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Unilateral Effects Screens for Partial Horizontal Acquisitions: The Generalized *HHI* and *GUPPI*[*]

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Abstract

Recent years have witnessed an increased interest, by competition agencies, in assessing the competitive effects of partial acquisitions. We propose a generalization of the two most traditional indicators used to screen unilateral anti-competitive effects - the Herfindahl-Hirschman Index and the Gross Upward Price Pressure Index - to partial horizontal acquisition settings. The proposed generalized indicators are endogenously derived under a probabilistic voting model in which the manager of each firm is elected in a shareholder assembly between two potential candidates who seek to obtain utility from an exogenous rent associated with corporate office. The model (*i*) can cope with settings involving all types of owners and rights: owners that can be internal to the industry (rival firms) and external to the industry; and rights that can capture financial and corporate control interests, can be direct and indirect, can be partial or full, (*ii*) yields an endogenous measure of the owners ultimate corporate control rights, and (*iii*) can also be used - in case the potential acquisition is inferred to likely enhance market power - to devise divestiture structural remedies. We also provide an empirical application of the two proposed generalized indicators to several acquisitions in the wet shaving industry, with the objective of providing practitioners with a step-by-step illustration of how to compute them in antitrust cases.

JEL Classification: L13, L41, L66

Keywords: Antitrust, Partial Horizontal Acquisitions, Oligopoly, Screening Indicators, HHI, GUPPI, Corporate Control, Banzhaf Power Index

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1 Introduction

Full acquisitions complete and permanently eliminate competition among the firms involved in the transaction. This constitutes the basic element of a merger analysis. *Partial acquisitions*, in contrast, do not completely and permanently eliminate competition among firms. Nevertheless, they may present - and recent empirical work confirms this - significant competitive concerns (see, e.g., Azar, Schmalz and Tecu, forthcoming; Azar, Raina and Schmalz, 2016).[1] As a consequence, competition agencies have taken an increased interest in assessing their anti-competitive effects.

Following the long theoretical literature in industrial organization, competition agencies have typically focused on acquisitions settings involving owners that are *internal* to the industry (rival firms), which induce a *cross-ownership* structure. Some recent examples include the UK Competition Commission assessment of the BskyB's proposed acquisition of a 17.9% stake in ITV and the European Commission assessment of the News Corporation's proposed acquisition of an approximately 25% stake in Premiere.

However, the phenomenal growth of private equity investment in recent years has led agencies to focus also on acquisitions settings involving owners that are *external* to the industry, but participate in more than one competitor firm, which induce a *common-ownership* structure. A recent example includes the FTC assessment of the Kinder Morgan buyout by (among others) private equity funds managed and controlled by the Carlyle Group and Riverstone Holdings LLC, which already held a significant partial ownership stake in Magellan Midstream, a major competitor of Kinder Morgan.

The anti-competitive effects of partial horizontal acquisitions giving rise to cross- or common-ownership structures depend heavily on whether the ownership rights involved in the acquisition capture *financial* or *corporate control* interests. The former refer to the rights of the (partial) owner to receive the stream of profits generated by the operations and investments of the acquired firm, while the latter refer to the rights of the (partial) owner to influence the decisions of the acquired firm. We need to identify and distinguish the two rights because partial acquisitions that do not result in effective control present competitive concerns distinct from partial acquisitions involving effective control. When a party (internal or external to the industry) acquires partial financial rights in a firm, it acquires a share of its profits. Such acquisition can lessen competition by reducing the incentive of the *acquiring party's firm* to compete aggressively, given it shares in the losses thereby inflicted on the acquired rival. On the other hand, when a party (internal or external to the industry) acquires corporate control rights in a firm, it acquires the ability to influence the competitive conduct of that firm. Such influence can lessen competition because it may be used to induce the *acquired firm* to compete less aggressively against the acquiring party's firm.

Brito, Ribeiro and Vasconcelos (2014) propose an empirical structural methodology to quantitatively assess the unilateral anti-competitive effects of partial horizontal acquisitions. However, competition agencies are typically given a very short period of time to analyze a potential acqui-

[1] Azar, Schmalz and Tecu (forthcoming) examine the U.S. airline industry and find that the interlinks in the ownership of the airlines matters for how the airlines compete. Azar, Raina and Schmalz (2016) find the same relation in the U.S. banking industry.

sition upon receiving its notification, with little data available before deciding whether to issue a second request. In this paper, we propose a generalization of the two most traditional indicators used to screen unilateral anti-competitive effects - the *Herfindahl-Hirschman Index* (*HHI*) and the *Gross Upward Price Pressure Index* (*GUPPI*) - to partial horizontal acquisition settings. The proposed indicators can be computed even within the time frame of phase I-type of investigations and with the data submitted in a typical notification. Further, they are endogenously derived under a probabilistic voting model in which the manager of each firm is elected in a shareholder assembly between two potential candidates who seek to obtain utility from an exogenous rent associated with corporate office. The model can cope with settings involving all types of owners and rights: owners that can be internal to the industry (rival firms) and external to the industry; and rights that can capture financial and corporate control interests, can be direct and indirect, can be partial or full.[2]

The contribution of the two proposed generalized indicators to the literature is three-folded. First, they combine two separate strands of the literature: the literature on cross-ownership and the literature on common-ownership. This is particularly important for competition agencies not only because real world industries can be characterized by both cross- and common-ownership structures (see, e.g., Azar, Raina and Schmalz, 2016), but also because external owners may use indirect partial ownership interests to evade antitrust rules that limit direct ownership in rivals. Such contribution is markedly significant to the *GUPPI*, for which the literature is almost inexistent. Second, they contribute to the literature that seeks to identify controlling owners within arbitrary ownership structures (see, e.g., Dorofeenko *et al.*, 2008) because the proposed indicators incorporate an endogenous measure, expressed by the normalized Banzhaf (1965) power index, of the owners' influence over the decision-making within a firm, which can cope with industries that can be characterized by both cross- and common-ownership structures. Third, they contribute to the literature on structural remedies because in addition to screen potential anti-competitive unilateral effects regarding partial horizontal acquisitions, the proposed indicators can also be used - in cases the potential acquisition is inferred to likely enhance market power - to devise divestiture structural remedies.

We also provide an empirical application of the two proposed generalized indicators to several acquisitions in the wet shaving industry, with the objective of providing practitioners with a step-by-step illustration of how to compute them in antitrust cases. On December 20, 1989, the Gillette Company, which had been the market leader for years and accounted for 50% of all razor blade units sales, contracted to acquire the wet shaving businesses of Wilkinson Sword in the U.S. (among other operations) to Eemland Management Services BV (Wilkinson Sword's parent company) for \$72 million. It also acquired a 22.9% non-voting stake in Eemland for about \$14 million. On January 10, 1990, the Department of Justice (DoJ) instituted a civil proceeding against Gillette. The complaint alleged that the effect of the acquisition by Gillette may have been substantially to lessen competition in the U.S. wet shaving razor blades' market. Shortly after the case was

[2] An owner has indirect partial ownership rights in firm B if it holds partial ownership rights in firm A and, in turn, firm A holds partial ownership rights in firm B.

filed, Gillette voluntarily rescinded the acquisition of Eemland's wet shaving razor blade business in the U.S., but went through with the acquisition of the 22.9% *non-voting* stake in Eemland. The DoJ approved the acquisition after being assured that this stake would be passive. These two acquisitions and two additional hypothetical ones, are screened below. An interesting result derived from this empirical application is that acquisitions that give rise to common-ownership structures in which external owners *partially* participate in more than one competitor firm *may* induce higher unilateral anti-competitive effects than acquisitions, involving the same rights, that give rise to cross-ownership structures.

This paper is organized as follows: Section 2 reviews the literature, Section 3 presents the theoretical framework, Section 4 develops the two proposed generalized indicators, Section 5 provides the above mentioned empirical application, and Section 6 concludes.

2 Literature Review

This section reviews *(i)* the theoretical frameworks employed in the literature to model cross- and common-ownership, and *(ii)* the generalized screening indicators proposed in the literature to evaluate the unilateral effects of partial horizontal acquisitions.

