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Mixed Duopoly in Education with Vouchers*

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Abstract

In a mixed duopoly environment, I study under which conditions the introduction of a voucher system for private schools can increase competition and as a result, social welfare. My model considers a market in which schools compete in qualities to attract students. Specifically, I consider two settings: one with two private profit-maximizing schools and one with a mixed duopoly, in which one of the schools maximizes social welfare. In the two situations, the quality level offered by the schools plays a crucial role in the students' enrollment decision. I find that in both private and mixed duopoly, the voucher reduces the tuition fee and the quality of the high quality school. It also increases own profits and decreases the ones of its competitor. Thus, the voucher reduces the incentives of the high-quality school to invest on its quality, and this weakens the competition in the market. In the mixed duopoly scenario, particularly for having an increasing consumer surplus and social welfare, the social planner needs to implement a low voucher. The contrary needs to be implemented in the private duopoly. Finally, the low voucher policy can be successful as a high voucher is costly.

Keywords: Mixed Duopoly, Voucher Programs, Educational System, Vertical and Horizontal Differentiation.

JEL classifications: D21, H52, I2, L13.

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1 Introduction

In the last decades, voucher programs that allow students to attend private schools have become increasingly common (see Epple et al., 2015, and Nechyba, 2000). Indeed, vouchers are already implemented in many countries such as: United States, Colombia, Chile, New Zealand, Denmark, Sweden, The Netherlands, India and Pakistan. The introduction of vouchers in the educational system and its impact on the quality provided by public and private schools have been widely discussed. One argument in favor of voucher programs is that public schools will improve their quality because they compete fiercely to attract students. However, it is also considered that voucher programs in private schools may have a negative impact on the quality provided by public schools. This is because they may draw away the most promising students from the public schools, leading to a reduction in the public schools' incentives to increase quality. Therefore, the impact of voucher programs is controversial and it is important to provide a comprehensive framework that is able to convey useful insights regarding the vouchers' role in a mixed duopoly environment.

To achieve this objective, I develop a model where schools compete in terms of quality and tuition fees. This model is built on a paper of vertical product differentiation of Cremer et al. (1997) which focuses on the service quality and competition in the postal sector. I adapt this model to consider a market in which schools compete in qualities to attract students. Specifically, I consider two settings: one with two private profit-maximizing schools and one with a mixed duopoly, in which one of the schools maximizes social welfare (public school). In the two situations, the quality level offered by the schools plays a crucial role in the students' registration decision. In this context, my analysis focuses on the impact of voucher programs on quality levels, tuition fees and in the enrollment of both private and public schools.

Quality and tuition fees are not the only variables that students consider when they decide which school to attend. Schools offer different educational services and this is especially true if one considers the educational services provided by public and private schools. For example, in several countries, most private schools have a strong religious connotation, while public schools are typically secular. I consider that each school offers a service of different quality, and students are heterogenous regarding their willingness to pay. If all students had the same willingness to pay, all of them would go the private high-quality school. But since this is not the case, students with the smaller willingness to pay enroll in the public low-quality school.

This article develops a model that studies the interaction between public and private schools.

The private school maximizes its own profits, while the public school maximizes social welfare which is a weighted function of its profits and the utilities of the other participants in the market.[1] The private school offers a higher quality to students who choose to go there and, consequently, it charges a higher tuition fee than the public school. Regarding the development of the model, I first analyze the education market without introducing the voucher. Then, once it is introduced, I show how the equilibrium changes in both settings: when both schools maximize profits and in a mixed duopoly setting. The voucher is introduced in the model as a reduction in the tuition fee of the private school, the quantity of which is decided by the government. I am interested on its impact on the model's variables such as: qualities, tuition fees and profits of schools and also consumer surplus and social welfare. The reason, I wanted to apply the voucher policy in my model is to see if it results successful. The main criteria for success is satisfying the students with low willingness to pay. This will imply that they cannot afford an expensive private school with a high tuition fee.

The results of the model show that in the context of a private duopoly, the introduction of the voucher to reduce the tuition fee of the high quality school reduces its quality. Interestingly enough, the voucher also reduces in the same amount the quality of the low-quality school. This suggest that the voucher reduces the incentives of the high-quality school to invest in its quality, and this situation weakens competition in the market and also leads the other school to reduce its quality. On the other hand, the voucher increases the enrollment in the high quality school, which increases its profits, and decreases the profits of the low-quality school. Taking this situation into account, I analyze which is the voucher policy that can increase consumer surplus. I show that the social planner has a trade-off when deciding the value of the voucher. On the one hand, the voucher increases the number of students that enroll in the high-quality school, which has a positive impact on consumer surplus. But on the other hand, the voucher leads to a reduction of the quality of both schools. As a consequence, students in the low-quality schools are necessarily made worse off. The optimal voucher policy takes this trade-off into account and depends on the marginal cost of the quality.

However, in the mixed duopoly context, the results differ compared to the previous case. The qualities for both schools decline with the implementation of a low-valued voucher. The high quality private school decreases its own quality because investing on it is expensive. Then, the low quality public school decreases it because of the weaker competition as in the previous

[1] A mixed oligopoly is defined as a market in which two or more firms with different objective functions co-exist (see Fraja and Delbono, 1990, and Nett, 1993, for a survey.)

case. Consequently, by implementing a low voucher it seems that the tuition fee of the high quality school decreases. On the other side, the tuition fee of the low-quality school stays constant intending to increase for higher values of voucher. As investing in quality is expensive, school competition strengthens through a competition through the tuition fees. Thus, in this education market the competition becomes strong. Moreover, the profits for private school 1 increase when the voucher tends to be low. Further, it is noted that profits of the public school decrease while private school gains a lot. Such result is generated because of the huge number of students getting enrolled there. Moreover, consumer surplus decreases with the voucher and the contrary happens with the social welfare. Social welfare increases due to the competition between schools. This satisfies both students and other agents. At the same time, the social planner may find this as a good policy. He may intend to maximize consumer surplus and social welfare by keeping the voucher really low. Finally, all agents in the education market may turn to be content with this policy.

Since the main results of the paper depend on the individuals' preferences for school quality and governmental decision of implementation, the analysis calls for additional empirical studies. Indeed, there is some empirical evidence showing voucher impacts after applying them on the outcome of poor children. It is shown that vouchers raise the competition between schools, increase their qualities, and poor students' test scores (see Neilson, 2013, for more details). Taking into account students' preferences, Gazmuri (2015) shows that after the SES[2] reform in Chile applied in private schools, low-SES students cared more about a school's quality. This result explains why students migrate from public to private schools, by affecting the competition between schools. But, in my case, these results are in contradiction with my findings. I find that the voucher does not raise the competition that much between schools in my model. Moreover, it decreases the qualities of both schools in the two education markets. Similarly to Gazmuri (2015), the students in my model care a lot about the school's quality. This is the reason why students choose to go to the private high-quality school when implementing the voucher.

The paper is organized as follows. In the next section, the related literature is reviewed; in Section 3 the model setup is presented; in Section 4 the optimal allocation and the equilibrium is characterized when both schools are private; Section 5 introduces the mixed equilibrium, when there is competition between a public and a private school; in Section 6 the voucher is introduced, applied to both previous settings, and its impact is studied; concluding remarks

[2]Subvencion Escolar Preferencial is a subsidizing reform for poor families.

and discussion of the results are given in Section 7.

2 Related Literature

This article is related to the literature on mixed duopoly and to the one on vouchers in education. First, when the analysis focuses on education, it is important to take into account that in many countries the educational services are provided by both public and private institutions. Moreover, voucher programs have been increasingly used over the last decades in many different countries.

Literature on mixed oligopoly. Since in numerous markets, for-profit firms compete with cooperative firms and non-profit organizations, there is a growing literature on mixed oligopoly (see for example Casadesus-Masanell and Ghemawat, 2006, and Marini et al., 2015). This literature focuses on the competition among firms with different objective functions (see Cremer et al., 1991, Grilo, 1994, and Delbono et al., 1996).[3] In this literature, some articles study the issue of competition in education markets when education providers can be public and private (see Cellini and Goldin, 2014, Deming et al., 2012, and Cremer and Maldonado, 2013). More specifically, Cellini and Goldin (2014) and Deming et al. (2012) empirically show that US education markets are effectively mixed, while Cremer and Maldonado (2013) study a mixed duopoly model in which the quality of education depends on "peer group" effects.

