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The Variance-Frequency Decomposition as an Instrument for VAR Identification: an Application to Technology Shocks

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Abstract: This paper proposes a new framework to study identification in structural VAR models. The framework is based on the variance-frequency decomposition and focuses on the contribution of the identified shock to the variance of model variables in a given frequency range. We use the hours-productivity debate as a connecting thread in our discussion since the identification problem has attracted a lot of attention in this literature. To start, we employ the framework to study the business cycle properties of a set of different identification schemes for technology shocks. Grounded on the simulation results, we propose a new model-based procedure which delivers a precise estimate of the response of hours. Finally, we put all the schemes to work with real data, obtaining substantial evidence in favor of plausible RBC parametrizations, especially from identification restrictions that perform better in simulations. This analysis also reveals that the schemes that recover a very strong response of hours (higher than the implied by typical RBC parameterizations) tend to overstate the contribution of the technology shock to the fluctuations of hours worked at business cycle frequencies.

Keywords: Business cycle, frequency domain, hours worked, productivity, vector autoregressions.

Classification: C1, E3

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1. INTRODUCTION

Estimated impulse responses from vector autoregressions (VAR) are often employed to validate macroeconomic models. For example, Gali (1999) open a large debate on the validity of the Real Business Cycle (RBC) model, finding a negative response of hours to a technology shock using a VAR identified with a long-run restriction. His work has been reexamined from several perspectives, including the treatment of the persistence of hours (Christiano et al., 2004, Pesavento and Rossi, 2005 or Lovcha and Perez-Laborda, 2015), or the influence of low-frequency cycles to the results (Fernald, 2007, Francis and Ramey, 2009, Canova et al., 2010 or Gospodinov et al., 2010). In these studies, the final conclusion is built on the sign of the estimated response of hours: a negative response is considered as evidence against the RBC model, and a positive response is an evidence in favor of the RBC model.

Obviously, the identification of the technology shock becomes a critical issue here. Simply put, in order to validate the model with the results that emerge from the VAR, one has to be sure that the technology shock in the model and the technology shock identified from the data is the same thing.[1] In this study, we propose a new methodology to study VAR identification which focuses directly on the business cycle (BC) properties of the identified shock. To study these properties, we make use of the variance-frequency (VF) decomposition (see e.g., Altig et al., 2011, DiCecio and Owyang, 2010). The VF decomposition measures the shares of the model's variables fluctuations that are attributed to each structural disturbance at a given frequency (or in a frequency range). Thus, it can be understood as the frequency-domain alternative to the usual forecast error variance (FEV) decomposition. However, it has a clear advantage for BC analysis over its time-domain counterpart, since the BC is naturally

[1] See Ramey (2016) for an excellent review on the identification of macroeconomic shocks.

defined in terms of frequency-domain concepts (usually cycles with periods ranging from 8 to 32 quarters) while there is not such a direct connection between the BC definition and the FEV decomposition.[2]

We organize the discussion as follows. To start, we employ Monte-Carlo methods to examine within this framework the small sample properties of a set of identification schemes proposed in the hours-productivity debate. The question we address is whether the identified shock in the VAR and the true technology shock in the model employed as data generating process (DGP) have similar contributions to the variance of the variables at BC frequencies. In our view, if these contributions diverge substantially, the identified shock should not be considered as a "technology shock" according to the model and consequently, the empirical and theoretical responses are not comparable no matter sign they have. Specifically, we study the following five schemes: the usual short-run and long-run restrictions, the modified long-run procedure (Christiano et al., 2006), the Max-Share method (Francis et al., 2014), and the identification in the frequency domain (DiCecio and Owyang, 2010). Consistent with previous findings, the short-run scheme performs remarkably well in simulations (see Christiano et al., 2007 or Lovcha and Perez-Laborda, 2015). Yet, the timing assumptions required for its application are seen by some researchers as too restrictive. The results obtained with other schemes are significantly worse. The sampling uncertainty in the estimated responses questions their practical use in many situations, and the BC properties of the identified shock usually differ substantially from the properties which the true shock has

[2] The forecast error variance decomposition computes how much of the (h-step) total forecast error variance of each of the variables can be explained by each shock. The typical use of this method implies a one-to-one mapping between forecast error and cyclical components, which is far from being true since the h-periods ahead forecast error contributes to the variability of the variables in all frequencies, not just at business cycle.

in the DGP. Among them, the Max-Share method performs noticeably better, although caution should still be used in its application.

Second, we propose a new identification strategy built on the grounds of the simulation study. The method is also based on the VF decomposition and uses RBC model values for the fractions of hours and productivity growth variances explained by the technology shock in several frequency ranges. Monte-Carlo results show that the proposed procedure provides a precise identification of the technology shock, reducing the uncertainty in the estimated responses. Obviously, our proposed scheme is model dependent. However, the degree of uncertainty associated with other methods turns precision a very valuable property. In fact, the idea to use RBC characteristics for identification is not new. Uhlig (2004) analyzes a theoretical model concluding that the contribution of technology shocks contribute most to the variance of the forecast revision of productivity peaks at intermediate horizons. Based on this, the author employs a middle-run scheme imposing that technology as the only shock affecting the FEV of productivity at a horizon of four years. Further, we are not the first researchers to employ the VF decomposition for identification. DiCecio and Owyang (2010) identify the technology shock in the frequency domain maximizing the contribution of this shock to productivity variance at several frequency ranges, although they do not evaluate their method in simulations. Unlike them, we reach identification by minimizing the difference between empirical and theoretical contributions, thus combining the ideas of the previous two works. This distinction is crucial since the DiCecio and Owyang (2010) procedure can only be justified when the maximization is carried out over the low-frequencies, and even in that range has still associated a large degree of uncertainty in the estimated responses.[3]

Finally, we put all the schemes to work with real data, finding substantial support to the RBC specifications of Christiano et al. (2006a), especially from the schemes that perform better in the Monte-Carlo study. Our results also accommodate quite well the recursive assumptions required for short-run identification. Besides, the empirical study reveals that schemes that recover very large responses of hours in the data (higher than those implied by typical RBC parametrizations) tend to overstate the BC contribution of the technology shock.

The remainder of this paper is organized as follows. Section 2 briefly describes the econometric framework. The Monte-Carlo study may be found in Section 3. Section 4 proposes a new identification scheme based on the VF decomposition. The real data analysis (RDA) is carried out in Section 5. Finally, Section 6 offers some concluding remarks. There is also a separate Appendix containing the description of all the identification schemes that were analyzed. A "user-friendly" code to compute the VF decomposition of a structural VAR is also available at the authors' site.[4]

2. ECONOMETRIC FRAMEWORK

2.1 The Structural VAR

For the sake of simplicity, consider the standard bivariate specification in the literature, where the first variable in the vector $X_t = \left(\ln l_t, \Delta \ln y_t\right)'$ is defined as the logarithm of hours worked and the second variable is defined as productivity growth.

[3] As shown in the Monte-Carlo study, their procedure does well in terms of the bias when maximization is carried out over low frequencies, yet with a large degree of uncertainty. However, when it is applied to other frequency ranges, the method is judged not appropriate to recover impulse-responses.

[4] The Appendix and the code can be downloaded at: https://sites.google.com/site/ylovcha/Research

The reduced-form $VAR(p)$ can be written in $MA(\infty)$ form as:

$$X_t = \left[I - F(L)\right]^{-1} u_t \tag{1}$$

where I is the identity matrix, $F(L)$ is a matrix polynomial of order p in the lag operator L, and the reduced form errors u_t have a zero mean and Ω variance-covariance matrix. The structural representation of the model is given by:

$$X_t = \left[I - F(L)\right]^{-1} A\varepsilon_t \tag{2}$$

where ε_t is a vector of uncorrelated, zero mean structural errors with identity variance-covariance matrix. The structural matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

maps the structural shocks into the reduced residuals, thus satisfying $\Omega = AA'$:

$$\begin{cases} \sigma_1^2 = a^2 + b^2 \\ \sigma_2^2 = c^2 + d^2 \\ \sigma_{12} = ac + bd \end{cases} \tag{3}$$

This implies a system of 3 equations and 4 unknowns, and therefore one additional restriction is needed to recover all the parameters.[5] The imposition of this restriction is a central issue in the hours-productivity debate, and several identifying assumptions had been proposed in the literature. Each of them has associated a different response of hours, which has fueled the debate.