2.1 Theoretical Frameworks

We begin by addressing the theoretical frameworks employed in the literature to model cross- and common-ownership.

2.1.1 Cross-Ownership

The theoretical framework typically employed in the literature to model cross-ownership is rooted either in the tradition of Reynolds and Snapp (1986) or in the tradition of Ellerman (1991). The former model cross-ownership in the perspective of firms (internal owners) while the latter models cross-ownership in the perspective of external owners. The two approaches complement each other.

Reynolds and Snapp (1986) are the first to model cross-ownership. They do so by focusing on cross-ownership structures of *direct financial rights*, which they model by introducing the distinction between a firm's operating and aggregated profit. The reason being that, in such settings, the aggregate profit of a firm includes not just the stream of profits generated by the firm's own operations, but also a share in the aggregate profits of the rivals in which it holds financial rights. This initial approach is afterwards extended in two directions. The first direction is taken by Bresnahan and Salop (1986), who maintain Reynolds and Snapp (1986)'s focus on direct rights but introduce the distinction between *financial and corporate control rights*. They model the latter by allowing holders of corporate control rights to influence the pricing and output decisions of the firm. However, they do not address the question of how to measure them. They consider, instead, limiting corporate control arrangements that involve either full influence or no influence at all in the firm's decisions.[3] The second direction is taken by Flath (1992), who maintains Reynolds and Snapp (1986)'s focus on financial rights but introduce the distinction between *direct and indirect* rights. He models the latter by noting that if *(i)* the aggregated profit of a firm is a function of the operating profit of the firm and the aggregated profit of the rival firms (in which it holds financial rights), and *(ii)* the same is true for all firms in the industry, then we can write a system of aggregate profit equations that implicitly determines the aggregate profit of each firm as a function of the operating profit of all the firms in the industry over which the firm holds financial rights, either direct or indirectly.

Ellerman (1991), in contrast with the above tradition that (by focusing on profits) takes the perspective of firms, models cross-ownership in the perspective of external owners. She does so by noting that a cross-ownership of financial and corporate control rights *changes the distribution of those rights* among external owners. In particular, she shows that cross-ownership changes that distribution in a way that induces common-ownership among external owners, even if this common-ownership is, in the absence of cross-ownership, non-existent. In order to see why, note - for example - that an external owner with sole *direct* rights in a firm (for example, firm A) has in fact *ultimate* rights in two rival firms, firm A and rival firm B, if firm A engages in cross-ownership and holds rights in firm B. As a consequence, Ellerman (1991) models cross-ownership as *induced* common-ownership, by computing the ultimate rights of external owners on the different firms that result from it.

2.1.2 Common-Ownership

The theoretical framework employed in the literature to model common-ownership is rooted in the tradition of O'Brien and Salop (2000). They argue that a common-ownership of rights *may induce a conflict* of objectives among the external owners of a firm and, for that reason, its manager must weight the (eventual) conflicting objectives of the different external owners according to the corporate control structure of the firm, which determines the influence of each of those owners over the decision-making. In order to see why, note - for example - that an external owner of firm A who also holds financial rights in a rival firm B typically wants firm A to pursue a less aggressive strategy than the strategy desired by an external owner which does not hold financial rights in firm B. To model this feature, O'Brien and Salop (2000) assume that (*i*) external owners seek to maximize the utility obtained from the return of their holdings of financial rights over the different firms, and (*ii*) all external owners have a linear utility function. This implies that the manager of the firm should maximize a corporate control weighted sum of the owners' returns. However, as Reynolds and Snapp (1986), they do not address the question of how to measure the owners' corporate control rights. Campos and Vega (2003) discuss this issue and present different measurement alternatives. However, the measures presented are exogenous to O'Brien and Salop

[3] Bresnahan and Salop (1986) also consider intermediate arrangements. However, those are modelled as a limiting case (influence or no influence) coupled with an ex-post redistribution of profits among the firms involved in the arrangement.

(2000)'s theoretical framework.

Azar (2017) extends O'Brien and Salop (2000)' framework by showing that their mathematical formulation can be microfounded through a probabilistic voting model in which owners vote to *either* (i) express whether they approve or not of a managerial change in the firm's status quo strategic plan *or* (ii) elect one of two potential managers. Further, he shows that if, in the former, the opposition against the new strategic plan imposes a cost to the manager that is linearly increasing in the share of votes against or, in the latter, the two candidates maximize the expected vote share within the firm, the corporate control rights of the owners can be endogenously measured by their holdings of voting stock. Furthermore, he also discusses (and in an earlier version of the article actually derives - see Azar, 2016) that if, in the former, the opposition against the new strategic plan imposes a cost to the manager that is zero below a certain threshold of the share of votes against, and a positive constant above the threshold or, in the latter, the two candidates maximize the probability of being elected, the corporate control rights of the owners can be endogenously measured by the Banzhaf (1965) power index that results from their holdings of voting stock.

2.1.3 Cross- and Common-Ownership

The theoretical framework typically employed in the literature to model both cross- and common-ownership is rooted in Brito, Ribeiro and Vasconcelos (2014). They combine a cross-ownership model in the lines of Reynolds and Snapp (1986) and Flath (1992) with a common-ownership model in the lines of O'Brien and Salop (2000). The resulting theoretical framework can cope with acquisition settings involving *internal and external owners* as well as *financial and corporate control rights* that can be *direct and indirect*, *partial or full*. Further, by doing so, they establish a link to Ellerman (1991)'s theoretical framework. Combining Reynolds and Snapp (1986), Flath (1992) and O'Brien and Salop (2000) is entirely equivalent to combining Ellerman (1991) and O'Brien and Salop (2000). However, as Reynolds and Snapp (1986) and O'Brien and Salop (2000), they do not address the question of how to measure the owners' corporate control rights and, following Campos and Vega (2003), just propose different exogenous measurement alternatives.

2.2 Screening Indicators

We now address the generalized screening indicators proposed in the literature to evaluate the unilateral effects of partial horizontal acquisitions. Table 1 summarizes the different proposals according to the indicator and the type of owners and rights involved.

2.2.1 Herfindahl-Hirschman Index

The *HHI* constitutes one of the most traditional indicators to screen the unilateral anti-competitive effects of a merger, due to its strong theoretical support. The change in *HHI* emerges as an appropriate measure of such effects under (i) the Cournot model of quantity competition among firms in homogeneous-product industries and (*ii*) some models of ordered bargaining (Moresi, Salop and Sarafidis, 2008).[4]

The literature on unilateral effects screening indicators for partial horizontal acquisitions is typically rooted on the Cournot model of quantity competition and focused initially on *cross-ownership* acquisitions. Reynolds and Snapp (1986) use their theoretical framework to develop a modified *HHI* to screen the unilateral effects of acquisitions that induce a *cross-ownership of direct financial rights*. Bresnahan and Salop (1986) maintain Reynolds and Snapp (1986)'s focus on direct rights and extend their proposal to corporate control rights. They do so by developing a set of modified *HHI*s to screen the unilateral effects of acquisitions inducing a set of alternative *cross-ownership* structures of *direct financial and corporate control rights* (established in their theoretical framework). Dietzenbacher, Smid and Volkerink (2000), on the other hand, maintain Reynolds and Snapp (1986)'s focus on financial rights and extend their proposal to indirect rights. They build on the theoretical framework of Flath (1992) to develop a modified *HHI* to screen the unilateral effects of acquisitions inducing a *cross-ownership of direct and indirect financial rights*.

O'Brien and Salop (2000) are the first to address an unilateral effects screening indicator for *common-ownership* acquisitions. They use their theoretical framework to develop a modified *HHI* to screen the unilateral effects of acquisitions that induce *a common-ownership of direct financial and corporate control rights*.

Finally, Azar, Raina and Schmalz (2016), building on the theoretical framework established in Ellerman (1991), O'Brien and Salop (2000), Brito, Ribeiro and Vasconcelos (2014), and on an earlier version of this article, develop a concurrent generalized *HHI* for acquisitions inducing *cross- and common-ownership* structures, which can cope with acquisitions of *direct and indirect, financial and corporate control rights*.

