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Yuliya Lovcha
Alejandro Pérez Laborda

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Universitat Rovira i Virgili
Facultat d'Economia i Empresa
Av. de la Universitat, 1
43204 Reus
Tel.: +34 977 759 811
Fax: +34 977 758 907
Email: sde@urv.cat

CREIP
www.urv.cat/creip
Universitat Rovira i Virgili
Departament d'Economia
Av. de la Universitat, 1
43204 Reus
Tel.: +34 977 758 936
Email: creip@urv.cat

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STRUCTURAL SHOCKS AND DYNAMIC ELASTICITIES IN A LONG MEMORY MODEL OF THE US GASOLINE RETAIL MARKET

Yuiya Lovcha[a,*], Alejandro Perez-Laborda[a]

[a] Universitat Rovira-i-Virgili and CREIP

Abstract: A structural multivariate long memory model of the US gasoline market is employed to disentangle structural shocks and to estimate the own-price elasticity of gasoline demand. Our main empirical findings are: 1) there is strong evidence of non-stationarity and mean-reversion in the real price of gasoline and in gasoline consumption; 2) accounting for the degree of persistence present in the data is essential to assess the responses of these two variables to structural shocks; 3) the contributions of the different supply and demand shocks to fluctuations in the gasoline market vary across frequency ranges; and 4) long memory makes available an interesting range of convergent possibilities for gasoline demand elasticities. Our estimates suggest that after a change in prices, consumers undertake a few measures to reduce consumption in the short- and medium-run but are reluctant to implement major changes in their consumption habits.

Keywords: fractional integration, gasoline demand, price elasticity, structural model
Classification: Q41, Q43, C32

*Corresponding author. Department of Economics and CREIP. Universitat Rovira-i-Virgili. Av. Universitat 1, Reus. 43204 Spain. Phone: (+34) 977759811; Fax: (+34) 977758907; Email: yuliya.lovcha@gmail.com.

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1. Introduction

Since the work of Kilian (2009), it is now well understood that energy prices are determined by both demand and supply conditions. The different demand and supply shocks have distinct effects on energy prices and cannot be treated alike. Thus, disentangling these shocks is crucial to understanding the relation between energy prices and the economy (see e.g. Kilian (2009, 2010), Kilian and Park (2009), or Edelstein and Kilian (2009)). A common feature of this empirical literature is that it relies on vector autoregressive models (VAR), treating energy prices as stationary. Implicitly, it is assumed that real prices have short memory (so-called I(0) processes) and thus the effect of shocks on prices vanishes fast in the short run. However, still there is little consensus about the stochastic behavior of real energy prices. On the basis of unit-root testing results, some other authors suggest that real energy prices are better characterized by unit root behavior (also known as I(1) processes). Some references include Serletis (1992), Carruth et al. (1999), or Narayan and Smyth (2007).

In this work, we base our analysis on Kilian (2010) and consider a structural model to separately identify demand and supply shocks in the US gasoline retail market, but unlike previous studies, we employ for that a fractionally integrated VAR (FIVAR). Our analysis incorporates several innovations relative to the existent literature. First, the FIVAR nests the traditional I(0)-I(1) alternatives but also accounts for situations where this dichotomy appears too restrictive, allowing the responses of variables to shocks to decay to zero at a slower rate than I(0) processes do. Moreover, the (possibly fractional) different orders of integration of all the series are not assumed but rather estimated together with the other parameters. Fractional integration (FI) models are widely employed to describe the dynamics of many economic and financial time series and have also been used to model energy variables, mostly in univariate contexts (see e.g. Gil-Alana (2001), Elder and Serletis (2008), Lean and Smyth (2009), or Choi and Hammoudeh (2009). A notable multivariate exception can be found in Haldrup et al. (2010)). Our results show that long memory is strongly supported by the data. In particular, both the real price of gasoline and gasoline consumption exhibit non-stationary but mean-reverting behavior. Thus, the effects of the different shocks on these two variables eventually disappear, but the degree of persistence is compatible with the results that emerge from unit-root testing.[1]

Our second innovation is to evaluate the influence of the different shocks to energy market fluctuations in the frequency domain. Previous literature has focused the analysis in the time domain only. However, interesting relationships may appear at different frequencies.[2] In order to address this issue, we apply a variance-frequency decomposition of the FIVAR model. We document that the relative importance of the different shocks for the fluctuations of gasoline market outcomes varies across frequency ranges. Thus, for instance, oil-market demand shocks explain the largest share of the gasoline price and gasoline consumption low-frequency movements, but this share declines rapidly with frequency in favor of gasoline market shocks.

Finally, we study the implications of fractional integration for the analysis of the gasoline demand own-price elasticity. This elasticity plays a central role in many policy issues, including national security, optimal taxation, or recently, climate change. As a result, hundreds of gasoline studies employing different models have provided a battery of short-run and long-run estimates (see the survey of Dahl and Sterner (1991), or the meta-analyses of Espey (1998), Brons et al. (2008), or Havranek et al. (2011)). In the standard I(0)/I(1) framework, the response of gasoline demand to a change in prices can only converge to a bounded nonzero value whenever price and demand are assumed to have exactly the same order of integration. If, for instance, demand is assumed to be I(1) and the real price is I(0), as in Kilian (2010), the long-run elasticity of gasoline demand is infinite. However, the inclusion of fractional integration opens a wide range of convergence possibilities that are studied in depth in Section 6. Our estimates indicate that gasoline demand is quite inelastic to gasoline price movements, which is consistent with the results from recent studies covering current data (Small and van Dender (2007) or Hughes et al. (2008)). Moreover, the elasticity presents a rather flat profile, with medium- and long-run values that do not differ much. Thus, our findings suggest that consumers undertake just a few measures aimed to reduce consumption after a rise in

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[1] Throughout this paper, we measure persistence by the speed of mean reversion, as in Diebold and Rudebush (1989). Note that if persistence is measured by the traditional infinite sum of impulse responses, two FI processes cannot be compared, since persistence would be either 0 for negative orders of fractional integration, or infinity for positive (see e.g. Hauser et al. (1999))

[2] In fact, the importance of frequency domain concepts in the relation between energy prices and the economy was already emphasized by Granger (1966). More recent studies include Granger and Lin (1995), Gronwald (2009), or Aguiar-Conraira and Soares (2011).

prices in the short and medium run, but are reluctant to make significant changes in their behavior.

This paper proceeds as follows. Section 2 provides the econometric framework; Section 3 describes the data and the set of identifying assumptions for the structural model. Section 4 presents the estimation results and conducts robustness checking. In Section 5, we evaluate the dynamic effects of structural shocks on the gasoline price and gasoline consumption. The relative contribution of the shocks at different frequencies is considered at the end of the section. We study the gasoline demand price elasticity in Section 6. Finally, Section 7 offers some concluding remarks. Details about the variance decomposition in the frequency domain and the derivation of the dynamic reaction function for gasoline demand may be found in the two appendices.