[5] See e.g. Rubio-Ramirez et al (2010) for a general analysis of identification. Also, note that sign restrictions to the responses of hours and productivity growth to their own shocks are also required. As is common in the literature, we assume that both are positive ($a, d > 0$).

2.2 The Variance-Frequency Decomposition

Consider the structural representation of the VAR model, and let $f(\omega)$ be the 2×2 spectral density matrix of X_t at a given frequency ω:

$$f(\omega)=\frac{1}{2\pi}\left(I-F\left(e^{i\omega}\right)\right)^{-1}AA'\left(I-F\left(e^{-i\omega}\right)'\right)^{-1} \tag{4}$$

where i denotes the imaginary unit, and $F\left(e^{-i\omega}\right)$ is the complex conjugate of $F\left(e^{i\omega}\right)$. The main diagonal of $f(\omega)$ contains the univariate spectral densities of (log) hours and productivity growth and the off-diagonal elements are the cross-spectral densities. The univariate spectrum of the n^{th} variable can be re-written from (4) as:

$$f_n(\omega)=\frac{1}{2\pi}\sum_{k=1}^{2}\left\|C_n\left(e^{i\omega}\right)A_{kn}\right\|^2 \tag{5}$$

C_n is the n^{th} row of the matrix $\left[I-F\left(e^{i\omega}\right)\right]^{-1}$, and A_{kn} is the k^{th} element of column n in the structural matrix A. This equation expresses the spectrum of the variable n at a given frequency ω as the sum of terms associated with each structural disturbance. Given that the spectrum of a process can be interpreted as the decomposition of the variance into a set of uncorrelated components at each frequency, the percentage contribution of the k^{th} shock to the variance of variable n attributed to cycles of frequency ω is given by:

$$w_{nk}(\omega)=\left\|C_n\left(e^{i\omega}\right)A_{kn}\right\|^2\left(f_n(\omega)\right)^{-1} \tag{6}$$

Thus, the fraction of the fluctuations of the variable n in the frequency range $R=[\omega_1,\omega_2]$ accounted for k^{th} shock can be computed easily from (6) as: [6]

[6] The definite integrals in (7) can be approximated by summations for the Fourier frequencies $\omega_j=2\pi j/T$, $j=0,1,...,T/2$ that belong to the given range (see e.g., Sargent, 1987, ch.11, equation (20)).

$$w_{nk}(R)=\left(\int_{\omega_1}^{\omega_2}\left\|C_n\left(e^{i\omega}\right)A_{kn}\right\|^2 d\omega\right)\left(\int_{\omega_1}^{\omega_2} f_n(\omega)\,d\omega\right)^{-1} \tag{7}$$

As can be deduced from the previous two expressions, the contribution of the different shocks to the variance of a variable at a given frequency (or in a given frequency range) depends crucially on the way the structural shocks are identified (it is, how the entries of the structural matrix A are obtained), and therefore it varies substantially with the particular identification scheme applied.

3. MONTE-CARLO ANALYSIS

In this section, we evaluate by means of simulations the short sample properties of a set of five identification schemes which have been proposed in the hours-productivity literature: the short-run (SR), the long-run (LR), the Christiano et al. (2006) modified long-run strategy (MLR), the Max-Share method (MS) of Francis et al. (2014), and the identification in the frequency domain (FD) of DiCecio and Owyang (2010). Details on the application of these schemes are provided in the separate Appendix to this work. We do not restrict the study to the analysis of impulse responses, as previously done in the literature and additionally we use (7) to evaluate the contribution of the identified shock to the variances of hours worked and productivity growth at BC frequencies.[7]

To do so, we simulate I=1,000 artificial quarterly datasets of 244 observations each, containing artificial data on hours worked and productivity from both the standard and the recursive versions of the two-shock RBC model of Christiano et al. (2007), using their benchmark parameterization.[8] This model provides a good fit to the data compared to other RBC specifications and has been already employed for the analysis of the IRFs

[7] As in the literature, we define the BC range as formed for cycles with periods between 2 and 8 years.

[8] The model solution and the state-space representation can be found in the technical Appendix of Lovcha and Perez-Laborda (2015), at: https://sites.google.com/site/ylovcha.

in Monte-Carlo studies (see Christiano et al., 2007 or Lovcha and Perez-Laborda 2015). The recursive version is included in the analysis to evaluate SR restrictions, which are not satisfied in the standard formulation. At each artificial dataset, we estimate a VAR(4), which is the standard lag-length in the literature, and we identify the technology shock using the five proposed schemes. Thus, each dataset represents what an econometrician would estimate based on a sample of 244 observations. For the MS method, the finite horizon *h* is fixed at 10 years, as in Francis et al. (2014). For FD identification, we employ two different bands for maximization: the BC band, with periods ranging from 2 to 8 years, as in DiCecio and Owyang (2010), and the low-frequency (LF) band, containing cycles with periods longer than 8 years.[9]

3.1 MonteCarlo Analysis of the Response of Hours

Figures 1 and 2 depict the mean of the estimated response of hours to the identified technology shock across the 1000 simulations, using both the standard and recursive versions of the RBC. The shadowed areas correspond to the 10th and 90th estimated percentile bands, which measure the sampling uncertainty associated with the particular identification method. We also include in the figures the true responses from the corresponding DGP. Note that for the recursive version, the positive response follows a contemporaneous zero response, which is a direct consequence of the timing assumptions which make this version compatible with SR restrictions.

As can be seen in Figure 2, SR identification performs outstandingly well with data generated from the recursive model. There is no evidence of bias in the estimated

[9] DiCecio and Owyang (2010) employ the same BC definition, but they split the LF band into medium-long (cycles from 8 to 20 years) and very-long (cycles from 20 to 200 years). However, with the standard lengths available in real datasets, the last band consists merely on a single frequency, so we merge the medium-long and very-long into a single band. Note also that, given the unit root of productivity, the spectrum of this variable is infinite at zero frequency. As standard practice, we exclude this frequency for identification using the LF range.

response and the sampling uncertainty is very small. However, this scheme does not work well when data come from the standard RBC (Figure 1) since this model does not satisfy the restrictions needed for SR identification. According to simulations, LR identification produces on average slightly over-estimated responses, for both standard and recursive versions of the DGP. More important is the fact that the percentile bands are always very wide, containing the zero response at all horizons. Thus, uncertainty associated with the LR scheme is so huge, that even a negative response for hours recovered from the real data can be perfectly compatible with the validity of RBC models in many situations.[10] The MLR procedure of Christiano et al. (2006) is able to slightly narrow the bands, but the associated uncertainty is still large. When the technology shock is identified with the MS method, the depicted bands are considerably narrower than those obtained with the LR and MLR schemes, and the response of hours is on average just slightly underestimated. Note however that the bands still include the zero at all-time horizons. Overall, the simulation analysis of the response of hours with the first four schemes is in line with the results obtained by Christiano et al. (2007), Lovcha and Perez-Laborda (2015) and Francis et al. (2014).[11] The figures also plot the results for FD identification, which has not yet been evaluated in Monte-Carlo studies. This scheme performs well in terms of the bias of estimated responses if maximization is produced over the LF band. In this case, the sampling uncertainty is smaller than with the LR and MLR schemes, but the band is still significantly wider than those of the SR and MS methods. However, when maximization is produced over the BC range of frequencies, the estimated response is not comparable to the theoretical response, no

[10] As noted in Christiano et al. (2007), as far as the confidence intervals for impulse responses reflect the true degree of uncertainty observed in simulations, the LR restrictions are still "appropriate", in the sense that an econometrician would not be misled in inference using this scheme. Yet, the impulse-responses would not be informative in most of the situations.