2.2.2 Gross Upward Price Pressure Index

The *GUPPI* constitutes also one of the most traditional indicators to screen the unilateral anti-competitive effects of a merger. It emerges as an appropriate measure of such effects under (*i*) the Bertrand model of price competition among firms in differentiated-product industries, (*ii*) the Cournot model of quantity competition among firms in homogeneous- and differentiated-product industries (Moresi, 2010), and (*iii*) bidding competition models (Moresi, 2010).

The literature on unilateral effects screening indicators for partial horizontal acquisitions is almost inexistent. To the best of our knowledge there are only two contributions in the literature, rooted on the Bertrand model of price competition among firms in heterogeneous-product industries.

Dietzenbacher, Smid and Volkerink (2000) build on Flath (1992) to examine the impact of acquisitions that induce a *cross-ownership of direct and indirect financial rights*. However, they do not propose an indicator to screen whether the analyzed acquisitions lead to unilateral anti-

[4] In particular, Moresi, Salop and Sarafidis (2008) show that in models of ordered bargaining in which a lead buyer bargains (through the Nash-bargaining solution) sequentially with several sellers, one-at-a-time, a merger of sellers decreases the buyer's equilibrium payoff linearly with the sellers' HHI if the buyer's utility function is quadratic.

competitive effects.

O'Brien and Salop (2000) use their theoretical framework to examine the impact of acquisitions that induce a *common-ownership of direct financial and corporate control rights*. They do so building on Shapiro (1996)'s diversion ratio approach. They screen the unilateral effects of such acquisitions using a summary measure of the economic pressure to respond to the acquisitions by changing prices. They refer to this measure as a Price Pressure Index (*PPI*). Afterwards, Farrell and Shapiro (2010) build on this proposed *PPI* to develop an Upward Pricing Pressure test to screen the unilateral anti-competitive effects of mergers. A test that gave rise, later on, to the *GUPPI*, proposed by Salop and Moresi (2009). However, to the best of our knowledge, it was never generalized to partial horizontal acquisition settings.[5]

3 The Theoretical Framework

This section introduces the theoretical framework under which the generalized indicators are derived. The general setting combines features from Brito, Ribeiro and Vasconcelos (2014) and Azar (2017).

3.1 The Setup

There are N single-product firms, indexed by $j \in \Im \equiv \{1, ..., N\}$. There are also K owners, indexed by $k \in \Theta \equiv \{1, \ldots, N, \ldots K\}$, who may include not just owners $\Theta \backslash \Im$ that are external to the industry (and can engage in common-ownership), but also owners from the subset $\Im$ of firms that are internal to the industry (and can engage in cross-ownership).[6]

As discussed above, the anti-competitive effects of partial horizontal acquisitions depend heavily on whether the ownership rights involved in the acquisition capture *financial* or *corporate control* interests. In order to express the distinction between these two rights, we consider that the total stock of each firm j is composed of voting stock and non-voting (preferred) stock. Both give the holder a share of the stream of profits generated by the firm's operations and investments, but only the former gives the holder the right to vote for the Board or to participate in other decisions.

The holdings of total stock of owner k in firm j, regardless of whether it be voting or non-voting stock, represented by $0 \leq \phi_{kj} \leq 1$ with $\sum_{k \in \Theta} \phi_{kj} = 1$, capture her *financial rights* to the firm's stream of profits. The holdings of voting stock of owner k in firm j, represented by $0 \leq \upsilon_{kj} \leq 1$ with $\sum_{k \in \Theta} \upsilon_{kj} = 1$, capture her *voting rights* in the firm, which may not necessarily coincide with her *corporate control rights* in the firm, which refer to the rights of owner k to influence the decisions of firm j. In particular, we are considering two types of decisions: those regarding the strategic

[5] The only exception, although of a different nature, is Brito, Cabral and Vasconcelos (2014). They build on both O'Brien and Salop (2000) and Dietzenbacher, Smid and Volkerink (2000) to extend the analysis to aquisitions in industries characterized by (eventually) both a *cross-* and a *common-ownership* of *financial and corporate control rights*, either *direct or indirect*. They propose sufficient statistics for the effects of partial ownership (and divestiture of partial ownership) on consumer welfare. However, they do so only for a duopoly setting.

[6] The set $\Theta \backslash \Im$ denotes the set Θ excluding the firms in the set $\Im$.

variables (price, quantity, R&D investment, etc.) and also the decision of how firm j will vote when it owns voting stock in other firm(s). We assume corporate control rights are a function not just of the owner's voting rights in the firm, but also of the voting rights of all the firm's other owners. For instance, an owner who holds 49% of the voting rights in a firm may have no control over the decision-making within the firm if one other owner holds 51%. In contrast, an owner who holds 10% of the voting rights in a firm may have effective control over the decision-making within the firm if each of the remaining owners hold a tiny amount of the firm's voting rights.

We make the following assumption regarding the holdings of external owners in the industry:

Assumption 1 *External owners hold voting rights in at least one firm of the industry.*

Assumption 1 implies that the firms in the industry are not entirely held by the firms themselves.[7] As such, we have that $\sum_{k\in\Theta\backslash\Im} \upsilon_{kj} > 0$ for at least one firm j. Since the financial rights of an owner in a firm denotes the owner's holdings of total stock in the firm, regardless of whether it be voting or non-voting stock, we also have that $\sum_{k\in\Theta\backslash\Im} \phi_{kj} > 0$ for at least one firm j.

3.2 Cross-Ownership

We now address the theoretical framework to capture cross-ownership. As discussed above, a cross-ownership of (financial and/or corporate control) rights among rival firms *changes the distribution of those rights* among external owners. As such, we follow Ellerman (1991) in modelling cross-ownership as *induced* common-ownership, by computing the ultimate rights of external owners that result from it. We begin this analysis by focusing on financial rights.

3.2.1 Financial Rights

The ultimate financial rights of external owner k in firm j, ϕ^u_{kj}, includes not just her direct financial rights in the firm, ϕ_{kj}, but also the indirect financial rights that may arise from having ultimate financial rights in a rival $g \in \Im\backslash j$ that holds, in turn, financial rights in firm j. This implies that for all $k \in \Theta\backslash\Im$ and $j \in \Im$, we have:

$$\phi^u_{kj} = \phi_{kj} + \sum\nolimits_{g\in\Im\backslash j} \phi^u_{kg}\phi_{gj}, \tag{1}$$

where $\Im\backslash j$ denotes the set $\Im$ not including firm j, which constitutes a set of equations that implicitly determines the ultimate financial rights of each external owner as a function of the direct financial rights of all owners (internal and external).

In order to see why, let $\mathbf{F}$ and $\mathbf{F}^u$ denote the $(K-N)\times N$ matrices capturing the direct and ultimate financial rights, respectively, of external owners, with typical elements ϕ_{kj} and ϕ^u_{kj} representing the direct and ultimate financial rights, respectively, of external owner k in firm j. Let also $\mathbf{F}^*$ denote the $N\times N$ matrix capturing the direct financial rights of internal owners, with zero

[7] Furthermore, it implies also that we can cope with settings in which a firm can hold 100% of the financial rights of a rival firm.

