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Frequency-Domain Estimation as an Alternative to Pre-Filtering External Cycles in Structural VAR Analysis

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Abstract: This paper shows that the frequency domain estimation of VAR models over a frequency band can be a good alternative to pre-filtering the data when a low-frequency cycle contaminates some of the variables. As stressed in the econometric literature, pre-filtering destroys the low-frequency range of the spectrum, leading to substantial bias in the responses of the variables to structural shocks. Our analysis shows that if the estimation is carried out in the frequency domain, but employing a sensible band to exclude (enough) contaminated frequencies from the likelihood, the resulting VAR estimates and the impulse responses to structural shocks do not present significant bias. This result is robust to several specifications of the external cycle and data lengths. An empirical application studying the effect of technology shocks on hours worked is provided to illustrate the results.

Keywords: Impulse-response, filtering, identification, technology shocks.
JEL Classification: C32, C51, E32, E37

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1. INTRODUCTION

Macroeconomic data often presents low-frequency movements, which influence considerably the business-cycle analysis but are thought to have little to do with the aim of the study. To get rid of this "harmful" influence, standard filters such as the Hodrick-Prescott (HP) or the Band-Pass (BP) are routinely applied and the filtered data is employed in the posterior analysis. Although the practitioner often believes that the filtered series conserve intact its business cycle properties, being free from the annoying low-frequency cycles, the econometric literature has emphasized that this is not true (King and Rebelo 1993; Harvey and Jaeger, 1993; Cogley and Nason, 1995). In fact, the use of filtering may result just as unattractive for structural VAR analysis as the presence of the contaminated cycle itself. Pre-filtering completely destroys the low-frequency range of the spectrum, over-subtracting the variances at zero and neighboring frequencies, which produces a strong dip in the periodogram. Obviously, this over-subtraction reduces significantly the persistence of the filtered series and changes the pattern of its responses to shocks. Further, filtering (or differencing) may also change significantly the way the variables interact in the VAR by removing their low-frequency co-movement (Fernald, 2007; Gospodinov et al., 2011). In particular Gospodinov et al (2011) show that this subtraction may lead to substantial biases in the impulse responses to structural shocks even if the low-frequency correlation is small. The effect is magnified when posterior analysis employs estimates of the spectrum at zero frequency, such as the use of long-run restrictions, and the true process for the contaminated variable is persistent.

In this paper, we turn to frequency domain analysis to circumvent the problems associated with the existence of an external low-frequency cycle, and we propose to estimate the VAR maximizing the Whittle likelihood (Whittle, 1964) over a frequency band. The intuition behind the proposed methodology is to exclude contaminated frequencies from the likelihood. Note that, unlike filtering, the use of a frequency band does not destroy the low-frequency range nor changes the data generating process (DGP). Simply put, although the method does not employ the information at the excluded frequencies, the estimation of the VAR is not driven towards an artificial dip, which implicitly imposes zero variability (and co-movement) at low frequencies. We show that this simple approach is able to overcome most of the problems associated with pre-filtering. Unfortunately, "free-lunch" is rare in econometrics and there is no exception here –in the selection of the frequency band, the researcher has to face a trade-off between the precision of the estimates and the bias due to neglected contaminated frequencies. Yet, standard filtering methods also require the selection of a band and result in strongly biased estimates. Besides, we show that even excluding the complete low-frequency range from estimation (corresponding to the standard calibration of filters for business cycle analysis), the proposed methodology performs remarkably better than pre-filtering.

We organize the discussion as follows. First, we conduct a Monte Carlo experiment to study how the omission of an external low-frequency cycle contaminates the ordinary least squares (OLS) estimation of the VAR. As expected, we find that the estimated process for the contaminated variable results significantly more persistent than is stated in the DGP and the impulse responses to shocks are strongly over-estimated. Consistent with previous findings, the use of either HP or BP filtering reduces drastically the persistence of the filtered series and gives rise to substantial biases in its responses to structural shocks, especially with the use of long-run restrictions. However, the proposed frequency-domain method largely attenuates these problems, and neither the VAR estimates nor the impulse responses present significant bias. These results are robust to several specifications of the external cycle and different sample lengths. Finally, we illustrate our analysis studying the response of hours worked to a positive technology shock. How hours should be treated in theVAR is a question that has generated a large debate in the macroeconomic literature. If hours are either differenced or filtered (as in Gali, 1999 or Canova et al., 2010), the estimated response recovered with a long-run restriction is negative. If hours enter in levels (as in Christiano, 2003), the estimated response recovered under the same scheme is positive. According to Gospodinov et al. (2011), most of the empirical discrepancy can be explained by the fact that filtering (or differencing) subtracts the low-frequency co-movement between hours and productivity, corrupting the application of the long-run scheme. Our empirical results support this view. In particular, the proposed frequency domain method recovers positive and very similar responses under both short-run and long-run identification restrictions.

We have organized the rest of the paper in the following way. In Section 2, we describe the frequency domain estimation of the VAR. We set the Monte Carlo experiment in Section 3. In Section 4, we present the simulation results and conduct robustness analysis. We also discuss the drawbacks of the method at the end of this section. The empirical example is provided in Section 5. Finally, Section 6 concludes. There is also a separate Appendix with the Tables and Figures from the robustness section. In addition, we have made available a "user-friendly" code to estimate a structural VAR model in the frequency domain. The Appendix and the code may be downloaded from the author's site.[1]

[1] https://sites.google.com/site/ylovcha/Research

2. ECONOMETRIC FRAMEWORK

Frequency domain methods are seldom employed in VAR estimation. The reason is simple. In the absence of further complications, the time domain provides an easy and reliable framework for the estimation of these models.[2] Yet, the frequency domain has proved to be useful when difficulties in estimation arise in the time domain, such as the estimation of fractal exponents in long-range dependent VARs (Hosoya, 1996; Shimotsu, 2007; Nielsen, 2011). In this work, we propose to turn to the frequency domain methods to circumvent the problems associated with the contamination of the data with an external low-frequency cycle.

The "Whittle" pseudo-maximum likelihood estimation of a VAR model

Consider the MA(∞) representation of a reduced-form VAR(p) of the $(N \times 1)$ vector Y_t:

$$Y_t = \left[I - F(L)\right]^{-1} u_t \tag{1}$$

where I is the $(N \times N)$ identity matrix, $F(L)$ is a finite order lag polynomial matrix with roots strictly inside the unit circle, and p is the number of lags. Reduced form errors have 0 mean and variance-covariance matrix **Ω**. The spectrum of the VAR process (1) at frequency *ω*, is a $N \times N$ matrix containing the spectra of the variables on the main diagonal, and the cross-spectra out of it:

$$f_y(\omega,\theta) = (2\pi)^{-1}\left(I - F_p\left(e^{i\omega}\right)\right)^{-1}\Omega\left(I - F_p\left(e^{-i\omega}\right)\right)^{-1} \tag{2}$$

where i is the imaginary unit, $F_p\left(e^{i\omega}\right) = F_1 e^{i\omega} + ... + F_p e^{pi\omega}$, $F_p\left(e^{-i\omega}\right)$ is its complex conjugate and the vector θ contains all the model parameters.

[2] A notable exception may be found in Christiano et al. (2003), where the authors stress the advantages of the frequency domain for structural VAR estimation and analysis.

