Finally, I consider that this economy can foster technological change. I specify this assumption as follows:

$$B_{i,t+1} = (k_{min}, k_{max}]A \qquad (15)$$

Following Arrow (1962), the rentiers can generate knowledge spillovers and foster technological change through the capital accumulation.

3 The equilibrium

Given the feature of technological change, Lemma 1 and Lemma 2 discussed previously, I can illustrate the equilibrium of the model.

Proposition 1: *There exists a unique equilibrium that depends on A and* $p_{i,t}$:

1. **Egalitarian case:** If $k_{max} < R\frac{y_{t+1}}{A_{min,0} p_{max} \, l_{i,t+1}}$ and $k_{i,t+1,} < k^{*}_{i,t+1} \; \forall \, i$ are satisfied, the economy is in the egalitarian case. All the individuals become workers. Finally, the working time of the workers is $l^{W}_{i,t+1} = 1$ and their leisure time is $z^{W}_{i,t+1} = 0$ since $z_{i,t+1} = (1 - l_{i,t+1})$.

2. **Intermediate case:** If $k_{min} \geq R\frac{y_{t+1}}{A_{max,0} p_{i,t} \, l_{i,t+1}}$ and $k_{min} < R\frac{y_{t+1}}{A_{max,0} p_{min} \, l_{i,t+1}}$ are satisfied, the economy is in the intermediate case. Individuals with $p_{i,t} > p_{min}$ and $k_{i,t+1} \geq k_{min}$ become rentiers and the agents with $p_{i,t} = p_{min}$ and $k_{i,t+1} \leq k_{min}$ become workers. Finally, the working time of the rentiers is $l^{r}_{i,t+1} < 1$ and their leisure time is $z^{r}_{i,t+1} > 0$. In turn, the working time of the workers is $l^{W}_{i,t+1} \leq 1$ and their leisure time is $z^{W}_{i,t+1} \geq 0$ since $z_{i,t+1} = (1 - l_{i,t+1})$.

3. **Polarized case:** If $k_{max} \geq R \frac{y_{t+1}}{A_0 p_{max} \; l_{i,t+1}}$ is satisfied, the economy is in the polarized case. Individuals with $k_{i,t+1} = k_{max}$ become rentiers and the remaining individuals become workers. Finally, the working time of the rentiers is $l^r_{i,t+1} < 1$ and their leisure time is $z^r_{i,t+1} > 0$. In turn, the working time of the workers is $l^W_{i,t+1} = 1$ and their leisure time is $z^W_{i,t+1} = 0$ since $z_{i,t+1} = (1 - l_{i,t+1})$.

Having defined the three plausible scenarios of this model, I now discuss and illustrate graphically the results of proposition 1 both at the aggregate and at the individual level. In the egalitarian case, having capital and a high innate potential does not guarantee becoming rentier because the low technological level does not give value to the individuals' productive potential. On the other hand, the technological change does not occur since all the agents become workers. Moreover, they have not leisure time and, consequently, their quality of life is poor. This scenario is typical of a poverty trap or Malthusian state.