[11] The reader is referred to these works for results on the accuracy of confidence intervals.

matter the DGP. Note that maximizing the contribution of the technology shock to the productivity variance in other frequency ranges lacks theoretical justification and is not satisfied in the DGP.

3.2 Monte-Carlo Analysis of the BC Contribution of the Technology Shock

3.2.1 The BC contribution of the technology shock in the RBC

Table 1 presents the percentage contribution of the technology shock to the variability of hours and productivity growth at BC frequencies in the model employed as DGP in our simulation study. As can be seen in the table, the percentages in the standard and recursive versions are virtually the same. The contribution of the technology shock for the variability of productivity growth at BC frequencies is around 80% in the two versions while the percentage for log hours is much smaller (around 8%). Thus, interestingly, the timing assumption does not affect the BC properties of the technology shock and, in fact, it can be shown that the differences between the two versions are only major at high frequencies (within the year).

In order to illustrate how much these numbers change with the model specification, Table 1 also reports the BC contribution of the technology shock in several alternative scenarios. Using the standard version, we first change the value of the parameter governing the Frisch elasticity of labor supply from the benchmark value of $\sigma = 1$ to $\sigma = 0$, which implies infinite Frisch elasticity. We also consider an alternative scenario with Frisch elasticity equal to 0.68 ($\sigma = 6$). Second, we change the specification of the model and compute the decomposition in the three-shock extension, which includes the investment tax shock.

As can be seen in the table, the contributions of the technology shock to the hours variance at BC frequencies in the alternative scenarios are always small and close to the benchmark value. On the other side, the contributions to productivity growth variance, although always large, vary a little more with the particular specification. If the utility is linear in leisure (indivisible labor), this contribution is slightly lower than in the benchmark case (62%) and rises to 92% if we consider the case $\sigma = 6$. As expected, the three-shock extension decreases the contribution of the technology shock to the two variances since variability is explained by one shock more. However, the decrease is not huge. As an overall, the results are similar to the percentages obtained by Christiano et al. (2007) by HP filtering model data.[12]

3.2.2 The BC contribution of the identified technology shock in the VAR

Once we have assessed the true contribution of the technology shock at BC frequencies in the two versions of the RBC model employed as DGP, we study the shares that account for the identified shock using each of the schemes in the Monte-Carlo study. Table 2 reports the average percentage contributions of the identified shock in the VAR to the variance of hours and productivity growth at BC frequencies across the 1,000 replications. The numbers inside parenthesis correspond to the 10th and 90th percentiles of the estimated percentages with each identification method.

When the artificial data come from the standard RBC, the SR method substantially understates the contribution of the technology shock to the hours' variance and overstates that of productivity growth. However, using the recursive model (where SR restrictions are satisfied), this identification performs well, obtaining average values that do not differ much from true values, with only a moderate degree of uncertainty. The LR and MLR schemes overstate significantly the share of the hour variance which is explained by the technology shock and understate the share of productivity growth. The percentile bands are huge, no matter the RBC employed to generate the data. Thus, in a

[12] Christiano et al. (2007) compute these values just to illustrate differences among model specifications. The authors neither employ these percentages for the evaluation of competing identification schemes nor to propose alternative identification methods.

large percentage of cases, these schemes are recovering shocks with completely different BC properties than those which the true shock has in the DGP. On average, the MS method recovers a lower contribution to the variance of hours than is explained by the true shock and a higher contribution to productivity growth variance.[13] Uncertainty, albeit large, is substantially smaller than that associated with the LR and MLR schemes. Note, however, that for data generated from the standard version, the true contribution to productivity growth variance lies outside the percentile bands, below the down bound. Note also that, while it is inside for data coming from the recursive version, the true value still lies very close to the down bound. Is this a strong pitfall of the method? In our opinion, it is not. Note that, unlike IRFs, the percentages explained by the technology shock are not usually the final object of the inference, and the econometrician has no need to determine with exactitude whether the real percentages are over or under a certain threshold. Our results suggest that using the MS scheme in empirical applications, the identified shock almost certainly overstates the contribution of the technology shock to the variance of productivity growth at BC frequencies. Yet, it is much more likely that the BC properties of the identified shocks are similar to the properties of the real shock using this scheme than when LR (or MLR) restrictions are applied. Turning to FD identification, when the contribution of the technology shock to productivity variance is maximized over the LF range, the average shares for the variances of hours and productivity growth at BC frequencies lie above and below the true percentages respectively, as with the LR method. The percentile bands, although somehow narrower than LR bands, are still too large to be confident on the properties of the identified shock in real data. Not surprisingly, when the targeted range for

[13] Note that the imposition of the MS and FD schemes requires productivity to enter in (log) level in the VAR. In the estimation of the spectral density we employ the proper filter $B(L) = [1 \quad 0; \quad 0 \quad (1-L)]$ to recover the contribution of the technology shock to productivity growth.

identification is the BC, the contribution of the technology shock to the productivity growth variance at these frequencies is largely overstated, and the true value lies below the percentile bands despite the version of the RBC used to generate the data.

Overall, the Monte-Carlo study shows that only the SR scheme performs well both in terms of the estimated responses and in terms of the percentages of model variances accounted for by the identified shock. However, there is still a requirement with respect to the timing of shocks that are often seen as too restrictive. The responses recovered with other identification schemes suffer from large sampling uncertainty and the schemes very often recover shocks with BC properties that differ strongly from those which the technology shock has in the DGP. Among them, the MS method does much better in simulations but still is associated with a considerable amount of uncertainty and is likely to understate the contribution of the technology shock to the variance of productivity growth at BC frequencies.

4. MODEL-BASED IDENTIFICATION

In this section, we propose a new identification method that is aimed at reducing uncertainty in the associated responses. Our model-based (MB) procedure finds the additional restriction needed to identify the system (3), minimizing the sum of the squared differences between theoretical and empirical contributions for all frequencies in a given range $R = [\omega_1, \omega_2]$. More specifically, MB identification is reached by:

$$\begin{aligned} &\min_{b,d} \quad \sum_{\omega_j \in R} \sum_{i=1}^{2} \left[w_{i,tech}^{VAR}(\omega_j) - w_{i,tech}^{RBC}(\omega_j) \right]^2 \\ &s.t. \quad \sigma_{12} = \sqrt{\sigma_1^2 - b^2}\sqrt{\sigma_2^2 - d^2} + bd \end{aligned} \tag{8}$$

where the restriction in the problem comes from the substitution of the first two equations of the system (3) into the third equation. The terms $w_{i,tech}^{VAR}(\omega_j)$ and $w_{i,tech}^{RBC}(\omega_j)$, can be computed from (6), and denote the fraction of the variance of the variable i attributed to the technology shock at the Fourier frequency $\omega_j \in R$ in the estimated VAR and in the theoretical RBC respectively.

Obviously, the previous procedure is model dependent since it requires the RBC contribution of the technology shock to the two model variances for all Fourier frequencies in the given range. In order to reduce model dependence, we propose a simplified version (SMB) which only requires the overall contribution in the range (only two model values):

$$\begin{aligned} &\min_{b,d} \quad \sum_{i=1}^{2}\left[w_{i,tech}^{VAR}\left(R\right) - w_{i,tech}^{RBC}\left(R\right)\right]^2 \\ &s.t. \quad \sigma_{12} = \sqrt{\sigma_1^2 - b^2}\sqrt{\sigma_2^2 - d^2} + bd \end{aligned} \tag{9}$$

To evaluate the proposed MB procedures, we carry out the same Monte-Carlo analysis of Section 3. For MB identification, we target various frequency ranges. As we did with the FD scheme, we chose the LF range (with periods ranging from 32 quarters on) and the BC range (with periods from 8 to 32 quarters). However, we also examine the MB procedure by targeting the whole frequency spectrum (ALL). For the SMB version of the method, we target the BC range only.