To estimate the process in (1), we propose to use the approximate frequency domain maximum likelihood (Whittle, 1963). To derive the likelihood function, we compute the finite Fourier transform of each series $y_{n,t}$ in the vector as:

$$x_n\left(\omega_j, y_{n,t}\right) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} y_{n,t} e^{-i\omega_j(t-1)} \tag{3}$$

for the Fourier frequencies: $\omega_j = \frac{2\pi j}{T}, j = 0,...,T/2$. An approximate log-likelihood function of θ based on Y_t is given (up to constant multiplication) by:

$$\ln L(\theta) = -\sum_{j=\tau}^{T/2} \left[\ln \det f_y\left(\omega_j, \theta\right) + tr f_y^{-1}\left(\omega_j, \theta\right) I_T\left(\omega_j, Y\right) \right] \tag{4}$$

The $N \times N$ periodogram matrix $I_T\left(\omega_j, Y\right)$ in the previous formula is defined as $I_T\left(\omega_j, Y\right) = x\left(\omega_j, Y\right) x\left(\omega_j, Y\right)^*$, where $x\left(\omega_j, Y\right)$ is a complex $N \times 1$ vector with entries given by (3), and $x\left(\omega_j, Y\right)^*$ is its complex conjugate. For each Fourier frequency ω_j, the elements of the main diagonal of $I_T\left(\omega_j, Y\right)$ are the periodograms of the different series evaluated at this frequency, which are real. The off-diagonal elements are cross-periodograms, which are complex.

Note that when $\tau = 0$, the estimation is carried out over all frequencies. However, if $\tau > 0$, the estimation is done over a frequency band that excludes the first $\tau - 1$ frequencies. Obviously, in the absence of contamination, excluding frequencies from estimation would result in a senseless loss of information. However, we show that when a low-frequency cycle contaminates some of the variables, the use of a sensible frequency band is a simple and effective way to circumvent the problems associated with pre-filtering.

The choice of the frequency band is not straightforward. Ideally, the researcher knows approximately which frequencies are contaminated after the inspection of the periodogram, and can select a band accordingly. However, this may not be always easy in practice. In either case, she should be aware of the trade-off between the precision of the method and the bias that may cause the neglect of some contaminated frequencies. Besides, the researcher may want to exclude the low-frequency range completely, and focus the estimation over the business cycle (and higher) frequencies only. This strategy is conservative in terms of the bias and it is comparable with the standard parameterization of the BP (and HP) filters for business cycle analysis. It can be reached by setting $\tau = T/8s$ in the band, where s is the number of data observations per year.[3] We show that even this conservative approach performs remarkably better than pre-filtering, leading to correct VAR inference.

3. THE MONTE CARLO EXPERIMENT

We design our simulation experiment to shed light on two specific questions. First, we assess how the existence of an external low-frequency cycle contaminates the ordinary least squares (OLS) estimation of the (structural) VAR model, and what are the short sample properties of two competing estimation alternatives: pre-filtering the data before OLS estimation, and the proposed frequency domain method. Second, we study whether the previous procedures are able to recover reliable estimates of the relation among variables, represented by the impulse responses to structural shocks.

3.1 Model Specification

We restrict our attention to a simple bivariate VAR(1) model with a persistent (but stationary) first variable which is observed contaminated by an external low-frequency

[3] Although there is not a single definition of the business cycle, most of the studies employ for that a cycle with periodicity that range between 1.5 and 8 years.

cycle. In this situation, we expect filtering to influence strongly the estimation of the model, and mimic the typical characteristics of the data - macroeconomic series are usually persistent but the fist-difference operator generally clears out the low-frequency cycles. The second variable is assumed to be I(1) and enters in the VAR in differences. This allows us to study the impulse responses to structural shocks recovered with a long-run restriction, as in Gospodinov et al. (2011). Overall, the hours-productivity literature provides a good example of this specification and, consequently, it is employed in Section 5 to illustrate our results.

3.1.1 The Structural VAR model and identification restrictions

Consider a reduced VAR model of order $p = 1$ for the vector $Y_t = \begin{bmatrix} y_{1,t} & \nabla y_{2,t} \end{bmatrix}'$ as in (1). The structural representation can be written as:

$$\begin{bmatrix} y_{1,t} \\ \nabla y_{2,t} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ \nabla y_{2,t-1} \end{bmatrix} + A \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{5}$$

where the structural errors $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are assumed to be orthogonal, with variances scaled to unity. The matrix A relates the structural and reduced-form shocks in the following way: $u_t = A\varepsilon_t$ and, as a result, $\Omega = AA'$. After simple algebraic transformations:

$$\begin{bmatrix} y_{1,t} \\ \nabla y_{2,t} \end{bmatrix} = \frac{1}{\det(F)} \begin{bmatrix} (1 - F_{22}L)a + F_{12}Lc & (1 - F_{22}L)b + F_{12}Ld \\ F_{21}La + (1 - F_{11}L)c & F_{21}Lb + (1 - F_{22}L)d \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \tag{6}$$

Finally, the impulse responses to structural shocks are given by the coefficients of the polynomials in (6).

To recover the structural parameters of the matrix A from the reduced-form estimation, it is necessary to impose one additional restriction.[4] The most popular schemes in the literature are the short-run (SR) and the long-run (LR) identification procedures. SR identification is a recursive scheme, usually imposing that the shocks to the second variable take at least one period to percolate to the first (i.e., it restricts A to be a lower-triangular matrix). This is usually attained by the Cholesky decomposition of the variance-covariance matrix of the reduced-form errors: $A = chol(\Omega)$. On the other side, the LR identification assumes that, in the long-run, the level of the non-stationary variable $(y_{2,t})$ is only affected by its own shock. Thus, the infinite sum of the responses of this variable in differences (as it enters the model) to the first shock $\varepsilon_{1,t}$ must be zero. The LR restrictions can be expressed from (6) as: $F_{21}a + (1 - F_{11})c = 0$ with $\det(F) \neq 0$.

The precision of the estimated impulse responses depends on the accuracy of both the estimated autoregressive coefficients and the recovered structural parameters. SR identification only requires the estimated variance-covariance matrix Ω to recover the matrix A from the reduced form estimation. On the contrary, LR requires both F and Ω. Consequently, the LR restriction usually leads to less precisely estimated responses presenting wider confidence intervals. Yet, the LR is the most popular scheme in the literature since it is satisfied in a wide variety of macroeconomic models. At the same time, the recursive assumptions required for SR identification are seen by some researchers as being too restrictive.

[4] See Ramey (2016) and Lovcha and Perez-Laborda (2016) for a review of different identification schemes for the technology shock in the literature, or Rubio-Ramirez et al. (2010) for a general treatment of identification of structural VAR models.

For the Monte Carlo exercise, we choose the following parameterization for the structural VAR:

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.1 & 0.5 \end{bmatrix} \text{ and } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \tag{7}$$

This parameterization satisfies the restrictions required for both SR and LR identification. In addition, the first variable is very persistent and there is a nontrivial correlation at low frequencies. These last two properties are clearly observable in Figure 1, which plots their spectral densities and the coherence. The spectral density of a process can be understood as the decomposition of its variance into a set of uncorrelated components at each frequency, and the coherence gives a measure of the correlation among variables at each frequency point. To help in interpretation, we have signaled the frequencies with 20-years and 8-years period with a vertical dotted line. Thus, the beginning of the business cycle frequencies starts to the right of the second vertical line.

3.1.2 The process for the contaminated variable

We assume that we do not observe the true value of the first variable, but a value contaminated with an external low-frequency cycle. To construct the contaminated series, we sum the external cycle to the first series generated by the VAR:

$$yc_{1,t} = y_{1,t} + C_t \tag{8}$$

where $yc_{1,t}$ is the contaminated series and C_t is the external low-frequency cycle. We employ three types of specifications for the cycle: deterministic trigonometric, stochastic trigonometric, and changes in the mean. These three specifications, together with the parameterization employed for the simulations, are described below.

The deterministic trigonometric cycle. We generate this cycle by a cosine wave with frequency ω:

$$C_{d,t} = B\cos\omega \tag{9}$$

where B is a parameter regulating the amplitude of the cycle. The trigonometric deterministic cycle ensures that the external cycle contaminates the first variable only at frequency ω. Although this assumption is rather restrictive, this type of cycle is interesting from the instructive point of view, and it is relaxed in the following two processes.

