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On the invertibility of seasonally adjusted series

Luis Gil Alana
Alejandro Pérez Laborda
Yuliya Lovcha

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ON THE INVERTIBILITY OF SEASONALLY ADJUSTED SERIES

Luis Gil-Alana
Departamento de Economia
Universidad de Navarra
Campus Universitario, 31009 Pamplona, Spain

Alejandro Perez-Laborda
Departament d'Economia i CREIP
Universitat Rovira-i-Virgili
Av. Universitat 1, 43204 Reus, Spain

Yuliya Lovcha*
Departament d'Economia,
Universitat Rovira-i-Virgili i CREIP,
Av. Universitat 1, 43204 Reus, Spain.
yuliya.lovcha@gmail.com.
Tel: (+34) 977 759 811 Fax: (+34) 977 758 907

Abstract

This paper examines the implications of the seasonal adjustment by an ARIMA model based (AMB) approach in the context of seasonal fractional integration. According to the AMB approach, if the model identified from the data contains seasonal unit roots, the adjusted series will not be invertible that has serious implications for the posterior analysis. We show that even if the ARIMA model identified from the data contains seasonal unit roots, if the true data generating process is stationary seasonally fractionally integrated (as it is often found in economic data), the AMB seasonal adjustment produces dips in the periodogram at seasonal frequencies, but the adjusted series still can be approximated by an invertible process. We also perform a small Monte Carlo study of the log-periodogram regression with tapered data for negative seasonal fractional integration. An empirical application for the Spanish economy that illustrates our results is also carried out at the end of the article.

JEL Classification: C15

Keywords: seasonality; invertibility; fractional integration; TRAMO-Seats; tapering;

* Corresponding autor.

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1. Introduction

Given the seasonal nature of many macroeconomic time series, seasonal adjustment is a widespread practice and millions of series are routinely adjusted, some of them are not even publicly available in the nonadjusted version. Seasonal adjustment is believed to remove undesirable fluctuations at seasonal frequencies without producing significant changes at other frequencies (especially at the low part of the spectrum) making the data easily tractable thereby simplifying posterior modeling and analysis. However, the properties of the adjusted series crucially depend on the method used for the adjustment and the initial properties of the series, and they may result just as unattractive for analysts as seasonality itself.

In this paper we examine one of the important features of the adjusted data: dips in the periodogram at seasonal frequencies and the resulting noninvertibility of the adjusted series. The spectral dips (or zeros) are produced by all seasonal adjustment methods used in practice, regardless of whether it is a naive adjustment by seasonal dummies or a sophisticated ARIMA-model based (AMB) signal extraction produced by specialized programs. Ooms and Hassler (1997) point out that the regression on seasonal dummies generates zeros in the periodogram at seasonal frequencies that can lead to the singularities in the log-periodogram regression. Nerlove (1965) applies Census X-11 and the modified 'Hannan' method and concludes that both methods remove more than just the seasonal component. Grether and Nerlove (1970) show that the phenomenon observed in Nerlove (1965), namely dips created near the seasonal frequencies after adjustment, is obtained as a result of 'optimal' adjustment procedure as well. The consequent seasonal adjustment routines of Census, X-11-ARIMA and X-12, produce the same result by construction. Gomez and Maravall (2001) call attention to the fact that TRAMO-Seats generates dips at seasonal frequencies whenever the model identified for the data contains seasonal unit roots. According to Gomez and Maravall (2001), the spectral zeros are the frequency counterpart of the unit MA roots and therefore the adjusted series is not invertible and does not accept autoregressive AR (or VAR) approximations to its Wold representation. Although often ignored, this is perhaps the most important practical implication of AMB adjustment, since AR (and VAR) approximations to seasonally adjusted data are typically carried out in the applied econometric work.

In this work we analyze in detail the dips at the seasonal frequencies and the apparent noninvertibility produced by the AMB approach, within the fractional integration (FI) framework, which admits a wider representation of the invertibility condition than the one applied by Gomez and Maravall (2001). In particular, a fractionally integrated (FI) process is (seasonally) invertible whenever the FI coefficients at seasonal frequencies are higher than -0.5. In addition, notice that the negative seasonal FI parameters correspond to the spectral zeros at seasonal frequencies. Thus, the process can have spectral zeros at seasonal frequencies, but still remain invertible.

We choose TRAMO-Seats for Windows (TSW) as the representative AMB seasonal adjustment program. TSW is a pair of the data adjustment programs developed by the Bank of Spain that have been intensively employed by Eurostat since 1994, and nowadays their use has been extended to various European countries (Gomez and Maravall (2001), ESS Guidelines on Seasonal Adjustment (2009)) [1].

To check for the invertibility of the series adjusted by TSW, we produce simulations for a set of processes. We do not restrict the analysis to processes with

[1] Recently, Eurostat has harmonised seasonal adjustment practices with the development of Demetra+ which currently includes both X-12-ARIMA and TRAMO-SEATS.

integer orders of integration at zero and the seasonal frequencies since it has been shown by many authors that FI at seasonal frequencies is also a widespread phenomenon in economics (Porter-Hudak, 1990; Gil-Alana and Robinson, 2001; etc.). However, we also make simulations for a set of Airline models, which are the default models in TSW. For each model, we simulated 500 series, we adjust them by TSW and then, we estimate the fractional differencing parameters at the seasonal frequencies in the adjusted series with the log-periodogram regression with tapered data.

We find that if the data generating process (DGP) follows the default Airline model, TSW always identifies seasonally nonstationary ARIMA models for the data, and the adjusted series produced by TSW are indeed noninvertible, which is in line with the results of Gomez and Maravall (2001). However, if the original series is fractionally integrated at the seasonal frequencies, which is less restrictive and very plausible in many cases according to the empirical evidence, the adjusted series may be approximated by an invertible process depending on the stationarity of the original series. If the DGP is a seasonally stationary FI model, TSW is less prone to identifying a seasonally nonstationary model. Moreover, even if the model chosen by the program was a seasonally nonstationary, the resulting series does contain dips at the seasonal frequencies but these dips correspond to negative seasonal FI with coefficients greater than -0.5. Hence the adjusted series still can be approximated by an invertible process. On the contrary, the adjustment of a series generated from a model with nonstationary FI seasonality results in noninvertible negative FI coefficients. Note that this last result is not straightforward since overdifferencing is expected to be larger for data generated from seasonal stationary models when a nonstationary model is employed for seasonality.

The paper is organized as follows. Section 2 describes the problem. Section 3 briefly introduces the ideas behind the concept of seasonal FI. The simulation set-up and the results are presented in Section 4. Section 5 contains a small empirical application illustrating the results reported in Section 4. Section 6 concludes the paper.

2. The Problem

SEATS is an "ARIMA-model-based" (AMB) seasonal adjustment routine. Within the AMB approach, the program TSW starts by identifying the ARIMA model to the observed data

$$\Phi(B)\,x_t \;=\; \Theta(B)\,a_t, \tag{1}$$

where B is backward shift operator, $B^i x_t = x_{t-i}$, a_t is an iid $N\left(0,\sigma^2\right)$ innovation, the polynomial $\Phi\left(B\right)=F\left(B\right)\left(1-B\right)^D\left(1-B^\tau\right)^{D_\tau}$ contains nonseasonal and seasonal roots respectively, τ is the number of observations per year, D is the (integer) order of integration at frequency zero, while D_τ is the (integer) seasonal order of differentiation. The polynomials $F\left(B\right)$ and $\Theta\left(B\right)$ are finite in B, the first one includes stationary seasonal and nonseasonal AR roots and the second is an invertible MA polynomial.

If the aim of the application of the TSW is the seasonal adjustment, the observed series x_t is decomposed into the mutually orthogonal seasonally adjusted (SA) component n_t and seasonal component s_t:

$$x_t = n_t + s_t$$

The processes for the seasonal and the SA components will follow ARIMA specifications:

$$\Phi_n(B)\,n_t \;=\; \Theta_n(B)\,a_{nt}, \quad a_{nt} \sim niid\,(0,\sigma_n^2) \tag{2}$$

$$\Phi_s(B)\,s_t \;=\; \Theta_s(B)\,a_{st}, \quad a_{st} \sim niid\,(0,\sigma_s^2) \tag{3}$$

such that $\Phi(B) = \Phi_n(B)\Phi_s(B)$ and $\Theta(B)\,a_t = \Phi_n(B)\Theta_s(B)a_{st} + \Phi_s(B)\Theta_n(B)a_{nt}$. In this way the seasonal component captures the peaks around the seasonal frequencies, which may be subtracted by the filter.