Ishibashi and Kaneko (2008) use a Hotelling model to argue that in a duopoly, the public firm provides lower quality than the private firm. Based on this paper's claim, I have assumed that the quality of private school is higher compared to public. Thus, the education system is considered a real market where schools are the ones that compete in my model. On the other hand, Fraja and Delbono (1990) and Delbono et al. (1996) use a duopoly model to show the existence of the qualities' equilibrium. Here, a public firm chooses the lower quality while private ones choose the higher quality. According to these authors' assumptions, I have chosen public school to have the lower quality. Matsumura and Matsushima (2003) use the sequential choice of location in a mixed duopoly, where a welfare-maximizing public firm stands as a competitor of a profit maximizing private firm. Additionally, they introduce the price regulation effect. Related to this paper, I introduce the voucher as a reduction of private school's price and I study

[3] Cremer et al. (1991) study price competition in a market represented by a Hotelling (1929) line in which private and public firms choose locations first and then prices. Then, Grilo (1994) analyzes a mixed competition model in which products are vertically differentiated and firms non-cooperatively choose qualities first and then prices. Finally, Delbono et al. (1996), using a model similar to Grilo (1994), introduce the possibility that the market might be uncovered.

its impact on quality, profits, consumer surplus and social welfare. However, the literature for mixed duopoly in education is scarce. The Romero and Rey (2004) paper examines the education market as a mixed duopoly through a sequential choice analysis and competition for optimal quality, prices and exams. Similarly to this paper, I study a mixed duopoly in education with a sequential choice in two scenarios. The first is when both public and private school maximize their profits. Then, the other case is when public school maximizes social welfare and the private one maximizes its profits.

Literature on vouchers. This article is also related to the literature of vouchers in the education system as a public policy. My paper is particularly related to Epple and Romano (1998). The authors deal with private and public school competition when students vary in ability and household income. Moreover, school quality increases in the peers' ability. Their model introduces a universal voucher where the increasing amount of vouchers make the average peer quality in the public schools decline. This happens because private entrants 'cream-skim' the higher income and ability students from public schools. Contrary to this paper, the students in my model vary in their preferences for school and their willingness to pay. Furthermore, I use a universal voucher for those students who choose to attend a private school. Though the quality of public school increases after applying the voucher in private school's price. Epple and Romano (2008) investigate the implication of voucher design for cream skimming. Moreover, they show that a voucher constrained by ability and tuition preserves benefits from competition. When eliminating cream skimming, there will be uniform benefits from across the distribution of student income and types of ability. Hence, I use a voucher constrained by the preferences of the students for schools. Once applying the voucher, there will be benefits for both schools and society.

Nechyba (1999, 2000) consider the effects of voucher programs in multi-district local economies. Tuition varies across private schools and, differently from Epple and Romano (1998, 2008), price discrimination is not allowed. More specifically, these papers study three voucher programs: (1) a general voucher applicable to any child in private school, (2) a voucher targeted only to low-income households, (3) a voucher targeted to poor districts. Their results change depending on the voucher program considered. Manski (1992) was a pioneer on using a theoretical and computational model to capture the features of the education environment. Students choose between public and private schools, differing by household income and motivation, with demand for education quality and a positive peer effect. Public sector is rent-seeking and private one

gets zero profits by setting tuition to maximize enrollments and a voucher spent on educational inputs. Even in the best case, a choice system would not reach to equalize educational opportunity across income groups. In my model, the contrary occurs. Specifically, the introduction of the voucher allows students to attend private schools even if their willingness to pay is low. Other articles try to better explain how different types of vouchers work in the education system (see Epple et al., 2015, and Brunner and Imazeki, 2008).[4] Neal (2002) gives a theoretical discussion on how vouchers can change the education market, giving examples from the U.S. This is also my aim for the paper, to see the impact of vouchers in the education system. Cellini and Goldin (2014) provide evidence that shows that federal student aid raises tuition fees for the case of profit colleges. In my model, it happens that it decreases the tuition fees while at the same time decreasing school quality. Therefore, these two effects cancel each other, as soon as social welfare increases. This is the case with a mixed duopoly education market, where all agents remain satisfied at the end.

3 Model Setup

I use a Mussa and Rosen (1978) vertical differentiation model similarly to Cremer et al. (1997), but extending and adapting it to the education sector. We consider a duopoly model where there is a private school ($i = 1$), and a public school ($i = 2$). The private school maximizes its profits, while public school may care not only about its own profits, but also about social welfare. Social welfare is measured by total surplus, which is equal to the sum of producer and consumer surplus. Consumer surplus represents the utility that students obtain by choosing the school they prefer. Schools decide the tuition fees and qualities they offer. As in Cremer et al. (1997), without loss of generality, I assume that the private school has higher quality and a higher tuition fee compared to the public school, i.e. $x_1 > x_2$ and $p_1 > p_2$.

Students make the choice depending on their willingness to pay for schools and their preferences for the quality of their school. The variable (c) represents the quality cost that schools need to pay once they increase their quality. The cost for increasing the quality is expensive. However, schools need to invest in order to satisfy the students preferences and at the same time increase their own profits.

There is a continuous uniform number of students whose types are identified by $\theta \in [\underline{\theta}, \bar{\theta}]$. θ represents the marginal willingness to pay for quality. The quality of a school i, $x_i > 0$, reflects

[4]These articles discuss Tiebout-choice for the universal vouchers.

the preferences of individuals and their utility while choosing between public and private schools. The student's surplus of type θ who asks for a school with quality x_i and tuition fee p_i is given by $\theta x_i - p_i$. All students have the same total demand which is perfectly inelastic and normalized to one.[5]

The timing of the game is as follows. In Stage 0, the voucher is selected by the government; in Stage 1, both schools decide their qualities. Then, in Stage 2, they choose the tuition fee that will apply; finally, in Stage 3, students decide which school to attend.

Since there is perfect information, I solve the model by backward induction. The following assumption guarantees an interior solution.

Assumption 1. *To guarantee an interior solution,* $\bar{\theta} > \underline{\theta} > \frac{\bar{\theta}}{5}$.

In this paper, I will consider two specific scenarios: First, I analyze the case in which both schools only care about their own profits (private duopoly); second, I consider the case where only the private school is a profit-maximizer, while the public school also takes into account the students' utility and the private school's profits (mixed duopoly). In general terms, the public school maximizes the following:

$$W = \alpha(\pi_1 + CS) + \pi_2. \tag{1}$$

where α represents the weight given to the utility of the other participants in the market. In the first scenario, the public school is a profit-maximizer and α takes the value of 0. In contrast, in the other extreme scenario where the public school maximizes social welfare, α is equal to 1.

In these two alternative settings, I will study whether the government finds it beneficial for the society to introduce a voucher that reduces the tuition fees students pay in the private school. If this is the case, I will study how the voucher impacts the quality provided by both schools, their profits, and social welfare in the two settings. Finally, I will discuss the policy implications of this analysis in the conclusions.

All the mathematical computations and proofs of the results are in the appendix.

4 Private duopoly

In this section, I consider the case in which both schools maximize their profits, i.e. $\alpha = 0$.

[5]Inelastic demand in economics is when people buy the same amount whether the price increases or decreases. In this case, it means that if the tuition fee drops, the quality demanded by students will not change.

The student's surplus is given by the following expression:

$$CS = \int_{\hat{\theta}}^{\bar{\theta}} (\hat{\theta}x_1 - p_1)d\hat{\theta} + \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta}x_2 - p_2)d\hat{\theta}. \tag{2}$$

Here, the variable $\hat{\theta} = \frac{p_1 - p_2}{x_1 - x_2}$ represents the marginal student. Students with a low willingness to pay ($\underline{\theta} < \hat{\theta}$) go to the cheapest and lowest quality school, i.e. the public school. In contrast, students with a high willingness to pay ($\overline{\theta} > \hat{\theta}$) choose the highest quality and more expensive school, i.e. the private school. Having the prices and qualities given, each student chooses the school that maximizes its own utility. Hence, a student chooses private school with the highest quality only if $[\hat{\theta}x_1 - p_1 > \hat{\theta}x_2 - p_2]$. Let's determine the optimal index of the marginal consumer given the qualities. Maximizing the CS function (2) with respect to $\hat{\theta}$, yields:

$$\hat{\theta} = \frac{c(x_1 + x_2)}{2} \tag{3}$$

Proposition 1 illustrates the results under private duopoly.

Proposition 1. *The optimal levels of quality for the two profit-maximizing private schools is unique and equal to the following:*

$$x_1^\circ = \frac{5\bar{\theta} - \underline{\theta}}{4c}. \tag{4}$$

$$x_2^\circ = \frac{5\underline{\theta} - \bar{\theta}}{4c}. \tag{5}$$

The equilibrium prices are:

$$p_1^\circ = \frac{25\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 49\bar{\theta}^2}{32c}. \tag{6}$$

$$p_2^\circ = \frac{49\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 25\bar{\theta}^2}{32c}. \tag{7}$$

After substituting the price and quality functions on each of the profit function, the equilibrium profits are:

$$\pi_1^\circ = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c}. \tag{8}$$

$$\pi_2^\circ = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c}. \tag{9}$$

It is possible to notice that both schools obtain the same profits in equilibrium.