For the Monte Carlo, we set the amplitude parameter B to 4 and the frequency ω to $2\pi/40s$, corresponding to an external cycle of 40 years period, being s the number of data observations per year.

The stochastic trigonometric cycle. Following Harvey (1989), we write the process for the stochastic trigonometric cycle as:

$$\begin{gathered} C_{s,t} = DS_t \\ \begin{bmatrix} S_t \\ S_t^* \end{bmatrix} = \begin{bmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} S_{t-1} \\ S_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix} \end{gathered} \tag{10}$$

where D governs the amplitude, and ξ_t and ξ_t^* are mutually independent zero-mean error terms with equal variance. This cycle contributes to the variance of the contaminated series at the selected frequency ω mostly, but the adjacent frequencies also result contaminated due to its stochastic behavior.

We parameterize the cycle with $\omega = 2\pi t/40s$ (the same than in the deterministic cycle), and we set the parameter D = 2 in order to have comparable amplitudes of the two trigonometric cycles. Finally, we set the variance of the errors terms to 0.001.

The changes in the mean cycle. Changes in the mean are relatively common in economic data and contribute to the low part of the spectrum of many economic time series. To generate this cycle, we divide the sample of T observations into three equal subsamples, and we assume different means for each. In particular, for the Monte-Carlo exercise, we specify:

$$C_{m,t} = \begin{cases} 4; & 1 \le t < T/3 \\ -2; & T/3 \le t < 2T/3 \\ 3; & 2T/3 \le t < T \end{cases} \tag{11}$$

which generates a cycle with similar amplitude to the previous two trigonometric cycles.

Figure 2 depicts examples of contamination by the three types of cycles for a realization of 500 quarterly observations from a structural VAR model parameterized as in (7). The figure also plots the periodogram of the true and the contaminated series. As can be seen in the figure, the deterministic cycle contributes only to the selected frequency, with the periodograms of the contaminated and non-contaminated series coinciding at any other frequency point. On the contrary, both, the stochastic trigonometric and the changes in the mean cycles contribute to a bunch of frequencies in the low-frequency range. The trigonometric stochastic cycle contaminates mostly the 40 years period frequency, although adjacent frequencies also result contaminated. As can be seen in the figure, the changes in the mean cycle contaminates mostly the very low-frequency range, but there is considerable presence of contamination in the whole low-frequency range.

3.2 Simulation Set-Up

To study the sample properties of the three estimation alternatives, we assume normality and employ a structural VAR parameterized as in (7) to generate I=1000 bivariate time series of the following lengths: i) $T = 500$ quarterly observations (125 years); ii) $T = 240$ quarterly observations (60 years) and; iii) 720 monthly observations (60 years). We contaminate the first variable of each simulated series as in (8), using for that the three different specifications for the cycle: the trigonometric deterministic (9), the trigonometric stochastic (10), and the changes in the mean (11). After, we collect the resultant contaminated first variable together with the differenced variable in a contaminated vector: $YC_{i,t} = \left[yc_{1,t} \quad \nabla y_{2,t} \right]_i$, for i=1,…,I.

For each simulated dataset, we estimate the parameters of a reduced form VAR(1) with the three estimation alternatives: i) OLS applied to non-filtered data; ii) OLS, pre-filtering the first variable with the HP and BP filters, and; iii) the "Whittle" estimation with non-filtered data using two different frequency bands for estimation. After, we recover the structural matrix A with both SR and LR restrictions and we compute the responses of the variables to structural shocks.

For the HP filter, we use a standard value for the penalty parameter (λ = 1600 for quarterly data, λ = 14400 for monthly data). For the BP, we do not allow passing frequencies with periods longer than 8 years, as typically done in the business cycle literature. As noted before, filtering the data completely subtracts the variance at low frequencies, creating a dip in the periodogram. This dip is clearly observable in Figure 3, which plots the periodogram of the filtered series of the illustrative example (depicted in Figure 2) contaminated with the trigonometric deterministic cycle. Figure 3 also depicts the periodogram of the true series to help in interpretation.

For the "Whittle" estimation, we employ two frequency bands. The first band omits all the frequencies with a period longer than 8 years. Thus, it employs only the information at business cycle and higher frequencies for estimation, which is the same parameterization that we use in the BP filter. The second band excludes the frequencies with a period longer than 20 years. We chose this band with a view to including as many frequencies as possible without allowing for strong contamination. The two vertical dotted lines included in the previous figures signal the beginning of these bands.

4. DESCRIPTION OF THE RESULTS

We organize the results as follows. We discuss below the results of the Monte-Carlo experiment for quarterly data of length T=500 contaminated by the trigonometric deterministic cycle. The results for the other two specifications of the external cycle and the results of the simulations with 60 years of data (both quarterly and monthly) are discussed as a robustness analysis. Finally, at the end of this section, we make an overall analysis of the methodology and we discuss its most important drawbacks.

Table 1 and Figure 4 summarize the simulation results for data contaminated by the deterministic trigonometric cycle. Table 1 presents the mean of estimated parameters across the 1000 replicas. The first half of the table (Table 1.a) contains results for the estimation of the autoregressive coefficients. The mean of the structural parameters recovered with both SR and LR restrictions across simulations is provided in Table 1.b. The numbers in parenthesis correspond to the 2.5^{th} and 97.5^{th} estimated percentiles. For the OLS with filtered data, we present only the results for the BP filter, since the results obtained with the HP are virtually identical, but are available upon request.

As can be seen in the table, neglecting the external cycle strongly influences the OLS estimates of the autoregressive parameters. The estimated process has to adjust for the additional exogenous persistence of the first variable, so the estimates of its own-autoregressive term result upward biased. Contamination also affects the estimates of the cross-autoregressive term of the second variable, which is biased downwards. Given that the variance-covariance matrix Ω is relatively well estimated, the parameters of the structural matrix A recovered with any of the two schemes do not present strong estimation bias. On the other side, when the contaminated variable is pre-filtered it becomes substantially less persistent. Consequently, the OLS estimation with filtered data produces negatively biased estimates of the autoregressive coefficients. On the top of that, the estimated parameters of the matrix A are also biased, especially when they are recovered with the LR scheme. Note that the bias in the contemporaneous effect of the second structural shock on the first variable is particularly strong. Finally, the table also reports the results from the proposed frequency domain method. The Whittle estimation over a frequency band performs very well in terms of the bias. Neither the autoregressive coefficients from the matrix F nor the structural parameters from A present significant short sample estimation bias, even if LR restrictions are employed for identification. Also, note that the use of the 8-years cycle band also results in non-biased estimates but as expected, the method becomes less precise than with the 20-years period band.

Figure 4 depicts the Monte-Carlo results for the impulse responses. We depict here only the responses of the first variable (i.e., the one observed contaminated) since these responses concentrate the larger disparities among the three estimation methods and have received more attention in the macroeconomic literature.[5] The figure 4.a depicts the responses of the first variable to its own structural shock recovered with SR and LR restrictions. The cross responses can be found in Figure 4.b. In both cases, the solid lines represent the mean responses across simulations and the dashed lines the 2.5 and the 97.5 percentiles. We collect the responses obtained from a VAR estimated with the OLS methods (with filtered and non-filtered data) in the first column of each figure, while the second column contains the analysis of the impulse responses from a VAR estimated in the frequency domain (using the two selected frequency bands). In order to interpret the results, we have included the true responses in each graph (dotted line).