Table 2 contains the average contribution of the identified shock for the variance of the two model variables at BC frequencies using the four proposed MB (and SMB) schemes across the replications. The table shows that the MB method performs really well using the two versions of the DGP. The average contribution is always very close to the actual shares and the percentile bands are very narrow. Thus, in a huge percentage of cases, the identified and the true technology shocks have virtually identical BC properties. More important, this result is irrespective of the frequency range employed for minimization. Note that even if the LF range is targeted, the average shares across replications are also very similar to theoretical values, with only a small amount of uncertainty. The simplified version of the procedure (SMB-BC) also performs very well in the simulation study.

Once we know that the MB schemes work well in identifying the technology shock in the proposed framework, we proceed to analyze the impulse-response of hours recovered with these schemes in the Monte-Carlo study. Results are depicted in the second part of Figures 1 and 2. As can be seen in these figures, the MB schemes do well in terms of the bias when compared to other methods. There is absolutely no evidence of this for data coming from the standard version and is, depending on the particular variation, either non-existent or very small for data coming from the recursive model. Interestingly, the simplified SMB variation does not show evidence of bias no matter whether the data come from the standard version or the recursive version. More important, the MB methods also present very good results in terms of the precision of the estimated responses. The depicted bands are always very narrow, only comparable to those from the SR and MS methods. However, unlike the MS bands, the percentiles obtained with the MB methods do not always contain zero. This occurs, for example, when minimization is carried out over the whole spectrum, irrespective of the DGP. Note that also the SMB-BC bands do not always include the zero for data coming from the recursive version.

5. REAL DATA ANALISYS

Throughout this section, we put all the identification schemes to work with real data. We recover the response of hours and assess the shares of the model variances at BC which account for the identified shock.

5. 1 Data Description.

We employ two datasets which differ in their measures of the hours and of productivity. The two datasets run from 1948:1 to 2009:4, thus covering a slightly longer period than other popular datasets in the literature, and were constructed by collecting data from the Federal Reserve Bank of St. Louis (FRED).[14]

Dataset A is similar to the dataset used in Christiano et al. (2003). It contains data from all sectors of the economy (including the farming sector). The total business productivity is measured as the log of the output per hour of all persons (OPHPBS), and hours worked as the log of the ratio of the business hours of all persons (HOABS) to the civilian non-institutional population over the age of 16 years (CNP16OV).

Dataset B is similar to the dataset used by Gali (1999). The only difference is the definition of hours worked. Gali (1999) employs the log of total employee hours in non-agricultural establishments. Given that hours worked in RBC models are usually defined in per capita terms, we perform the analysis using hours per capita. In particular, the "non-agricultural business sector productivity" is the log of the OPHNFB series in the FRED dataset. Hours worked are constructed by subtracting the log of the civilian non-institutional population over the age of 16 years (CNP16OV) from the log of the non-farming business sector hours of all persons (HOANBS).

In both datasets, "the civilian non-institutional population over the age 16" is converted to quarterly by taking simple averages of monthly observations. Except for population, all the series are seasonally adjusted.

5.2 The Response of Hours to Technology Shock in Real Data

The estimated responses of hours to a positive technology shock in Dataset A and Dataset B are plotted in Figures 3 and 4, respectively. The shadowed areas correspond

[14] We have not included the last years of the data in order to eliminate the influence of the last big crisis. Also, in this way the results are directly comparable to those of Lovcha and Perez-Laborda (2015).

to the estimated 10% confidence interval computed with non-parametric bootstrap.[15] Together with the estimated responses, we also depict the theoretical values from the two versions of the RBC that we have employed as DGP in the Monte-Carlo study, in order to study the empirical evidence of the RBC specification employed in the Monte-Carlo study.

As can be seen in the figures, the response of hours worked recovered with the SR identification is positive in the two datasets (following the zero contemporaneous response) and statistically significant in the short-run. The LR identification also returns positive responses, but of significantly higher magnitude, especially in the Dataset A. The confidence intervals are much wider, reflecting the high sampling uncertainty associated with the method. In fact, in Dataset B, the response is not significant at any horizon. Overall, the responses obtained with SR and LR methods are consistent with the findings of Christiano et al. (2007) and Chari et al. (2008). The MLR procedure returns positive but not statistically significant responses of hours in both datasets. To the best of our knowledge, this scheme has not been applied to real data, and our results cannot be compared with the previous literature. The contemporaneous responses retrieved with MS identification are negative, followed by positive responses after 3 quarters (Dataset A), and 1 quarter (Dataset B). However, they are not statistically significant at any horizon. Our results differ slightly from those reported in Francis et al. (2014). Note that the authors estimate a VAR with five variables while we employ the standard bivariate specification. Also, they made a special treatment in order to remove low-frequency movements in hours. Responses from FD identification in the LF range are similar to those obtained with LR identification, also with very wide confidence intervals. When this identification is carried out over the BC range, the responses are

[15] For that we employ 1,000 bootstrap replications.

initially negative, turning positive in the short-run, although not significant at any horizon (recall however that this scheme is judged to be inappropriate for the study of impulse-responses). Finally, the MB (and SMB) identification produces positive responses in the short-run regardless of the frequency range used for identification. Responses are very precisely estimated with small confidence intervals. The positive response is usually statistically significant in the short-run, departing from a contemporaneous non-significant response.

Note that the responses uncovered with the SR, MS, and the MB schemes are of a similar magnitude, and also similar to those in the theoretical model. This is especially relevant in Dataset A, where the responses of hours uncovered with other schemes are much stronger. Finally, note that the timing assumptions required for SR identification find quite considerable support within the data, in the sense that estimated responses with most of the schemes usually grow following a zero (or non-statistically significant) contemporaneous value.

5.3 The BC Contribution of the Identified Shock in the Real Data

Table 3 reports the shares of the variance in hours and productivity growth at BC frequencies attributed to the identified technology shock in Dataset A and Dataset B. The numbers in parenthesis are the 10% confidence intervals of the estimated percentages computed with a non-parametric bootstrap.

As can be seen in the table, the percentages obtained with the LR, MLR and FD-LF identification schemes differ from the theoretical values in the models employed in the simulation study, especially for hours worked. This is particularly evident using Dataset A, where the estimated values are huge and model-based shares lie outside the estimated confidence intervals. Note however that our Monte-Carlo results show that these schemes tend to overestimate drastically the BC contribution of technology shock to hours variance. Other identification strategies return results that are compatible with the RBCs. The results of the proposed MB identification schemes are characterized by estimated shares of the BC variances attributed to the technology shock that are very similar to the RBC-based values, with relatively narrow confidence intervals. Note that this result holds even if the BC is not the targeted range for identification. The SR and MS estimated percentages are similar to the shares obtained with MB strategies, also supporting RBC values.

Overall, we find substantial evidence in favor of the RBC model in the real data, especially from the identification schemes that perform better in the Monte-Carlo study. The SR, MS, and MB methods recover responses that are similar to responses from the RBC specification and the identified shocks explain similar percentages of model variances at BC frequencies.

5.4 Analysis of Empirical Regularities

There is an interesting regularity in the empirical results. If an identification scheme recovers a technology shock that counts for a large share of the fluctuations in hours at BC frequencies (larger than implied by the RBC specifications), the estimated response of hours is positive and very strong (stronger than RBC responses). This occurs for the LR, MLR, and FD-LF identification schemes, especially in Dataset A. Opposite, when the estimated contribution of the identified shock to the fluctuations in hours is similar to the RBC value, the response of hours is also positive but highly moderated. Further, if in addition the estimated contribution to productivity growth variance at BC frequencies is lower than the contribution implied by RBC specifications, the estimated response even become negative in the middle-run, as with the response that is recovered with the MS scheme.

To study this issue further, we performed a small experiment. Using the two sets of data, we identify the technology shock with the SMB-BC method. Recall that this scheme identifies the technology shock by minimizing the difference between the targeted contributions of the shock to the fluctuations of the two model variables at BC frequencies and the shares that are estimated in the data. As a result of minimization, the targeted and the estimated percentages, albeit unequal, are very similar. Thus, we can target different values to see how responses change with the contributions identified from the data. Specifically, we set the targeted contribution to the variance of hours at 8, 20 and 50%. For each of these values, we gradually change the targeting contribution to productivity growth variance from 50 to 90%, identify the shock and compute the response of hours. All the estimated responses are plotted in Figure 4.