For the seasonal adjustment, the purpose is, given X_t, to obtain the estimator of $\hat{n}_t$ such that $E\left[\left(n_t - \hat{n}_t\right)^2 \mid X_t\right]$ is minimized, i.e. the MMSE estimator of n_t.

Define $n_t = \Psi_n(B)a_{nt}$, with $\Psi_n(B) = \Theta_n(B)/\Phi_n(B)$; x_t and s_t are defined in the same way. As shown in Whittle (1963), $\hat{n}_t$ is obtained by means of the Wiener-Kolmogorov (WK) filter as the MMSE estimator of the signal given the observed series:

$$\hat{n}_t = \left[\frac{\sigma_n^2\,\Psi_n(B)\,\Psi_n(F)}{\sigma^2\Psi(B)\,\Psi(F)}\right]x_t = \nu(B,F)x_t \tag{4}$$

where F is a forward-shift operator (i.e. $F^i x_t = x_{t+i}$). Estimator given by (4) is called historical estimator. The WK filter can be expressed after simplification as

$$\nu(B,F) = \frac{\sigma_n^2}{\sigma^2}\frac{\Theta_n(B)\,\Phi_s(B)\Theta_n(F)\,\Phi_s(F)}{\Theta(B)\,\Theta(F)} \tag{5}$$

From (1), (4) , and (5) it can be obtained that

$$\Phi_n(B)\,\hat{n}_t = \Theta_n(B)\frac{\Theta_n(F)\,\Phi_s(F)\,\sigma_n^2}{\Theta(F)\;\sigma^2}a_t \tag{6}$$

It is clear that the process for the SA component (2) is different from the process for its historical estimator (6). If the process (1) is seasonally stationary, the seasonal component (3) will not contain the seasonal unit roots and, as a result, the polynomial $\Phi_s(F)$ will be stationary. In this case $\hat{n}_t$ given by (6) is going to be invertible. If the seasonal component is nonstationary I(1) (i.e. it contains unit roots at seasonal frequencies) $\Phi_s(F) = 1 + F + F^2 + ... + F^{r-1} = S(F)$, these unit roots will show up as

MA in the model generating $\hat{n}_t$ and will produce spectral values of zeros for the associated seasonal frequencies.

If the seasonal component identified within AMB approach is nonstationary the historical estimator of the series will not be invertible. An important implication of this result according to Gomez and Maravall (2001) is that the estimator of the SA series " ... *will not accept, in general, an AR (or VAR) approximation to its Wold representation*"[2].

3. Seasonal fractional integration

The AMB approach assumes that the data follows an ARIMA-type process. This assumption restricts the DGP to be either stationary I(0) or, alternatively integrated of order one, I(1), at zero and/or the seasonal frequencies. In this article we extend the seasonal I(1)/I(0) approach to the fractional case, and examine cases where the original series have noninteger orders of integration at seasonal frequencies. In such a case, the process is said to be seasonally fractionally integrated or seasonal I(d).

For the purpose of the present work, we first define an I(0) process as a covariance stationary process with a positive and bounded spectral density at all frequencies in the spectrum. Then, we say that a process x_t is seasonal I(d) if it can be represented as:

$$(1 - B^{\tau})^{d} x_t \; = \; a_t, \tag{7}$$

where B^{τ} is the seasonal lag operator (i.e., $B^{\tau}x_t = x_{t-\tau}$) and τ represents the number of periods per year (e.g., $\tau = 4$ with quarterly data, $\tau = 12$ in case of monthly data, etc.), d is a real value and a_t is an I(0) process that may include seasonal and nonseasonal

[2] The Note 1 is applied to the historical or final estimator of the SA series. However, only the central observations of the SA series are produced by the historical estimator. For the periods close to the beginning or the end, the filter cannot be completed and some preliminary estimator has to be used. The filters for preliminary estimator are different from each other, generating different models for each period they are applied, and all of them are different from the final estimator (6) and from the model of the SA component (2). If the series is long its properties are dominated by the final estimator. However, if it is short it will have a highly nonlinear structure.

weakly autocorrelated (e.g., ARMA) terms. If $d > 0$ in (1), x_t is said to be a seasonal long memory process, so-named because of the strong degree of association between observations widely (seasonally) separated in time. It may be shown that for this process, the spectral density function is unbounded at the zero and the seasonal frequencies, which is a characteristic of seasonal long memory processes. However, the specification in (9) is rather restrictive in the sense that it imposes the same degree of integration at all frequencies, noting that $(1 - B^{\tau})$ can be decomposed into $(1 - B)S(B)$ where $S(B) = 1 + B + B^2 + \ldots + B^{\tau-1}$ refers exclusively to the seasonal frequencies. Thus, for example, in case of the polynomial $(1 - B^4)^d$, it can be expressed as $(1 - B)^d(1 + B + B^2 + B^3)^d = (1 - B)^d (1 + B)^d (1 + B^2)^d$ implying the same degree of integration, d, at zero and the seasonal frequencies π, $\pi/2$ ($3\pi/2$) (of a 2π cycle). Extending this model, we may consider a more general specification that permits different degrees of integration at each of the frequencies. In particular, for the case of quarterly data, in the paper we will examine models of the form:

$$(1 - B)^{d_0} (1 + B)^{d_2} (1 + B^2)^{d_1} x_t \; = \; a_t, \tag{8}$$

where d_0 refers to the order of integration at the long run or zero frequency; d_2 is the order of integration at the semiannual frequency π, and d_1 corresponds to the annual frequencies $\pi/2$ and $3\pi/2$. Applications using the flexible model (10) can be found in Arteche and Robinson (2000), Gil-Alana and Robinson (2001), Arteche (2003) and Hassler, Rodrigues and Rubia (2009).

If the true process for (quarterly) data is fractionally integrated and is given by (10), then TSW will find the best possible integer framework approximation to model seasonality, which can be stationary or not. Theoretically, the process for the signal will have the following form:

$$\nabla^{d_0} \hat{N}_t = \frac{\sigma_n^2}{\sigma^2} \Theta_n(B)\, \Theta_n(F) \frac{\Phi_s(B)}{\Theta(B)} \frac{\Phi_s(F)}{\Theta(F)} (1+B)^{-d_2} \left(1+B^2\right)^{-d_1} a_t \qquad (9)$$

where the terms $\frac{\Phi_s(B)}{\Theta(B)}$ and $\frac{\Phi_s(F)}{\Theta(F)}$ are parts of two-sided WK filter (with backward and forward operators correspondingly) aimed to subtract the seasonal component given by $(1+B)^{d_2}\left(1+B^2\right)^{d_1}$. In other words, $\frac{\Theta(B)}{\Phi_s(B)}$ is the TSW integer framework approximation to the fractionally ingenerated seasonal component.

Note that the spectrum of the process in (9) is the same as that of

$$\nabla^{d_0} \hat{N}_t = \frac{\sigma_n^2}{\sigma^2} \Theta_n^2(B) \left[\frac{\Phi_s(B)}{\Theta(B)}\right]^2 (1+B)^{-d_2} \left(1+B^2\right)^{-d_1} a_t$$

If TSW choose a nonstationary SARIMA to fit the data, then $\Phi_s(B) = (1+B)\left(1+B^2\right)$ and therefore:

$$\nabla^{d_0} \hat{N}_t = \frac{\sigma_n^2}{\sigma^2} \Theta_n^2(B) \frac{(1+B)^{2-d_2}\left(1+B^2\right)^{2-d_1}}{\Theta^2(B)} a_t \qquad (10)$$

Therefore, if $d_1, d_2 < 1$, the adjusted series should have a seasonal fractional integrated MA polynomial with coefficients larger than 1 being, hypothetically, not invertible. In the following section we investigate this issue in practice by means of simulations.