Consistent with the analysis of the estimated coefficients, the OLS estimation with non-filtered data produces largely overstated responses of the first variable to system shocks that are considerably more persistent than the true responses due to the neglected contamination. Bothe the own- and cross-estimated responses are upward biased irrespective of the restrictions employed to identify the shocks. The percentile bands recovered with the LR scheme are substantially wider, reflecting the larger uncertainty associated with this identification method.

As can be seen in the figure, filtering does not improve the results at all. The responses are strongly biased downward, with the true response lying always above the percentile bands. Note that the sampling uncertainty in the estimated responses is substantially smaller than in OLS estimation with non-filtered data, especially those responses recovered with LR restrictions. However, it is important to highlight that the reduction in uncertainty comes from the removal of a sizable portion of the true variance of the first variable by filtering the data. In addition, given that the responses

[5] None of the estimation methods shows very strong bias in the responses of the second (i.e., observed non-contaminated) variable. Yet, the proposed methodology still performs better than the other estimation methods in the Monte Carlo study. These responses are available upon request to the authors.

are strongly biased, the narrow bands not containing the true values indicate that, with very high probability, the estimated response is going to be far the true response regardless of the identification method applied. Yet, the analysis of the response to the second shock (figure 4.b) identified with an LR restriction is especially relevant. Recall that this response is not only affected by a large bias in the estimated autoregressive coefficients, but also by a strong bias in the estimated structural parameter governing the contemporaneous effect. As can be seen in the figure, the mean of the estimated responses becomes negative in the short-run, with the zero value lying outside the percentile bands. This result is especially important because it provides an example of how pre-filtering may lead towards misleading inference in the VAR, not only about the magnitude and the persistence of the responses, but also about their sign. Finally, the right column of the two figures collect the mean responses from a VAR estimated in the frequency domain with the two selected bands (20-years and 8-years period). As can be seen in the figure, none of the estimated responses presents significant bias, neither with LR nor with SR identification schemes. Note that, in particular, that there is no estimation bias in the response to the second shock recovered with LR restrictions. As expected, the sampling uncertainty grows with the number of frequencies excluded from the estimation band, but even the 8-years period band performs very well in terms of the bias and appears precise enough for inference. More important, as far as the confidence intervals for impulse responses reflect the true degree of uncertainty observed in simulations, using the proposed frequency domain method the econometrician would not be misled in inference regardless the identification scheme she uses. Yet, our results suggest a close inspection of the periodogram in order to select a sensible band prior estimation in order to increase the precision of the method.

4.1 Robustness Analysis

We check for robustness along two different directions. First, we carry out additional simulations for the other two specifications for the external cycle, the trigonometric stochastic and the changes in the mean. After that, we study if the results of the previous section are robust to changes in the sample size or the frequency of the data. The tables and the figures from this section are provided in a separate appendix to this work to conserve space.

4.1.1. Alternative processes for the external cycle

In the previous section, we contaminate the first variable with a trigonometric deterministic cycle, which only alters a single frequency in the spectrum (40 years period). Note that in this way, the two selected bands for frequency domain estimation contain only non-contaminated frequencies. We relax this assumption with the trigonometric stochastic and the changes in the mean specifications for the external cycle. As can be observed in Figure 2, we have parameterized these cycles such that the bunch of contaminated frequencies is still situated to the left of the frequency of 20-years period. However, there is also presence of contamination at (relatively) higher frequencies. This contamination is mostly located in between the 20-years and the 8-years period frequencies, which allow us to study the trade-off between bias and precision.

Overall, the results of the simulation study are very similar to those obtained with the trigonometric deterministic cycle. A summary is provided in Table A1 and Figures A1 and A2 in the separate Appendix to this work. Again, the proposed frequency domain method performs remarkably better than the two OLS alternatives, with the two selected bands. Yet, now the use of an 8-years period band usually leads to slightly better results in terms of the bias, but at the cost of penalizing precision. Obviously, the larger the amount of contamination neglected by the band, the larger the bias incurred by the method. Yet, our results seem to indicate that the proposed methodology is not extremely sensible to a small amount of contamination, which again advocates for the selection of a sensible band upon the study of the periodogram.

4.1.2. Modifying the length and the frequency of the artificial data

The empirical studies of the business cycle are usually based on quarterly or monthly data, which often leads to the use of relatively short datasets. To see how the proposed methodology behaves with the standard lengths found in business cycle analysis, we carry out additional simulations for shorter artificial datasets of T=240 quarterly observations (corresponding to 60 years of data). Results are summarized in Table A2 and Figure A3 in the Appendix. Overall, we find no significant differences with respect to the use of T=500 quarterly observations, except an expected small increase in the sampling uncertainty associated with all the estimation methods.

Finally, we study if the use of higher frequency data improves the precision of the frequency domain estimates. A larger number of observations per year increase the resolution of the periodogram in a very particular way, since it makes available a larger set of Fourier frequencies mostly situated at the medium and the high-frequency range of the spectrum (thus not influenced by the external low-frequency cycle). To study this issue in deep, we draw artificial datasets of T=720 monthly observations (which correspond to the same short span of 60 years). Figure A4 and Table A2 (right) in the Appendix collect these results. Again, the OLS alternatives are penalized either by contamination or by filtering, while the frequency domain estimates do not show significant bias. However, the most interesting result is that, although the use of monthly data reduces the sampling uncertainty of all methods, the frequency domain alternatives result much more benefited from it. This is especially relevant for the estimation with the 8-years period band, which now presents unbiased and very precise estimated responses, comparable to those obtained with the 20-years band.

4.3 Overall Evaluation and Limitations

In the simulation study, we show that the proposed frequency domain method is a useful alternative to deal with contamination at low frequencies in structural VAR analysis. It outperforms significantly the OLS estimation with non-filtered data and overcomes the strong problems associated to pre-filtering. However, like all econometric methodologies, it has several drawbacks and limitations, which should be taken into account in real data applications.

The first of them is that the method requires the selection of a frequency band for estimation. The choice of the band can sometimes be ambiguous, and there is a trade-off between the bias due to the possible neglected contaminated frequencies and the precision of the estimates. In most situations, a close inspection of the periodogram may help to select a sensible band for estimation. On the contrary, if the researcher wants to restrict the estimation to the business cycle and higher frequencies, the precision of the method suffers, especially if the true process is very persistent. As it is shown in the robustness section, the use of monthly data may help a lot in these situations. Yet, the estimation shows no significant bias and, as far as the confidence intervals capture the true uncertainty, the researcher is not going to make mistakes inferring with the estimated model no matter the identification scheme she applies. On the other hand, standard filtering methods also require the selection of a band, and their use leads to strong estimation bias and erroneous VAR inference.[6]

[6] With the BP filter, the choice of the frequencies to be removed is done exactly in the same way. The issue is more problematic with the HP filter since it is not always easy to establish a one to one relation between the choice of the penalty parameter and the frequencies removed.

The second limitation concerns the use of maximum likelihood estimators. Unlike OLS methods, it may be necessary to enforce stationarity if the process is very persistent. In addition, in the presence of quasi-unit roots, the precision of the estimated error variance may be affected. As shown in the literature, the use of a taper may alleviate this problem.[7] We illustrate the application of these strategies in the empirical example of the next section. Besides, note that these problems are not exclusive of the proposed frequency-domain method, and may arise as well using maximum likelihood estimation in the time domain.