When the targeting contribution to the fluctuations in hours is set at 8% (a value close to the percentage implied by RBC parameterizations), the response of this variable turns positive if the share for productivity growth is targeted at 70% or higher (Dataset A) or 60% or higher (Dataset B). This implies that the SMB scheme reports positive responses using all parametrizations of the RBC specification of Christiano et al. (2007), except for the indivisible labor case using Dataset A (see Table 1).

We now turn to the cases where the share of the hours worked variance at BC frequencies accounted by the technology shock is set to 20 and 50%. These percentages are much higher than those implied by RBC parametrizations but are of the magnitude of the values recovered from the data using the LR, MLR, or FD-LF restrictions. When the percentage is fixed at 20%, the responses are stronger in magnitude. Note that in this case the contemporaneous values for positive responses are also positive. A further increase of this percentage to 50% (or higher), turns all the responses positive and strong, no matter the percentage targeted for productivity growth variance.[16]

Generalizing these results helps to understand the behavior of identification schemes in real data. For instance, the LR, CH, and FD-LF schemes overstate substantially the contribution of the technology shock to the fluctuations of hours at BC frequencies in the Monte-Carlo exercise. As a result, the estimated shares of the variance in hours accounted by the identified shock in the real data are so huge (especially in Dataset A) that, even if the share of productivity growth variance at BC frequencies attributed to the technology shock is estimated to be relatively small, the recovered responses of hours are much stronger than the responses implied by RBC parameterizations.[17]

6. CONCLUSIONS

In this work, we have proposed a framework, based on the VF decomposition, for VAR identification. The idea behind this framework is to study the properties of the identified shock in terms of its contribution to the variance of model variables in a given frequency range. We have shown how to perform a Monte-Carlo study within this framework in a similar manner to what is usually carried out for the analysis of IRFs. Also, we have explained how to adapt the framework to reach identification in the VAR, leading to precise estimates of the response of hours worked. Finally, the application of the framework to real data has revealed interesting regularities in the response of hours.

Throughout this paper, we have employed the hours-productivity debate as a connecting thread in the narration since identification has been at the core of the

[16] In fact, all the responses are positive and strong once the targeted share for hour variance is over 30%.

[17] Note also that these three schemes require a reliable estimate of the low frequency movements for identification, which is difficult to reach.

discussion in this body of literature, and also because RBC models give a natural frequency range to focus on (i.e., the BC). Note however that the proposed framework can be applied to another type of theoretical model that can be cast in state-space form, or be employed in the study of other questions to be addressed with the help of structural VARs, as is usually the case in the analysis of monetary or fiscal policy issues. We consider these subjects to be interesting avenues for future research.

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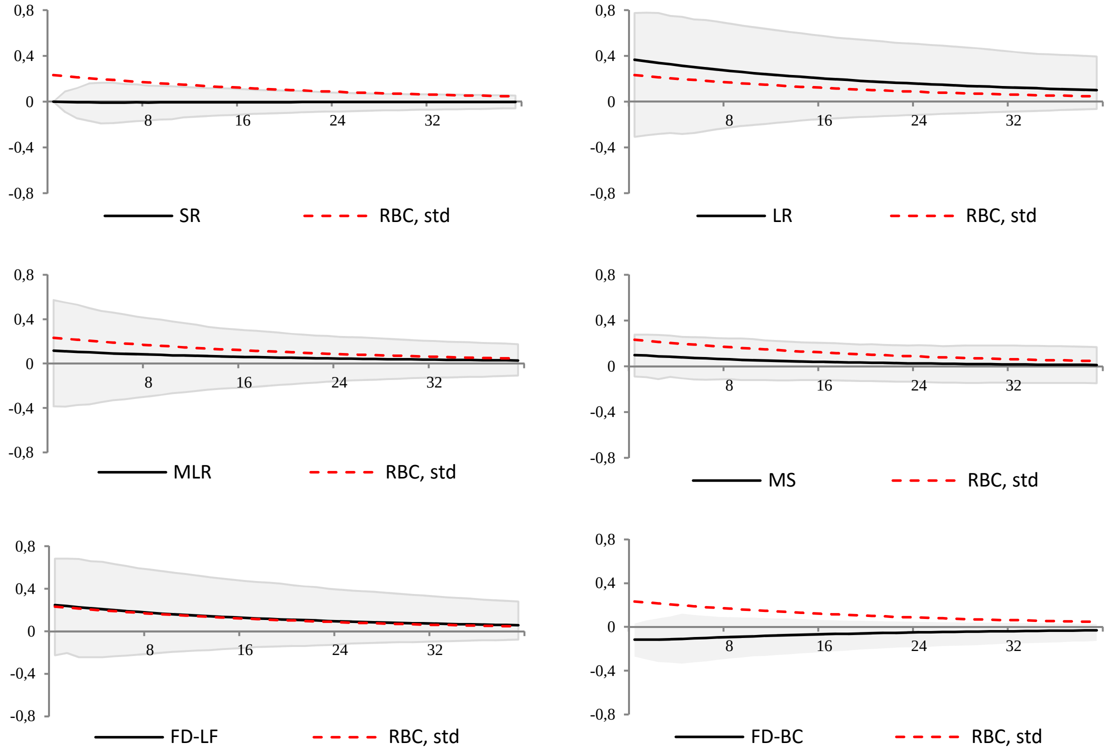
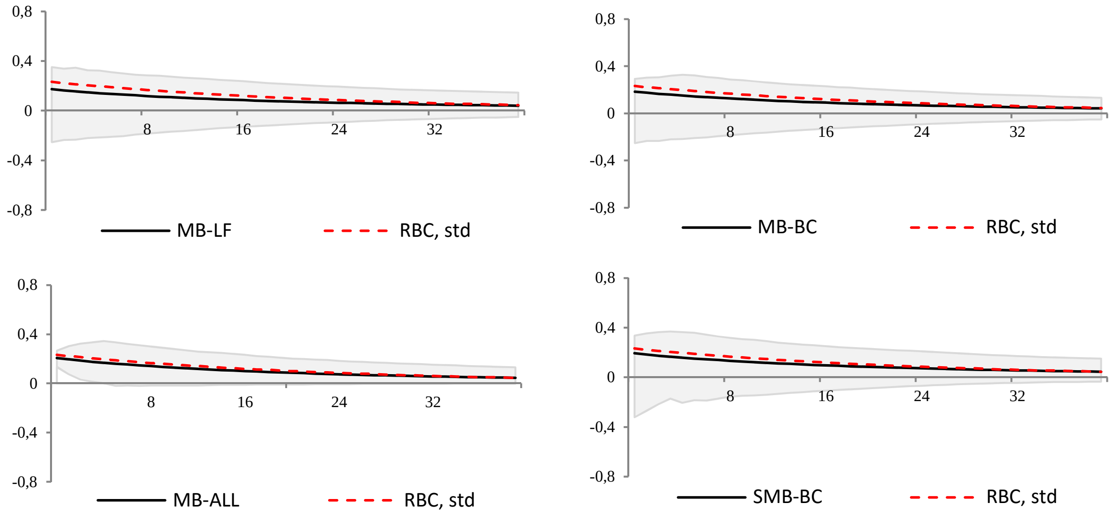
FIGURES AND TABLES

Fig. 1 – Monte-Carlo response of hours to a positive technology shock; Standard RBC