4. Simulation study

4.1. Simulation set-up

To study invertibility of the time series adjusted by TSW, we generate quarterly data from different specifications of SARFIMA models. The parameters for the simulated SARFIMA processes of the form as in (2) are d_0 = {0.3, 0.7, 1, 1.5}, d_i =

{0.1, 0.3, 0.5, 0.7}, i = {1, 2}, i = {1, 2} and $\sigma^2 = 1$. For $d_0 = 1$ we simulate additionally $d_i = 1$.

The choice of the values is justified by the empirical evidence. The number of observations for each series is set T = 500.

To generate the data, the long memory polynomials in (2) have to be expanded. We choose the lag truncation 1000 for each polynomial. Thereafter, we multiply expanded long memory polynomials (the resulting polynomial has 3000 lags) and following Bhardwaj and Swanson (2006), we truncate the resulting polynomial when the coefficients become smaller than 1.0e-004 (the truncation lag is always smaller than 1000). All observations are generated using standard normal errors. For each process and each replication, we generate 3000 observations and we use just the last T observations to avoid the initial values problem, especially important when taking into account long-memory properties of the DGP.

To each simulated series we apply TSW. If TSW chooses a seasonally nonstationary ARIMA model for this series, we collect it for the future analysis. If the model chosen by TSW contains stationary seasonality we discard the simulated series. We proceed until we have I = 500 simulated series for each specification identified by TSW as seasonally nonstationary.

In addition to SARFIMA, we produce simulations for a set of quarterly Airline models of Box and Jenkins (1970) that are believed to approximate reasonably well the stochastic properties of many series

$$(1 - B)(1 - B^{\tau})x_t \;=\; (1 + Q_1 B)(1 + Q_{\tau} B^{\tau})\, a_t$$

with $\tau = 4$ and negative values for Q_1 and Q_4: $Q_i = \{-0.8, -0.6, -0.3\}$, $i = 1, 4$ and $[Q_1, Q_4] = \{[-1, -0.8], [-1, -0.6], [-1, -0.3]\}$[2]. In the same way as for SARFIMA, we collect I = 500 series for each specification identified by TSW as seasonally nonstationary.

Thereafter, each selected series for each specification is adjusted by TSW and coefficients of FI at seasonal frequencies are estimated[3]. It is important to remark that, even if several series are simulated from the same SARFIMA specification, the TSW can choose distinct ARIMA models for each of the simulated series. Moreover, even if the model chosen is the same the estimated ARIMA parameters under misspecification may be very different. Since seasonal filters applied to the data are based on the identified ARIMA models, different filters may be applied to each of the series simulated from the same SARFIMA process. In this way, the mean of the estimated parameters of FI at seasonal frequencies of the adjusted series does not have statistical meaning[4]. Therefore, to build conclusions on the invertibility of the adjusted series we propose the following testing procedure. After estimating the FI parameters at seasonal frequencies we test if the adjusted series is statistically noninvertible, i.e. whether we can reject the null hypothesis $d_i = -0.5$ in favor of the alternative $d_i < -0.5$ at least for one of the parameters of seasonal FI. If it is not the case, we test if the series is statistically invertible: i.e. we can reject the null hypothesis $d_i = -0.5$ in favor of the

[2] For example, in Maravall (2009) 50% of the 500 monthly exports and imports series of 15 European Union countries analyzed accept the Airline model as appropriate.
[3] We exclude 40 observations from the beginning and from the end of series to eliminate nonlinearities produced by preliminary estimator of SA.
[4] Even if we restrict TSW to choose always the default Airline model, the estimated coefficients of the model will differ at each replication and the seasonal filters will be distinct. Obviously, we could fix an ARIMA model with a given set of parameters to use for the adjustment, but this would produce results not interesting from a practical point of view.

alternative $d_i > -0.5$ for both estimated parameters of seasonal FI[5]. As a result, for each Airline and SARFIMA specification, we can compute both the percentage of statistically noninvertible and the percentage of statistically invertible series (in the adjusted I = 500 series chosen by TSW to be seasonally nonstationary before adjustment).

To estimate the coefficients of FI at the seasonal frequencies we use the log-periodogram regression with a complex-valued taper proposed by Hurvich and Chen (2000):

$$h_t = 0.5\left[1 - \exp\left\{\frac{i2\pi\left(t-0.5\right)}{T}\right\}\right], t = 1, \ldots, T .$$

The choice of the log-periodogram regression is justified by several reasons. Since we do not know what the correct specification after adjustment is, we avoid the parameterization of the whole spectrum by choosing a local estimation method. Tapering is particularly suitable when the estimated coefficients of FI are expected to be negative, possibly smaller than -0.5. In these circumstances, the estimation results based on the nontapered data will have a strong positive bias, making the method not appropriate for the purposes of this work. As Hurvich and Ray (1995) and Velasco (1999) point out, the use of a taper can alleviate the negative effects of overdifferencing, reducing the bias in FI estimates. Finally, tapering also reduces the bias that appears due to contamination of the periodogram from the short memory component of the spectral density and allows for a less restrictive trimming of frequencies in presence of asymmetries as it happens at frequency $\pi/2$. A comprehensive discussion of the performance of the method for seasonal and cyclical time series with asymmetric long memory properties is presented in Arteche and Velasco (2005).

[5] We use 10% significance level. Note also that our procedure is very conservative.

Nevertheless, we also perform a small Monte Carlo study to check the performance of the estimation method in the presence of negative seasonal fractional integration at seasonal frequencies in the following way. For each specification, after simulating the data and before applying the TSW (i.e. when we still know the true DGP), we take yearly differences, making sure that the resulting series are over-differenced at seasonal frequencies having negative FI coefficients. We estimate these coefficients computing the mean for each specification to assess the estimation bias.

All the simulations and estimations were produced in Matlab. The programs are available from the authors upon request. For seasonal adjustment we use the last release of the TSW for Matlab developed by the Bank of Spain. The programs with instructions can be downloaded from the web-site of the Bank of Spain[6].

4.2. Simulation Results

The results of the simulation study for the different Airline and SARFIMA specifications are presented in Tables 1 and 2 respectively. In both tables, the particular specification from which the data is simulated appears in the first column (i.e., the values for the MA parameters Q_1 and Q_4 for the Airline model and the parameters of FI at seasonal frequencies d_1 and d_2 for SARFIMA).

4.2.1 Monte Carlo results for the tapered log-periodogram regression for seasonal frequencies with negative FI

The results of the Monte Carlo study are presented in the following two columns. The coefficients in the table are $\hat{d}_i = 1 + \hat{d}_i^*$, $i = 1,2$ in columns two and three

[6] http://www.bde.es/servicio/software/interfacese.htm.

respectively, where $\hat{d}_i^*$ is the mean of the estimates of the coefficient with yearly differenced data[7].

To study the performance of FI estimators it is important to distinguish between the actual bias of the estimation procedure and the bias due to the contamination of periodogram frequencies by components attributed to other frequencies. To do so, we first analyze simulation results for SARFIMA processes with {$d_0 = 1$, $d_1 = 1$, $d_2 = 0.1, 0.3, 0.5, 0.7$} and {$d_0 = 1$, $d_1 = 0.1, 0.3, 0.5, 0.7$, $d_2 = 1$}. Note that after taking yearly difference these processes will be characterized by negative FI at one seasonal frequency only. As can be observed in Table 2, the bias for these processes is very small; the mean of the estimated values $\hat{d}_i$ over the replicas are always very close to the values used for simulation.

If the process contains FI at other frequencies, either seasonal or zero frequency, the estimated parameters are slightly positively biased. The bias is higher when the negative FI at other frequencies is greater in absolute value. Thus, the highest bias is observed for the process {$d_0 = 0.3$, $d_1 = 0.1$, $d_2 = 0.1$}. Recall that after taking yearly difference it becomes {$d_0^* = -0.7$, $d_1^* = -0.9$, $d_2^* = -0.9$}.