diagonal elements, $\phi_{jj} = 0$, and off-diagonal elements, $0 \leq \phi_{jg} \leq 1$ (if $j \neq g \in \Im$), representing the direct financial rights of firm j in firm g. We can then use matrices $\mathbf{F}$, $\mathbf{F}^u$ and $\mathbf{F}^*$ to write the set of equations (1) in vector notation, as follows:

$$\mathbf{F}^u = \mathbf{F} + \mathbf{F}^u \mathbf{F}^*, \tag{2}$$

an equation that is related to the notion of eigenvector centrality used to investigate power and influence both in social and economic network (see, e.g., Vitali, Glattfelder and Battiston, 2011, and references therein). In order to solve for $\mathbf{F}^u$ explicitly we can rewrite it as:

$$\mathbf{F}^u \left(\mathbf{I}_N - \mathbf{F}^* \right) = \mathbf{F}, \tag{3}$$

where $\mathbf{I}_N$ denotes a $N \times N$ identity matrix. Assumption 1 implies that $\sum_{k \in \Im} \phi^*_{kj} \leq 1$ for all firms j with strict inequality for at least one firm. This constitutes a sufficient condition for the Frobenius root of the non-negative square matrix $\mathbf{F}^*$ to be less than unit (see Theorem 12, Chapter 4, in Murata, 1977). This implies, in turn, that the absolute value of any eigenvalue of $\mathbf{F}^*$ is less than unit and, thus, its spectral radius. As a consequence, matrix $\left(\mathbf{I}_N - \mathbf{F}^* \right)^{-1}$ exists and can be expressed as a power series of $\mathbf{F}^*$, i.e., $\left(\mathbf{I}_N - \mathbf{F}^* \right)^{-1} = \sum_{t=0}^{\infty} \left(\mathbf{F}^* \right)^t$ (see Theorem 11, Chapter 4, in Murata, 1977). We can, thereby, solve for $\mathbf{F}^u$ explicitly as follows:

$$\mathbf{F}^u = \mathbf{F} \left(\mathbf{I}_N - \mathbf{F}^* \right)^{-1}, \tag{4}$$

which establishes, as postulated, that the ultimate financial rights of each external owner can, in fact, be written as a function of the direct financial rights of all owners.

The ultimate rights of external owners implied by matrix $\mathbf{F}^u$ have the properties established in Proposition 1 below.

Proposition 1 *The ultimate financial rights of external owners have the following properties:*

(i) $\phi^u_{kj} \geq 0$ *for all* $k \in \Theta \backslash \Im$ *and all* $j \in \Im$*.*

(ii) $\sum_{k \in \Theta \backslash \Im} \phi^u_{kj} = 1$ *for all* $j \in \Im$*.*

Proof. See Appendix A.

Proposition 1 ensures that the ultimate financial rights of external owners implied by matrix $\mathbf{F}^u$ are non-negative and sum up to one for any given firm j, making clear that a cross-ownership of financial rights changes the distribution of those rights among external owners, as the *ultimate* financial rights of external owner k in firm j, ϕ^u_{kj}, is not necessarily equal to the *direct* financial rights of external owner k in that firm, ϕ_{kj}, but the sum of all financial interests in the firm, is: $\sum_{k \in \Theta} \phi_{kj} = \sum_{k \in \Theta \backslash \Im} \phi^u_{kj} = 1$.

3.2.2 Voting Rights

We now address the computation of voting rights in a cross-ownership setting. Following Dorofeenko *et al.* (2008), the ultimate voting rights of external owner k in firm j, υ_{kj}^{u}, includes not just her direct voting rights in the firm, υ_{kj}, but also the indirect voting rights that may arise from having ultimate corporate control rights in a rival $g \in \Im \backslash j$ that holds, in turn, voting rights in firm j. This implies that for all $k \in \Theta \backslash \Im$ and $j \in \Im$, we have:

$$\upsilon_{kj}^{u} = \upsilon_{kj} + \sum\nolimits_{g \in \Im \backslash j} \gamma_{kg}^{u} \upsilon_{gj}, \tag{5}$$

where γ_{kg}^{u} denotes the ultimate corporate control rights of external owner k in firm g. In order to see why, consider the following example, borrowed from Levy (2011). If an external owner fully controls firms A and B and each of the firms holds in turn 30% of the voting rights in firm C, then the external owner ultimately holds 60% of the voting rights in firm C.

We make the following assumption regarding the ultimate corporate control rights of external owners.

Assumption 2 *The ultimate corporate control rights of external owners capture their ultimate influence within the firm, and have the following properties:*

(i) γ_{kj}^{u} *is a function of the vector of ultimate voting rights in firm* j*.*

(ii) $\gamma_{kj}^{u} \geq 0$ *for all* $k \in \Theta \backslash \Im$ *and all* $j \in \Im$*.*

(iii) $\sum_{k \in \Theta \backslash \Im}^{u} \gamma_{kj}^{u} = 1$ *for all* $j \in \Im$*.*

Assumption 2 ensures that the ultimate corporate control rights implied by the vector of ultimate voting rights of any given firm are non-negative and sum up to one. Further, it implies that the set of equations (5) implicitly determine the ultimate voting rights of each external owner as a function of the direct voting rights of all owners (internal and external). In order to see why, let $\mathbf{V}$, $\mathbf{V}^{u}$ and $\mathbf{C}^{u}$ denote the $(K - N) \times N$ matrices capturing the direct voting rights, ultimate voting rights and ultimate corporate control, respectively, of external owners, with typical elements υ_{kj}, υ_{kj}^{u} and γ_{kj}^{u} representing the direct voting rights, ultimate voting rights and ultimate corporate control rights, respectively, of external owner k in firm j. Let also $\mathbf{V}^{*}$ denote the $N \times N$ matrix capturing the direct voting rights of internal owners, with zero diagonal elements, $\upsilon_{jj} = 0$, and off-diagonal elements, $0 \leq \upsilon_{jg} \leq 1$ (if $j \neq g \in \Im$), representing the direct voting rights of firm j in firm g. We can use matrices $\mathbf{V}$, $\mathbf{V}^{u}$, $\mathbf{C}^{u}$ and $\mathbf{V}^{*}$ to write the set of equations (5) in vector notation, as follows:

$$\mathbf{V}^{u} = \mathbf{V} + \mathbf{C}^{u}\mathbf{V}^{*}. \tag{6}$$

Finally, let $\mathcal{F}(\cdot)$ denote the function which maps the ultimate voting rights of external owners implied by matrix $\mathbf{V}^{u}$ into the corresponding ultimate corporate control rights established in matrix

$\mathbf{C}^u$. This implies we rewrite the set of equations (5) above as:

$$\mathbf{V}^u = \mathbf{V} + \mathcal{F}(\mathbf{V}^u)\,\mathbf{V}^*. \tag{7}$$

The literature suggests several ad-hoc alternatives for the mapping $\mathbf{C}^u = \mathcal{F}(\mathbf{V}^u)$, which include capturing the corporate control rights of the different owners by, for instance, their voting rights (which capture proportional corporate control), their Shapley-Shubik (1954) power index or their Banzhaf (1965) power index. However, we allow the mapping to be, for now, as general as possible. In section 3.3, we show that under a probabilistic voting model in which the manager of each firm is elected in a shareholder assembly between two potential candidates who seek to obtain utility from an exogenous rent associated with corporate office, the ultimate corporate control rights of each owner is captured by the normalized Banzhaf (1965) power index.

In order to solve for $\mathbf{V}^u$ explicitly, we make the following assumption.

Assumption 3 *There exists a unique matrix* $\mathbf{V}^u$ *that solves* $\mathbf{V}^u = \mathbf{V} + \mathcal{F}(\mathbf{V}^u)\,\mathbf{V}^*$.

Assumption 3 ensures the existence of a unique solution for $\mathbf{V}^u$ and establishes, as postulated, that the ultimate voting rights of each external owner are, in fact, a function of the direct voting rights of all owners.[8] Note also that since, under Assumption 1, $\mathbf{V}$ is not a null matrix, the fixed point iterates given by $\mathbf{V}^{u(i+1)} = \mathbf{V} + \mathcal{F}\left(\mathbf{V}^{u(i)}\right)\mathbf{V}^*$ converge to $\mathbf{V}^u$ as $i \to \infty$ from *any* initial condition $\mathbf{V}^{u(0)}$.

The ultimate voting rights of external owners implied by the unique solution for $\mathbf{V}^u$ have the properties established in Proposition 2 below.

Proposition 2 *The ultimate voting rights of external owners have the following properties:*

(i) $\upsilon^u_{kj} \geq 0$ *for all* $k \in \Theta \backslash \Im$ *and all* $j \in \Im$.

(ii) $\sum_{k \in \Theta \backslash \Im} \upsilon^u_{kj} = 1$ *for all* $j \in \Im$.

Proof. See Appendix A.