5. EMPIRICAL EXAMPLE

In this section, we assess the response of hours to a (positive) technology shock to illustrate the results of the simulation study. This response has been the subject of a strong debate in the literature since the work of Gali (1999), who cornered the real business cycle models (RBCs) finding a negative response of hours in a structural VAR identified with an LR restriction. The heart of the debate centers on how hours should enter the VAR. If hours are either differenced or filtered, and the technology shock is identified with an LR restriction (Gali, 1999; Canova et al., 2010), the response of hours is negative. However, when the hours enter in levels (Christiano et al. 2003), the response is positive even if the LR identification is applied.[8] Fernald (2007), Francis and Ramey (2009) and Canova et al. (2010), analyze the low-frequency movements in hours worked arguing that the series contains some noise at very low frequencies that

[7] Dahlhaus (1988) shows that tapering reduces the leakage effect of the periodogram as estimate of the true spectrum. He finds that the new estimate competes well with the Burg estimate for an AR(14) model where roots of the characteristic equation are complex and close to the unit circle. There are several other studies concerning the use of data tapers in univariate AR models (Pukkila and Nyquist, 1985; Kang, 1987; Hurvich , 1988; or Zhang, 1991). The general conclusion of these works is that tapering should be conducted. However, the specific taper and amount applied appear not to be of much importance.

[8] SR restrictions recover a positive response of hours irrespective of how this series enters the VAR.

must be removed prior to VAR estimation. For Fernald (2007) and Canova et al. (2010), the noise is the consequence of some changes in the mean of the process. Francis and Ramey (2009) attribute these changes to demographic and sectoral movements affecting the US labor force. However, Gospodinov et al (2011) have called into attention that differencing or filtering this noise corrupts the application of the LR scheme, which may lead to erroneous inference.[9]

In order to assess the response of hours to a technology shock, we construct a dataset similar to that of Christiano et al. (2003), collecting data from the Federal Reserve Bank of St. Louis (FRED). The dataset runs from 1948:1 to 2009:4, thus covering a similar period than other popular datasets in the literature, and contains data from all sectors (including the farm sector). The total business productivity is defined as the log of the output per hour of all persons (OPHPBS), and the hours worked as the log of the ratio of the business hours of all persons (HOABS) to the civilian non-institutional population over the age 16 (CNP16OV). The last series are converted to quarterly by taking the average of the monthly observations inside the quarter. Except for population, all the series are seasonally adjusted.

Figure 5 presents the estimated impulse responses up to 20 quarters (5 years) recovered from a VAR(4) identified with both SR and LR restrictions together with one standard deviation bootstrapped bands.[10] We also include in the figure the periodogram of the first variable and its estimated spectrum from the VAR. The results from OLS estimation with rough data are collected in the first row of the figure. Consistent with the previous analysis, the periodogram of the non-filtered hours presents a very strong

[9] A different stream of the literature has considered alternative identification assumptions (Uhlig, 2004; Francis et al., 2010; Lovcha and Perez-Laborda, 2016) or other model specifications that internally account for the degree of persistence (Pesavento and Rossi, 2005; Lovcha and Perez-Laborda, 2015).

[10] We employ 1000 nonparametric bootstrap replicas.

peak at the very low part of the spectrum, corresponding to a cycle of about 60 years period. Consequently, the estimated process is very persistent (the highest eigenvalue of the companion form matrix equals 0.98). As can be seen in the figure, the responses recovered with SR and LR restrictions are both positive, but the response recovered with LR restrictions is much stronger presenting wider confidence bands. As a second step of the analysis, we BP filter hours before OLS estimation.[11] As standard in the business cycle literature, we filter all the frequencies with a period longer than 8 years, but the results are robust to the use of a 20 years period band in the filter.[12] The responses estimated with filtered data are depicted in the second row of the figure. Filtering the data over-subtracts the variance at low frequencies, producing a strong dip in the periodogram, which is visible in the first plot. Consequently, the estimated model attempts to reproduce that dip, as can be seen in the spectrum from the estimated VAR. As a result, the model is significantly less persistent than was estimated with rough data. The SR identification returns positive responses, but smaller than those obtained without filtering the data. On the other hand, the LR identification returns negative and significant responses, in compliance to the well-known puzzle in the literature. Confidence bands for LR responses are of the same magnitude as SR responses since filtering removes a substantial part of the variance. Finally, we employ the proposed frequency domain methodology for estimation of the VAR. We assume that the 60 years period cycle in hours is exogenous and employ the 20 years band used for

[11] Again, the analyses of the HP and BP filtered series do not differ, so we present in the paper the results from the BP only to save space.

[12] The periodogram shows another peak on the border of business cycle range, which may contain useful information for business cycle analysis. However, the results do not change significantly including or excluding this frequency with BP filter.

simulations, in order to exclude the lesser number of frequencies as possible.[13] Due to the strong persistence of the data found in OLS estimation, we ensure stationarity and apply a cosine taper before estimation (Tukey, 1967).[14] Results from the frequency domain estimation are depicted in the last row of Figure 5. The dots in the periodogram of tapered hours signal the part corresponding to the excluded frequencies. Bootstrapped bands for impulse responses are computed with non-parametric bootstrap in the frequency domain (Berkowitz and Diebold, 1998). As can be seen in the table, the estimated process is slightly less persistent than one estimated with OLS using raw data but substantially more than the one estimated with filtered hours. Note that the estimated spectrum is not driven towards zero in the low-frequency range. The responses of hours recovered with the SR and the LR schemes are both positive and very similar to each other, but of smaller magnitude than those estimated by OLS with non-filtered data. Furthermore, the confidence bands for LR contain the responses recovered with the SR scheme. Note that with the other estimation methods, the responses recovered with the LR and the SR schemes are not compatible.

To sum up, the proposed methodology backs up positive and similar responses under LR and SR restrictions, but smaller than those found neglecting the external noise. On the other hand, if the data is filtered, the response recovered with an LR scheme is negative and significant, even if the same 20-years cycle band used for frequency domain estimation is employed in the filter. In the view of the results of the Monte-Carlo study, we believe that the negative response found with filtered data under LR restrictions is a direct consequence of the filter, unless in our dataset.

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[13]Although including more frequencies for estimation would be useful to reduce the variance associated to persistence, this is not feasible due to the small amount of observations.

[14]The ratio of taper to constant sections (the parameter α) is taken equal to 0.125. The results are robust to the choice of the taper and to other parameterizations.

5. CONCLUSIONS

In this paper, we propose the frequency estimation of the VAR as an alternative of pre-filtering when some of the variables are contaminated by an external low-frequency cycle. In these situations, the OLS is largely penalized by contamination, giving rise to a substantial bias in the impulse responses to shocks. We show that the use of filtering can be even more harmful that the neglect of contamination, which is consistent with other analyses in the literature. In particular, not only the estimates of the autoregressive parameters appear downward biased, but also the estimates of the structural parameters are biased under the LR scheme. As a result, the true responses in the DGP and the ones recovered from the model may diverge completely, not only in magnitude and persistence but also in sign. This can easily lead to erroneous inference based on the estimated model. The increase in the number of observation does not improve the situation. On the contrary, the proposed frequency-domain method takes into account contamination overcoming the problems of filtering. If a sensible band is employed for estimation, neither model estimates nor the impulse responses present signs of significant bias. This result is robust to several specifications of the external cycle and data lengths. We have shown the utility of the method in applied work assessing the response of hours to a productivity shock, which we have found positive under both SR and LR identification schemes, proposing a solution to a largely debated puzzle in the literature.

Like any empirical methodology, our approach suffers from several shortcomings that we have discussed in the text. In our view, the most important drawback is that the researcher has to face a trade-off between the bias arising from the neglect of contamination and the precision of the method when selecting a frequency band for estimation. Yet, filtering also requires the selection of a frequency band and, although the precision of the proposed method suffers, it works much better than pre-filtering, even excluding the low-frequency range completely. As a result, the researcher cannot go wrong using the estimated model no matter the identification scheme she applies.

References

Berkowitz J, Diebold FX (1998) Bootstrapping multivariate spectra. *The Review of Economics and Statistics* 80: 664-666.

Canova F, Lopez-Salido D, Michelacci C (2010) The effects of technology shocks on hours and output: a robustness analysis. *Journal of Applied Econometrics* 25: 755-773.