Therefore, the marginal consumer obtained from these equalities is:

$$\hat{\theta}^{\circ} = \frac{\bar{\theta}}{2} + \frac{\underline{\theta}}{2}. \tag{10}$$

By comparing the quality levels, it results that $x_1^{\circ} < x_2^{\circ}$, if Assumption 1 is satisfied. Moreover, the tuition fee of private school 1 is higher than the tuition fee paid to attend the public school, i.e. $(p_1^{\circ} > p_2^{\circ})$. Finally, schools obtain the same profits.

5 Mixed duopoly

Here, I consider a mixed duopoly environment in which a public and a private school compete in terms of quality and tuition fees. The private school continues to maximize profits while the public school cares only about social welfare (this is the case in which $\alpha = 1$ in (1)). The game is solved by backward induction, as in the previous section. The following proposition illustrates the results under mixed duopoly:

Proposition 2. *The equilibrium of qualities in the second stage of the game for a mixed duopoly market in which private school maximizes profits and the public one maximizes social welfare is unique and is given by:*

$$x_1^m = \frac{\underline{\theta} + 3\bar{\theta}}{4c}. \tag{11}$$

$$x_2^m = \frac{3\underline{\theta} + \bar{\theta}}{4c}. \tag{12}$$

The equilibrium prices are:

$$p_1^m = \frac{9\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 17\bar{\theta}^2}{32c}. \tag{13}$$

$$p_2^m = \frac{17\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 9\bar{\theta}^2}{32c}. \tag{14}$$

The indifferent student will be equal to the one in private duopoly, where:

$$\hat{\theta}^m = \frac{\underline{\theta}}{2} + \frac{\bar{\theta}}{2}. \tag{15}$$

After substituting these price and quality equilibrium functions in the each of the profit

functions, the generated profits of each school in equilibrium;

$$\pi_1^m = \frac{(\bar{\theta} - \underline{\theta})^3}{8c}. \tag{16}$$

$$\pi_2^m = \frac{(\bar{\theta} - \underline{\theta})^3}{8c}. \tag{17}$$

and both schools will have the same profit function in equilibrium.

Here, the welfare function is different from the private duopoly case. It will have a higher value compared to the private duopoly. Thus, it's function is represented as below:

$$W^m = \frac{(-\underline{\theta} - \bar{\theta})(5\underline{\theta}^2 + 6\underline{\theta}\bar{\theta} + 5\bar{\theta}^2)}{32c}. \tag{18}$$

By comparing the quality levels that depend on $\bar{\theta}$, $\underline{\theta}$ and c, it obvious that $x_1^m > x_2^m$ if Assumption 1 is satisfied. Then, the tuition fee paid to attend the private school is higher than the one in the public school, i.e. $p_1^m > p_2^m$. Interestingly, profits continue to be the same in the private and public school. But, they are lower in this case compared with the private duopoly market. Similarly to the previous case, irrespective of the value of θ and the schools' objective functions, they obtain the same profits.[6]

To provide an intuition for the results, it is important to understand the public school's incentives on choosing an efficient quality level. To understand this, let us recall that the high quality choice of a private school is the result of two opposing effects. On the one hand, the private school wants to move closer to its competitor and cover the largest possible market share. On the other hand, it prefers to choose a quality far away from its competitor's to reduce the intensity of the tuition fee's competition. If the competitor is also private, the second effect tends to dominate and schools will over-differentiate in equilibrium. Moreover, if the competitor is public, the price competition will have a different nature. This happens because the public school is interested more on allocating the students efficiently. Hence, it has less aggressive behaviour in undercutting its competitor. Also, the public school no longer has incentives to choose a very low quality. This is an interesting explanation on why the public school (the low quality school) finds it optimal to increase it own quality. In such a way, it can move closer to the social optimum. However, in my education markets I first introduce only two private schools and to the other case; one public school and one private school. I assume that private

[6] A similar result has been obtained by Cremer et al. (1997) by using a horizontal (Hotelling-type) differentiation model.

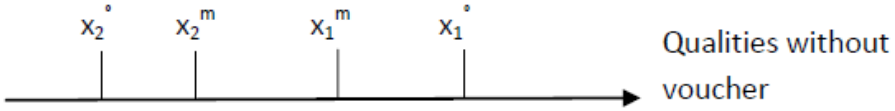

Figure 1: Private and Mixed Duopoly Qualities

school has a higher quality compared to public one. Thus, I observe what happens with schools' qualities in equilibrium. [7]

Finally, I compare the results in the private duopoly with those obtained under mixed duopoly. It is obvious that the quality chosen by the private school in private duopoly is higher than in mixed duopoly. The quality of public school in private duopoly is lower than that on mixed duopoly. Then, the private school has a higher quality compared to public one in the mixed duopoly market, but it results lower than that of the private school in private duopoly. These results are represented in Figure 1. To obtain them, I use a numerical simulation by assigning values to the parameters they depend on. Thus, the cost is ($c = 0.5$), the lowest willingness to pay ($\underline{\theta} = 1$), and ($\bar{\theta} = \underline{\theta} + 1) = 2$.

Dealing with the tuition fees, they are lower in mixed duopoly case for both schools. Furthermore, it results that the public school has the lowest tuition fee in both education markets compared to private on. Now, considering the profits of each school, they are equal inside the market. Thus, I compare the profits of private school in the private duopoly case with those of private school in mixed duopoly. The private schools's profits of the private duopoly market are lower than in the mixed duopoly one. The same result is generated for the public school. However, the social welfare and consumer surplus in mixed duopoly is higher. Hence, in the mixed duopoly education market all agents are satisfied.

6 Vouchers

In general, a voucher is defined as a government-supplied coupon that is used to offset tuition at an eligible private school. Programs that distribute such vouchers exhibit variation in di-

[7] A crucial assumption of the paper is that the demand is inelastic. This explains that public school can always achieve an efficient allocation of students, even when it involves setting a "high" tuition fee. If the demand were elastic, these results would not be true, as the public school may face quality and quantity distortions. Therefore, it may not be able to set a high price and also achieve an efficient segmentation of the market.

mensions including who is eligible to receive them, their source of funding and the criteria for private school participation. In this section, I introduce a voucher (v) as a reduction in private school's price. Similarly to Epple and Romano (1998) and Nechyba (1999), I use a general voucher applicable to any child that decides to go to a private school after the policy starts to get implemented.

The equilibrium is achieved in the same way as in the previous sections. I start implementing the voucher in an education market when both schools are profit-maximizers. Then, I introduce the voucher in a mixed duopoly setting where the public school maximizes the social welfare. The aim of this analysis is to study the impact of the introduction of the voucher on quality, school profits, and social welfare.

6.1 Private duopoly with voucher

The objective function, I consider for the public school in this case is the following:

$$W_r = \alpha(\pi_1 + CS_r) + \pi_2, \quad \text{with} \quad 0 \leq \alpha \leq 1. \tag{19}$$

where $\pi_1 = (p_1 - \frac{cx_1^2}{2})(\bar{\theta} - \hat{\theta})$ is the profit function of private school and $\pi_2 = (p_2 - \frac{cx_1^2}{2})(\hat{\theta} - \underline{\theta})$ is the profit function of public school. This is the case of the model when $\alpha = 0$. This generates $W_r = \pi_2$ and the public school maximizes its profits behaving like a private school. Different from previous cases, the indifferent student, taking the voucher into account, will be represented by: $\hat{\theta} = \frac{-p_1+p_2+\bar{\theta}v}{v-x_1+x_2}$. Thus, when I do the difference between the functions obtained here with those of Cremer et al. (1997),I assume $\bar{\theta} = \underline{\theta} + 1$. Then, if I substitute the results for $v = 0$, I go back to the results of the benchmark model without vouchers. I also consider that the quality of private school 1 is higher than the quality of public school 2. The student's surplus function (CS) is equal to:

$$CS_r = \int_{\hat{\theta}}^{\bar{\theta}} (\hat{\theta}x_1 - p_1 + v(\bar{\theta} - \hat{\theta}))d\hat{\theta} + \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta}x_2 - p_2)d\hat{\theta}. \tag{20}$$

The equilibrium for the case of $\alpha = 0$ is obtained by backward inductions.[8]. The results are provided in Proposition 3.