Proposition 2 ensures that the ultimate voting rights of external owners implied by matrix $\mathbf{V}^u$ are non-negative and sum up to one for any given firm j, making clear that a cross-ownership of voting rights changes the distribution of those rights among external owners, as the *ultimate* voting rights of external owner k in firm j, υ^u_{kj}, is not necessarily equal to the *direct* voting rights of external owner k in that firm, υ_{kj}, but the sum of all voting rights in the firm, is: $\sum_{k \in \Theta} \upsilon_{kj} = \sum_{k \in \Theta \backslash \Im} \upsilon^u_{kj} = 1$.

[8] In the particular case of proportional corporate control, where the corporate control rights of the different owners are captured by their corresponding voting rights, we have that $\mathbf{C}^u = \mathcal{F}(\mathbf{V}^u) = \mathbf{V}^u$. This implies that $\mathbf{V}^u = \mathbf{V} + \mathbf{V^u}\mathbf{V}^*$, which - under Assumption 1 - yields $\mathbf{V}^u = \mathbf{V}(\mathbf{I}_N - \mathbf{V}^*)^{-1}$. As a consequence, Assumption 3 is always satisfied.

3.3 Common-Ownership

Having established that we can model cross-ownership as induced common-ownership by computing the ultimate rights of external owners on the different firms that result from it, we now address the theoretical framework to capture common-ownership. We follow Azar (2017) in assuming a standard theory of probabilistic voting. In particular, we assume, in the lines of Lindbeck and Weibull (1987), that the manager of each firm is elected among potential candidates who compete for the owners' votes to obtain utility from an exogenous rent associated with corporate office.[9,10] The details are as follows.

We consider that the manager of each firm $j \in \Im$ is elected in a shareholder assembly between two potential candidates, an incumbent a_j and a challenger b_j, who seek to obtain utility from an exogenous rent Ξ associated with corporate office. The candidates compete for the ultimate voting rights of external owners by proposing a strategy $x_j \in \{x_{aj}, x_{bj}\}$ for the firm, where x_{aj} and x_{bj} denote the strategy proposals of the incumbent and the challenger for firm j, respectively. Let $\mathbf{x} = (x_1, \ldots, x_j, \ldots, x_N)^\top$ denote the $N \times 1$ vector of strategy proposals for all the firms in the industry.

External owners and managerial candidates are assumed to play the following two-stage game. In the first stage, the two candidates to each firm j simultaneously choose the strategy proposals x_{aj} and x_{bj}, which can refer to any decision variable (e.g., quantity, price, R&D investment, etc.) of the firm. We make the following assumption regarding the strategy space Ω_j available to the candidates of each firm j.

Assumption 4 *The strategy space Ω_j of each firm j* is a *nonempty compact subset of $\Re$.*

Assumption 4 implies that the candidates quantity, price or R&D investment proposals in x_{aj} and x_{bj} are chosen from a closed and bounded interval in $\Re$ (and, therefore, convex). In the second stage of the game, the shareholder assemblies of all firms are simultaneously held and external owners vote to elect the manager of each firm. The candidate that receives the majority of each firm j's ultimate voting rights is elected manager of the firm and her identity is denoted $m_j \in \{a_j, b_j\}$. Let $\mathbf{m} = (m_1, \ldots, m_j, \ldots, m_N)^\top$ denote the $N \times 1$ vector of elected managers for all the firms in the industry.

We begin by addressing the second stage. To do so, we make the following assumption regarding the voting behavior of external owners.

Assumption 5 *External owners are conditionally sincere.*

[9] Alternatively, we could use a standard theory of costly voting in which each ultimate external owner experiences a cost of voting at the shareholder assembly such that she votes for a candidate if and only if the voting cost is smaller than the perceived gain from doing so, abstaining from voting, otherwise. Kamada and Kojima (2013) show that the two voting theories are equivalent if owners estimate the probability of being pivotal to be a fixed number.

[10] It would also be possible to use, as Azar (2017), a probabilistic voting model in which owners vote to express whether they approve or not of a managerial change in the firm's status quo strategic plan. The two voting theories are equivalent if we assume that the opposition against the new strategic plan imposes a cost to the manager that is linearly increasing in the probability of losing the voting.

Assumption 5 implies, following Alesina and Rosenthal (1995), that external owners are assumed to vote, in each firm's shareholder assembly, for the *candidate* whose strategy proposals *maximize* their *utilities,* given the *equilibrium strategy proposals* of the candidates to the *remaining firms*, *randomizing* between the two in case of *indifference*. We consider that the utility of each external owner k is a function of the winning strategies of all firms in the industry and involves two elements, assumed additively separable, as follows:

$$\begin{aligned} u_k(\mathbf{x}, \mathbf{m}) &= R_k(\mathbf{x}) + \sum\nolimits_{g \in \Im} 1(m_g = b_g)\, \xi_{kg} \\ &= \sum\nolimits_{g \in \Im} \phi_{kg}^u \pi_g(\mathbf{x}) + \sum\nolimits_{g \in \Im} 1(m_g = b_g)\, \xi_{kg}. \end{aligned} \tag{8}$$

The first utility element follows from O'Brien and Salop (2000) and captures (assuming a linear utility function) the utility associated to the *return of owner* k*'s ultimate financial rights holdings*, where $R_k(\mathbf{x}) = \sum_{g \in \Im} \phi_{kg}^u \pi_g(\mathbf{x})$ denotes the return of owner k's financial rights holdings in all the firms in the industry, and $\pi_g(\mathbf{x})$ denotes the operating profit of firm g. We make the following technical assumption regarding the return $R_k(\mathbf{x})$ of each external owner k.

Assumption 6 *The return* $R_k(\mathbf{x})$ *of external owner* k *is continuous and twice differentiable in* $\mathbf{x}$*, with continuous second derivatives.*

The second utility element follows from Kramer (1983) and captures the utility associated to the *credibility (or lack of credibility) attached to the challenger' strategy proposal*, where $1(m_g = b_g)$ denotes a dummy variable that takes the value 1 if the challenger is elected manager of firm g and ξ_{kg} denotes the utility that owner k obtains from such event.

The two utility elements above imply that the manager choice is deterministic and it is a discontinuous function of the difference in the utilities obtained from the strategy proposals of each candidate. That is, each external owner k will vote for firm j's incumbent with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)$, will vote for firm j's challenger with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) < u_k(\mathbf{x}_b, \mathbf{m}_b)$, and will vote for the two candidates with probability 1/2 if $u_k(\mathbf{x}_a, \mathbf{m}_a) = u_k(\mathbf{x}_b, \mathbf{m}_b)$, where $\mathbf{x}_a = (x_1, \ldots, x_{aj}, \ldots, x_N)^\top$, $\mathbf{x}_b = (x_1, \ldots, x_{bj}, \ldots, x_N)^\top$, $\mathbf{m}_a = (m_1, \ldots, a_j, \ldots, m_N)^\top$ and $\mathbf{m}_b = (m_1, \ldots, b_j, \ldots, m_N)^\top$.

Having described the second stage of the game, we now address the first stage, in which candidates simultaneously choose strategy proposals. To do so, we follow Lindbeck and Weibull (1987) in assuming that the utility associated to the credibility (or lack of credibility) attached to the challenger' strategy proposal, while known to external voters, is unobserved by candidates, which treat it as a random utility shock. Further, we assume that these random utility shocks are independent and identically distributed across firms and external owners according to a symmetric probability distribution with mean zero and cumulative distribution $G(\cdot)$. As a consequence, from the perspective of the candidates, voting by external owners is probabilistic.

We make the following assumption regarding the candidates choice of strategy proposals.

Assumption 7 *Candidates choose strategy proposals so to maximize the expected utility from corporate office.*

Assumption 7 implies that candidates choose strategy proposals so to maximize the product of the probability that they are elected in the second stage and the utility obtained from the rent associated with corporate office they expect to accrue conditional upon being elected.

For simplicity of exposition, we describe solely the incumbent's problem since the maximization problem of the two candidates to firm j is symmetric. She chooses x_{aj} so to solve:

$$\max_{x_{aj}} \varpi_{aj} = \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) \Xi, \tag{9}$$

where ϖ_{aj} denotes her objective function, i.e., her expected utility from corporate office and $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)$ denotes the probability with which she is, from the candidates perspective, elected manager of the firm in the second stage.