2. Methodology

2.1 The FIVAR Model and its Estimation Procedure

Vector autoregressive models with fractional integration (FIVAR) are the multivariate version of the well-known autoregressive ARFIMA model (see e.g. Baillie (1996)). Recent contributions in economics have been developed mainly in the context of fractional cointegration, which imposes a particular long-run equilibrium relationship among the variables (see e.g. Jøhansen (2008) and Jøhansen and Ørregaard (2012)). Given the observed different degree of persistence of the variables employed, we consider a simple unrestricted specification that allows for a (possibly) different order of integration of all the variables, as in Abritti et al. (2015), Golinski and Zaffaroni (2015), or Lovcha and Perez-Laborda (2015).[3] This makes available an interesting range of possibilities for the pattern of the dynamic own-price elasticities of gasoline demand (discussed in Section 6) and eases comparison with previous VAR studies.

More specifically, the model can be written as:

$$D(L)X_t = v_t \tag{1}$$

$$\left(I - F_p(L)\right)v_t = w_t \tag{2}$$

where X_t is an $N \times 1$ vector of variables for $t = 1,...,T$; L is the lag operator; I is an $N \times N$ identity matrix; and w_t is an $N \times 1$ vector of i.i.d errors with 0 mean and $N \times N$ variance-covariance matrix Ω. The VAR(p) process in (2) is assumed to be stationary.

[3] Fractional cointegrated models, as Johansen (2008) or Johansen and Nielsen (2011), require equal coefficients of fractional integration for all variables.

$D(L)$ is a diagonal $N \times N$ matrix with fractional integration polynomials on the main diagonal given by:

$$D^{(n)}(L) = (1-L)^{d_n}, \; n = 1,...,N \tag{3}$$

The scalar parameter $d_n \in [0,1]$ is the order of (fractional) integration of the series x_t^n in the model. In this context, this parameter plays a crucial role as an indicator of persistence. The higher the d_n, the more persistent x_n. If $d_n = 0$ or $d_n = 1$, the series x_t^n exhibits standard I(0) or I(1) properties, respectively. Instead, if $0 < d_n < 0.5$, the series is covariance stationary, but the response of the variable to a shock takes more time to disappear than if $d_n = 0$[4]. Finally, if $0.5 \le d_n < 1$, the series is no longer covariance stationary but still mean-reverting, with the effect of the shocks dissipating slowly in the long run. These parameters d_n, $n = 1...N$ are not assumed but estimated jointly with the other parameters of the model.

The $MA(\infty)$ representation of the reduced form model (1) is:

$$X_t = D(L)^{-1}\left[I - F(L)\right]^{-1} w_t \tag{4}$$

This last expression can be found by arranging terms after substitution of equation (1) into 2.

To estimate the model, we employ the approximate frequency domain maximum likelihood estimator proposed by Boes et al. (1989).[5] The discussion of the multivariate version of the procedure can be found in Hosoya (1996). An advantage of this method is that it is relatively simple and allows circumventing the problems associated with the complicated likelihood function arising in the time domain. Moreover, the method estimates the orders of integration of all the series jointly with the other parameters, which is a clear advantage over two-step procedures that suffer from lack of efficiency and do not yield standard √n-asymptotics. See e.g. Lovcha and Perez-Laborda (2015) for further details about the estimation procedure.

[4] The autocorrelation function of I(0) processes decay at an exponential rate but that of FI processes presents slow hyperbolic decay.

[5] Given that we expect large orders of integration (as they are indeed), we difference the series prior to estimation and we subsequently transform them back by adding 1 to the estimated FI orders. Computing the periodogram, we taper the data with the cosine bell taper. See Velasco and Robinson (2000) for details.

2.2 The Structural Model

The FIVAR model given by (1) and (2) is a reduced form model. The structural form of the model incorporates the contemporaneous relationships between variables. The structural model is given by:

$$AD(L)X_t = u_t \quad (5)$$

$$(I - G(L))u_t = \varepsilon_t \quad (6)$$

where A is an $N \times N$ matrix of contemporaneous relationships; $D(L)$ is the diagonal matrix containing the orders of integration of all the series in the model as in (3); the matrix $G(L)$ contains the short memory autoregressive polynomials; and ε_t is a vector of uncorrelated structural errors with 0 mean and diagonal variance-covariance matrix V. Substitution of (5) into (6) and pre-multiplication of both sides by A^{-1} leads, after arranging, to the $MA(\infty)$ infinite representation of the structural model:

$$X_t = D(L)^{-1}\left[I - A^{-1}G(L)A\right]^{-1} A^{-1}\varepsilon_t \quad (7)$$

It follows from (4) and (7) that the equations relating the autoregressive polynomials and the reduced and structural shocks are given by:

$$F(L) = A^{-1}G(L)A \quad (8)$$

$$w_t = A^{-1}\varepsilon_t \quad (9)$$

To identify the structural parameters, we apply Sims' (1989) short–run identification scheme (SR)[6]. We further assume that the matrix A of contemporaneous relationships is lower triangular with ones in the main diagonal. That is, a variable y_i is not contemporaneously influenced by any shock to a variable y_{i+k}, $k > 0$ situated down in the vector, but it may be influenced contemporaneously by shocks to the variables y_j, $j \leq i$ situated before. Once the variance-covariance matrix of the reduced form model errors Ω has been estimated, the entries of the contemporaneous responses A can be easily found from (9).

[6] The effects of long memory on other identification procedures have been discussed in Tschering et al. (2013) and Lovcha and Perez-Laborda (2015).

3. Data Description and Identification Assumptions

To construct a small structural model of the US gasoline retail market, we follow Kilian (2010) and employ monthly data for five key variables. These five variables are defined as a vector:

$$X_t = [os_t, rea_t, rpo_t, rpg_t, gd_t]^T \tag{10}$$

where os_t is the world oil supply, rea_t is the real economic activity index developed by Kilian (2009), rpo_t is the real imported price of crude oil, rpg_t is the real regular gasoline retail prices, and gd_t is the US regular gasoline consumption.[7] Data span from 1978:01 to 2015:06. The initial date is dictated by the availability of the monthly gasoline price. All series, except the activity index, were downloaded from the E.I.A. Monthly Energy Review and are expressed in natural logarithms[8]. The activity index can be obtained directly from Kilian's site, and it is expressed in percentage deviations from the trend as provided by the author[9]. To transform nominal prices to real, we employ the US Consumer Price Index (CPI) from the FRED database.[10] Also, we employ the X12 Census to seasonally adjust the monthly gasoline consumption.

The structural FIVAR model in (5) and (6) is therefore driven by five structural disturbances, which are defined as unanticipated changes in supply or demand: aggregate oil supply shocks; global demand shocks; oil-market specific demand shocks; gasoline supply shocks; and gasoline demand shocks. Aggregate oil supply shocks capture changes in the global oil production that may occur, for example, as a consequence of political events such as the civil disorder events in Venezuela in 2002. Global demand shocks are mostly related to cyclical factors, but may also reflect unexpected shifts in the demand of commodities from new emerging economies. Oil-specific demand shocks capture variations in the precautionary demand for oil. An example of this shock is the sudden increase in demand that can be observed immediately before the Iraq War in 2003. Gasoline supply shocks are unexpected disruptions in the supply of gasoline as a consequence, for instance, of shutting down operations of US refiners after Hurricanes Rita or Katrina in 2005. Finally, US gasoline

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[7] The gasoline demand is the sum of gasoline consumption in industrial, commercial, and transportation sectors.