Proposition 3. *By using Assumption* 1*, together with the condition:* $\bar{\theta} = \underline{\theta} + 1$ *for the case of* $\alpha = 0$*, I find the equilibrium under private duopoly. Taking into account that the public school*

[8]All the steps and mathematical computations are in the appendix

only maximizes profits, the equilibrium quality levels are:

$$x_1^r = \frac{5\bar{\theta} - \underline{\theta}}{4c} - \frac{v}{3 + 4cv}. \tag{21}$$

$$x_2^r = \frac{5\underline{\theta} - \bar{\theta}}{4c} - \frac{v}{3 + 4cv}. \tag{22}$$

The tuition fees students pay to attend the private or the public school are given by:

$$p_1^r = \frac{25\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 49\bar{\theta}^2}{32c} + \frac{-63v - 36\underline{\theta}v + 138cv^2 + 48\underline{\theta}cv^2 - 64c^2v^3}{12(-3 + 4cv)^2}. \tag{23}$$

$$p_2^r = \frac{49\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 25\bar{\theta}^2}{32c} + \frac{-81v - 36\underline{\theta}v + 210cv^2 + 48\bar{\theta}cv^2 - 128c^2v^3}{12(-3 + 4cv)^2}. \tag{24}$$

Thus, the marginal student after having implemented the voucher in private school's price will be as following:

$$\hat{\theta}^r = \frac{\bar{\theta} + \underline{\theta}}{2} + \frac{2cv}{3(-3 + 4cv)}. \tag{25}$$

Finally, the schools' profits are:

$$\pi_1^r = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c} + \frac{27v + 24cv^2 - 64c^2v^3}{36(-3 + 4cv)^2}. \tag{26}$$

$$\pi_2^r = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c} + \frac{-189v + 456cv^2 - 256c^2v^3}{36(-3 + 4cv)^2}. \tag{27}$$

I use the label 'r' to denote the equilibrium. It is noticeable that voucher plays a positive role on increasing social welfare and school quality. Moreover, it also increases the competition on tuition fees and quality between the schools.

Comparing the equilibrium results, it is easy to show that the quality, the tuition fee, and the profits obtained by the private school are higher than those obtained by the public school. Thus, $x_1^r > x_2^r$, $p_1^r > p_2^r$, and $\pi_1^r > \pi_2^r$. Differing from the results of Cremer et al. (1997)'s model, the voucher reduces the school quality and the tuition fees for both school. It also reduces the profits of public school. The contrary happens with the profits of private school, which increase with the voucher.

By deriving the consumer surplus function (CS) with respect to the voucher (v) and then equaling the first order condition to zero, I find the optimal voucher $(v)_{optimal}$ in this setting.

Proposition 4 illustrates this result.

Proposition 4. *The optimal voucher is equal to:*

$$v^r_{optimal} = \frac{66c^4 + (33(-396 + \sqrt[2]{160941}))^{\frac{1}{3}} c^2 (c^6)^{\frac{1}{3}} - (33(396 + \sqrt[2]{160941}))^{\frac{1}{3}} (c^6)^{\frac{2}{3}}}{88c^5}. \tag{28}$$

It is intuitive that the voucher will increase the number of students going to the private school. The voucher is negatively affected by the quality cost (c). The higher is the quality cost, the lower should be the voucher. Moreover, the indifferent student will be better off when the voucher increases. The introduction of the voucher to reduce the tuition fee of the high quality school reduces its quality. At the same time, the voucher also reduces in the same amount, the quality of the low-quality school. As a consequence, students in the low-quality school are necessarily worse off.

6.2 Mixed duopoly with unconstrained equilibrium

Similarly to the previous subsection, the marginal student will be different from the simple case due to the voucher. Moreover, in this section, we consider the case in which $\alpha = 1$. In this setting, the public school maximizes the social welfare that is equal to:

$$W_v = CS_v + \pi_1 + \pi_2. \tag{29}$$

The game is solved again by backward induction. Stage 2 of the game determines the price equilibrium for any given pair of qualities and then, stage 1 analyzes the quality choices. The function of consumer surplus is the same as in the previous case ($CS_r = CS_v$). Thus, I state the following proposition:

Proposition 5. *Considering Assumption* 1*, and also* $\bar{\theta} = \underline{\theta} + 1$*, the equilibrium of qualities is obtained in the Stage* 2 *of the game for unconstrained mixed duopoly education market. By adding the voucher in this market, in which private school maximizes profits and public school maximizes social welfare, the equilibrium is unique and is given by:*

$$x_1^v = \frac{\underline{\theta} + 3\bar{\theta}}{4c} - \frac{v}{1 + 4cv}. \tag{30}$$

$$x_2^v = \frac{3\underline{\theta} + \bar{\theta}}{4c} - \frac{v}{1 + 4cv}. \tag{31}$$

The prices that students have to pay in equilibrium, depending on the school they choose, are as following:

$$p_1^v = \frac{9\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 17\bar{\theta}^2}{32c} + \frac{-v - 4\underline{\theta}v - 6cv^2 + 16\underline{\theta}cv^2}{4(-1+4cv)^2}. \tag{32}$$

$$p_2^v = \frac{17\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 9\bar{\theta}^2}{32c} + \frac{v - 4\underline{\theta}v - 2cv^2 + 16\underline{\theta}cv^2}{4(-1+4cv)^2}. \tag{33}$$

Thus, the marginal student after having implemented the voucher in private school's price will be as following:

$$\hat{\theta}^v = \frac{\underline{\theta} + \bar{\theta}}{2} + \frac{2cv}{-1+4cv}. \tag{34}$$

After substituting all these previous results in the profits' functions, they are equal to:

$$\pi_1^v = \frac{(-\underline{\theta} + \bar{\theta})^3}{8c} + \frac{3v - 8cv^2}{4(-1+4cv)^2}. \tag{35}$$

$$\pi_2^v = \frac{(-\underline{\theta} + \bar{\theta})^3}{8c} + \frac{-v}{4(-1+4cv)^2}. \tag{36}$$

Now, let's see what happens to the social welfare function in equilibrium.

$$W_v = \frac{(-\underline{\theta} + \bar{\theta})(5\underline{\theta}^2 + 6\underline{\theta}\bar{\theta} + 5\bar{\theta}^2)}{32c} + \frac{-v}{8(-1+4cv)^2}. \tag{37}$$

Here, I label the variables with a 'v' to indicate the equilibriums of mixed duopoly when I apply the voucher. Now, let's compare the equilibrium results with each other. It is obvious that the quality of the private school is higher than that of public school. Hence, we have: $x_1^v > x_2^v$. But the tuition fee and the profits of private school are lower than those of public one. So, we have: $p_1^v > p_2^v$ and $\pi_1^v > \pi_2^v$. This is true because a part of the tuition fee of private school is covered by the voucher. It means that the private school will generate low profits, by putting a higher quality and a lower tuition fee. Even though the public school's aim is not to maximize own profits, it still generates higher profits compared to private school.

Next, the qualities and profits of both schools are lower with the voucher compared to the model without voucher. The tuition fees are higher with the voucher due to the competition between schools. Here, the indifferent student is better off. Anyway, the social welfare is higher with the voucher.

Furthermore, it is the turn to obtain the optimal voucher's function. I find it by deriving the consumer surplus function (CS) with respect to the voucher (v) and equaling the obtained result to zero. The sign of the derivative function is negative which means that consumer surplus decreases. However, its value from positive changes to negative at low levels of voucher. Thus, it is constant and positive at high levels of voucher. This is translated into a higher voucher compared to the private duopoly case for having a positive CS.

Finally, I do a comparison of the equilibrium results between the two different education markets of both schools. It is obvious, that the quality of private school in private duopoly case is lower than in mixed duopoly market (i.e. $x_1^r < x_1^v$). A similar result is obtained when I consider the quality provided by the public school which results in higher quality in the mixed duopoly market (i.e. $x_2^r < x_2^v$). These results are illustrated in Figure 2. Here, I represent the qualities of each of schools in the private and mixed duopoly with voucher. The results are generated after doing simulations and giving values to the variables they depend on. Thus, the cost is ($c = 0.5$), the lowest willingness to pay ($\underline{\theta} = 1$), ($\bar{\theta} = \underline{\theta} + 1$), and the voucher is very low.