In order to solve the above maximization problem, we must beforehand derive $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)$. To do so, let ℓ_j denote the number of external owners with ultimate voting rights in firm j, $\wp_j$ denote all the $2^{\ell_j - 1}$ possible subsets of those owners that can award the majority of votes to a candidate and $\Theta_j^\imath \in \wp_j$ denote a particular subset of those owners. Given that her election is ensured with the votes of all owners in each subset in $\wp_j$, we have that the probability $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)$ with which the incumbent is elected manager of firm j just sums the probabilities $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right)$ with which she is elected in each subset $\Theta_j^\imath$, as follows:

$$\begin{aligned} \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right) \\ &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k \in \Theta_j^\imath} \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{k \notin \Theta_j^\imath} \Pr_{kb_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right), \end{aligned} \tag{10}$$

where $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ and $\Pr_{kb_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ denote the probability that external owner k votes for the incumbent and the challenger, respectively. It remains to derive $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ and $\Pr_{kb_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$. Under the assumptions described above for the second utility element, the probability that external owner k votes for the incumbent is given by:

$$\begin{aligned} \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) &= \Pr\left(u_k\left(\mathbf{x}_a, \mathbf{m}_a\right) > u_k\left(\mathbf{x}_b, \mathbf{m}_b\right)\right) \\ &= \Pr\left(R_k\left(\mathbf{x}_a\right) > R_k\left(\mathbf{x}_b\right) + \xi_{jg}\right) \\ &= \Pr\left(\xi_{jg} < R_k\left(\mathbf{x}_a\right) - R_k\left(\mathbf{x}_b\right)\right) \\ &= G\left(R_k\left(\mathbf{x}_a\right) - R_k\left(\mathbf{x}_b\right)\right), \end{aligned} \tag{11}$$

where the second equality makes use of the fact that the term $\sum_{g \in \Im \backslash j} 1\left(m_g = b_g\right) \xi_{kg}$ enters the utility obtained from both strategy proposals. In turn, the probability that external owner k votes for the challenger is given by:

$$\begin{aligned}\Pr_{kb_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b) &= 1-\Pr_{ka_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b) \\ &= 1-G\left(R_k(\mathbf{x}_a)-R_k(\mathbf{x}_b)\right) \\ &= G\left(R_k(\mathbf{x}_b)-R_k(\mathbf{x}_a)\right),\end{aligned} \tag{12}$$

where the last equality makes use of the fact that the probability distribution of the random utility shocks is symmetric. Substituting the probabilities (10), (11) and (12) into problem (9), we can rewrite the incumbent's problem as follows:

$$\max_{x_{aj}} \varpi_{aj} = \left(\sum\nolimits_{\Theta_j^\imath\in\wp_j}\prod\nolimits_{k=1}^{\ell_j} G\left((2d_k-1)\left(R_k(\mathbf{x}_a)-R_k(\mathbf{x}_b)\right)\right)\right)\Xi, \tag{13}$$

where d_k takes the value one if external owner $k\in\Theta_j^\imath$ and takes the value zero otherwise. We make the following assumption regarding this maximization problem under the Assumptions 4 to 7.

Assumption 8 *The maximization problem of the incumbent has an unique maximum.*

Assumption 8 is satisfied if, for instance, the cumulative distribution $G(\cdot)$ is positive, continuous and twice differentiable, with continuous second derivatives, strictly quasi-concave and uniformly monotone in firm j's strategy $x_j\in\{x_{aj},x_{bj}\}$, conditional on the strategies of the remaining firms. In order to see why note that under Assumption 5, external owners are conditionally sincere, which implies that the incumbent candidate to firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. Given that (i) the product of positive strictly quasi-concave and uniformly monotone functions is a strictly quasi-concave function, and (ii) the sum of strictly quasi-concave and uniformly monotone functions is also a strictly quasi-concave function (see Prékopa, Yoda and Subasi, 2011; Kopa and Lachout, 2012; Lachout, 2016), we have that the objective function of the incumbent candidate to firm j is strictly quasi-concave (and uniformly monotone) conditional on the strategies of the candidates to the remaining firms. Finally, given that the strategy proposal x_{aj} is, under Assumption 4, defined in a convex set, we have that her maximization problem has an unique maximum.

Assumptions 4 to 8 ensure the existence of a pure-strategy Nash equilibrium for the candidates strategy proposals' game $(x_{a1},x_{b1},\ldots x_{aj},x_{bj},\ldots,x_{aN},x_{bN})$, characterized as follows.

Proposition 3 *There exists a pure-strategy Nash equilibrium for the candidates strategy proposals' game. In the equilibrium, the candidates of every firm converge to the same strategy $x_{aj}=x_{bj}=x_j$, which maximizes a corporate control weighted average of the returns of the firm's external owners, conditional on the strategies of the candidates to the remaining firms:*

$$\max_{x_j} \varpi_j = \sum\nolimits_{k\in\Theta\setminus\Im}\alpha_{kj}R_k(\mathbf{x}), \tag{14}$$

where α_{kj} denotes the weight assigned by firm j's manager to the return of external owner k, *measured by the normalized Banzhaf (1965) power index of external owner k in firm j:*

$$\alpha_{kj} = \frac{\lambda_{jk}^{p}/2^{\ell_j - 1}}{\sum_{h\in\Theta\backslash\Im}\left(\lambda_{jh}^{p}/2^{\ell_j-1}\right)}, \tag{15}$$

where λ_{jk}^{p} denotes the number of subsets in $\wp_j$ in which external owner k enters and is pivotal.
Proof. See Appendix A.

Proposition 3 establishes, in line with O'Brien and Salop (2000), that the manager of each firm j must *weight* the (eventual) conflicting objectives of the different external owners. These weights satisfy the conditions of Assumption 2 and, as a consequence, may be used to measure the ultimate corporate control of each external owner in the firm: $\gamma_{kj}^{u} = \alpha_{kj}$ for all $k \in \Theta\backslash\Im$. In order to see why, note that the weights capture the ultimate importance (and, thus, influence) of each external owner (via their return) over the decision-making within the firm. Note also that the weights are a function of the vector of ultimate voting rights in the firm as established by the *normalized Banzhaf (1965) power index* (an index that tends to assign more than proportional weight to an external owner who holds large blocks of ultimate voting rights in the firm, with the former converging to 100% as the latter approach 50%). Finally, note also that the weights are non-negative, $\alpha_{kj} \geq 0$, since the number of subsets in $\wp_j$ in which a external owner k enters and is pivotal is, by definition, non-negative, and sum up to one, $\sum_{k\in\Theta\backslash\Im}\alpha_{kj} = 1$, since $\sum_{k\in\Theta\backslash\Im}\alpha_{kj} = \left(\sum_{k\in\Theta\backslash\Im}\left(\lambda_{jk}^{p}/2^{\ell_j-1}\right)\right)/\left(\sum_{h\in\Theta\backslash\Im}\left(\lambda_{jh}^{p}/2^{\ell_j-1}\right)\right) = 1$. Naturally, alternative formulations of Assumption 7 would yield different ultimate corporate control measures. For instance, Azar (2017) shows that when candidates choose strategy proposals so to maximize their vote share, the influence of owners over the decision-making of the firm is measured by their voting rights.

Proposition 3, combined with $\gamma_{kj}^{u} = \alpha_{kj}$ for all $k \in \Theta\backslash\Im$, also establishes that the ultimate corporate control rights of each external owner in a particular firm is given by the solution to following system of $2(K-N)$ equations:

$$\begin{aligned} \upsilon_{kj}^{u} &= \upsilon_{kj} + \sum\nolimits_{g\in\Im\backslash j}\gamma_{kg}^{u}\upsilon_{gj} \\ \gamma_{kj}^{u} &= \frac{\lambda_{jk}^{p}/2^{\ell_j-1}}{\sum_{h\in\Theta\backslash\Im}\left(\lambda_{jh}^{p}/2^{\ell_j-1}\right)}, \end{aligned} \tag{16}$$

for all $k \in \Theta\backslash\Im$ and $j \in \Im$. The definition of the conditions on $\mathbf{V}$ and $\mathbf{V}^{*}$ that ensure that Assumption 3 is satisfied, so that there exists a unique solution constitutes a very interesting potential area for future research.