[8] http://www.eia.gov/totalenergy/data/monthly/

[9] http://www-personal.umich.edu/~lkilian/reaupdate.txt.

[10] http://research.stlouisfed.org/fred2/

demand shocks come as a result of unanticipated changes in consumer preferences or demographic structure.

As noted in Section 2, the SR identification implies a set of restrictions on the contemporaneous responses to shocks: a variable is not contemporaneously influenced by shocks to variables situated down in the vector, but it may be contemporaneously influenced by shocks to variables situated above. Thus, the ordering in (10) restricts the global oil production to be contemporaneously influenced by its own shock only, implicitly assuming that oil producers set their production based on the expected trend in the demand and not based on unexpected high-frequency movements. Also, aggregate oil supply and global demand shocks may have a contemporaneous effect on the US gasoline market, but not vice-versa. The reason for this assumption is to place fewer restrictions on smaller and thus more agile markets. Finally, the gasoline demand shock does not percolate through the gasoline price in the given month, which implicitly assumes that gasoline distributors have sufficient storage to supply the required quantities in the given month. Refer to Kilian (2009, 2010) for further details about the definition of the shocks and the identification strategy.

As a preliminary stage, we apply standard ADF unit root tests. The unit root hypothesis cannot be rejected for any of the series but the activity index. Kilian (2010) assumes that both the oil supply and the gasoline demand contain a unit root and, consequently, he includes these variables in the VAR in first differences. However, he leaves the two real price series and the activity index in levels. It is important to note that the strong persistence of the price series is not ignored by Kilian. As the author states, it is not clear whether the real price series have a unit root since unit root tests have very minimal power against persistent stationary processes in short datasets, and falsely imposing a unit root will render the estimates inconsistent. As noted in the introduction, the main advantage of our framework is that it allows us to identify the structural disturbances without imposing any additional assumptions on the order of integration of any of the variables included in the model.

4. Estimation Results

Selected results are presented in Table 1. The table reports the estimated orders of fractional integration, which measure the persistence of the variables to the system shocks. For the autoregressive part, we have selected one lag according to the Schwarz Information Criterion (SIC). The standard errors for these coefficients are computed by numerical evaluation of the Hessian matrix and are presented in parentheses.

As can be seen in the table, there is evidence of long memory in the data. The estimated orders of integration of the real prices of oil and gasoline are 0.860 and 0.602 respectively, and they are statistically different from 0 and 1. Thus, both the I(0) and I(1) assumptions are rejected by the data. Note that although our results sustain the mean reversion hypothesis, the two series are so persistent that stationarity ($d < 0.5$) cannot be supported at usual significance levels. However, evidence suggests that the US consumption of retail gasoline is not I(1), as assumed in Kilian (2010), but also a non-stationary mean reverting process. This last result is in line with the evidence provided by Lean and Smith (2009) on US petroleum consumption.

We test the VAR specification against the FIVAR alternative by bootstrapping the empirical distribution of the likelihood ratio test statistic. [11] The VAR null is rejected at usual significance levels. Finally, we also test the hypothesis of equal order of FI in all of the series that were also rejected at 5%.

4.1 Robustness Analysis

It has been argued that fractional integration may appear as a spurious phenomenon caused by the presence of breaks in the data (see e.g. Cheung (1993) or Diebold and Inoue (2001)). Nevertheless, the opposite effect is also well documented (see e.g. Nunes et al. (1995) or Hsu (2001)). An important advantage of the multivariate model over univariate approaches is that we explicitly account for the key demand and supply factors driving the dynamics of the series. Thus, as far as the parameter variation present in some univariate studies is caused by changes in the composition of demand and

[11] For testing this hypothesis, we assume one autoregressive lag in the VAR, ensuring that the two models are nested. As in previous VAR literature, the 1st and 5th variables are assumed to be I(1) and enter to the model in differences while the other variables enter in levels. Given that we pre-difference data prior to FIVAR estimation (see footnote (5)), to make the models comparable, we transform back to levels only the 2nd, 3rd, and 4th variables, leaving the 1st and 5th in differences (with negative orders of integration). For the LR statistic, we estimate both models in the frequency domain and compute the values of the likelihood function. We bootstrap the empirical distribution of this statistic using both residual-based and frequency-domain bootstrap methods. We generate 500 bootstrap replications in each case.

supply shocks, this is not a concern in our multivariate model. Although, in principle, other factors might cause parameters to change, empirical evidence shows that their contribution is rather small at the monthly frequency for the sample period considered here, as noted by Edelstein and Kilian (2009).

Nevertheless, in order to assess if the presence of fractional integration is robust to the existence of breaks, we perform the estimation of the FIVAR in a subsample characterized by its stability[12]. This subsample runs from 1986:04 to 2004:02[13]. The starting date is motivated by Baumeister and Peersman (2013) who find a break on the oil demand curve in the first quarter of 1986 in a time-varying SVAR framework showing that parameters remained stable afterward. The date also coincides with the collapse of the OPEC cartel and the beginning of the 'Great Moderation', and is often selected for sample splits in the oil literature; the end is February 2004. This date coincides with a period of violent oil price fluctuations prior to the global economic crises.[14]

The second row of Table 1 presents the estimated orders of integration of the FIVAR model in the selected subsample. As can be observed, long memory is also present in the data. The estimated orders of fractional integration of the two real prices and the demand of gasoline are very similar to the ones obtained with the whole sample and are again statistically different from 1 and 0.5, confirming the non-stationary but mean reverting behavior of these series. However, we find a statistically significant increase of the persistence of the global activity index in the selected subsample, with an estimated order of integration not statistically different from 1. This last result is consistent with evidence on unit root provided by standard ADF testing procedure in the selected subsample.

[12] Although there are some techniques to distinguish between fractional integration and short memory processes containing trends and/or breaks, most of them deal with a single series and have not been extended to the multivariate case.

[13] We have also taken April 1991 as the initial date (just after the oil price shock and the end of the early 90s recession) and November 2007 as the final date (before the beginning of the Great Recession). The main conclusions are robust.

[14] Yet, most of the literature finds those movements explained by fundamental factors. We thank a referee for pointing this out.

5. The Effect of Demand and Supply Shocks

5.1 Impulse-Response Analysis

Once the reduced FIVAR model is estimated, we can employ the structural representation defined in (5) and (6) to track the responses of gasoline market prices and quantities to system shocks. Figure 1 plots the impulse responses (IRFs) of these two variables to one-standard-deviation shocks up to 8 years horizon. The figure also reports two-standard-deviation confidence bands computed by multivariate non-parametric bootstrap in the frequency domain (Berkowitz and Diebold (1998))[15]. As in Kilian (2010), we have normalized the supply shocks to represent supply disruptions, and the demand shocks to represent demand expansions.

As can be seen in the figure, demand expansions and supply disruptions cause the real price of gasoline to increase. Thus, the responses of this variable to the five disturbances computed from the FIVAR model evolve according to economic theory. In line with recent studies, the response of gasoline prices to an oil supply contraction is positive but not statistically significant, which calls into question the quantitative importance of this shock (see e.g. Kilian (2008)). The magnitudes of the global and the oil-market-specific demand shocks are greater, especially the latter, which has a persistent effect that remains significant for more than eight years. Unexpected gasoline supply disruptions cause the gasoline price to rise in the very short run, with an effect that also remains significant for a long period. Finally, gasoline consumption expansions peak around the sixth month, but in line with preceding studies, are not significant at any horizon.