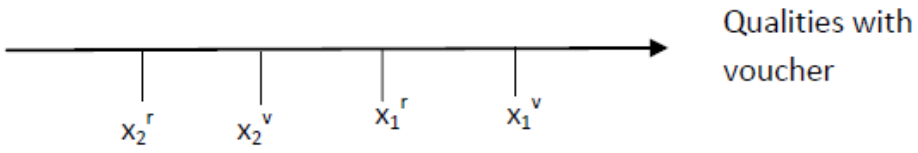

Figure 2: Private and Mixed Duopoly Qualities

The tuition fees of both private and public school in private duopoly education market are higher than in mixed duopoly (i.e. $p_1^r > p_1^v$ and $p_2^r > p_2^v$). Then, the profits of private school in private duopoly case are lower than in mixed duopoly one (i.e. $\pi_1^r < \pi_1^v$). For public school, the contrary happens ($\pi_2^r < \pi_2^v$). Furthermore, the social welfare is higher in a mixed duopoly education market (i.e. $W_v > W_r$). Finally, consumer surplus is higher in private duopoly market (i.e. $CS_r > CS_v$). It is positive and constant when the voucher is high in a mixed duopoly.

6.3 Discussion of the voucher impact

In this subsection, I provide some intuition and comment on the impact of the voucher on the different variables in both settings considered.

I start dealing with the private duopoly. In this case, the quality of both private and public school decreases with the voucher. This happens because investing in quality is expensive. It

is interesting to note that both schools' tuition fees decrease with the voucher. Note that the voucher is introduced as a reduction in the tuition fee of the private school. Because of the weak competition among schools, the tuition fee of the public school decreases too. Then, the profits of public school behaving as private together with the social welfare decrease when voucher increases. I restrict the value of voucher not more than 0.8 and especially for consumer surplus, I use a really high voucher. It makes sense, as by lowering the quality and also the tuition fee, probably the profits will decrease. In the model, the profits of public school are equal to social welfare. This is the reason why it decreases too when the voucher increases. The contrary happens with consumer surplus which increases only with higher values of voucher. The social planner has to take this problem into account. It's aim is to maximize the consumer surplus, but it results quite impossible and really expensive in this case. This may be a reason for not implementing the voucher in such an education market.

In order to see the impact of the voucher in this education market, I show it also with graphs. In this way, it is easier to see the variable's evolution and notice the impact that the voucher has on each variable. The graph is represented below:

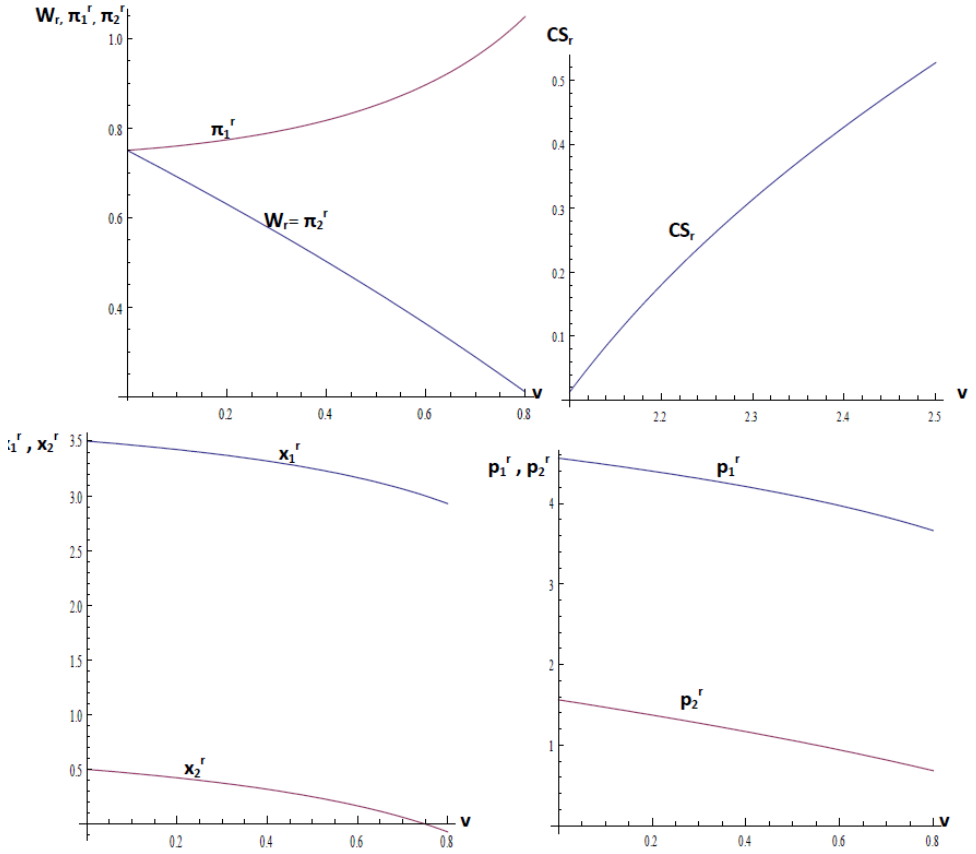

Figure 3: Voucher's Impact in Private Duopoly

Now, it is the turn of mixed duopoly. The quality of high and low quality schools decreases with the voucher only for low values of the voucher. In the simulation, I restrict the voucher not more than 0.35 as for higher values of it, some variables make no economic sense. By implementing a low voucher it seems that the tuition fee of the high quality school decreases. On the other side, the tuition fee of the low-quality school stays constant and increases for higher values of the voucher. As investing in quality is expensive, schools strengthen the competition by competing in tuition fees. Thus, in this education market the competition is strong. Moreover, the profits for the private school increase when the voucher tends to be low. It is obvious that the profits of public school decrease while private school gains a lot. Such a result is because of the huge number of students getting enrolled there. Moreover, consumer surplus decreases with the voucher and the contrary happens with the social welfare. Social welfare increases due to the competition between schools. This makes students happy. At the same time, the social planner may find this as a good policy. He may intend to maximize the consumer surplus and also the social welfare by keeping the voucher low. Finally, all agents in the education market may turn to be content with this policy.

In order to see the impact of the voucher in the mixed duopoly education market, I represent the following graphs:

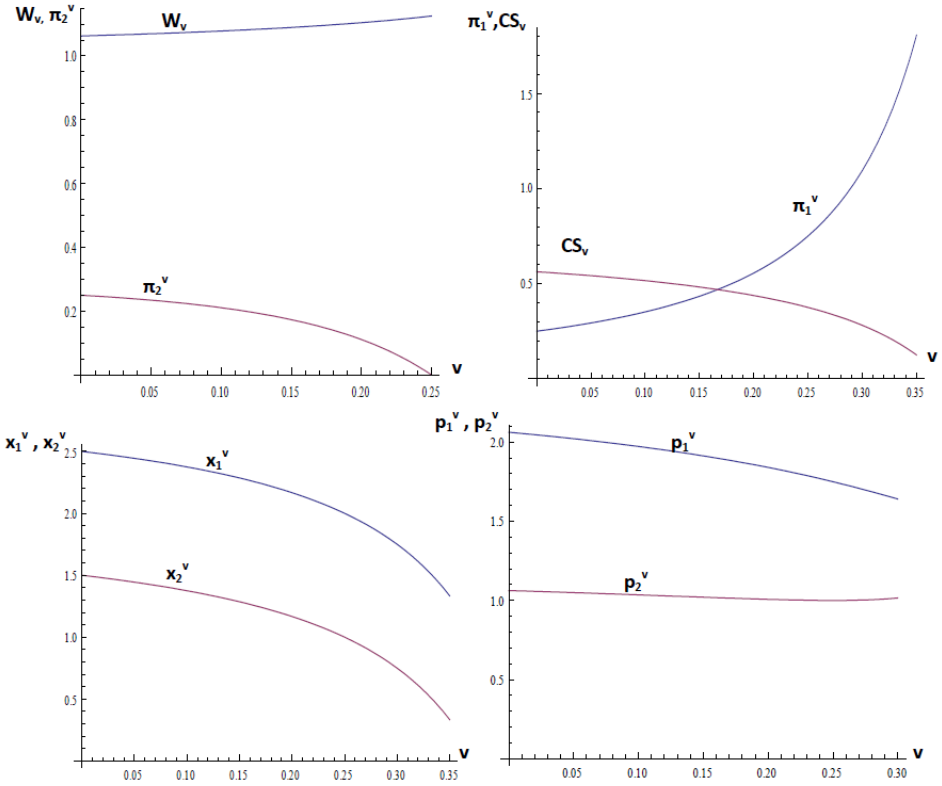

Figure 4: Voucher's Impact in Mixed Duopoly

7 Concluding Remarks

In this paper, I have modeled a mixed duopoly in education market when the public policy of vouchers is implemented. The economics literature has stressed how schools benefit from vouchers, but I have shown several issues beyond that. Let me summarize the steps I followed and then the results. If students get a voucher as a reduction in the private school tuition fee, they will probably go to a private school. This increases the number of students going to private school even though their willingness to pay is low. This also creates a competitive environment for schools. On the other hand, this policy has its own impacts on the quality and tuition fee of both schools. For this reason, I take into account two different scenarios of the education market. Firstly, we consider a private duopoly market where both schools behave like private ones and maximize their profits. Secondly, there is a mixed duopoly case when the private school maximizes profits and public one maximizes the social welfare. The final results are not the same for both cases so I deal with them separately. Further, the model is followed by a discussion of the voucher's policy impacts on the tuition fees, qualities and profits of each school. By the end, I comment its effect on consumer surplus and social welfare and find the

optimal voucher.