Simulation results from Airline specifications are useful to study the performance of the method in the presence of short memory components. Results are presented in the Table 1. The estimation method always detects seasonal unit roots. The precision of the estimates depends on the value of the seasonal MA parameter Q_4: the greater this value in absolute terms, the greater the bias.

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[7] Recall that since we expect to have negative seasonal fractional integration after adjustment, the original data was yearly differenced in the Monte Carlo analysis to ensure that the resulting series are overdifferenced at seasonal frequencies with negative coefficients of fractional integration. For the processes with the fractional order of integration at zero $d_0 = 1.5$, we take also a first difference in addition to the yearly difference. The same applies to the Airline model.

Finally, note that no matter the specification, the coefficient d_1 tends to be estimated less precisely than the coefficient d_0 due to the asymmetries presented in the periodogram around frequency $\pi/2$.

Overall, the performance of the method at seasonal frequencies is similar to its performance at frequency zero, documented in previous studies. The log-periodogram regression with tapered data performs well for negative seasonal FI even for coefficients from the noninvertible region and also for the estimation of the parameter at frequency $\pi/2$, where the spectrum is not symmetric. The method works also well in the presence of short memory components, as shown in Table 1. On the whole, the results from the Monte Carlo study confirm that the log-periodogram regression with tapered data is appropriate for the purposes of the present work.

4.2.2 Results on the invertibility of the adjusted series

The column four of Tables 1 and 2 presents the percentage of cases for which TSW chooses a seasonally nonstationary model to fit the data for each of the simulated processes. As can be seen in Table 1, TSW always chooses a nonstationary model when the true DGP follows the Airline model. As expected for the SARFIMA specifications (Table 2) this percentage increases together with the magnitude of both d_1 and d_2.

Next three columns of Tables 1 and 2 contain results of the statistical testing as described in the simulation set-up. Column five presents the percentage of cases in which the seasonal adjusted processes is estimated invertible, i.e., with the two estimated coefficients of FI at seasonal frequencies greater than -0.5. Thereafter we compute the percentage of replications in which the SA series have at least one estimated coefficient of seasonal FI statistically smaller than -0.5 - that is to say, the series is statistically noninvertible (column six). If the hypothesis of statistical noninvertibility is rejected, we test statistical invertibility: both estimated coefficients are statistically greater than -0.5. The percentage of statistically invertible results is given in column seven. When the data are simulated from the Airline model (Table 1), the estimated coefficients of FI at frequencies $\pi/2$ and π are almost always smaller than -0.5, which indicates the (possible) noninvertibility of the corresponding SA series. Moreover, in a high percentage of cases this noninvertibility is statistically significant. This result is not surprising and it is completely in line with the implications of the TSW for this class of models (Gomez and Maravall (2001)). For the SARFIMA specifications (Table 2), the result of the application of TSW depends on the initial properties of the simulated data. Thus, if the two coefficients of the seasonal FI are within the stationary region ($d_i < 0.5$, $i = \{1, 2\}$), even if the TSW identifies a nonstationary seasonal model (this occurs in a relatively small percentage of cases), the estimated coefficients of seasonal FI of the SA series are greater than -0.5 in most of the cases. Only a very small percentage of series are (possibly) statistically noninvertible. The percentage of statistically invertible results decreases as the seasonal FI coefficients of the original series approach the nonstationary region. For example, for $d_0 = 0.3$, if the original series have both coefficients of seasonal FI $d_i = 0.1$, TSW only selects a seasonal nonstationary representation in 31.7% of the cases. In addition, even if this is the case and a nonstationary ARIMA is chosen, the estimated coefficients of the SA series almost always lie (99.4%) in the invertible region. Moreover, in 91.4% of the cases both parameters are statistically greater than -0.5 and the series are statistically invertible. The percentage of statistically invertible results decreases to 78.6% if d_1 and d_2 are equal to 0.3. Still, the estimated parameters are greater than -0.5 in 97% of the cases and the percentage of statistically noninvertible results is just 0.2%.

On the contrary, if one of the coefficients of seasonal FI in the DGP is greater than 0.5, TSW fits a seasonally nonstationary model in a relatively higher percentage of cases, and for those cases the SA series are often estimated to be noninvertible. Once more, the percentage of statistically noninvertible results increases with the parameters of seasonal FI of the original series. Thus, (again for d_0 = 0.3) if d_i = 0.7 for the two seasonal FI coefficients in the original DGP, TSW selects a nonstationary representation in 99.8% of the cases. Only in 13.2% of the cases the SA series are invertible (and only in 7.6% the invertibility is statistically significant) whereas in 50.2% of the replicas the SA series were found to be statistically noninvertible. It is also interesting to note that, although the parameter of fractionally integration at zero is not neutral, the same conclusions are obtained for all simulated $d_{0.}$

The previous results indicate that invertibility may not be a severe issue in many circumstances. However, they contradict the theoretical findings derived in Section 3. Recall that the simulations are based on data identified by TSW as seasonally nonstationary and according to equation (10) the adjusted series should always contain seasonal unit MA roots. To aid in the explanation of this apparent puzzle, the columns eight and nine in Tables 1 and 2 report the median with the 16th and 84th percentiles (68% band) of the estimated seasonal FI parameters after adjustment. Several results emerge from these columns:

1. If the DGP contains stationary seasonality with relatively low parameters of FI, the adjusted series is not only usually estimated (statistically) invertible, but also the estimated parameters of FI are indeed very small in magnitude, with no sign of unit MA roots. For example, if the data is generated from an SARFIMA with $d_0 = 0.3$, $d_1 = 0.1$ and $d_2 = 0.1$, the median estimated values after adjustment are $\tilde{d}_1 = -0.116$ and $\tilde{d}_1 = -0.096$.

2. If the DGP contains seasonal FI with equal parameters ($d_1 = d_2$), the medians of parameter estimates after adjustment are also close to each other and very similar to

the values employed to generate the data but with opposite sign. This can also be observed in the former example.

3. If d_1 and d_2 in the DGP are very different in magnitude, the median values after adjustment are negative, but smaller in magnitude than the larger parameter in the DGP . For example, if parameters in the DGP are $d_0 = 0.3$, $d_1 = 0.7$ and $d_2 = 0.1$, the median estimated values after adjustment are $\tilde{d}_1 = -0.476$ and $\tilde{d}_1 = -0.465$.

An extremely (positive) bias of the estimation method for the type of processes generated by TSW could probably explain the first finding. However, it is in clear contradiction with the Montecarlo analysis and, especially, with the results from Airline specifications (Table 1) since the estimation method clearly detects the noninvertible unit MA roots in the adjusted data. The alternative explanation is that the integer framework approximation for stationary FI seasonal processes employed by TSW is good enough to be virtually not distinguishable from FI when the seasonal filter is applied. Note that, if $\frac{\Theta(B)}{\Phi_s(B)} \approx \left(1+B^2\right)^{d_1}\left(1+B\right)^{d_2}$, then (10) becomes:

$$\nabla^{d_0}\hat{N}_t \approx \frac{\sigma_n^2}{\sigma^2}\Theta_n^2(B)\left(1+B\right)^{d_2}\left(1+B^2\right)^{d_1} a_t \qquad (11)$$

Thus, if $d_i > -0.5,\ i = 1, 2$, the process can be approximated by an invertible process.

This last explanation can also accommodate the second and third findings. The integer SARIMA framework where TSW operates is not flexible enough to fit seasonality with different level of persistence at seasonal frequencies. Therefore, TSW finds an approximation where $\frac{\Theta(B)}{\Phi_s(B)} \approx \left(1+B^2\right)^{d}\left(1+B\right)^{d}$, $\min\{d_1, d_2\} < d < \max\{d_1, d_2\}$. In this case: $\nabla^{d_0}\hat{N}_t \approx \frac{\sigma_n^2}{\sigma^2}\Theta_n^2(B)\left(1+B\right)^{2d-d_2}\left(1+B^2\right)^{2d-d_1} a_t$ which is trivial if $d_1 = d_2$.