Christiano LJ, Vigfusson RJ (2003) Maximum likelihood in the frequency domain: the importance of time-to-plan. *Journal of Monetary Economics* 50: 789-815.

Cogley T, Nason JM (1995) Effects of the hodrick-prescott filter on trend and difference stationary time series: Implications for business cycle research. *Journal of Economic Dynamics and Control* 19: 253–278.

Dahlhaus R (1988) Small sample effects in time series analysis: A new asymptotic theory and a new estimate. *The Annals of Statistics* 16: 808-841.

Fernald JG (2007) Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics* 54: 2467-2485.

Francis N, Owyang MT, Roush JE, DiCecio R (2014) A flexible finite-horizon alternative to long-run restrictions with an application to technology shocks. *The Review of Economics and Statistics* 96: 38-647.

Francis N, Ramey VA (2009) Measures of per capita hours and their implications for the technology-hours debate. *Journal of Money, Credit and Banking* 41: 1071-1097.

Gospodinov N, Maynard A, Pesavento E (2011) Sensitivity of impulse responses to small low-frequency comovements: reconciling the evidence on the effects of technology shocks. *Journal of Business and Economic Statistics* 29: 455-467.

Harvey AC (1989) Forecasting, structural time series models and the Kalman Filter. Cambridge, UK: Cambridge University Press

Harvey AC, Jaeger A (1993) Detrending, stylized facts and the business cycle. *Journal of Applied Econometrics* 8: 231–247.

Hurvich CM (1988) A mean squared criterion for time series data windows. *Biometrika*, 75: 485-490.

Kang HJ (1987) The tapering estimation of the first-order autoregressive parameters. *Biometrika* 74: 643-645.

King RG, Rebelo ST (1993) Low frequency filtering and real business cycles. *Journal of Economic Dynamics and Control* 17: 207–231.

Lovcha Y, Perez-Laborda A (2015) Hours worked - productivity puzzle: identification in fractional integration settings. *Macroeconomic Dynamics* 19: 1593-1621.

Lovcha Y, Perez-Laborda A (2016) The Variance-Frequency Decomposition as an Instrument for VAR Identification: an Application to Technology Shocks. CREIP WP #9.2016.

Nielsen FS (2011) Local Whittle estimation of multi-variate fractionally integrated processes. *Journal of Time Series Analysis* 32: 317–335

Pesavento E, Rossi B (2005) Do technology shocks drive hours up or down?A little evidence from an agnostic procedure. *Macroeconomic Dynamics* 9: 478-488.

Pukkila T, Nyquist H (1985) On the frequency domain estimation of the innovation variance of a stationary univariate time series. *Biometrika* 72: 317-323.

Rubio-Ramirez JF, Waggoner DF, Zha T (2010) Structural vector autoregressions. Theory of identification and algorithms for inference. *The Review of Economic Studies* 77: 665-696.

Shimotsu K (2007) Gaussian semiparametric estimation of multivariate fractionally integrated processes. *Journal of Econometrics* 137: 277-310.

Tukey, JW (1967) An introduction to the calculations of numerical spectrum analysis. In *Advanced Seminar on Spectral Analysis of Time Series*. Ed. B. Harris. pp. 25-46. New York: Wiley.

Uhlig H (2004) Do technology shocks lead to a fall in total hours worked? *Journal of the European Economic Association* 2: 361-371.

Whittle P (1963) On the fitting of multivariate autoregressions, and the approximate canonical factorization of a spectral density matrix. *Biometrika* 50: 129-134.

Zhang HC (1991) Reduction of the asymptotic bias of autoregressive and spectral estimators by tapering. *Journal of Time Series Analysis* 13: 451-469.

TABLES AND FIGURES

Table 1 Monte Carlo results: trigonometric deterministic; T=500-quarters

Tab. 1.a Autoregressive parameters from the matrix F

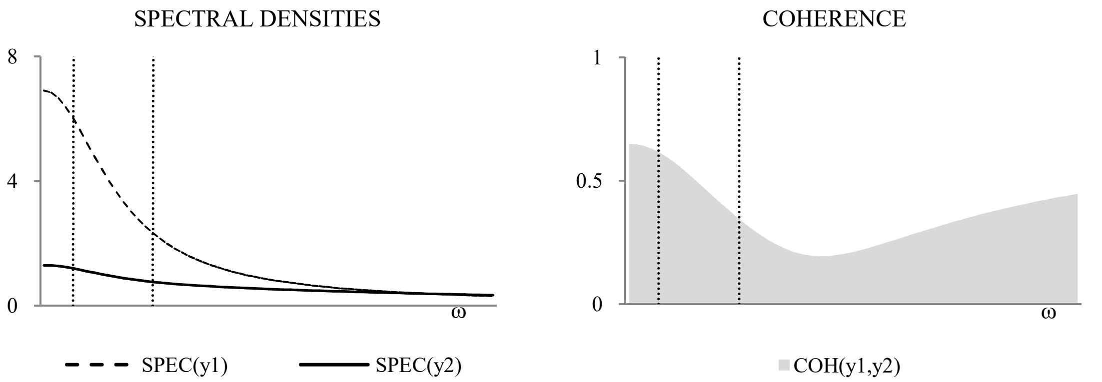
	True	OLS, non-filtered	OLS, BP	Whittle, 20	Whittle, 8
F_{11}	0.8	0.94[0.92; 0.95]	0.61 [0.52; 0.69]	0.79 [0.72; 0.85]	0.79 [0.67; 0.91]
F_{12}	0.3	0.30[0.22; 0.38]	0.19 [0.12; 0.26]	0.29 [0.23; 0.37]	0.30 [0.20; 0.40]
F_{21}	0.1	0.02[-0.01; 0.05]	0.08 [0.00; 0.16]	0.10 [0.02; 0.17]	0.10 [-0.00; 0.21]
F_{22}	0.5	0.49[0.40; 0.57]	0.52 [0.43; 0.60]	0.49 [0.40; 0.57]	0.49 [0.38; 0.60]

Tab 1.b Structural parameters from the matrix A

	True	OLS, non-filtered	OLS, BP	Whittle, 20	Whittle, 8
			SR IDENTIFICATION		
a	1.0	1.05 [0.97; 1.11]	0.95 [0.89; 1.01]	0.99 [0.93; 1.06]	1.00 [0.92; 1.07]
c	-0.5	-0.52 [-0.61; 0.43]	-0.55 [-0.64; -0.45]	-0.49 [-0.59; -0.40]	-0.49 [-0.59; -0.39]
d	1.0	0.99 [0.93; 1.05]	0.98 [0.91; 1.03]	0.99 [0.93, 1.06]	1.00 [0.92; 1.07]
			LR IDENTIFICATION		
a	1.0	1.02 [0.82; 1.10]	0.90 [0.81; 0.97]	0.98 [0.89; 1.05]	0.95 [0.71; 1.05]
b	0.0	-0.01 [-0.64; 0.37]	-0.31 [-0.48; -0,13]	-0.01 [-0.35; 0.33]	-0.02 [-0.50; 0.74]
c	-0.5	-0.50 [-0.84; 0.18]	-0.19 [-0.38; -0.00]	-0.48 [-0.79; -0.12]	-0.47 [-1.05; 0.03]
d	1.0	1.00 [0.74; 1.15]	1.10 [1.02; 1.18]	1.00 [0.78; 1.13]	1.00 [0.38; 1.14]