To compare our results with the existent literature, we also recover the IRFs from a standard VAR model. As in previous literature, oil supply and gasoline demand are both assumed to be I(1) in the VAR and are included in differences, while the two real prices and the activity index enter in levels. Following the same lag-length criterion as for the FIVAR case (SIC), we select two lags in the autoregressive part for the VAR. VAR impulse responses with recursive wild bootstrap error bands are depicted in Figure 1 together with FIVAR responses. Interesting results emerge from the comparison between the two models. Consistent with the estimated order of fractional integration

[15] To compute confidence intervals, we produce 500 bootstrap replications, treating the estimated model as the true data generating process. Conditions on the spectral density of the VARFIMA process for the application of the bootstrap are satisfied for all frequencies except for frequency 0. Consistent with standard practice, the 0 frequency is excluded from estimation and bootstrap.

for the gasoline price (0.683), the response is mean-reverting in both models, but the responses computed with the FIVAR converge to zero much more slowly, remaining significant for a considerably longer period.[16]

The second column of Figure 1 depicts the FIVAR and VAR responses of gasoline demand. FIVAR responses conform once more with economic theory. Consistent with the increase in prices, the (normalized) structural disturbances lower the demand of gasoline on impact, except the response to its own shock, which raises demand by assumption. With the exception of the first two shocks, all of the responses are significant. Note than in this case, the pattern of FIVAR and VAR impulse-responses are not at all similar. Recall that gasoline demand is assumed to be I(1) in the VAR and enters to this model in differences, as in Kilian (2010). Therefore, shocks have permanent effects on its level, and impulse-responses do not necessarily converge to zero.

5.2 Variance Decomposition in the Frequency Domain

Impulse responses demonstrate the reaction of a variable to a shock over time, according to the behavior described by a statistical model. However, IRFs are not an appropriate instrument to study the contribution of the different shocks to the variation of the variables (or driving forces of this variation). A standard instrument to pursue this type of analysis is the forecast error variance decomposition. This decomposition, however, requires stationarity of all variables in the model and lacks a one-to-one mapping between forecast errors at different horizons and the different cyclical components. In this paper, we decompose the variance in the frequency domain. This decomposition is an easy way to analyze a contribution of shocks at different frequency ranges (as business cycle), and it does not require stationarity of the variables in the system if one is interested in business cycle or higher frequency ranges. Details about this decomposition can be found in Appendix A.

Figure 2 depicts shock percentage contributions to the volatility of gasoline price and gasoline demand across frequencies[17]. Note that for a given frequency, the contribution

[16] Kilian (2010) selects 14 lags for the VAR (without using a formal criterion) to yield sufficient persistence in the responses of real prices to shocks. Note that this model requires the estimation of 365 parameters. As a robustness check, we also recover IRFs from a VAR(14). It turns out that price responses to shocks from the FIVAR(1) model also converge to 0 more slowly than those of a VAR(14). This is because FIVAR models exhibit hyperbolic decay of the autocorrelations, while autocorrelations in VARs decay at a faster exponential rate. In fact, the IRFs of the VAR(14) and the VAR(2) do not differ much, especially if one is guided by their statistical significance. In this sense, parsimony is another justification of the FIVAR model. We thank a referee for pointing this last issue out.

of the different shocks sums to 100%. Thus, a peak in the figure implies an important contribution of the given shock to the volatility of the particular series in a neighborhood of the corresponding frequency. To facilitate interpretation, we have shaded the area corresponding to the standard definition of the business cycle range[18]. Figure 2 clarifies two points: first, not all shocks have the same contribution to the variance; second, the relative contribution of a particular shock is not constant across frequencies. For gasoline prices, the variability at low frequencies is mostly explained by oil-market demand shocks, with a rather small contribution of the other shocks. However, the importance of oil market shocks declines as frequency increases in favor of gasoline supply disruptions. The variability of gasoline consumption at low frequencies is mostly explained by both oil-market and gasoline demand shocks. Once more, the importance of the former shock vanishes as frequency increases.

Yet, these numbers can be misleading if one is interested in fluctuations across an entire frequency range because this variability may not be distributed evenly within its component frequencies. Figure 4 depicts the estimated spectral densities of gasoline price and gasoline demand. Consistent with their degree of persistence, the estimated densities decline sharply with frequency. Thus, fluctuations around the first frequencies of a given range contribute more to the variability in the range than the fluctuations around the remaining frequencies. In order to correctly account for this fact, we compute the relative contribution of a given shock in a particular range as the ratio of the total variance attributable to this shock in the range to the total variability in the range. Table 2 reports the variance decomposition at two selected frequency ranges: business cycle and fluctuations inside a year. As expected from Figure 2, we find that the share attributable to the oil market demand shock declines when moving from the business cycle to the higher frequency range. However, since the contribution of oil-market shocks is higher precisely in frequencies contributing more to variability, the decline is much smaller than the one expected from solely inspecting the figure.

Table 2 also reports the decomposition for a standard VAR. Again, the contribution changes from one shock to another and shares are not constant across frequency ranges. The variance decomposition for the gasoline real price is similar

[17] If the order integration is strictly positive at 0 frequency, the spectrum tends to infinity at this frequency. Consistent with standard procedure, we have excluded the 0 frequency for the estimation and also for posterior analysis.

[18] *Business cycle* corresponds to a range of frequencies with period from 1.5 to 8 years; *high frequencies* with a period smaller than or equal to 1 year.

between the two models. However, VAR tends to understate the contribution of oil market-specific shocks to the variability of gasoline consumption. Overall, results show the importance of frequency domain tools to study the contribution of the different shocks.

6. The own-price elasticity of gasoline demand

In this section, we use the structural model to obtain gasoline price elasticities of gasoline demand. Using the structural model, we can overcome the well-known problem of estimating demand equations, that is, that prices and quantities are jointly determined. This results in biased estimates when nonstructural models are employed unless valid instruments are found. A similar strategy is followed by Baumaister and Peersman (2012) as well as Kilian and Murphy (2014) for oil demand. More specifically, dynamic price elasticities of demand can be derived from the reaction function of the demand of gasoline to the other variables in the system. The reaction function of gasoline demand for the structural FIVAR can be written as (see Appendix B):

$$gd_t = \sum_{m=1}^{3} C_m(L)(1-L)^{d_{x_m}-d_{gd}} x_{m,t} + C_{rpg}(L)(1-L)^{d_{rpg}-d_{gd}} rpg_t + G(L)\varepsilon_t^{gd} \qquad (11)$$

where the term $x_{m,t}$ refers to the variables other than gasoline price (os_t, rea_t, and rpo_t). Since all variables, except the real economic activity index, are expressed in natural logarithms, the resulting coefficients in (11) can be interpreted as dynamic elasticities[19]. The short-run price elasticity of gasoline demand is the first coefficient of the polynomial $\Phi(L) = C_{rpg}(L)(1-L)^{d_{rpg}-d_{gd}}$, and measures the contemporaneous % change in gasoline consumption as a result of a 1% increase in gasoline prices. To compute the dynamic reaction to a *permanent* change in prices, the coefficients of the polynomial $\Phi(L)$ should be summed to the lag of interest. In this way, the total cumulative % change (long-run elasticity) can be obtained as a limit. While the short-run elasticity reflex initial measures adopted by consumers after a change in price (an increase in the efficiency of driving, for example), the long-run elasticity is also linked to fundamental changes of consumption patterns that usually require more time to be adopted (for