Overall, simulation results are in line with Gomez and Maravall (2001), but using a more flexible definition of invertibility: if the process contains strong nonstationary seasonality (including FI) then the SA series estimated by TSW will be in general noninvertible. However, if the original series were stationary fractionally integrated at seasonal frequencies, TSW will choose a nonstationary representation in a smaller percentage of cases and, even if a nonstationary seasonal model is chosen, the resulting SA series is likely not distinguishable from an invertible process. This result is important because an econometrician never knows what the DGP for the real data is, and always works with approximations which fit the data reasonably well according to the results of statistical testing. We illustrate our results with real data in the following section.

5. Empirical examples

To illustrate the simulation results, we consider several quarterly series of the Spanish economy: Industrial Production Index (IPI), airline passengers (AIR), employment (EMP) and three quarterly cyclical economic indicators, namely: cement consumption (CC), car registrations (CR) and housing starts (HS). IPI and these three indicators are considered to be the cycle drivers for an economy in Leamer (2009) and have been recently used by Bujosa et al. (2012) to construct a composite leading indicator for the Spanish economy. All series are strongly seasonal, and cover a span starting from the beginning of the 70s. Monthly data for IPI, CC, CR, HS and AIR can be obtained from the Bank of Spain. To convert the IPI to quarterly, we use the simple average of the observations inside each quarter. Other series are converted to quarterly by adding the observations inside the quarter. Employment has been obtained from the

OECD stats database. We exclude the last years of observations to avoid the influence of the current crisis[8].

Figure 1 (left panels) plots the original series (not seasonally adjusted) and the series after adjustment by TSW. Nonstationary in the mean and a strong seasonal pattern is observed in all the original series.

[Insert Figure 1 about here]

The right panel in Figure 1 depicts the periodograma of both differenced series: original and adjusted by TSW. As expected, the periodograma of the differenced original series have strong peaks at the two seasonal frequencies, while those of the differenced adjusted series present dips at the same frequencies.

TSW identifies the following models for the original series:

Variable	ARIMA model chosen by TSW
ln(IPI)	$\left(1-0.2275B\right)\nabla\nabla^4 y_t = \left(1-0.7146B^4\right)\varepsilon_t$ changed by Seats to: $\nabla\nabla^4 y_t = \left(1-0.2275B\right)\left(1-0.7146B^4\right)\varepsilon_t$
CC	$\nabla\nabla^4 y_t = \left(1-0.7054B^4\right)\varepsilon_t$
CR	$\nabla\nabla^4 y_t = \left(1-0.6775B^4\right)\varepsilon_t$
HS	$\left(1+0.4836B^4\right)\nabla y_t = \left(1-0.5360B\right)\varepsilon_t$
ln(AIR)	$\nabla\nabla^4 y_t = \left(1-0.1686B\right)\left(1-0.6158B^4\right)\varepsilon_t$
EMP	$\left(1+0.7923B\right)\nabla\nabla^4 y_t = \left(1-0.6320B^4\right)e_t$

As can be seen from the above, all the series except IPI, HS and EMP follow a standard Airline model. For CC and CR the trend is very strong and Q_1 is equal to zero. The model identified for the IPI does not accept the admissible decomposition and is modified by SEATS. Given that AR(1) polynomials with Φ_1 in the interval (-0.2, -0.4) are practically indistinguishable from the MA(1) with $Q_1 = -\Phi_1$, SEATS replaces the

[8] For IPI, CC, and CR we exclude the last years of data. For the case of housing starts (HS) an additional year had to be taken into account since the effects of the crisis manifest earlier for this indicator. On the other hand, an additional year was able to be added for airline passengers (AIR) and employment (EMP).

NA model with the corresponding Airline model (Maravall (2009)). TSW chooses a stationary seasonal model for HS and SARIMA for EMP.

We estimate the coefficients of FI at seasonal frequencies before and after the adjustment by TSW. The results are presented in Table 3.

[Insert Table 3 about here]

It is interesting to note that the examined series follow neither the Airline nor the SARIMA model, since the estimated coefficients of FI at the seasonal frequencies before the adjustment (columns four and five) are all statistically different from one. As can be seen in the table CC, CR and EMP seem to be seasonally stationary before adjustment, especially CC, for which the null hypothesis $H_0 : d_i = 0.5$ is rejected in favor to the alternative $H_1 : d_i < 0.5$ at conventional significance levels for both parameters of seasonal FI. For the other two series, we can reject the null in favor of the alternative at 15% significance level which seems to be appropriate give the short length of the data. In line with our simulation results, even though TSW has selected a seasonally nonstationary models for the three series, the estimated coefficients after the adjustment are substantially higher than -0.5, suggesting that the adjusted series can be approximated by an invertible process. That also seems to be the case of HS, for which TSW has selected a stationary representation before adjustment. For the IPI and AIR series, the estimated coefficients of FI at frequency $\pi/2$ are larger than 0.5, albeit we cannot reject the null of $d_1 = 0.5$ at any significance level. After adjustment, one of the estimated coefficients is smaller than -0.5 and the adjusted series may then be noninvertible, although we cannot reject null of $d_1 = -0.5$ in favor of $d_1 < -0.5$. Overall, the empirical results are in line with the results of the simulation study.

6. Conclusions

In this paper we have examined the invertibility property of seasonal series adjusted by TSW. According to Gomez and Maravall (2001) whenever the process chosen by TSW to fit the data contains seasonal unit roots, the adjusted series estimated by the program has MA unit roots and, as a result, it is not invertible and cannot be approximated by an AR (VAR) process as is ordinarily done in practice.

In the simulation study carried out in this work we found that the invertibility issue may not be in many circumstances a strong concern. In particular we found that if the true DGP follows the default of the program Airline model (ARIMA with unit roots at seasonal frequencies), the adjusted series produced by TSW are indeed noninvertible. However, if the series is fractionally integrated at the seasonal frequencies, which is less restrictive and very plausible in some cases according to the empirical evidence, the adjusted series still can be approximated by invertible processes, depending on the stationarity of the original series. Thus, if the original series is seasonally stationary with coefficients of FI at seasonal frequencies smaller than 0.5, the SA series estimated by TSW is likely to be statistically invertible or undistinguishable from an invertible process even if the model chosen by the program to model seasonality was nonstationary , therefore still admit AR (or VAR) approximation. This approximation is more plausible the further the seasonal FI parameters of the original series are from the nonstationary region. On the contrary, if the original series is seasonally nonstationary, the resulting adjusted series are expected to be noninvertible. As shown in the empirical examples, these results are interesting since stationary FI seasonality is not a rare event in economic data.

References

Arteche J (2003) Semiparametric robust tests on seasonal or cyclical long memory time series. J Time Ser Anal 23:251-285

Arteche J, Robinson PM (2000) Semiparametric inference in seasonal and cyclical long memory processes. J Time Ser Anal 21:1-25

Arteche J, Velasco C (2005) Trimming and tapering semi-parametric estimates in asymmetric long memory time series. J Time Ser Anal 26:581-611

Bhardwaj G, Swanson NR (2006) An empirical investigation of the usefulness of ARFIMA models for predicting macroeconomic and financial time series. J Econometrics 131:539-578

Box GEP, Jenkins GM (1970). Time Series Analysis: Forecasting and Control, San Francisco:Holden-Day

Bujosa M, García-Ferrer A, de Juan A (2013) Predicting recessions with factor linear dynamic harmonic regressions. J Forecasting 32: 481-499.