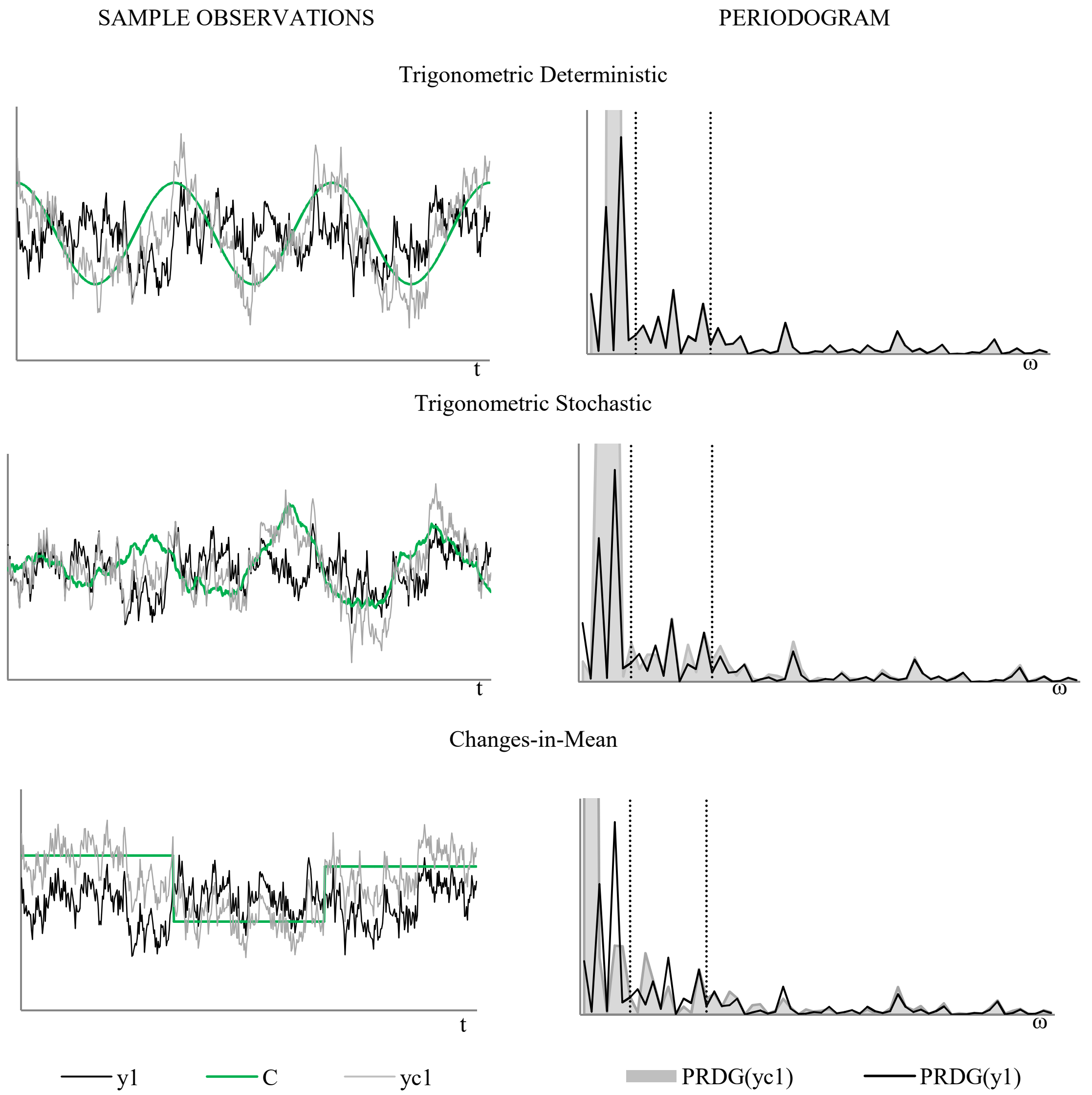
Notes: OLS- OLS estimation with non-filtered data; OLS, BP - OLS estimation data filtered with the BP; Whittle, 20 and Whittle, 8 denote the simulation results from the "Whittle" estimation of the VAR excluding frequencies with a period longer than 20 and 8 years, respectively.

Figure 1 Spectral densities and coherence of the parameterized VAR



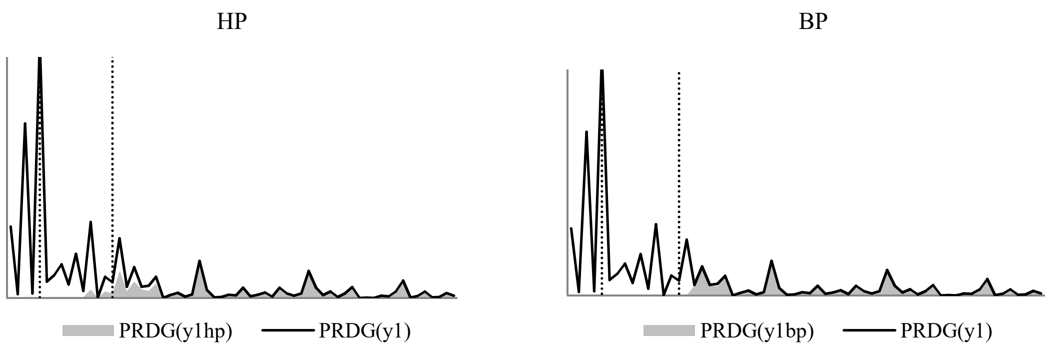
Notes: a) the spectral densities and the coherence are depicted up to 1.5 year period frequency; b) the vertical dotted line signals the frequencies with periods 20-years (left) and 8-years (right)

Figure 2 Examples of contamination by low-frequency cycles



Notes: a) periodograms are depicted up to the 1.5 years period frequency; b) the vertical dotted lines signal frequencies with periods 20-years (left) and 8.years (right); c) y1 - non-contaminated first variable, yc1 -contaminated value by the external low-frequency cycle C .

Figure 3 Periodograms of the filtered series; trigonometric deterministic cycle



Notes: y1: non-contaminated fist variable, y1hp and y1bp: HP and BP filtered series respectively.