Finally, Proposition 3 establishes also that the manager of each firm j must *weight* the operating profits of (potentially) all the firms in the industry. In order to see why, note that we can rewrite the objective function of the manager as follows:

$$\varpi_j = \sum\nolimits_{k\in\Theta\backslash\Im}\gamma_{kj}^{u}R_k(\mathbf{x}) = \sum\nolimits_{g\in\Im}\sum\nolimits_{k\in\Theta\backslash\Im}\gamma_{kj}^{u}\phi_{kg}^{u}\pi_g(\mathbf{x}) = \sum\nolimits_{g\in\Im}l_{jg}\pi_g(\mathbf{x}), \tag{17}$$

where the weight $l_{jg} = \sum_{k\in\Theta\backslash\Im} \gamma_{kj}^{u}\phi_{kg}^{u} \geq 0$ for any $j, g \in \Im$ denotes the typical element of the $N \times N$ matrix $\mathbf{L} = (\mathbf{C}^{u})^{\top} \mathbf{F}^{u}$ and incorporates two rights: the *ultimate corporate control rights* of the firm's external owners (that capture the corporate control structure of the firm) and the *ultimate financial rights* of the firm's external owners across the different firms (that capture the return of external owners).[11] This objective function has the properties established in Proposition 4 below.

Proposition 4 *The objective function of the manager of a firm is characterized by the following attributes:*

(i) The weight associated to the ultimate financial rights of an external owner with no ultimate corporate control rights in the firm is null.

(ii) The weight associated to the ultimate financial rights of an external owner is increasing with the ultimate corporate control rights of that owner in the firm.

Proof. See Appendix A.

Attribute *(i)* implies that the manager of the firm does not weight the ultimate financial rights of an external owner with no ultimate corporate control rights in the firm, since the influence of such owner over the decision-making within the firm is null. Attribute *(ii)* implies that as the ultimate corporate control rights of an external owner in the firm increases, the manager of the firm must weight more the ultimate financial rights of that owner.

Without loss of generality, we normalize the weight on the own-operating profit to be one by dividing ϖ_j by l_{jj}. This implies that the manager of firm j maximizes the following, entirely equivalent, objective function:

$$\varpi_j' = \sum\nolimits_{g\in\Im} \frac{l_{jg}}{l_{jj}} \pi_g(\mathbf{x}) = \sum\nolimits_{g\in\Im} w_{jg}\pi_g(\mathbf{x}), \tag{18}$$

where $w_{jg} = l_{jg}/l_{jj} \geq 0$ for any $j, g \in \Im$ denotes the typical element of the $N \times N$ *normalized* weight matrix $\mathbf{W} = diag(\mathbf{L})^{-1}\mathbf{L}$, and $diag(\mathbf{L})$ is the $N \times N$ matrix formed by substituting zeros for all off-diagonal elements of $\mathbf{L}$.[12]

3.3.1 Industry Ownership Structures

The normalized weight function ϖ_j' established by our theoretical framework can cope with a multitude of general industry ownership structures, involving owners that can be internal (represented in matrices $\mathbf{F}^{*}$ and $\mathbf{V}^{*}$) and external (represented in matrices $\mathbf{F}$ and $\mathbf{V}$) to the industry; and rights

[11] In order to see why the weights l_{jg} are non-negative, note that $\gamma_{kj}^{u} \geq 0$ and $\phi_{kj}^{u} \geq 0$ for all $k \in \Theta$ and all $j \in \Im$.

[12] In order to see why the weights w_{jg} are non-negative for any $j, g \in \Im$, note that - as discussed above - an owner can not hold corporate control rights in a firm without holding financial rights in that firm. This implies that $l_{jj} > 0$ and, in turn, that $w_{jg} \geq 0$ (since, as discussed above, $l_{jg} \geq 0$).

that can capture financial interests (represented in matrices $\mathbf{F}^*$ and $\mathbf{F}$) and corporate control interests (defined as a function of the voting rights represented in matrices $\mathbf{V}^*$ and $\mathbf{V}$), can be direct and indirect, partial or full. Moreover, it nests several particular industry ownership structures, of which we highlight the following.

First, in structures in which *cross- and common-ownership interests are absent*, it reduces to the operating profit of the firm. In order to see why, note that, in those cases, we have that $\mathbf{F}^u = \mathbf{F}$ and $\mathbf{V}^u = \mathbf{V}$ (since in the absence of cross-ownership $\mathbf{F}^*$ and $\mathbf{V}^*$ constitute null matrices). This implies that the objective function of the manager of firm j is given by:

$$\varpi_j = \left(\sum\nolimits_{k\in\Theta\backslash\Im} \gamma_{kj}\phi_{kj}\right)\pi_j\left(\mathbf{x}\right), \tag{19}$$

where γ_{kj} denotes the corporate control rights of external owner k in firm j, established by matrix $\mathbf{C} = \mathcal{F}\left(\mathbf{V}\right)$. In turn, this implies that the weight matrix $\mathbf{L}$ is a diagonal matrix and that the normalized weight matrix $\mathbf{W} = \mathbf{I}$. As consequence, we have that $\varpi'_j = \pi_j$. In other words, with no partial ownership interests of any kind, the objective function yields that the manager of each firm should maximize the firm's own operating profit.

Second, in structures characterized by a *cross-ownership of financial rights*, it reduces to the objective function in Flath (1992) and Dietzenbacher, Smid and Volkerink (2000). In order to see why, note that, in those cases, we have that $\mathbf{F}^u = \mathbf{F}\left(\mathbf{I}-\mathbf{F}^*\right)^{-1}$ and $\mathbf{V}^u = \mathbf{V}$ (since in the absence of a cross-ownership of voting rights $\mathbf{V}^*$ constitutes a null matrix). This implies that the weight function of the manager of firm j is given by:

$$\varpi_j = \sum\nolimits_{g\in\Im}\left(\sum\nolimits_{k\in\Theta\backslash\Im} \gamma_{kj}\phi^u_{kg}\right)\pi_g\left(\mathbf{x}\right), \tag{20}$$

where the corporate control rights are, as before, established by matrix $\mathbf{C} = \mathcal{F}\left(\mathbf{V}\right)$. In turn, this implies that the weight matrix $\mathbf{L}$ yields weights, for each firm j, that are proportional to those proposed by Flath (1992) and Dietzenbacher, Smid and Volkerink (2000).[13] As a consequence, we have that the normalized weight matrices implied by the two approaches coincide, establishing that: $\mathbf{W} = diag\left(\left(\mathbf{I}-\mathbf{F}^*\right)^{-1}\right)^{-1}\left(\mathbf{I}-\mathbf{F}^*\right)^{-1}$, where $diag\left(\left(\mathbf{I}-\mathbf{F}^*\right)^{-1}\right)$ is the $N\times N$ matrix formed by substituting zeros for all off-diagonal elements of $\left(\mathbf{I}-\mathbf{F}^*\right)^{-1}$.

Third, in structures characterized by a *common-ownership of financial and/or voting rights*, it reduces to the objective function in O'Brien and Salop (2000). In order to see why, note that, in those cases, we have that $\mathbf{F}^u = \mathbf{F}$ and $\mathbf{V}^u = \mathbf{V}$ (since in the absence of cross-ownership $\mathbf{F}^*$ and $\mathbf{V}^*$ constitute null matrices). This implies that the objective function of the manager of firm j is given by:

$$\varpi_j = \sum\nolimits_{g\in\Im}\sum\nolimits_{k\in\Theta\backslash\Im} \gamma_{kj}\phi_{kg}\pi_g\left(\mathbf{x}\right), \tag{21}$$

where the corporate control rights are, as before, established by matrix $\mathbf{C} = \mathcal{F}\left(\mathbf{V}\right)$. In turn, this im-

[13] The factor of proportionality is given, for each firm j, by $\sum_{k\in\Theta\backslash\Im}\gamma_{kj}\phi^u_{kj}$.

plies that the weight matrix $\mathbf{L} = \mathbf{C}^\top \mathbf{F}$ and the normalized weight matrix $\mathbf{W} = diag\left(\mathbf{C}^\top \mathbf{F}\right)^{-1} \mathbf{C}^\top \mathbf{F}$, both of which coincide with the ones proposed by O'Brien and Salop (2000).