Eurostat (2009) ESS Guidelines on seasonal adjustment. European Communities, Luxembourg

Gil-Alana, LA, Robinson PM (2001) Testing of seasonal fractional integration in the UK and Japanese consumption and income. J Appl Econom 16:95-114

Gomez V, Maravall A (2001) Seasonal adjustment and signal extractions in economic time series. A course in Advanced Time Series analysis. In: Peña D, Tiao DG, Tsay RS (eds.), Wiley, New York, pp 171-201

Grether DM, Nerlove M (1970) Some properties of optimal seasonal adjustment. Econometrica 38:682-703

Hassler U, Rodrigues P, Rubia A (2009) Testing for general fractional integration in the time domain. Economet Theor 25:1793-1828

Hurvich CM, Ray BK (1995) Estimation of the memory parameter for non-stationary or non-invertible fractionally integrated processes. J Tim Ser Anal 16:17-41

Hurvich CM, Chen WW (2000) An efficient taper for potentially overdifferenced long-memory time series. J Tim Ser Anal 21:155-180

Leamer EE (2009) Macroeconomic patterns and stories. Springer-Verlag, Berlin

Maravall A (2009) Identification of Reg-ARIMA models and of problematic series in large scale applications program TSW. Mimeo, Bank of Spain, Statistic and Econometric Software

Nerlove M (1965) A comparison of a modified 'Hannan' and the BLS seasonal adjustment filters. J Am Stat Ass 60:442-91

Ooms M, Hassler U (1997) On the effect of seasonal adjustment on the log-periodogram regression. Econ Lett 56:135-141

Porter-Hudak S (1990) An application of the seasonal fractionally differenced model to the monetary aggregates. J Am Stat Ass 85:338-344

Velasco C. (1999) Non-stationary log-periodogram regression. Journal of Econometrics 91: 325-371

Whittle P (1963) Prediction and regulation by linear least-square methods. London, English Universities Press

Appendix: Tables and Figures

Table 1: Simulation results, Airline model

True specification	Before adjustment		After adjustment					
$\{Q_1, Q_4\}$	$\hat{d}_1$	$\hat{d}_2$	%, NS	%, I	%, SNI	%, SI	$\tilde{d}_1$ [68 band]	$\tilde{d}_2$ [68 band]
{-0.8,-0.3}	1.043	1.062	100.0	16.4	81.8	10.6	-0.790 [-0.734, -0.883]	-1.004 [-0.843,-1.100]
{-0.8,-0.6}	1.124	1.089	100.0	14.6	76.2	11.0	-0.713 [-0.613,-0.801]	-1.102 [-0.828,-1.258]
{-0.8,-0.8}	1.195	1.112	100.0	10.8	87.2	9.8	-0.760 [-0.677,-0.980]	-1.214 [-1.069,-1.320]
{-0.6,-0.3}	1.040	1.071	100.0	6.2	92.6	4.8	-0.827 [-0.778,-0.870]	-1.069 [-0.982,-1.133]
{-0.6,-0.6}	1.113	1.110	100.0	25.2	72.6	20.2	-0.740 [-0.591,-0.814]	-1.067 [-0.787,-1.213]
{-0.6,-0.8}	1.203	1.096	100.0	19.5	72.4	14.0	-0.648 [-0.530,-0.730]	-1.002 [-0.726,-1.327]
{-0.3,-0.3}	1.024	1.056	100.0	4.0	94.4	3.4	-0.837 [-0.791,-0.878]	-0.963 [-0.878,-1.036]
{-0.3,-0.6}	1.116	1.071	100.0	6.0	92.2	4.4	-0.785 [-0.732,-0.859]	-0.909 [-0.774,-1.118]
{-0.3,-0.8}	1.192	1.083	100.0	15.0	68.6	7.8	-0.601 [-0.515,-0.675]	-0.766 [-0.638,-0.902]
{-1,-0.3}[b]	1.025	1.064	100.0	4.0	95.0	2.6	-0.889 [-0.839,-0.935]	-1.002 [-0.943,-1.066]
{-1,-0.6}[b]	1.114	1.089	100.0	4.2	95.4	4.0	-1.008 [-0.963,-1.044]	-1.013 [-0.950,-1.075]
{-1,-0.8}[b]	1.213	1.106	100.0	3.8	95.6	3.4	-1.096 [-1.045,-1.137]	-1.008 [-0.944,-1.074]

Notes: (a) Airline model: $(1-B)(1-B^4)y_t = (1+Q_1B)(1+Q_4B^4)\varepsilon_t$. If $Q_4 \to -1$, the seasonality is small or stable; if $Q_1 \to -1$, the trend is small. (b) These models correspond to the case when $Q_1 = -1$ in the standard Airline model. ∇ and $(1+Q_1L)$ will be canceled out in this case and the model will have a special form $\nabla^4 y_t = (1+Q_4L^4)\varepsilon_t$. (c) Since the Airline model contains seasonal unit roots $d_1 = d_2 = 1$.(d) $\hat{d}_1$ and $\hat{d}_2$ are estimated means of the coefficients of FI at seasonal frequencies before adjustment. (e) NS – percentage of cases the TSW identifies a seasonally nonstationary model for the simulated data for a given process; I – percentage of cases both estimated coefficients of seasonal fractional integration are greater than -0.5; SNI – percentage of cases at least one coefficient of seasonal fractional integration is statistically smaller than -0.5 (statistically noninvertible); SI - percentage of cases both coefficients of seasonal fractional integration are statistically greater than -0.5 (statistically invertible); (f) $\tilde{d}_1$ and $\tilde{d}_2$ are medians of the estimated coefficients of FI after adjustment and [68 band] refers to the 16th and 84th percentiles. (g)The bandwidth parameter in the estimation is $T^{0.5}$;