1. **Egalitarian case**

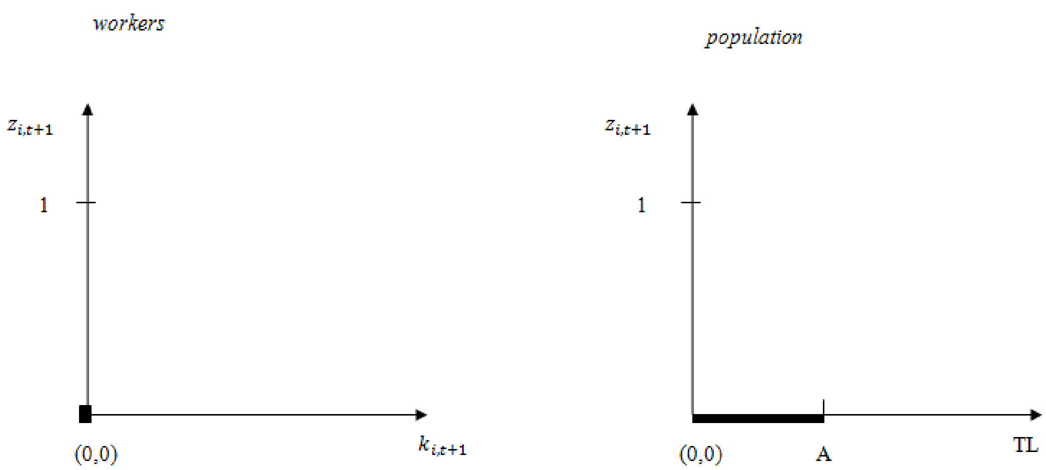

Figure 1

Regarding the intermediate case, economies with a very favorable technological level allow becoming rentier with low levels of capital. Nevertheless, this condition is not met when the individuals' innate potential is poor. The technological change occurs both at the aggregate and the individual level because some agents can access to a new type of technology. This scenario consists in a situation where the rentiers and two types of workers coexist. Note that having capital is not a sufficient condition for becoming rentier since the individuals must have a minimum level of capital to generate technological change. Otherwise, they continue accessing the traditional technology. Regarding the quality of life, this equilibrium allows some workers to have leisure time and improve their well-being. This is a typical scenario of an economy of the technological age.