[19] The coefficients of real economic activity index can be interpreted in the following way: if the economic activity index increases 1 unit (1% since this index is expressed in %), gasoline demand increases $100 \times C_2(L)\%$.

instance, a change of residence to reduce commuting or a switch to an alternative energy source). As noted in the introduction, the literature has provided a battery of different estimates of these values. The average short- and long-run elasticities across studies found by most recent meta-analyses of the literature were −0.26 and −0.58 (Espey (1998)), −0.34 and−0.84 (Brons et al. (2008)), or −0.09 and −0.31 (Havranek et al. (2012)). Yet, there is a lot of variation from one study to another. Interestingly, studies covering more recent data tend to report much lower estimates (see e.g. Small and van Dender (2007) or Hughes et al. (2008))[20].

As can be deduced from equation (11), the inclusion of fractional integration has strong implications for the pattern of dynamic estimates. If, as found in the data, the gasoline real price is less persistent than demand ($-1 < d_{rpg} - d_{gd} < 0$), the dynamic price elasticities will converge to some nonzero value. The speed of convergence depends on the magnitude of the difference between the two orders of integration, being more slowly the larger the difference. Conversely, if the real price is more persistent than demand ($0 < d_{rpg} - d_{gd} < 1$), the elasticity is going to be 0 in the long run. In this case, larger differences boost convergence. Note that in the traditional I(0)/I(1) framework, the own-price elasticity of gasoline demand can converge to a nonzero bounded value only if demand an prices have exactly the same order of integration. If, for instance, demand is I(1) while prices I(0), as assumed in Kilian (2010), dynamic elasticities will explode because the two processes are unbalanced.

Figure 3 plots FIVAR dynamic elasticities up to a horizon of fifteen years. The short-run elasticity is estimated to be -0.06, reaching -0.10 during the first month. Consistent with the estimated orders of integration, the dynamic response converges very quickly to a long-run value of -0.16, somewhat smaller than values usually reported in the literature but in line with those reported by studies covering recent data.

As a matter of comparison, the figure also plots dynamic elasticities computed from two competing VAR models, each one with a different assumption on the order of integration of gasoline demand (either I(0) or I(1)). As can be seen in the figure, the estimated short-run elasticities are similar than the FIVAR estimate, albeit slightly

[20] Using data over the period 1966-2001, Small and van Dender (2007) find short-run and long-run elasticities of 0.04 and -0.22, respectively. These numbers fall to -0.02 and -0.10 for the period 1997-2001. Hughes et al. (2008) also document a decrease in the short-run elasticity. For the period 1975-1980, they find estimates ranging from -0.21 to -0.34 depending on the model, but these values fall to -0.034 to -0.077 for the period 2001-2006.

smaller in magnitude (-0.04 and -0.02, respectively). However, the dynamic patterns are completely different in the three models. Note that if gasoline demand is assumed I(0), the elasticity converges to a very large long-run value (close to -1). As noted above, if demand is I(1) while prices I(0), the elasticity does not even converge.

In summary, fractional integration provides an interesting range of convergence possibilities to long-run elasticities with mean reverting prices. We find that the demand for gasoline is highly inelastic, showing a relatively flat pattern, with no large differences between medium-run and long-run values. Our results indicate that consumers undertake a few measures to reduce gasoline consumption in the short and middle-run, but they are reluctant to adopt strong measures that significantly change their consumption habits. In this sense, our results emphasize the importance of short-run estimates for policy analysis.

7. Concluding Remarks

In this paper, we model the US gasoline retail market as a structural fractional integrated VAR. We find strong evidence of non-stationary mean-reverting behavior in the real prices and in the demand for gasoline, which reconciles previous VAR analyses with evidence from unit root testing. The estimated FIVAR model produces impulse-responses to structural shocks that are consistent with economic theory, but much more persistent than previously predicted. We also provide new findings on the asymmetric effect of the different demand and supply shocks. Their contribution to the volatility of gasoline market outcomes is different, and the share attributable to each shock changes with the different frequencies of the spectra. Finally, we show that fractional integration has interesting implications for the convergence pattern of dynamic price elasticities of gasoline demand.

Like all empirical work, our approach suffers from several shortcomings, many of which have been discussed in the main body of the paper. The most important, in our opinion, is that a long memory model is not well-suited for the analysis of short data samples. Thus, we cannot answer the question of whether the price elasticity of gasoline demand has fallen in recent years, or how gasoline consumption will respond to the sudden decline in prices recently observed in the data. Also, our study maintains the assumption that consumption equals production, abstracting from the possibility that gasoline distributors may run out of gasoline. Inventory movements have proved useful in the analysis of oil demand (Kilian and Murphy (2014)). Therefore, it would be interesting to determine whether considering gasoline inventories changes our results. Yet, the importance of long memory for the convergence pattern of demand elasticity calls for an adequate treatment of persistence in models intended to estimate long-run values. Finally, it would also be interesting to assess the influence of long memory on the relationship between the gasoline market and the rest of the economy. We consider these issues interesting avenues for future research.

References

Abbriti, M., A. Moreno, L. Gil-Alana and Y. Lovcha, 2015. Term structure persistence. Journal of Financial Econometrics (First View). doi:10.1093/jjfinec/nbv003.

Aguiar-Conraira, L., and M.J. Soares, 2011. Oil and the macroeconomy: using wavelets to analyse old issues. Empirical Economics 40, 645-655.

Baillie, R.T., 1996. Long memory processes and fractional integration in econometrics. Journal of Econometrics 73, 5-59.

Berkowitz, J., and F.X. Diebold, 1998. Bootstrapping multivariate spectra. The Review of Economics and Statistics 80, 664-666.

Baumeister, C., and G. Peersman, 2013. Time-varying effects of oil supply shocks on the US Economy. American Economic Journal: Macroeconomics 5, 1-28.

Braun, P.A., and S. Mittnik, 1993. Misspecifications in vector autoregressions and their effects on impulse responses and variance decompositions. Journal of Econometrics 59, 319-341.

Brons, M., P. Nijkamp, E. Pels, and P. Rietveld, 2008. A meta-analysis of the price elasticity of gasoline demand. A SUR approach. Energy Economics 30, 2105–2122.

Boes, D.C., R.A. Davis, and S.N. Gupta, 1989. Parameter estimation in low order fractionally differenced ARMA processes. Stochastic Hydrology and Hydraulics 3, 97-110.

Carruth, A., M. Hooker, and A. Oswald, 1998. Unemployment equilibria and input prices: Theory and evidence for the United States. The Review of Economics and Statistics 80, 621-628.

Cheung, Y.W., 1993. Tests for fractional integration. A Monte Carlo investigation. Journal of Time Series Analysis 14, 331-345.