Regarding the private duopoly case, the results seem to be quite unexpected from the students point of view. This is because the voucher implemented as a reduction in the tuition fee of the private school causes a decrease in its own quality. Such result may disappoint the students who expected to receive higher quality from the private school. On the other hand, there is public school who competes with the private by decreasing quality too. Note that quality is expensive to invest on. Then, the tuition fees of both schools decrease when voucher increases. This later impact may be positive from the point of view of the student and negative for the education market side. The students may be satisfied if they would pay less. But, for schools it may result the contrary as they earn less per student. Consequently, the profits of the private school increase within the restricted value of the voucher. The contrary happens with the profits of the public school which decrease because its quality decreases. Note that $W = \pi_2$ in this case. Thus, the social welfare will decrease because of the weak competition between schools. Further, the consumer surplus tends to increase with high values of the voucher, resulting in it being expensive for the social planner to implement this high voucher policy. If he chooses the low voucher policy, he may not satisfy its aim of maximizing consumer surplus. Finally, I can say that such policy may not be appropriate in this case.

Now, it's time to the most important case, the mixed duopoly one. Here, the results are the same compared to the previous case. The qualities for both schools decline with the implementation of a low voucher. The high quality school decrease its own quality because investing on it is expensive. The low quality school decreases it because of the competition and the same reason as before. Consequently, by implementing a low voucher it seems that the tuition fee of the high quality school decreases. On the other side, the tuition fee of the low-quality school stays constant intending to increase for higher values of voucher. As investing in quality is expensive, schools strengthen the competition by competing in tuition fees. Thus, in this education market the competition is strong. Moreover, the profits for the private school increase when the voucher tends to be low. Further, it is noted that profits of public school decrease while private school gains a lot. Such result is generated because of the huge number of students getting enrolled there. Moreover, consumer surplus decreases with the voucher but social welfare increases. Social welfare increases due to the competition between schools. This satisfies both students and other agents. At the same time, the social planner may find this as a good policy. He may intend to maximize the consumer surplus and also the social welfare by

keeping the voucher really low. Finally, all agents in the education market may turn out to be content with this policy.

Further, I will state several limitations of the model. Firstly, it is the government of the country who decides on implementing a voucher policy or not by the end. Then, there is always a limit on the value of the voucher. The government may intent to maximize the consumer surplus by lowering the voucher's value. So, the students will go to private school as the quality is higher. But, this may not be the case for all of them because there exist some capacity restrictions of private school. Secondly, I restrict in my model the cost that each of schools has to afford implementing the voucher. On the other hand, there is the government which may not do that. Finally, there are several externalities, that I do not consider. I say that more students go to private school and this causes externality. Moreover, private school is the one that invests less on quality. This creates the effect of segregation and the public school remains the poorest, because of the reduction of own quality.

Finally, it is important to understand the impact of voucher as a public policy in the education market. For further research, the paper can be extended to use different models or social welfare functions. However, to be completely sure about my results, I would suggest proving them with more empirical works in order to attempt to estimate a value for the optimal voucher.

A Appendix

A.1 Private duopoly. Proof of Proposition 1.

I start by considering the case in which there are two private schools in the education market. In order to guarantee strictly positive equilibrium qualities for both schools, I use Assumption 1. The equilibrium of the sequential game is determined by backward induction. The student choosing a school in the private duopoly obtains the following utility: $\theta x_i - p_i$. We have just two private schools, and a student choosing the high quality one, means that this school gives him more utility: $\theta x_1 p_1 > \theta x_2 p_2$.

The profit function of the private school is as below:

$$\pi_1 = (p_1 - \frac{cx_1^2}{2})(\bar{\theta} - \frac{p_1 - p_2}{x_1 - x_2}). \tag{38}$$

Then, the public school's profit function is as following:

$$\pi_2 = (p_2 - \frac{cx_2^2}{2})(\frac{p_1 - p_2}{x_1 - x_2} - \underline{\theta}). \tag{39}$$

Here, it is the case that both schools maximize profits.

In stage 2, by deriving each of the profit functions with respect to prices; I obtain p_1 and p_2 respectively equal to:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow \frac{-4p_1 + 2p_2 + 2\bar{\theta}x_1 + cx_1^2 - 2\bar{\theta}x_2}{2(x_1 - x_2)} = 0. \tag{40}$$

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Leftrightarrow \frac{2p_1 - 4p_2 - 2\underline{\theta}x_1 + 2\underline{\theta}x_2 + cx_2^2}{2x_1 - 2x_2} = 0. \tag{41}$$

After solving the equations for p_1 and p_2, there are obtained respectively the following:

$$p_1 = \frac{1}{6}[4\bar{\theta}(x_1 - x_2) + 2\underline{\theta}(-x_1 + x_2) + c(2x_1^2 + x_2^2)]. \tag{42}$$

$$p_2 = \frac{1}{6}[2\bar{\theta}(x_1 - x_2) + 4\underline{\theta}(-x_1 + x_2) + c(x_1^2 + 2x_2^2)]. \tag{43}$$

In stage 1, I apply the derivative of each profit function with respect to qualities. The choice is made sequentially and solving the game by backward induction yields:

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow \frac{1}{36}[4\underline{\theta}^2 + 16\bar{\theta}^2 - 16\bar{\theta}cx_1 + \underline{\theta}(-16\underline{\theta} + 8cx_1) + c^2(3x_1^2 + 2x_1x_2 - x_2^2)] = 0. \tag{44}$$

$$\frac{\partial \pi_2}{\partial x_2} = 0 \Leftrightarrow \frac{1}{36}[-16\underline{\theta}^2 - 4\bar{\theta}^2 - 8\bar{\theta}cx_2 + 16\underline{\theta}(\bar{\theta} + cx_2) + c^2(x_1^2 - 2x_1x_2 - 3x_2^2)] = 0. \tag{45}$$

Let's solve the obtained equations for x_1 and x_2 respectively, in order to get a final expression for each of them:

$$x_1^\circ = \frac{5\bar{\theta} - \underline{\theta}}{4c}. \tag{46}$$

$$x_2^\circ = \frac{5\underline{\theta} - \bar{\theta}}{4c}. \tag{47}$$

As a final step, having these final expressions for qualities, I substitute each of them in the price and profit functions. The expressions generated depend only on cost (c) and willingness to pay ($\theta's$), same as with the quality functions. Thus, it gives us the following:

$$p_1^\circ = \frac{25\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 49\bar{\theta}^2}{32c}. \tag{48}$$

$$p_2^\circ = \frac{49\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 25\bar{\theta}^2}{32c}. \tag{49}$$

Hence, the equilibrium profits are as below:

$$\pi_1^\circ = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c}. \tag{50}$$

$$\pi_2^\circ = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c}. \tag{51}$$

A.2 Mixed duopoly-Simple case with unconstrained equilibrium. Proof of Proposition 2.

Let's see what happens in the case in which there is one private and one public school in education market. This is the case when $\alpha = 1$. Thus, the private school is a profit maximizer one, while public school aims maximizing social welfare. In this case to guarantee strictly positive equilibrium qualities for both schools, I use Assumption 1. The equilibrium of the sequential game is determined by backward induction. The utility obtained by the student when choosing in the private duopoly is equal to: $\theta x_i - p_i$. There are two schools (public and private) and the students that choose the school that give them more utility.