2. **Intermediate case**

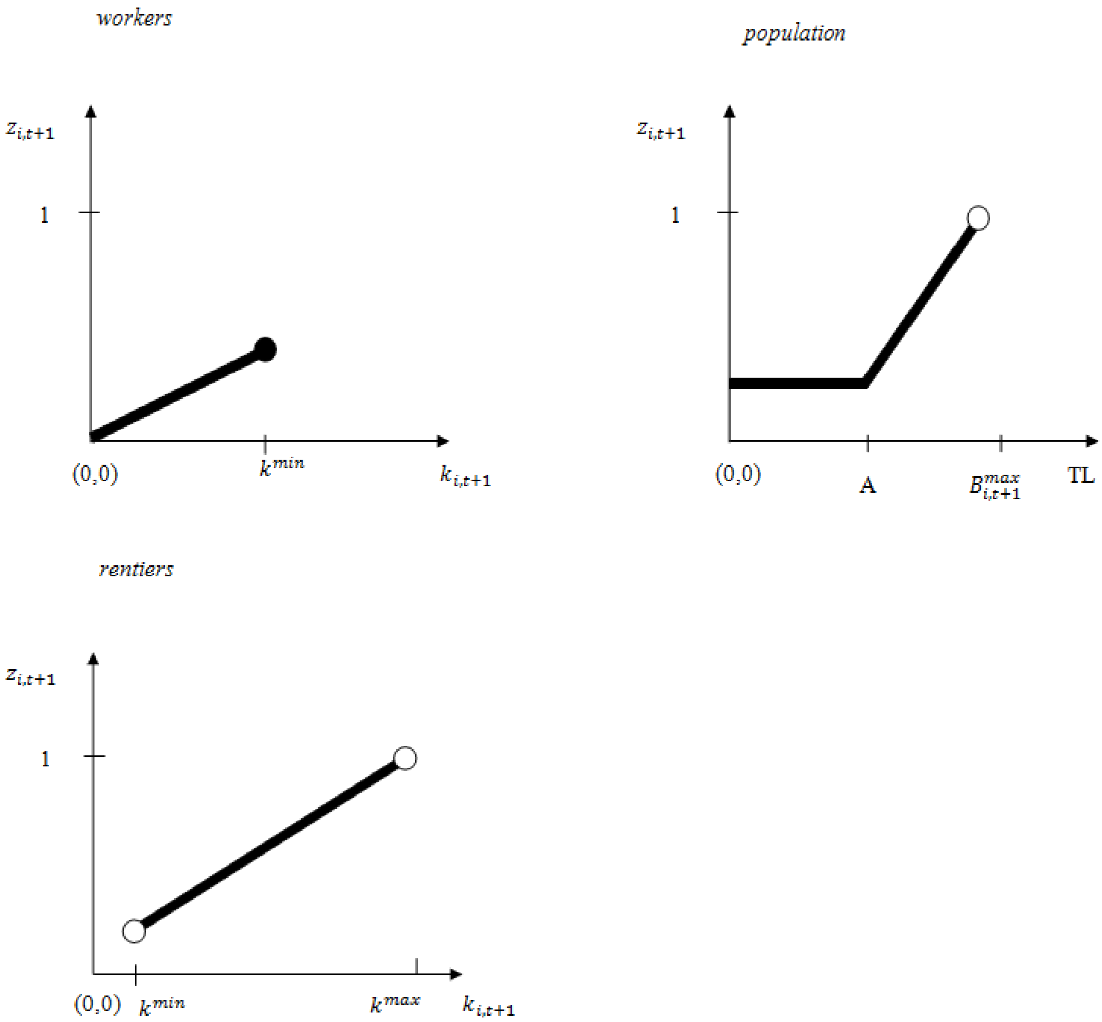

Figure 2

In the polarized case, having capital and a high innate potential guarantee become rentier because the high technological level of the economy provides value to the productive potential of individuals. On the other hand, technological change occurs since some agents becoming rentiers. Finally, rentiers have leisure time but workers do not. Comparatively, both