Table 2: Simulation results, SARFIMA model

True specification	Before adjustment		After adjustment					
$\{d_1, d_2\}$	$\hat{d}_1$	$\hat{d}_2$	%, NS	%, I	%, SNI	%, SI	$\tilde{d}_1$ [68 band]	$\tilde{d}_2$ [68 band]
$d_0 = 0.3$								
{0.1,0.1}	0.173	0.191	31.7	99.4	0.2	91.4	-0.116 [-0.041,-0.178]	-0.096 [-0.028,-0.162]
{0.1,0.3}	0.166	0.366	49.1	97.0	0.2	78.6	-0.240 [-0.176,-0.285]	-0.119 [-0.055,-0.180]
{0.1,0.5}	0.173	0.539	58.1	84.6	1.6	46.0	-0.272 [-0.212,-0.341]	-0.273 [-0.207,-0.356]
{0.1,0.7}	0.168	0.734	35.8	45.0	15.8	11.0	-0.258 [-0.192,-0.327]	-0.511 [-0.434,-0.583]
{0.3,0.1}	0.360	0.169	60.5	86.2	1.0	46.8	-0.200 [-0.147,-0.260]	-0.327 [-0.249,-0.400]
{0.3,0.3}	0.353	0.362	64.7	92.4	0.0	56.0	-0.274 [-0.227,-0.328]	-0.259 [-0.197,-0.335]
{0.3,0.5}	0.348	0.540	80.5	69.8	4.2	30.6	-0.333 [-0.275,-0.404]	-0.302 [-0.231,-0.397]
{0.3,0.7}	0.352	0.731	74.7	27.2	27.6	6.4	-0.340 [-0.272,-0.407]	-0.563 [-0.452,-0.625]
{0.5,0.1}	0.544	0.158	95.8	66.8	9.4	26.2	-0.301 [-0.257,-0.349]	-0.403 [-0.312,-0.497]
{0.5,0.3}	0.529	0.342	94.9	68.2	6.2	22.6	-0.341 [-0.284,-0.401]	-0.399 [-0.329,-0.481]
{0.5,0.5}	0.533	0.533	96.9	63.4	9.0	17.2	-0.401 [-0.339,-0.457]	-0.404 [-0.333,-0.476]
{0.5,0.7}	0.530	0.730	99.6	32.8	30.0	10.2	-0.478 [-0.397,-0.561]	-0.440 [-0.375,-0.531]
{0.7,0.1}	0.741	0.156	100.0	38.8	26.0	11.6	-0.476 [-0.420,-0.526]	-0.465 [-0.340,-0.574]
{0.7,0.3}	0.736	0.332	100.0	30.0	28.2	10.4	-0.489 [-0.427,-0.548]	-0.531 [-0.447,-0.613]
{0.7,0.5}	0.720	0.530	99.8	27.0	33.4	8.0	-0.518 [-0.461,-0.575]	-0.536 [-0.456,-0.628]
{0.7,0.7}	0.721	0.708	99.8	13.2	50.2	7.6	-0.590 [-0.528,-0.658]	-0.598 [-0.519,-0.680]
$d_0 = 0.7$								
{0.1,0.1}	0.184	0.187	16.7	98.2	0.2	91.0	-0.031 [0.012,-0.096]	-0.046 [0.014,-0.118]
{0.1,0.3}	0.174	0.363	22.1	96.2	0.4	86.6	-0.116 [-0.056,-0.172]	-0.069 [0.004,-0.155]
{0.1,0.5}	0.142	0.569	72.4	81.0	2.4	41.2	-0.186 [-0.104,-0.269]	-0.298 [-0.237,-0.373]
{0.1,0.7}	0.145	0.745	77.8	47.8	16.0	18.6	-0.290 [-0.181,-0.389]	-0.490 [-0.410,-0.575]
{0.3,0.1}	0.358	0.164	73.3	76.4	8.2	42.2	-0.236 [-0.172,-0.300]	-0.335 [-0.238,-0.421]
{0.3,0.3}	0.348	0.349	76.9	80.8	5.2	51.0	-0.286 [-0.201,-0.368]	-0.278 [-0.200,-0.380]
{0.3,0.5}	0.335	0.534	74.2	73.6	7.2	39.0	-0.275 [-0.189,-0.364]	-0.325 [-0.263,-0.400]
{0.3,0.7}	0.332	0.731	97.3	28.2	36.4	11.0	-0.527 [-0.412,-0.628]	-0.488 [-0.401,-0.560]
{0.5,0.1}	0.534	0.159	98.2	52.0	15.4	19.2	-0.400 [-0.320,-0.475]	-0.457 [-0.373,-0.556]
{0.5,0.3}	0.522	0.335	99.2	30.0	38.8	12.2	-0.465 [-0.395,-0.548]	-0.585 [-0.475,-0.680]
{0.5,0.5}	0.527	0.532	99.0	31.0	28.4	9.2	-0.524 [-0.453,-0.590]	-0.493 [-0.424,-0.575]
{0.5,0.7}	0.524	0.726	99.2	17.2	59.2	8.8	-0.638 [-0.549,-0.727]	-0.535 [-0.457,-0.617]
{0.7,0.1}	0.728	0.142	100.0	22.2	56.4	10.4	-0.617 [-0.523,-0.731]	-0.457 [-0.327,-0.618]
{0.7,0.3}	0.717	0.324	100.0	13.6	59.0	4.8	-0.590 [-0.523,-0.652]	-0.590 [-0.451,-0.736]
{0.7,0.5}	0.712	0.522	100.0	8.0	66.8	3.0	-0.610 [-0.563,-0.678]	-0.689 [-0.583,-0.770]
{0.7,0.7}	0.712	0.718	100.0	5.6	76.0	3.8	-0.677 [-0.625,-0.731]	-0.639 [-0.569,-0.715]
$d_0 = 1$								
{0.1,0.1}	0.164	0.176	24.8	100.0	0.0	98.4	-0.042 [-0.003,-0.083]	-0.035 [0.019,-0.095]
{0.1,0.3}	0.177	0.356	26.3	98.6	0.0	90.8	-0.134 [-0.072,-0.191]	-0.025 [0.038,-0.089]
{0.1,0.5}	0.165	0.545	28.9	88.8	1.2	56.4	-0.098 [-0.045,-0.155]	-0.280 [-0.175,-0.375]
{0.1,0.7}	0.148	0.748	57.6	38.2	27.4	15.2	-0.086 [-0.034,-0.173]	-0.562 [-0.453,-0.658]
{0.1,1.0}	0.117	1.037	64.6	9.6	86.6	9.0	-0.056 [-0.005,-0.110]	-0.876 [-0.814,-0.937]
{0.3,0.1}	0.355	0.168	58.1	94.0	1.2	71.0	-0.145 [-0.093,-0.195]	-0.212 [-0.143,-0.295]
{0.3,0.3}	0.335	0.348	70.0	92.4	1.8	70.0	-0.211 [-0.144,-0.262]	-0.214 [-0.139,-0.288]
{0.3,0.5}	0.339	0.533	74.4	86.8	1.6	48.2	-0.211 [-0.140,-0.297]	-0.280 [-0.215,-0.345]
{0.3,0.7}	0.328	0.728	89.9	38.6	18.4	12.6	-0.208 [-0.145,-0.281]	-0.514 [-0.438,-0.578]
{0.3,1.0}	0.316	1.037	91.4	18.2	75.8	14.0	-0.459 [-0.244,-0.607]	-0.799 [-0.715,-0.888]
{0.5,0.1}	0.533	0.159	96.2	69.0	10.2	29.8	-0.355 [-0.286,-0.415]	-0.351 [-0.247,-0.443]
{0.5,0.3}	0.527	0.338	97.8	40.4	30.0	14.8	-0.474 [-0.379,-0.561]	-0.503 [-0.394,-0.607]
{0.5,0.5}	0.514	0.530	98.0	42.6	30.0	14.2	-0.484 [-0.405,-0.580]	-0.483 [-0.381,-0.566]
{0.5,0.7}	0.516	0.722	98.4	35.8	31.8	11.0	-0.386 [-0.323,-0.513]	-0.530 [-0.452,-0.603]
{0.5,1.0}	0.505	1.017	100.0	11.4	86.6	9.2	-0.768 [-0.496,-0.866]	-0.812 [-0.727,-0.887]
{0.7,0.1}	0.728	0.142	100.0	29.6	36.0	11.8	-0.557 [-0.485,-0.628]	-0.351 [-0.221,-0.460]
{0.7,0.3}	0.711	0.327	100.0	15.8	66.6	7.6	-0.654 [-0.578,-0.727]	-0.630 [-0.513,-0.722]
{0.7,0.5}	0.714	0.516	100.0	15.2	75.8	6.6	-0.721 [-0.633,-0.801]	-0.792 [-0.637,-0.916]

{0.7,0.7}	0.700	0.716	100.0	8.2	80.4	3.4	-0.773 [-0.670,-0.851]	-0.727 [-0.637,-0.820]
{0.7,1.0}	0.699	1.010	100.0	8.6	90.2	5.8	-0.942 [-0.866,-1.003]	-0.824 [-0.757,-0.895]
{1.0,0.1}	1.017	0.117	100.0	7.6	91.2	6.8	-0.947 [-0.888,-0.992]	-0.153 [-0.66,-0.254]
{1.0,0.3}	1.009	0.314	100.0	8.6	90.8	5.0	-0.944 [-0.901,-0.989]	-0.403 [-0.278,-0.587]
{1.0,0.5}	1.004	0.506	100.0	6.2	93.6	4.8	-0.942 [-0.900,-0.979]	-0.596 [-0.479,-0.738]
{1.0,0.7}	1.010	0.691	100.0	7.2	90.8	4.6	-0.914 [-0.867,-0.968]	-0.795 [-0.655,-0.983]
{1.0,1.0}	1.005	0.991	100.0	6.4	93.2	4.6	-0.919 [-0.878,-0.962]	-0.899 [-0.833,-0.956]
$d_0 = 1.5$								
{0.1,0.1}	0.177	0.188	18.3	94.8	1.4	76.4	-0.155 [-0.104,-0.215]	-0.171 [-0.095,-0.248]
{0.1,0.3}	0.142	0.367	49.1	96.2	0.4	77.0	-0.208 [-0.142,-0.269]	-0.147 [-0.083,-0.220]
{0.1,0.5}	0.137	0.541	72.5	82.0	4.4	41.0	-0.307 [-0.239,-0.375]	-0.279 [-0.214,-0.363]
{0.1,0.7}	0.132	0.739	69.2	55.2	13.8	19.2	-0.304 [-0.245,-0.372]	-0.464 [-0.384,-0.548]
{0.3,0.1}	0.332	0.153	75.1	63.0	11.0	31.8	-0.269 [-0.200,-0.336]	-0.410 [-0.302,-0.531]
{0.3,0.3}	0.331	0.343	69.8	73.4	4.4	34.0	-0.345 [-0.277,-0.415]	-0.341 [-0.279,-0.427]
{0.3,0.5}	0.325	0.535	84.5	63.4	9.4	20.2	-0.408 [-0.359,-0.475]	-0.358 [-0.285,-0.437]
{0.3,0.7}	0.318	0.728	80.4	38.6	30.0	12.8	-0.430 [-0.356,-0.502]	-0.486 [-0.390,-0.622]
{0.5,0.1}	0.519	0.147	89.6	53.6	22.6	19.2	-0.344 [-0.272,-0.410]	-0.444 [-0.322,-0.573]
{0.5,0.3}	0.516	0.336	96.5	49.2	21.2	13.0	-0.383 [-0.330,-0.433]	-0.489 [-0.404,-0.576]
{0.5,0.5}	0.505	0.526	96.3	50.8	12.2	8.8	-0.427 [-0.382,-0.481]	-0.441 [-0.380,-0.528]
{0.5,0.7}	0.502	0.717	97.8	31.4	27.2	6.8	-0.497 [-0.435,-0.560]	-0.477 [-0.391,-0.565]
{0.7,0.1}	0.709	0.134	99.6	32.4	45.8	14.6	-0.479 [-0.388,-0.574]	-0.436 [-0.241,-0.682]
{0.7,0.3}	0.705	0.323	99.8	26.8	44.2	7.4	-0.456 [-0.386,-0.554]	-0.582 [-0.444,-0.749]
{0.7,0.5}	0.703	0.508	100.0	20.8	42.4	5.0	-0.488 [-0.427,-0.546]	-0.601 [-0.516,-0.705]
{0.7,0.7}	0.700	0.717	100.0	16.4	46.8	4.8	-0.547 [-0.490,-0.616]	-0.605 [-0.510,-0.681]