Part 1

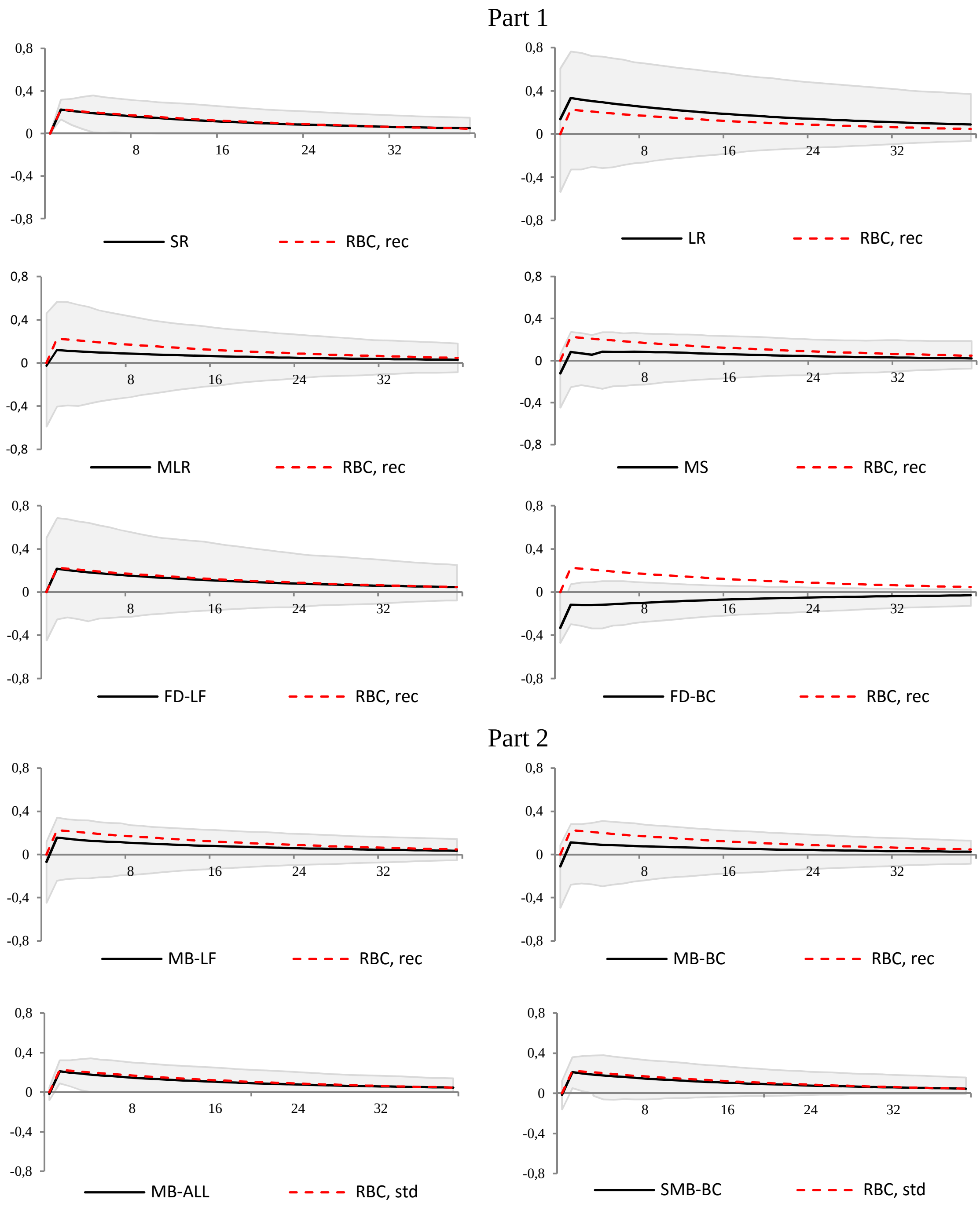


Part 2



Notes: a) The figures depict the mean response of hours work to a positive technology shock across simulations together with the true RBC response; b) the shadowed area corresponds to the 10^{th}-90^{th} percentile band; c) The DGP is the two-shock standard RBC of Christiano et al. (2007).

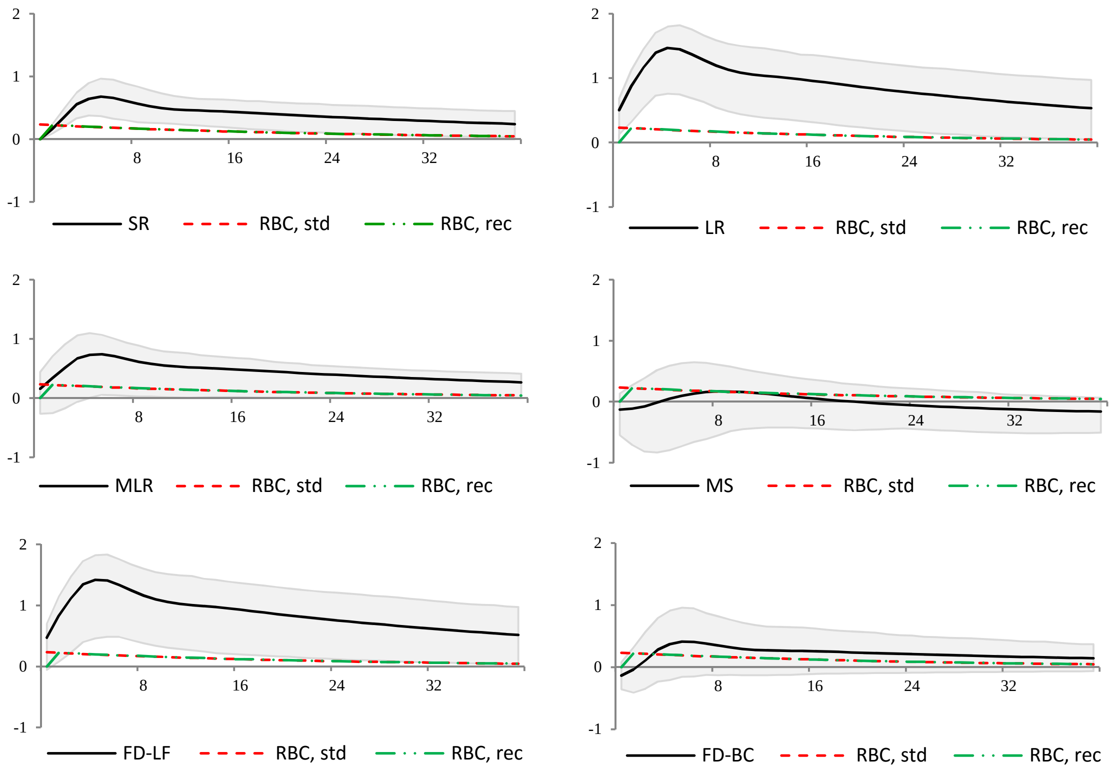
Fig. 2 – Monte-Carlo response of hours to a positive technology shock; Recursive RBC



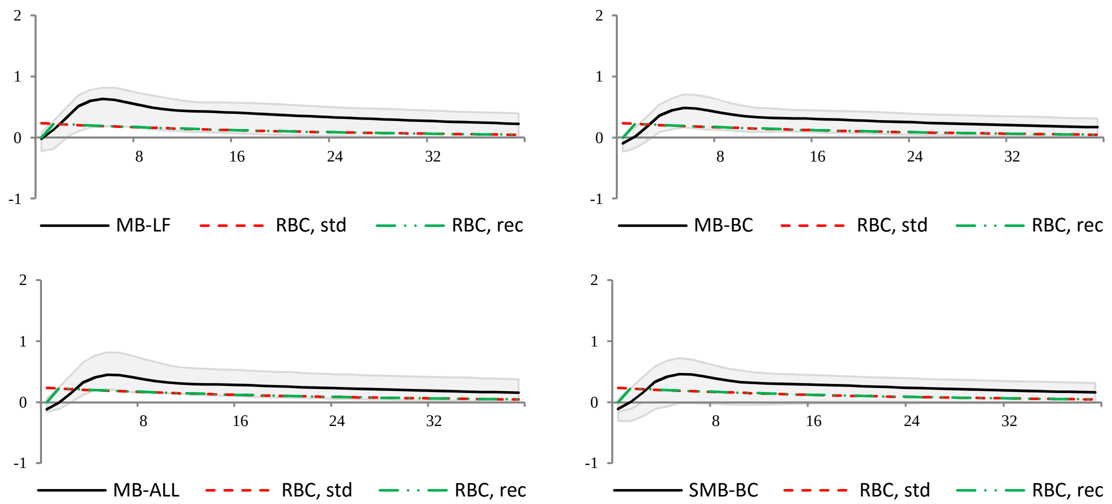
Notes: a) The figures depict the mean response of hours work to a positive technology shock across simulations together with the true RBC response; b) the shadowed area correspond to the 10^{th}-90^{th} percentile band; c) The DGP is the two-shock recursive RBC model of Christiano et al. (2007).