level of life are unequal. This scenario is typical of the Industrial Revolution.

3. <u>Polarized case</u>

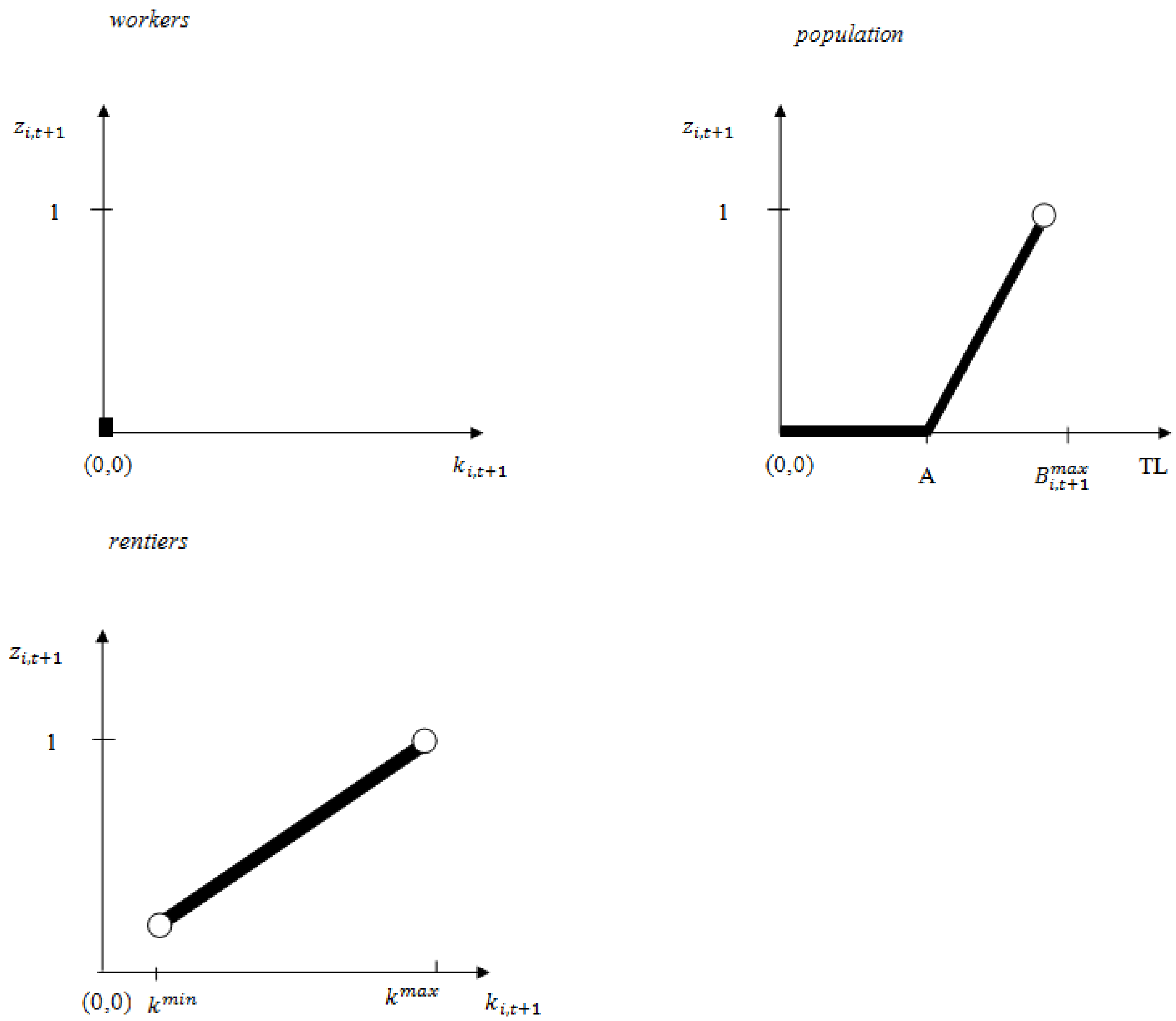

Figure 3

In order to clarify the above result, we can ask the model this question: "Has individual *i* capital?". That is, depending on how the capital is distributed, there is a certain socioeconomic order in the economy. Figure 4, Figure 5 and Figure 6 illustrate this result for each case.