Here, it is considered the consumer surplus (CS) and social welfare function (W). Their

functions are represented by the following equations:

$$CS_m = \int_{\hat{\theta}}^{\bar{\theta}} (\hat{\theta}x_1 - p_1)d\hat{\theta} + \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta}x_2 - p_2)d\hat{\theta}. \tag{52}$$

$$W_m = \alpha(CS_m + \pi_1) + \pi_2. \tag{53}$$

The profit function of the private school is as below:

$$\pi_1 = (p_1 - \frac{cx_1^2}{2})(\bar{\theta} - \frac{p_1 - p_2}{x_1 - x_2}). \tag{54}$$

Then, the public school's profit function is as following:

$$\pi_2 = (p_2 - \frac{cx_2^2}{2})(\frac{p_1 - p_2}{x_1 - x_2} - \underline{\theta}). \tag{55}$$

In stage 2, deriving the profit function of private school and the social welfare function for public school with respect to prices; hence, p_1 and p_2 are respectively equal to:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow \frac{-4p_1 + 2p_2 + 2\bar{\theta}x_1 + cx_1^2 - 2\bar{\theta}x_2}{2(x_1 - x_2)} = 0. \tag{56}$$

$$\frac{\partial W_m}{\partial p_2} = 0 \Leftrightarrow \frac{2p_1 - 2p_2 - 2cx_1 + cx_2^2}{2x_1 - 2x_2} = 0. \tag{57}$$

After solving the equations for p_1 and p_2, there are obtained respectively the following:

$$p_1 = \bar{\theta}(x_1 - x_2) + \frac{cx_2^2}{2}. \tag{58}$$

$$p_2 = \bar{\theta}(x_1 - x_2) + c(\frac{-x_1^2}{2} + x_2^2). \tag{59}$$

In stage 2, there are applied the same steps as in private duopoly case, but now with quality functions. As the choice is made sequentially, but the game is solved by backward induction, it yields:

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow \frac{1}{4}[4\bar{\theta}^2 - 8\bar{\theta}cx_1 + c^2(3x_1^2 + 2x_1x_2 - x_2^2)] = 0. \tag{60}$$

$$\frac{\partial W_m}{\partial x_2} = 0 \Leftrightarrow \frac{1}{4}[-4\underline{\theta}^2 + 8\underline{\theta}cx_2 + c^2(x_1^2 - 2x_1x_2 - 3x_2^2)] = 0. \tag{61}$$

Let's solve the obtained equations for x_1 and x_2 respectively, in order to get a final expression

for each of them;

$$x_1^m = \frac{\underline{\theta} + 3\bar{\theta}}{4c}. \tag{62}$$

$$x_2^m = \frac{3\underline{\theta} + \bar{\theta}}{4c}. \tag{63}$$

As a final step, having these expressions for qualities, we substitute each of them in the price, profit, consumer surplus and social welfare functions. The functions generated depend only on cost and $\theta's$. Thus, it gives us the following:

$$p_1^m = \frac{9\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 17\bar{\theta}^2}{32c}. \tag{64}$$

$$p_2^m = \frac{17\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 9\bar{\theta}^2}{32c}. \tag{65}$$

Hence, each schools' profit functions result to be equal:

$$\pi_1^m = \frac{(\bar{\theta} - \underline{\theta})^3}{8c}. \tag{66}$$

$$\pi_2^m = \frac{(\bar{\theta} - \underline{\theta})^3}{8c}. \tag{67}$$

The indifferent student will be equal to the one in private duopoly, where:

$$\hat{\theta}^m = \frac{\underline{\theta} + \bar{\theta}}{2}. \tag{68}$$

Then, the consumer surplus and social welfare functions are represented by the following expressions:

$$CS^m = \frac{(\underline{\theta} - \bar{\theta})(3\underline{\theta}^2 - 22\underline{\theta}\bar{\theta} + 3\bar{\theta}^2)}{32c}. \tag{69}$$

$$W^m = \frac{(-\underline{\theta} + \bar{\theta})(5\underline{\theta}^2 + 6\underline{\theta}\bar{\theta} + 5\bar{\theta}^2)}{32c}. \tag{70}$$

A.3 Private duopoly behaviour with binding constraints (Vouchers). Proof of Proposition 3.

I start considering the case in which there are vouchers in the education market. In this case to guarantee strictly positive equilibrium qualities for both schools, it has to be fulfilled the Assumption 1. The equilibrium of the sequential game is determined by backward induction.

The student choosing a school in the private duopoly obtains an utility equal to: $\theta x_i p_i$. This is the case when $\alpha = 0$. We have a mixed duopoly, where public firm behaves as a private schools. Thus, it seems that both schools are private and their aim is to maximize own profits. In stage 0, the voucher is applied as a reduction in the price of the private school (p_1).

The social welfare and consumer surplus, I consider for this case are as following:

$$W_r = \alpha(\pi_1 + CS_r) + \pi_2, \quad \text{with} \quad 0 \leq \alpha \leq 1. \tag{71}$$

$$CS_r = \int_{\hat{\theta}}^{\bar{\theta}} (\hat{\theta}x_1 - p_1 + v(\bar{\theta} - \hat{\theta}))d\hat{\theta} + \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta}x_2 - p_2)d\hat{\theta}. \tag{72}$$

Hence, if $\alpha = 0$, it gives us $W_r = \pi_2$ and the public school maximizes its profits, behaving like a private school. Then, I represent the profit functions of the schools (private and public) as below:

$$\pi_1^r = (p_1 - \frac{cx_1^2}{2})(\bar{\theta} - \hat{\theta}). \tag{73}$$

$$\pi_2^r = (p_2 - \frac{cx_2^2}{2})(\hat{\theta} - \underline{\theta}). \tag{74}$$

To derive the mixed equilibrium, I follow the steps as in the simple cases without voucher. However, there are some differences in this case because the equations are more complex.

In stage 1, I derive the profit function of private school and the social welfare function for public school with respect to prices. I obtain p_1 and p_2 functions respectively equal to:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow -\frac{-4p_1 + 2p_2 + 2\bar{\theta}x_1 + cx_1^2 - 2\bar{\theta}x_2}{2(v - x_1 + x_2)} = 0. \tag{75}$$

$$\frac{\partial W_r}{\partial p_2} = 0 \Leftrightarrow -\frac{2p_1 - 4p_2 + 2\underline{\theta}v - 2\bar{\theta}v - 2\underline{\theta}x_1 + 2\underline{\theta}x_2 + cx_2^2}{2(v - x_1 + x_2)} = 0. \tag{76}$$

After solving the equations for p_1 and p_2, there are obtained respectively the following:

$$p_1 = \frac{1}{6}[2\underline{\theta}(v - x_1 + x_2) - 2\bar{\theta}(v - 2x_1 + 2x_2) + c(2x_1^2 + x_2^2)]. \tag{77}$$

$$p_2 = \frac{1}{6}[4\underline{\theta}(v - x_1 + x_2) - 2\bar{\theta}(2v - 2x_1 + 2x_2) + c(x_1^2 + 2x_2^2)]. \tag{78}$$

Similarly, there are applied the same steps as in simple mixed duopoly case, but now with quality functions. As the choice is made sequentially, and the game is solved by backward

induction, in stage 1 it yields:

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow \tag{79}$$

$$\frac{\bar{\theta}(-6v+4x_1-4x_2)+2\underline{\theta}(v-x_1+x_2)+c(4vx_1-3x_1^2+4x_1x_2-x_2^2)(2\underline{\theta}(v-x_1+x_2)-2\bar{\theta}(v-2x_1+2x_2)+c(-x_1^2+x_2^2))}{36(v-x_1+x_2)^2} = 0.$$

$$\frac{\partial W_r}{\partial x_2} = 0 \Leftrightarrow \tag{80}$$

$$\frac{(4\underline{\theta}(v-x_1+x_2)-2\bar{\theta}(2v-x_1+x_2)+c(x_1^2-x_2^2))(2\bar{\theta}(-x_1+x_2)-4\underline{\theta}(v-x_1+x_2)+c(x_1^2+4vx_2-4x_1x_2+3x_2^2))}{36(v-x_1+x_2)^2} = 0.$$

Let's solve the obtained equations for x_1 and x_2 respectively. By using the Assumption 1 and the condition $\bar{\theta} = \underline{\theta} + 1$ we get a final expression for each of them:

$$x_1^r = \frac{5\bar{\theta} - \underline{\theta}}{4c} - \frac{v}{3+4cv}. \tag{81}$$

$$x_2^r = \frac{5\underline{\theta} - \bar{\theta}}{4c} - \frac{v}{3+4cv}. \tag{82}$$

Then, having these final expressions for qualities, we substitute each of them in the price, profit, consumer surplus and social welfare functions. The functions generated depend only on cost and $\theta's$ as qualities' functions. Thus, it gives us the following:

$$p_1^r = \frac{25\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 49\bar{\theta}^2}{32c} + \frac{-63v - 36\underline{\theta}v + 138cv^2 + 48\underline{\theta}cv^2 - 64c^2v^3}{12(-3+4cv)^2}. \tag{83}$$

$$p_2^r = \frac{49\underline{\theta}^2 - 58\underline{\theta}\bar{\theta} + 25\bar{\theta}^2}{32c} + \frac{-81v - 36\underline{\theta}v + 210cv^2 + 48\bar{\theta}cv^2 - 128c^2v^3}{12(-3+4cv)^2}. \tag{84}$$