Fig. 3 – Response of hours worked to a positive technology shock. Dataset A.

Part 1



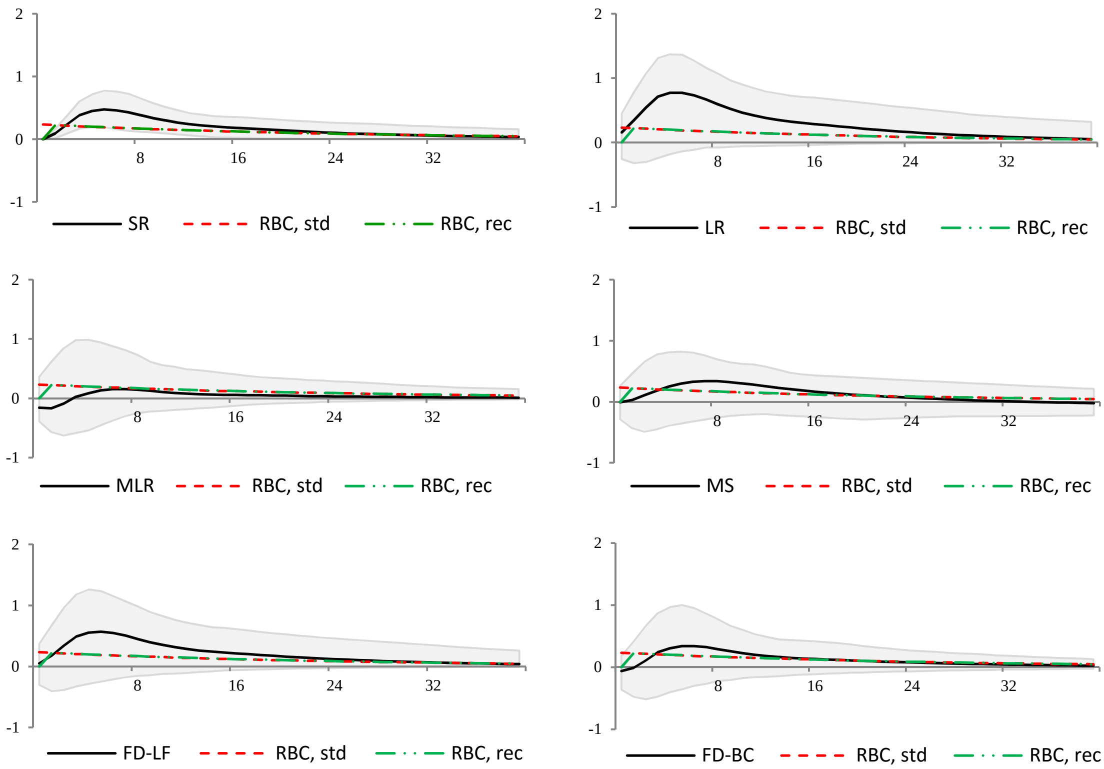
Part 2



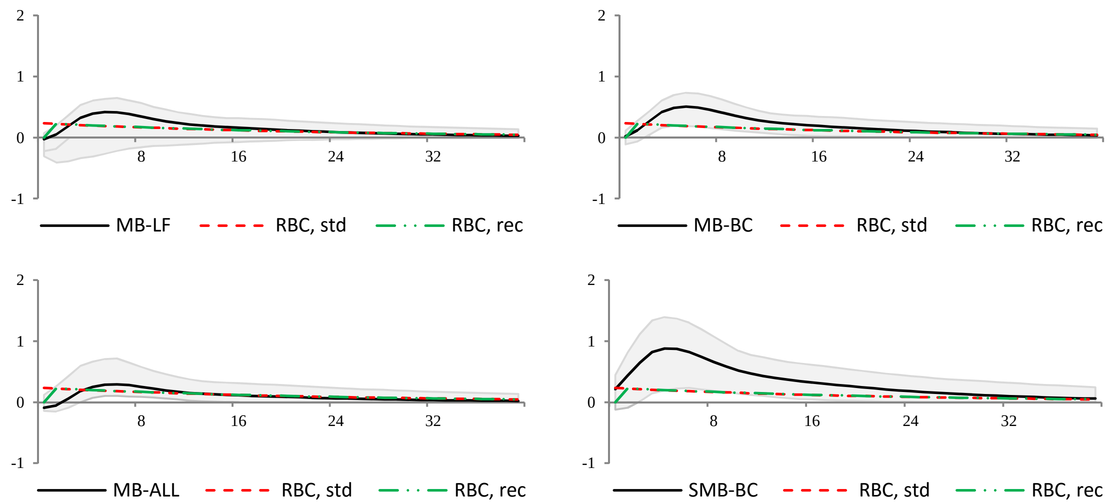
Notes: a) The solid lines depict the response of hours worked to a positive technology shock recovered from Dataset A. This dataset is similar to that of Christiano et al. (2003), and contains data for all sectors; b) the shadowed area corresponds to the 10% CI computed with nonparametric bootstrap; c) the dashed and dash-dotted lines are the theoretical responses from the standard and recursive versions of the two-shock RBC of Christiano et al. (2007) which is employed in the Monte-Carlo analysis.

Fig. 4 – Response of hours worked to a positive technology shock. Dataset B.