Choi, K., and S. Hammoudeh, 2009. Long memory in oil and refined products markets. The Energy Journal 30, 97-116.

Dahl, C., and T. Sterner, 1991. Analyzing gasoline demand elasticities: a survey. Energy Economics 13, 203-210.

Diebold, F.X., and G.D. Rudebusch, 1989. Long memory and persistence in aggregate output. Journal of Monetary Economics 24, 189-209.

Edelstein, P., and L. Kilian, 2009. How sensitive are consumer expenditures to retail energy prices? Journal of Monetary Economics 56, 766-779.

Elder, J., and A. Serletis, 2008. Long memory in energy futures prices. Review of Financial Economics 17, 146-155.

Espey, M., 1998. Gasoline demand revisited: an international meta-analysis of elasticities. Energy Economics 20, 273–295.

Fan, Y., and J.H. Xu, 2011. What has driven oil prices since 2000? A structural change perspective. Energy Economics 33, 1082-1094.

Ghoshray, A., and B. Johnson, 2010. Trends in world energy prices. Energy Economics 32, 1147-1156.

Gil-Alana, L.A., 2001. A fractionally integrated model with a mean shift for the US and the UK real oil prices. Economic Modelling 18, 643-658.

Golinski, A., and P. Zaffaroni, 2015. Long memory affine term structure models. Journal of Econometrics (First View). doi:10.1016/j.jeconom.2015.09.006.

Granger, C.W.J., 1966. The typical spectral shape of an economic variable. Econometrica 34, 150-161.

Granger, CWJ, and J. Lin, 1995. Causality in the long run. Econometric Theory 11, 530-536.

Gronwald, M.. 2008. Reconsidering the macroeconomics of oil price in Germany: testing for causality in the frequency domain. Empirical Economics 36, 441-453.

Haldrup, N., F.S. Nielsen, and M.Ø. Nielsen, 2010. A vector autoregressive model for electricity prices subject to long memory and regime switching. Energy Economics 32, 1044-1058.

Hauser, M.A., M.B. Pötscher and E. Reschenhofer, 1999. Measuring Persistence in Aggregate Output: ARMA models, fractionally integrated ARMA models and nonparametric procedures. Empirical Economics 24, 243-269.

Havranek, T., Z. Irsova, and J. Karel, 2012. Demand for gasoline is more price-inelastic than commonly thought. Energy Economics 34, 201-207.

Hosoya, Y., 1996. The quasi-likelihood approach to statistical inference on multiple time-series with long-range dependence. Journal of Econometrics 73, 217-236.

Hsu, C.C., 2001. Change point estimation in regressions with I(d) variables. Economics Letters 70, 147-155.

Hughes, J.E., C.R. Knittel, and D. Sperling., 2008. Evidence of a shift in the short-run price elasticity of gasoline demand. The Energy Journal 29, 113-134.

Johansen, S. , 2008. A representation theory of a class of vector autoregressive models for fractional processes, Econometric Theory 24, 651-676.

Johansen, S., and M.Ø. Nielsen, 2012. Likelihood inference for a fractionally cointegrated vector autoregressive model. Econometrica 80, 2667-2732.

Kilian, L. 2008. Exogenous oil supply shocks: how big are they and how much do they matter for the U.S. economy? Review of Economics and Statistics 90, 216-240.

Kilian, L., 2009. Not all oil price shocks are alike: disentangling demand and supply shoks in the crude oil-market. American Economic Review 99, 1053-1069.

Kilian, L., 2010. Explaining fluctuations in gasoline prices: a joint model of the global crude oil-market and the US retail gasoline-market. The Energy Journal 31, 87-112.

Kilian, L., and D.P. Murphy, 2014. The role of inventories and speculative trading in the global market for crude oil. Journal of Applied Econometrics 29, 454–478.

Kilian, L., and C. Park, 2009. The impact of oil price shocks on the US stock market. International Economic Review, 50, 1267-1287.

Lean, H.H., and R. Smyth, 2009. Long memory in the US disaggregated petroleum consumption: Evidence from univariate and multivariate LM tests for fractional integration. Energy Policy 37, 3205-3211.

Lovcha, Y., and A. Perez-Laborda, 2015. Hours-worked productivity puzzle: Identification in fractional integration settings. Macroeconomic Dynamics 19, 1593-1621.

Narayan, P., R. Smyth, 2007. Are shocks to energy consumption permanent or temporary? Evidence from 182 countries. Energy Policy 35, 333-341.

Nunes, L.C., C.M. Kuan, and P. Newbold, 1995. Spurious breaks. Econometric Theory 11, 736-49.

Robinson, P.M. and C. Velasco, 2000. Whittle pseudo-maximum likelihood estimation for nonstationary time series. Journal of the American Statistical Association 95, 1229-1243.

Serletis, A. 1992. Unit root behaviour in energy futures prices. The Energy Journal, 13, 119-128.

Tschering, R., E. Webber, and R. Weigand, 2013. Long-run identification of a fractionally integrated system. Journal of Business and Economic Statistics 31, 438-450.

Sims, C.A. 1986. Are forecasting models usable for policy analysis? Minneapolis Federal Reserve Bank Quarterly Review 10, 2-16.

Small, K.A., and K. van Dender, 2007. Fuel efficiency and motor vehicle travel: the declining rebound effect. The Energy Journal 28, 25-51.

Tables and Figures

Table 1 – FIVAR: Estimated Long Memory Coefficients

Sample:	os	rea	rpo	rpg	gd
1978:01 - 2015:06	0.747 (0.079)	0.657 (0.080)	0.860 (0.065)	0.683 (0.057)	0.753 (0.032)
1986:04 - 2004:02	0.794 (0.105)	0.981 (0.091)	0.765 (0.109)	0.642 (0.094)	0.674 (0.052)