Figure 4 Monte Carlo results: trigonometric deterministic, T=500 quarterly

Fig.4.a Impulse response of the first variable to the first shock

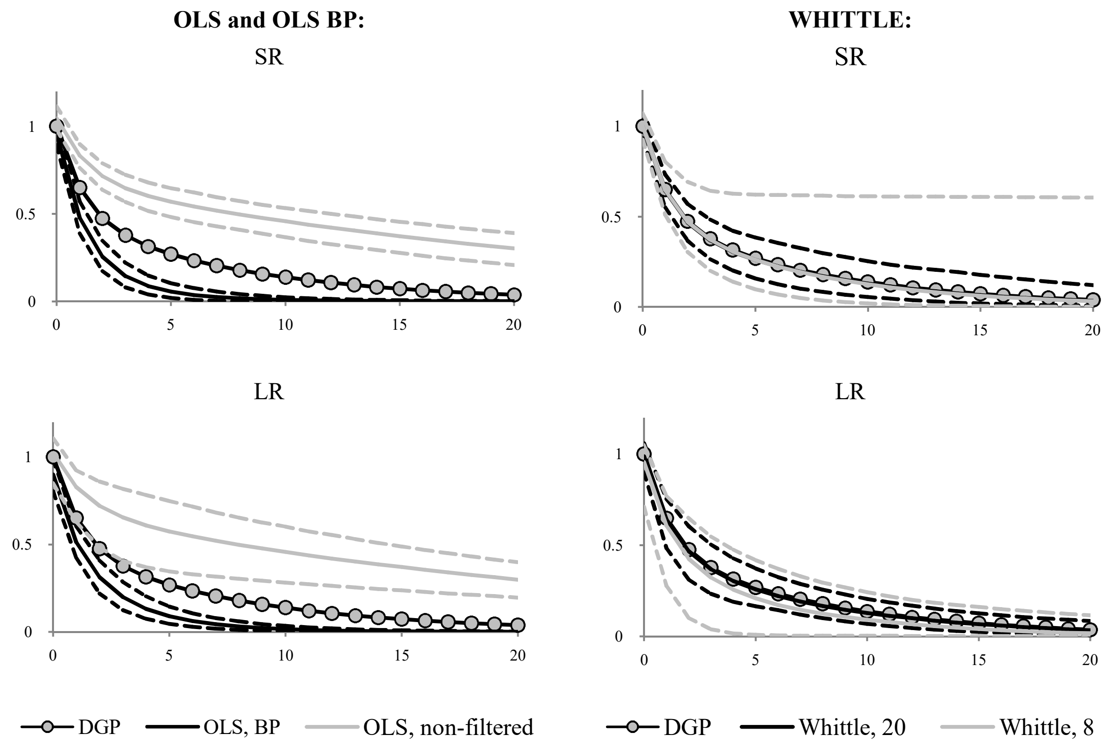
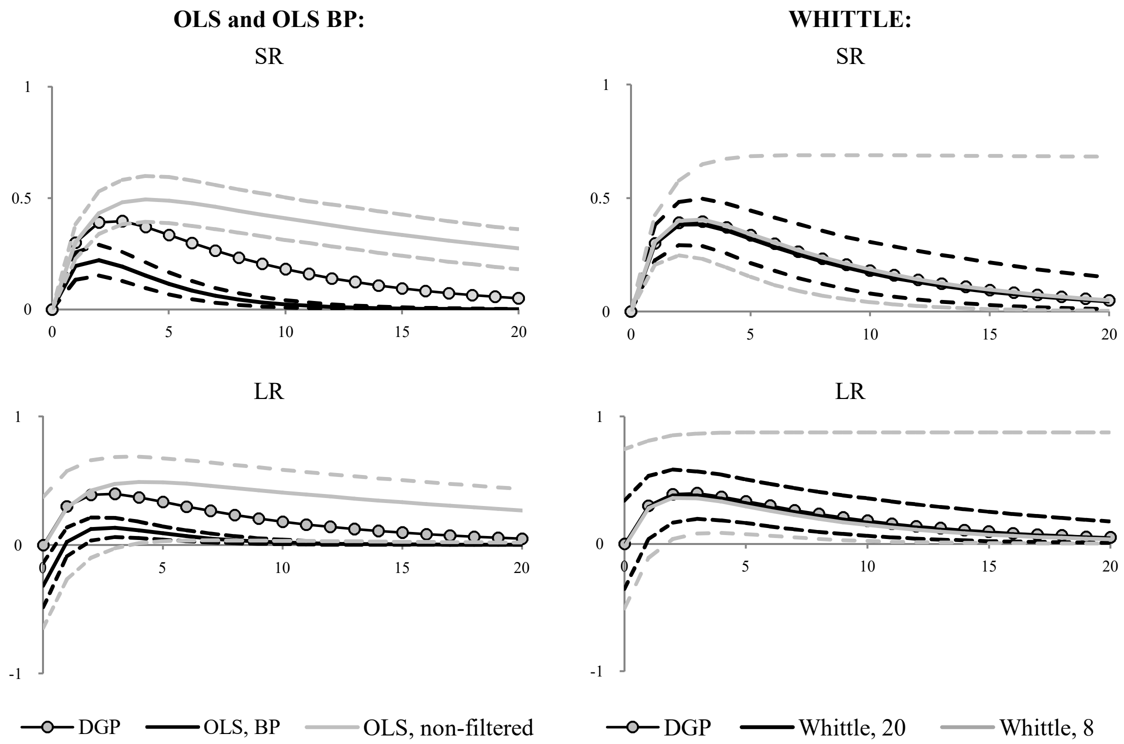
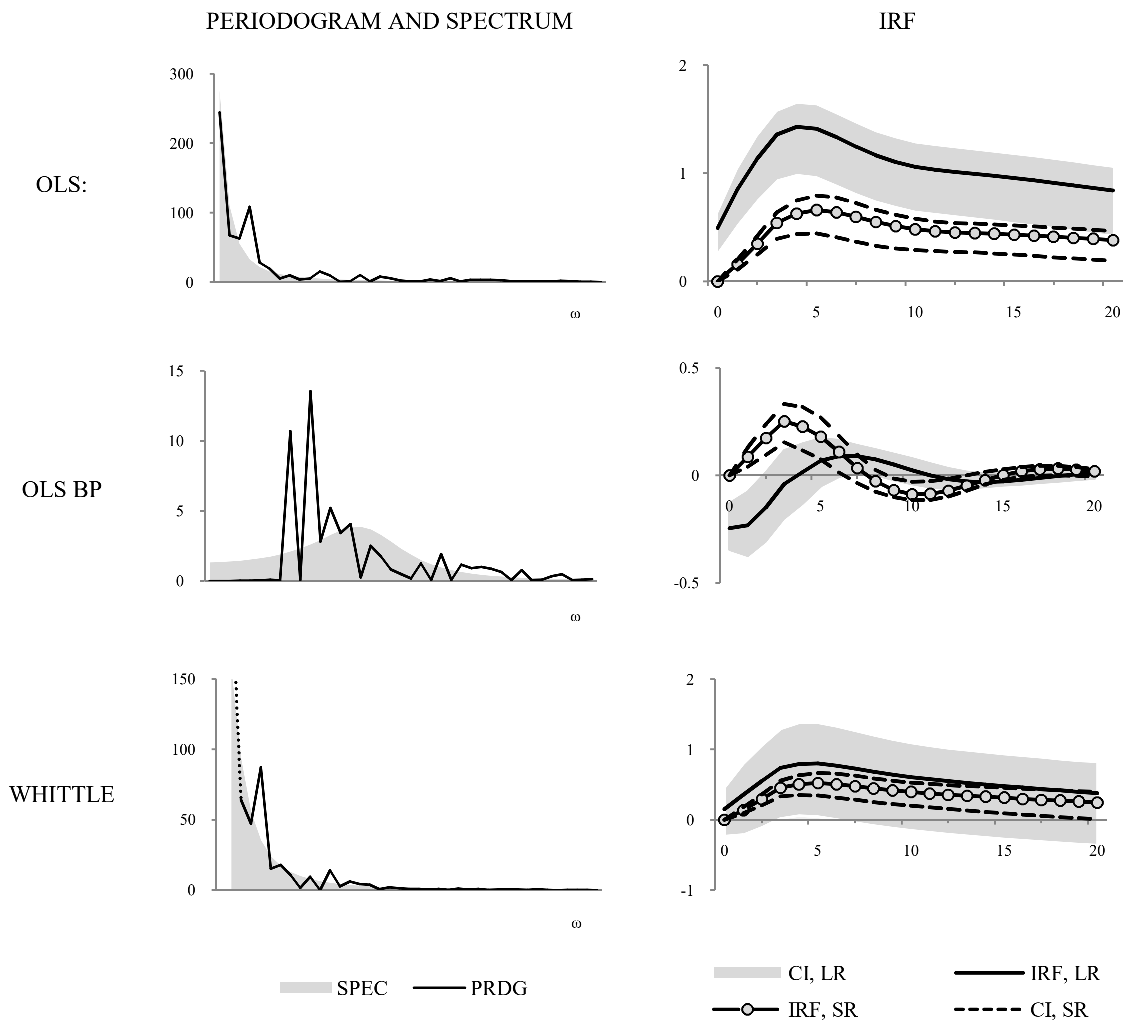


Fig. 4.b Impulse response of the first variable to the second shock



Notes: a) impulse responses are depicted up to a 5 years horizon; b) solid lines depict the average response across simulations. Dashed lines the percentiles 2.5 and 97.5%.

Figure 5 Empirical example: the response of hours to a positive technology shock



Notes: CI denotes one standard deviation confidence intervals for impulse responses computed with non-parametric bootstrap.