Part 1

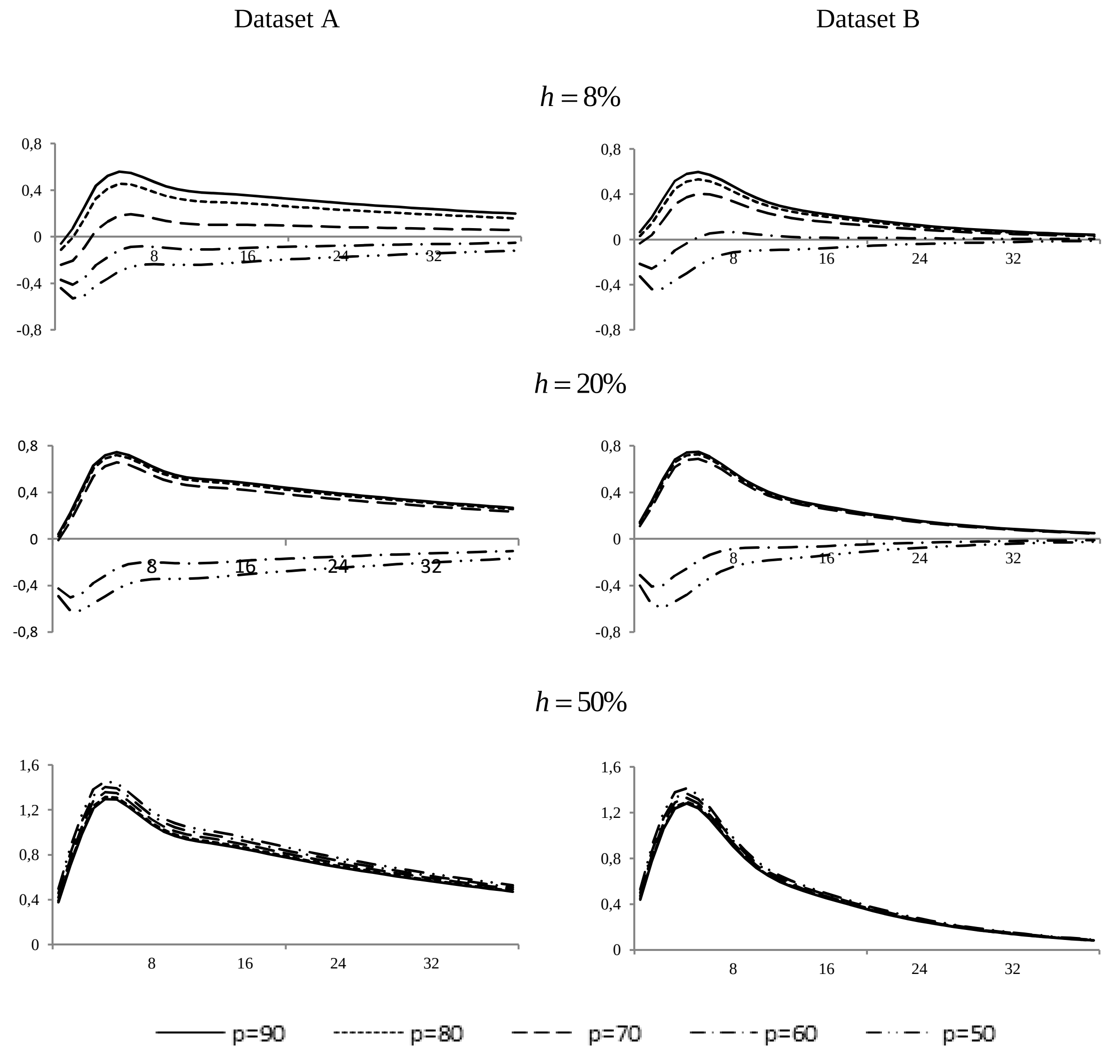


Part 2



Notes: a) The solid lines depict the response of hours worked to a positive technology shock recovered from Dataset B. This dataset is similar to that of Gali (2003), and contains data for the non-farming business sector only; b) the shadowed area corresponds to the 10% CI computed with nonparametric bootstrap; c) the dashed and dash-dotted lines are the theoretical responses from the standard and recursive versions of the two-shock RBC of Christiano et al. (2007) which was employed in the Monte-Carlo analysis;

Fig. 5 Empirical regularity: SMB response with different parameterizations



Notes: $h = w^{RBC}_{\log h, tech}(BC)$ and $p = W^{RBC}_{\Delta \log y, tech}(BC)$ are the targeting contributions of the technology shock to the (log) hours and productivity growth variances at BC frequencies using the SMB identification procedure. These values are *targeting* because the SMB minimizes the difference between these percentages and those obtained from the VAR, and as a result of minimization these two values do not coincide with exactitude.

Tab. 1 –BC contribution of the technology shock in different RBC specifications.

RBC	$\ln l_t$	$\Delta \ln y_t$
Simulation study: 2 shocks: benchmark		
Standard version:	7.49	80.36
Recursive version:	7.12	80.55
Alternative (non-recursive) specifications:		
2 shocks: $\sigma = 0$ (Indivisible labor)	7.95	62.80
2 shocks: $\sigma = 6$ (Frisch elasticity = 0.63)	7.07	92.49
3 shocks: benchmark	5.34	73.28

Notes: a) the numbers in the table are the % contribution of the technology shock to the variances of hours and productivity growth at BC frequencies; b) the RBC specification corresponds to the model of Christiano et al. (2007); c) the benchmark uses the parameterization proposed by the authors (σ =1).

Tab. 2 – BC contribution of the identified technology shock; Monte-Carlo.

% contribution to the BC variance of:	$\ln l_t$	$\Delta \ln y_t$
	STANDARD RBC	
Model Value:	**7.48**	**80.36**
SR	1.67 [0.05, 6.58]	94.75 [88.22, 99.09]
LR	34.50 [0.98, 34.50]	56.09 [6.13, 98.70]
MLR	36.09 [1.46, 83,64]	66.81 [4.80, 99.54]
MS	3.72 [0.18, 12.05]	94.37 [83.74, 99.57]
FD-LF	20.70 [0.43, 66.07]	71.43 [15.34, 99.34]
FD-BC	4.94 [0.16, 15.36]	97.81 [93.28, 99.86]
MB-LF	7.32 [1.49, 15.08]	81.55 [67.17, 94.91]
MB-BC	7.66 [1.90, 14.28]	81.85 [78.59, 87.23]
MB-ALL	7.25 [1.84, 15.09]	82.75 [72.94, 91.06]
SMB-BC	9.69 [1.54, 22.53]	80.36 [80.36, 80.36]
	RECURSIVE RBC	
Model Value:	**7.12**	**80.55**
SR	8.51 [1.57, 17.69]	79.79 [67.60, 90.48]
LR	32.01 [0.99, 78.92]	57.12 [7.65, 97.73]
MLR	38.32 [2.16, 88.40]	64.09 [4.38, 99.43]
MS	4.58 [0.33, 12.77]	92.74 [80.36, 99.01]
FD-LF	19.38 [0.57, 66.95]	72.75 [14.85, 98.83]
FD-BC	6.99 [1.04, 17.18]	97.48 [93.04, 99.86]
MB-LF	8.47 [1.44, 15.13]	81.70 [65.25, 96.70]
MB-BC	8.10 [1.11, 16.13]	82.25 [79.00, 91.33]
MB-ALL	7.81 [1.07, 16.82]	81.26 [71.44, 90.85]
SMB-BC	9.55 [0.88, 24.79]	80.55 [80.55, 80.55]

Notes: a) The numbers in the table are the average % contribution of the technology shock to the variance of (log) hours and productivity growth at BC frequencies across simulations, identified with the corresponding scheme. The parenthesis corresponds to the 10th and 90th percentile percentages; b) The DGP is the standard and the recursive versions of the RBC model of Christiano et al. (2007).

Tab. 3 –BC contribution of the identified technology shock; Datasets A and B

% contribution to the BC variance of:	$\ln l_t$	$\Delta \ln y_t$
	DATASET A	
SR	17.29 [6.27, 31.33]	82.82 [66.83, 92.21]
LR	80.70 [28.08, 94.59]	59.68 [36.07, 85.35]
MLR	89.61 [9.62, 93.72]	88.67 [13.41, 96.23]
MS	9.07 [1.48, 30.66]	68.71 [34.16, 94.54]
FD-LF	75.73 [12.54, 96.20]	64.01 [33.93, 88.46]
FD-BC	7.82 [1.78, 31,87]	77.71 [59.81, 91.14]
MB-LF	15.45 [4.91, 23.60]	82.29 [60.17, 91.94]
MB-BC	9.86 [3.17, 17.70]	79.50 [66.38, 86.71]
MB-ALL	8.79 [3.09, 24.14]	78.64 [66.76, 89.65]
SMB-BC	9.16 [4.02, 16.53]	78.95 [67.55, 82.27]
	DATASET B	
SR	8.80 [1.70, 22.07]	73.13 [55.17, 86.77]
LR	23.42 [2.20, 65.89]	78.02 [49.52, 87.80]
MLR	10.07 [2.13, 79.47]	90.72 [39.60, 97.40]
MS	5.53 [0.90, 32.64]	66.03 [39.14, 89.26]
FD-LF	12.63 [1.35, 56.36]	75.25 [47.18, 87.18]
FD-BC	5.06 [1.24, 35.30]	
MB-LF	8.24 [1.30, 15.66]	67.95 [43.44, 85.73]
MB-BC	10.05 [3.02, 18.56]	72.61 [58.63, 83.18]
MB-ALL	8.11 [0.85, 18.04]	70.32 [57.34, 81.23]
SMB-BC	12.70 [4.42, 18.66]	76.56 [65.31, 80.36]

Notes: a) The numbers in the table are the % contribution of the identified technology shock to the variance of (log) hours and productivity growth; b) the numbers in parenthesis are the 90% confidence intervals computed with nonparametric bootstrap. Dataset A is similar to that of Christiano et al. (2003) and contains data for all sectors. Dataset B is similar to that of Gali (2003) and contains data for the non-farming business sector only.