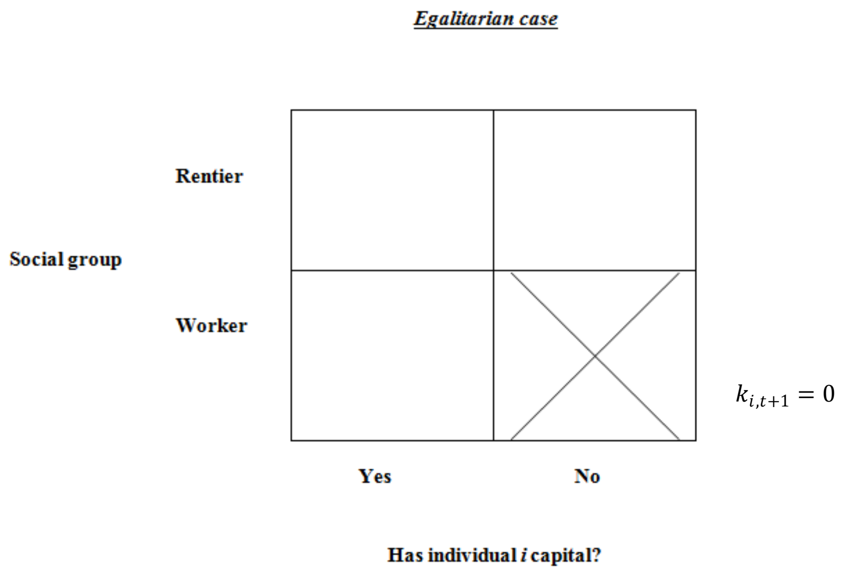

Figure 4

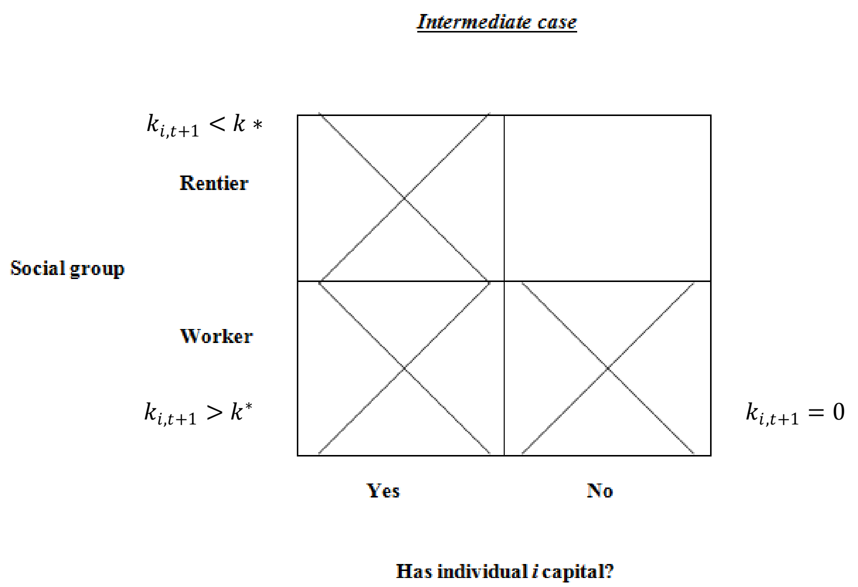

Figure 5

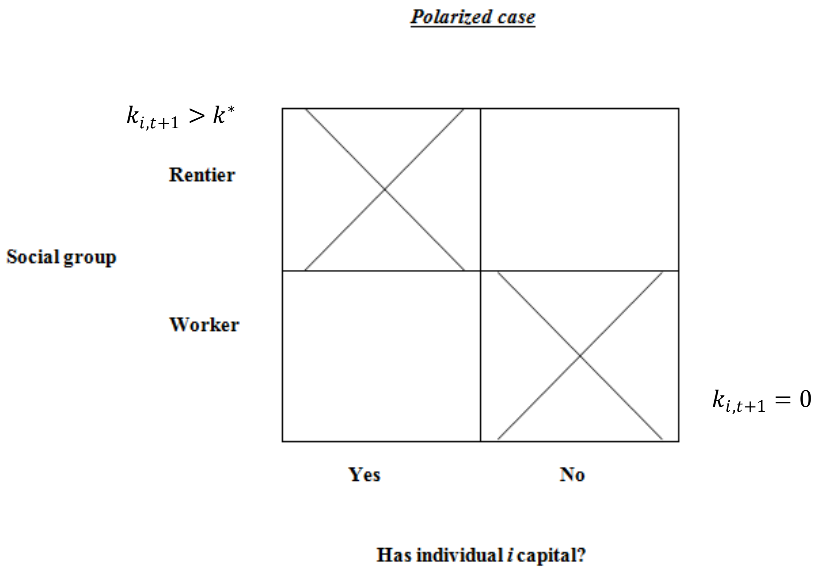

Figure 6

To sum up, this model showed that the innate potential and the technological level determine the type of technology that can be accessed by each agent through the capital accumulation and, consequently, the technological change. That is, depending on the individuals' characteristics and the economic environment, the agents have different capital levels. In turn, the distribution of the capital among the individuals, at the individual level, ends up determining their quality of life and, at the aggregate level, the different socioeconomic orders.

4 Conclusions

In this paper I theorically illustrated that individual i's innate potential and the technological level in the initial period determine the individual i's investment amount in formal education and, consequently, the capital level of each agent. Furthermore, I proposed a new approach about how agents can generate positive economic outcomes in terms of income levels and life quality, with respect to leisure. In turn, these events which are privately beneficial for each agent end up determining the several socioeconomic orders at the aggregate level. In contrast to Sato et al. (2008), the model generates a not so obvious prediction. In economies with sufficiently high technological levels, the investment in formal education of more skilled individuals allows them foster technological change through access to capital and becoming rentiers. Accordingly, the higher the capital level, the higher the leisure time. Hence, rentiers obtain a higher life quality. Conversely, less skilful agents become workers since their investment in formal education do not allow them foster technological change because of their low capital levels. Therefore, workers cannot improve their life quality.

5 Appendices

5.1 Appendix A

Proof of Lemma 1:

From Eqs. (6) and (11), the assumption that $p_{i,t} \geq p_{min}$, and the existence of technological change, it may simply verified that $z^{E}_{i,t+1} > z^{W}_{i,t+1}$ holds for all individuals i, if

$$B_{i,t+1}\, p_{min}\, l_{min} + e > A\, p_{i,t}$$

From Assumption 1 where $B_{i,0}\, p_{min}\, l_{min} > A\, p_{max}$, it is straightforward to verify that the above condition holds. □

5.2 Appendix B

Proof of condition (14):

$$W^{r}_{i,t+1} \geq W^{w}_{i,t+1} =$$

$$= (B_{i,t+1}\, p_{i,t+1} l_{i,t+1})^{\beta\gamma} > (A\, p_{i,t+1})^{\beta\gamma}$$

$$= (Ak_{i,t+1}\, p_{i,t+1} l_{i,t+1})^{\beta\gamma} > (A\, p_{i,t+1})^{\beta\gamma}$$

$$= (k_{i,t+1} y_{t+1})^{\beta\gamma} \geq (y_{t+1})^{\beta\gamma}. \ \square$$

5.3 Appendix C

Proof of Lemma 2:

If we have $k^{*} = 1$ and $y_{t+1} = A\, p_{i,t+1} l_{i,t+1}$,

By substituting the equations $p_{it+1} = p_{i,t} \alpha e_{i,t}$ and $k_{i,t+1}/R = \alpha e_{i,t}$ into $y_{t+1} = A\, p_{i,t+1} l_{i,t+1}$, we obtain:

$$y_{t+1} = A\, p_{i,t} \alpha e_{i,t} l_{i,t+1}$$

$$= y_{t+1} = \frac{A\, p_{i,t} k^*_{i,t+1} l_{i,t+1}}{R}$$

$$= k^*_{i,t+1} = R \frac{y_{t+1}}{A p_{i,t} l_{i,t+1}}. \quad \square$$

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