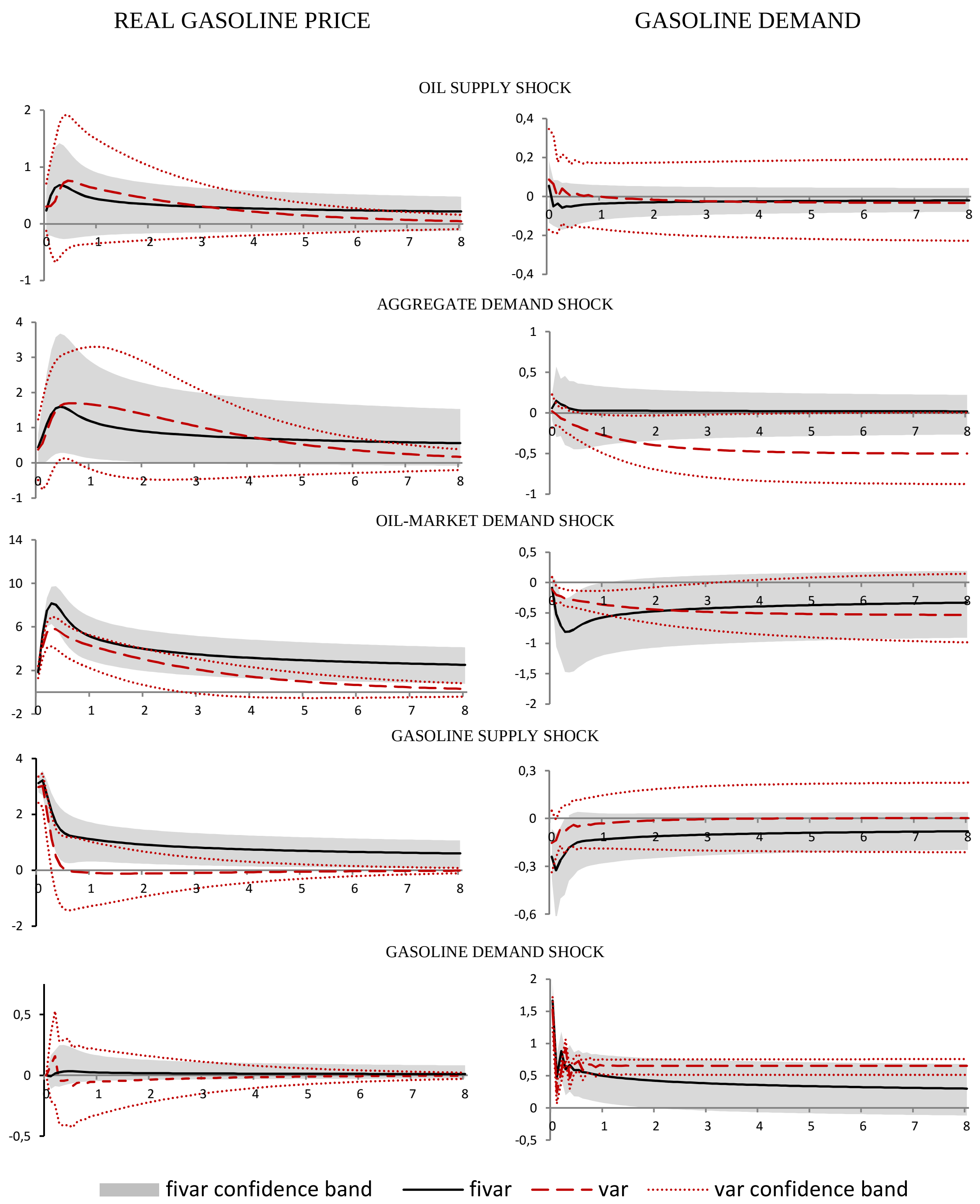
Notes: Estimation results of the FIVAR parameters with standard errors in parentheses. A value of d_i between [0.5, 1) implies that process is non-stationary but still mean reverting. The lag order of the autoregressive part has been chosen by SIC criterion.

Table 2 – Variance Decomposition at Frequency Ranges; FIVAR and VAR

	REAL PRICE OF OIL				GASOLINE DEMAND			
Shock	BC		HF		BC		HF	
	FI	VAR	FI	VAR	FI	VAR	FI	VAR
Oil supply shock	0.64	1.49	0.46	0.33	0.22	0.27	0.31	0.47
Aggregate demand shock	4.01	8.79	1.09	1.03	0.41	11.62	0.69	0.14
Oil-market demand shock	88.92	84.12	70.02	49.50	51.01	15.57	10.53	0.88
Gasoline supply shock	6.42	5.58	28.43	48.97	3.64	1.06	3.18	1.15
Gasoline demand shock	0.00	0.02	0.00	0.17	44.72	71.48	85.30	97.36

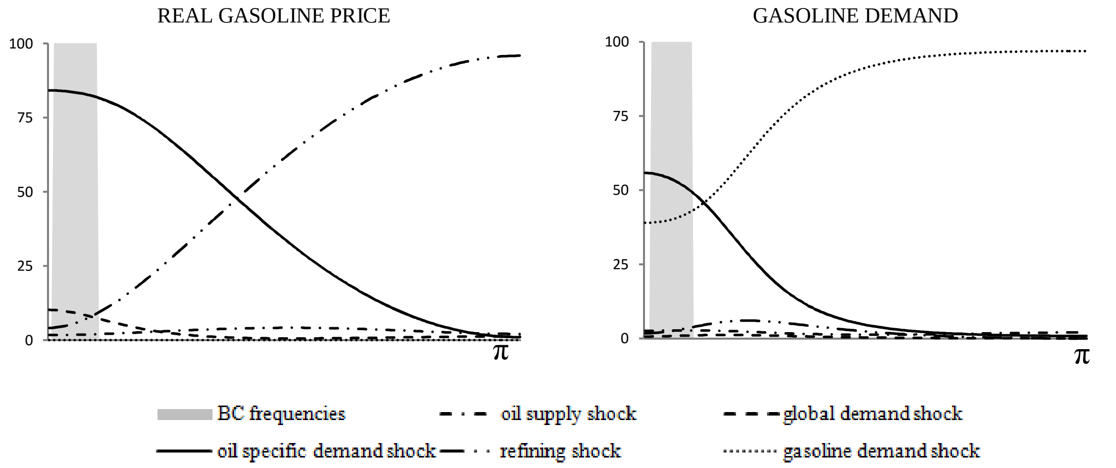
Notes: The statistics are from the spectral analysis of impulse responses in the structural FIVAR and VAR models. The sum of contributions by all types of shocks to the volatility of a variable over a frequency range equals 100%. BC - Business Cycle (1.5 to 8 years period) and HF - High Frequencies (period smaller than 1 year).

Figure 1 – IRFs to One Standard Deviation Shocks



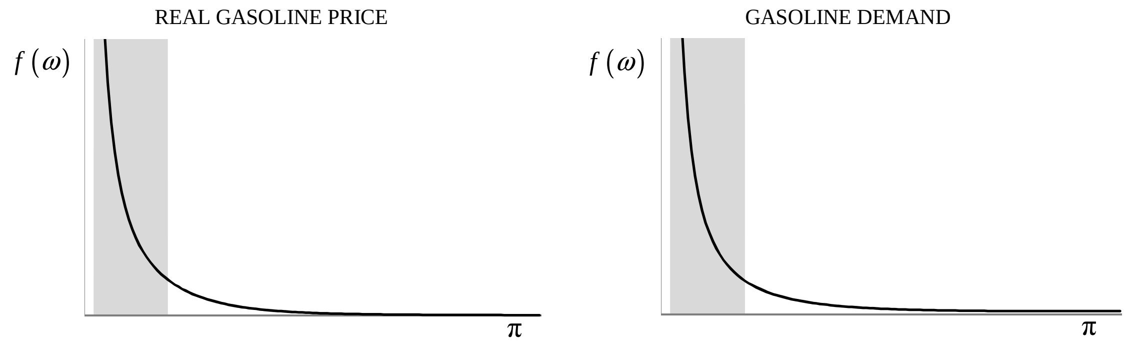
Notes: FIVAR and VAR confidence intervals for the IRFs are computed by multivariate non-parametric bootstrap in the frequency domain and recursive wild bootstrap respectively. The reported bands correspond to two standard deviations. The orders of the autoregressive parts (one for FIVAR and two for VAR) are selected by SIC criterion. Gasoline demand is seasonally adjusted.