The profit functions result to be as below:

$$\pi_1^r = \frac{-3(\underline{\theta} - \bar{\theta})^3}{8c} + \frac{27v + 24cv^2 - 64c^2v^3}{36(-3+4cv)^2}. \tag{85}$$

$$\pi_2^r = -\frac{3(\underline{\theta} - \bar{\theta})^3}{8c} + \frac{-189v + 456cv^2 - 256c^2v^3}{36(-3+4cv)^2}. \tag{86}$$

Hence, the marginal student after having implemented the voucher in private school's price will be as following:

$$\hat{\theta}^r = \frac{\bar{\theta} + \underline{\theta}}{2} + \frac{2cv}{3(-3+4cv)}. \tag{87}$$

Then, consumer surplus and social welfare function are as following:

$$CS_r = \frac{11}{18v} + \frac{5}{32(3c-4c^2v)} - \frac{3}{4c} + \frac{\underline{\theta} + \underline{\theta}^2}{2c} - \frac{1}{96c(3-4cv)^2}. \tag{88}$$

$$W_r = \pi_2^r = -\frac{3(\underline{\theta} - \bar{\theta})^3}{8c} + \frac{-189v + 456cv^2 - 256c^2v^3}{36(-3+4cv)^2}. \tag{89}$$

A.4 Private duopoly (Optimal Voucher). Proof of Proposition 4.

In order to find the optimal voucher, it suffices doing the first order condition of consumer surplus (CS_r) with respect to voucher (v):

$$\frac{\partial CS_r}{\partial v} = 0 \Leftrightarrow \frac{11}{18} + \frac{5}{8(3-4cv)^2} + \frac{3}{2(-3+4cv)^3} = 0 \tag{90}$$

Finally, by solving the equation above, it gives us the optimal voucher function:

$$v^r_{optimal} = \frac{66c^4 + (33(-396 + \sqrt[2]{160941}))^{\frac{1}{3}} c^2 (c^6)^{\frac{1}{3}} - (33(396 + \sqrt[2]{160941}))^{\frac{1}{3}} (c^6)^{\frac{2}{3}}}{88c^5}. \tag{91}$$

A.5 Mixed duopoly with unconstrained equilibrium (Vouchers). Proof of Proposition 5.

Let's consider the case, in which there is one private and one public school in education market. This is the case when $\alpha = 1$. Private school is a profit maximizer one, while public school aims maximizing social welfare. In this case to guarantee strictly positive equilibrium qualities for both schools, I use Assuption 1 and also the condition $\bar{\theta} = \underline{\theta} + 1$. The equilibrium of the sequential game is determined by backward induction. The student choosing a school in the private duopoly obtains an utility equal to: $\theta x_i p_i$. We have two schools (public and private) and the student choosing the school that gives him more utility. Moreover, it is added the voucher as a reduction in the price of private school. Thus, we expect different results from the Mixed duopoly case without vouchers.

Here, it is considered the consumer surplus (CS_v) and the social welfare function (W_v). Their functions are represented by following equations:

$$CS_v = \int_{\hat{\theta}}^{\bar{\theta}} (\hat{\theta}x_1 - p_1 + v(\bar{\theta} - \hat{\theta}))d\hat{\theta} + \int_{\underline{\theta}}^{\hat{\theta}} (\hat{\theta}x_2 - p_2)d\hat{\theta}. \tag{92}$$

$$W_v = \alpha(\pi_1 + CS_v) + \pi_2, \quad \text{with} \quad 0 \leq \alpha \leq 1. \tag{93}$$

The profit function of the private school is as below:

$$\pi_1^v = (p_1 - \frac{cx_1^2}{2})(\bar{\theta} - \frac{-p_1 + p_2 + \bar{\theta}v}{v - x_1 + x_2}). \tag{94}$$

Then, the public school's profit function is as following:

$$\pi_2^v = (p_2 - \frac{cx_2^2}{2})(\frac{-p_1 + p_2 + \bar{\theta}v}{v - x_1 + x_2} - \underline{\theta}). \tag{95}$$

In stage 1, by deriving the profit function of private and public school with respect to prices: p_1 and p_2, they are respectively equal to:

$$\frac{\partial \pi_1^v}{\partial p_1} = 0 \Leftrightarrow -\frac{-4p_1 + 2p_2 + 2\bar{\theta}x_1 + cx_1^2 - 2\bar{\theta}x_2}{2(v - x_1 + x_2)} = 0. \tag{96}$$

$$\frac{\partial \pi_2^v}{\partial p_2} = 0 \Leftrightarrow -\frac{2p_1 + 2p_2 + c(x_1^2 - x_2^2)}{2(v - x_1 + x_2)} = 0. \tag{97}$$

Now, in stage 2, I apply the same steps as in private duopoly case, but with quality functions. It is already known that the choice is made sequentially and the game is solved by backward induction. Thus, it yields:

$$\frac{\partial \pi_1^v}{\partial x_1} = 0 \Leftrightarrow \tag{98}$$

$$\frac{(x_1 - x_2)(-2\bar{\theta} + c(x_1 - x_2))(2\bar{\theta}(2v - x_1 + x_2) + c(-4vx_1 + 3x_1^2 - 4x_1x_2 + x_2^2))}{4(v - x_1 + x_2)^2} = 0$$

.

$$\frac{\partial \pi_2^v}{\partial x_2} = 0 \Leftrightarrow \tag{99}$$

$$\frac{(2\bar{\theta}v - 2\underline{\theta}(v - x_1 + x_2) + c(-x_1^2 + x_2^2))(2\bar{\theta}v + 2(v - x_1 + x_2) - c(x_1^2 + 4vx_2 - 4x_1x_2 + 3x_2^2))}{8(v - x_1 + x_2)^2} = 0$$

.

Let's solve the obtained equations for x_1 and x_2 respectively, in order to get a final expression for each of them:

$$x_1^v = \frac{\underline{\theta} + 3\bar{\theta}}{4c} - \frac{v}{1+4cv}. \tag{100}$$

$$x_2^v = \frac{3\underline{\theta} + \bar{\theta}}{4c} - \frac{v}{1+4cv}. \tag{101}$$

As a final step, I substitute each of the above expressions in the price, profit, consumer surplus and social welfare functions. The functions generated depend only on cost and $\theta's$. Thus, it gives us the following:

$$p_1^v = \frac{9\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 17\bar{\theta}^2}{32c} + \frac{-v - 4\underline{\theta}v - 6cv^2 + 16\underline{\theta}cv^2}{4(-1+4cv)^2}. \tag{102}$$

$$p_2^v = \frac{17\underline{\theta}^2 - 10\underline{\theta}\bar{\theta} + 9\bar{\theta}^2}{32c} + \frac{v - 4\underline{\theta}v - 2cv^2 + 16\underline{\theta}cv^2}{4(-1+4cv)^2}. \tag{103}$$

Then, the marginal consumer has a function as below:

$$\hat{\theta}^v = \frac{\underline{\theta} + \bar{\theta}}{2} + \frac{2cv}{-1+4cv}. \tag{104}$$

Each schools' profit functions result to be equal:

$$\pi_1^v = \frac{(-\underline{\theta} + \bar{\theta})^3}{8c} + \frac{3v - 8cv^2}{4(-1+4cv)^2}. \tag{105}$$

$$\pi_2^v = \frac{(-\underline{\theta} + \bar{\theta})^3}{8c} + \frac{-v}{4(-1+4cv)^2}. \tag{106}$$

Then, consumer surplus and social welfare function are as following:

$$CS_v = \frac{(\underline{\theta} - \bar{\theta})(3\underline{\theta}^2 - 22\underline{\theta}\bar{\theta} + 3\bar{\theta}^2)}{32c} + \frac{3v}{8(-1+4cv)^2}. \tag{107}$$

$$W_v = \frac{(-\underline{\theta} + \bar{\theta})(5\underline{\theta}^2 + 6\underline{\theta}\bar{\theta} + 5\bar{\theta}^2)}{32c} + \frac{-v}{8(-1+4cv)^2}. \tag{108}$$

In order to find the optimal voucher, it suffices doing the first order condition of consumer surplus (CS_v) with respect to voucher (v):

$$\frac{\partial CS_v}{\partial v} = 0 \Leftrightarrow -\frac{3}{8(1-4cv^2)} = 0. \tag{109}$$

The derivative function results negative. Thus, it can not be obtained the exact function of the optimal voucher. This means that when voucher increases the consumer surplus will be positive but will decrease.

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