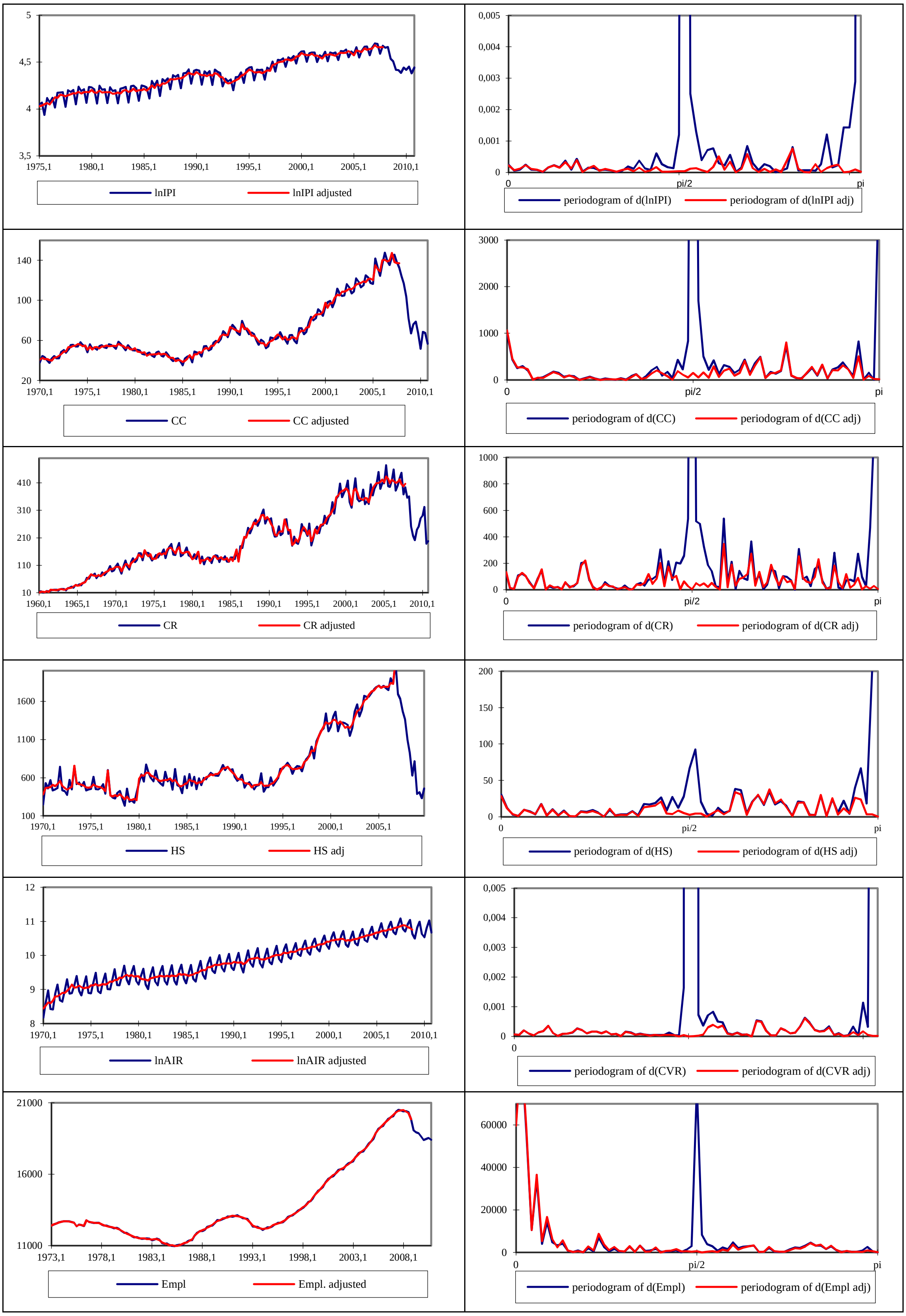
Notes: (a) SARFIMA model: $(1-L)^{d_0}(1+L)^{d_2}(1+L^2)^{d_1} y_t = e_t$, if $d_1, d_2 > 0.5$ the seasonality is nonstationary; (b) $\hat{d}_1$ and $\hat{d}_2$ are estimated means of the coefficients of FI at seasonal frequencies before adjustment. (c) NS – percentage of cases the TSW identifies a seasonally nonstationary model for the simulated data for a given process; I – percentage of cases both estimated coefficients of seasonal fractional integration are greater than -0.5; SNI – percentage of cases at least one coefficient of seasonal fractional integration is statistically smaller than -0.5 (statistically noninvertible); SI - percentage of cases both coefficients of seasonal fractional integration are statistically greater than -0.5 (statistically invertible). (d) $\tilde{d}_1$ and $\tilde{d}_2$ are medians of the estimated coefficients of FI after adjustment and [68 band] refers to the 16th and 84th percentiles. (e) The bandwidth parameter in the estimation is $T^{0.5}$.

Table 3: Empirical results

	T	m_1 / m_2	Before adjustment		After adjustment	
			$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_1^a$	$\hat{d}_2^a$
ln(IPI)	128	32/16	0.617 (0.144)	0.231 (0.229)	-0.153 (0.144)	-0.646 (0.229)
CC	148	36/18	0.276 (0.133)	0.075 (0.209)	-0.127 (0.133)	-0.284 (0.209)
CR	188	48/24	0.385 (0.112)	0.060 (0.171)	-0.075 (0.112)	-0.330 (0.171)
HS	144	36/18	0.391 (0.134)	0.474 (0.210)	-0.266 (0.134)	-0.132 (0.2100)
ln(AIR)	148	36/18	0.532 (0.133)	0.244 (0.209)	-0.195 (0.133)	-0.537 (0.209)
EMP	140	36/28	0.380 (0.134)	0.054 (0.210)	-0.232 (0.134)	-0.226 (0.210)

Notes: (a) HS- Houses started (1970:1-2006:4); CR – Cars registered (1960:1-2007:4); CC – Cement consumption (1970:1-2007:4); ln(IPI) – natural logarithm of IPI (1975:1-2007:4); ln(AIR) – Airline passengers (1970:1-2008:4); ICR – industrial cars registration - (1964:1-2007:4); (b) m_1 / m_2 - bandwidth parameters for the estimation at frequencies(π/2)/π. For each time series the bandwidth parameters (for both frequencies) were selected based on an examination of the log-log plot of the tapered periodogram of the data in differences.

Figure 1. Nonadjusted and TSW adjusted data of the empirical application and their respective sample periodogram



Note: Left panel: The original and adjusted by TSW variables; right panel: estimated sample periodograma of the original and adjusted by TSW series.