Figure 2 – Spectral Decomposition of Volatility across Frequencies



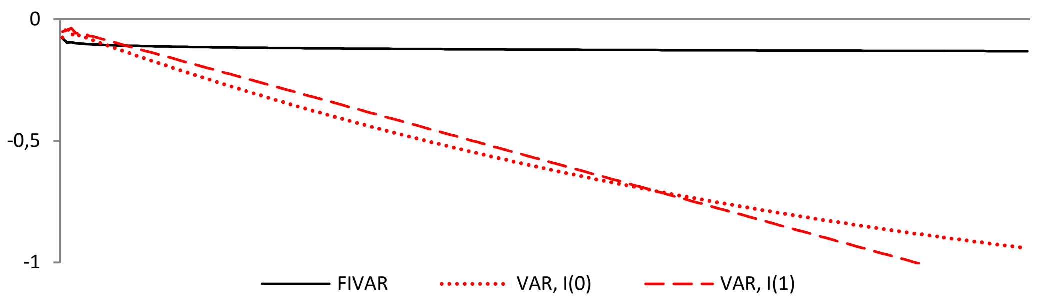
Notes: The sum of contribution by all types of shocks to the volatility of a variable equals 100% at a given frequency. The shaded area corresponds to the business cycle range.

Figure 3 – Estimated Spectral Densities; FIVAR



Notes: The spectral densities are computed parametrically from the estimated FIVAR. The shaded area corresponds to the business cycle range.If the order integration is strictly positive at zero frequency, the spectrum tends to infinity at this point. As standard procedure, we have excluded the zero frequency for the estimation and also for posterior analysis.

Figure 4 – Own-Price Elasticity of Gasoline Demand



Notes: The figure plots the dynamic gasoline price elasticities of US gasoline consumption up to a horizon of fifteen years. The I(0)-I(1) in VAR specification refer to the assumed order of integration for gasoline demand. As in previous literature, prices in the two VAR models are assumed I(0).

Appendix A: Variance Decomposition of the Structural FIVAR in the Frequency Domain.

Let $f(\omega)$ denote the $N \times N$ spectral density matrix of the structural FIVAR process at the frequency ω. Employing the same notation than in the Section 2, the multivariate spectrum $f(\omega)$ of the FIVAR model is given by the expression:

$$f(\omega) = \frac{1}{2\pi} B\left(e^{i\omega}\right) V B\left(e^{-i\omega}\right)$$

$$B\left(e^{i\omega}\right) \equiv D\left(e^{i\omega}\right)^{-1} \left(I - F\left(e^{i\omega}\right)\right)^{-1} A^{-1}$$

where i denotes the imaginary unit; $B\left(e^{-i\omega}\right)$ is the complex conjugates of $B\left(e^{i\omega}\right)$; $D\left(e^{i\omega}\right)$ is a $N \times N$ diagonal matrix with terms $\left(1-e^{i\omega}\right)^{d_n}$ on the main diagonal; and $F\left(e^{i\omega}\right)$ is given by $F_1 e^{i\omega} + ... + F_p e^{pi\omega}$. The main diagonal of the matrix $f(\omega)$ contains the univariate spectra $f_n(\omega)$ of all the series of the model.

We can re-write the univariate spectrum of the series y_n as:

$$f_n(\omega) = \frac{1}{2\pi} \sum_{j=1}^{N} \left\| b_{nj} \right\|^2 v_j$$

where b_{nj} is the (n, j) element of the matrix B and v_j is the jth diagonal term of the variance-covariance matrix V of the uncorrelated structural disturbances. This equation allows us to decompose the spectrum $f_n(\omega)$ at a given frequency ω as the sum of the terms $k_n^j(\omega) = \frac{1}{2\pi} \left\| b_{nj} \right\|^2 v_j$ associated to each structural disturbance. Given that the spectrum can be interpreted as the decomposition of the variance of the process into a set of uncorrelated components at each frequency, the term $\int_{\omega_1}^{\omega_2} k_n^j(\omega) d\omega$ represents the contribution of the j^{th} structural disturbance to the fluctuations of the series y_n attributable to cycles with frequencies in the interval (ω_1, ω_2).

Appendix B: Reaction Functions of US Gasoline Demand from the Structural FIVAR

Reaction functions of gasoline demand can be computed from the $MA(\infty)$ infinite representation of the structural FIVAR model given by equation (7) in the text.

Define $(I-F(L))^{-1}A=Q(L)$. The last equation from this system is related to the demand of gasoline:

$$gd_t=(1-L)^{-d_{gd}}\,Q_{(5,1:4)}(L)\,\varepsilon_{(1:4),t}+(1-L)^{-d_{gd}}\,Q_{55}(L)\,\varepsilon_t^{gd}$$

where $\varepsilon_{(1:4),t}$ is a vector containing the elements from 1 to 4 of the vector ε_t; $Q_{(5,1:4)}(L)$ is a sub-matrix with elements 1 to 4 of the last row of the matrix $Q(L)$.

The remaining first four equations from the system can be grouped together as:

$$X_{(1:4)t}=D^{-1}_{(1:4,1:4)}(L)Q_{(1:4,1:4)}(L)\,\varepsilon_{(1:4)t}+D^{-1}_{(1:4,1:4)}(L)Q_{(1:4,5)}(L)\,\varepsilon_t^{gd}$$

where $X_{(1:4)t}$ is a vector containing elements from 1 to 4 of the vector X_t; $D_{(1:4,1:4)}(L)$ and $Q_{(1:4,1:4)}(L)$ are sub-matrices of the matrices $D(L)$ and $Q(L)$, respectively, containing rows from 1 to 4 and columns from 1 to 4; $Q_{(1:4,5)}(L)$ is a sub-matrix with elements 1 to 4 of the last column of the matrix $Q(L)$.

From these four equations we get first four elements of the vector of structural shocks:

$$\varepsilon_{(1:4)t}=Q^{-1}_{(1:4,1:4)}(L)\,D_{(1:4,1:4)}(L)X_{(1:4)t}+Q^{-1}_{(1:4,1:4)}(L)\,Q_{(1:4,5)}(L)\,\varepsilon_t^{gd}$$

that can be substituted in the equation of the demand of gasoline:

$$gd_t=(1-L)^{-d_{gd}}\,Q_{5,1:4}(L)\,Q^{-1}_{(1:4,1:4)}(L)\,D_{(1:4,1:4)}(L)X_{(1:4)t}+G(L)\,\varepsilon_t^{gd}$$

Rearranging the last equation we obtain:

$$gd_t=Q_{5,1:4}(L)\,Q^{-1}_{(1:4,1:4)}(L)\,D_d(L)X_{(1:4)t}+G(L)\,\varepsilon_t^{gd}$$

where $D_d(L)$ is 4×4 diagonal matrix with diagonal elements given by $(1-L)^{d_m-d_{gd}}$, Finally, defining $C(L)=Q_{5,1:4}(L)\,Q^{-1}_{(1:4,1:4)}(L)$, the reaction function of gasoline demand is given as:

$$gd_t=\sum_{m=1}^{4}C_i(L)(1-L)^{d_{x_m}-d_{gd}}\;x_{m,t}+G(L)\,\varepsilon_t^{gd},\quad \text{where } x_{m,t}=os_t,rea_t,rpo_t,rpg_t$$