Finally, in structures characterized by a *cross- and common-ownership of financial and/or voting rights*, it reduces to the objective function in Brito, Ribeiro and Vasconcelos (2014) and Azar, Raina and Schmalz (2016). In order to see why, note - as before - that when managerial candidates choose strategy proposals so to maximize their vote share, the influence of owners over the decision-making of the firm is measured by their voting rights (Azar, 2017). This implies that $\mathbf{V}^u = \mathbf{V} + \mathbf{C}^u \mathbf{V}^* = \mathbf{V} + \mathbf{V}^u \mathbf{V}^*$, which under Assumption 3 yields that $\mathbf{V}^u = \mathbf{V}\left(\mathbf{I}_N - \mathbf{V}^*\right)^{-1}$. This implies that the objective function of the manager of firm j is given by:

$$\varpi_j = \sum\nolimits_{g \in \Im} \sum\nolimits_{k \in \Theta \setminus \Im} \upsilon_{kj}^u \phi_{kg}^u \pi_g\left(\mathbf{x}\right). \tag{22}$$

In turn, this implies that the weight matrix $\mathbf{L} = \left(\mathbf{V}^u\right)^\top \mathbf{F}^u$ and that the normalized weight matrix $\mathbf{W} = diag\left(\left(\mathbf{V}^u\right)^\top \mathbf{F}^u\right)^{-1} \left(\mathbf{V}^u\right)^\top \mathbf{F}^u$, both of which coincide with the ones applied by Brito, Ribeiro and Vasconcelos (2014) and Azar, Raina and Schmalz (2016).

4 The Proposed Generalized Indicators

In this section, we use the theoretical framework described above to develop two generalized screening indicators proposals for partial horizontal acquisitions. To do so, let the operating profit of firm j be given by:

$$\pi_j = \left(p_j - mc_j\right) q_j - c_j, \tag{23}$$

where p_j, mc_j, q_j, and c_j denote the price, the (assumed constant) marginal cost, the quantity, and the fixed cost, respectively, of firm j.

We derive the generalized *HHI* under the Cournot model of quantity competition among firms in homogeneous-product industries and derive the generalized *GUPPI* under the Bertrand model of price competition among firms in differentiated-product industries. However, as discussed above, we could have derived both generalized screening indicators under a variety of other economic models.

4.1 Cournot Homogeneous-Product Industries

In a Cournot homogeneous-product industry, we have that $x_j = q_j$ and $p_j = p$ for all $j \in \Im$, with p being determined by the downward sloping inverse market demand function, $p\left(Q\right)$, where $Q = \sum_{g \in \Im} q_g$ denotes the industry output level. Under this setting, we propose a generalized *HHI*, structurally constructed as follows. The manager of each firm j solves:

$$\max_{q_j} \varpi_j' = \sum\nolimits_{g \in \Im} w_{jg} \pi_g\left(Q\right) = \sum\nolimits_{g \in \Im} w_{jg}\left(\left(p\left(Q\right) - mc_g\right) q_g - c_g\right). \tag{24}$$

The Cournot-Nash equilibrium in quantities, $\left(q_1, \ldots, q_j, \ldots, q_N\right)$ and consequently $Q = \sum_{g \in \Im} q_g$, for an interior solution is characterized by the following system of first-order conditions, for all $j \in \Im$:

$$(p(Q) - mc_j) + \frac{\partial p(Q)}{\partial Q} q_j + \sum_{g \neq j \in \Im} w_{jg} \frac{\partial p(Q)}{\partial Q} q_g = 0, \tag{25}$$

which makes use of the fact that $w_{jj} = 1$ and $\partial Q / \partial q_j = 1$. This result establishes that a decision by the manager of firm j to produce an extra unit of output has several impacts on her objective function. First, it increases the operating profit of the firm (which the manager weights by $w_{jj} = 1$) by $(p(Q) - mc_j)$, which corresponds to the operating profit derived from this extra unit. Second, it decreases the market price by $\partial p(Q) / \partial Q$, which, in turn, *(i)* decreases the operating profit of the firm (which, again, the manager weights by $w_{jj} = 1$) by $(\partial p(Q) / \partial Q) q_j$, since the new price impacts the operating profit derived from the units previously produced by the firm; and *(ii)* decreases the operating profit of each rival firm g (which the manager weights by $w_{jg} > 0$ for $j \neq g$ if the owners of firm j hold, direct or indirectly, financial and/or a corporate control rights in firm g) by $(\partial p(Q) / \partial Q) q_g$, since the new price also impacts the operating profit derived from the units previously produced by the rival firms.

Multiplying both sides of the above first-order condition by $Q / p(Q) Q$ yields:

$$(p(Q) - mc_j) \frac{Q}{p(Q) Q} + \sum_{g \in \Im} w_{jg} \frac{\partial p(Q)}{\partial Q} \frac{Q}{p(Q) Q} q_g = 0, \tag{26}$$

which after some rearranging becomes:

$$\frac{p(Q) - mc_j}{p(Q)} = -\sum_{g \in \Im} w_{jg} \frac{(q_g / Q)}{(\partial Q / \partial p(Q)) (p(Q) / (Q))} = \frac{1}{\eta} \sum_{g \in \Im} w_{jg} s_g, \tag{27}$$

where the second equality makes use of the fact that $\eta = -(\partial Q / \partial p(Q)) (p(Q) / (Q))$ denotes the (assumed constant) absolute value of the elasticity of demand, and $s_g = (q_g / Q)$ denotes the output market share of firm g, for all $g \in \Im$. This implies that the (price-cost) margin to price ratio of firm j is proportional to a weighted sum of the own-output share and the output share of all the other rival firms in which the owners of firm j have an interest in.

Multiplying both sides of the first-order condition of each firm j by s_j and summing over all firms, we can express the simultaneous market solution as:

$$\sum_{j \in \Im} \left(\frac{p(Q) - mc_j}{p(Q)} \right) s_j = \frac{1}{\eta} \sum_{j \in \Im} \sum_{g \in \Im} w_{jg} s_g s_j \tag{28}$$

which establishes that the output share-weighted margins to price ratio of all the firms in the industry is proportional (with the scale factor being $1/\eta$) to a measure of concentration. This measure establishes our proposal.

Definition 1 *The generalized HHI is given by:*

$$GHHI = \sum_{j \in \Im} \sum_{g \in \Im} w_{jg} s_g s_j = \mathbf{s}^\intercal \mathbf{W} \mathbf{s}, \tag{29}$$

where $\mathbf{s}$ *is the* $N \times 1$ *vector of output market shares and* $\mathbf{W}$ *denotes the normalized weight matrix described above.*

By separating out the terms for which $j = g$, we have that:

$$GHHI = \mathbf{s}^{\intercal}\mathbf{W}\mathbf{s} = \mathbf{s}^{\intercal}\mathbf{s} + \mathbf{s}^{\intercal}\left(\mathbf{W} - \mathbf{I}\right)\mathbf{s} = HHI + \mathbf{s}^{\intercal}\left(\mathbf{W} - \mathbf{I}\right)\mathbf{s}, \tag{30}$$

which for $\mathbf{W} \neq \mathbf{I}$ implies that *GHHI* differs from the standard *HHI*. The *GHHI* is equal to the *HHI* plus a set of terms that reflect the impact of cross- and common-ownership on the concentration of the industry. In other words, using the standard *HHI* to measure the concentration of an industry in which firms engage in cross- or common-ownership, induces a bias. Further, since the cross-terms in $\mathbf{W}$ are non-negative, we have that this bias is downward ($HHI \leq GHHI$).

The *GHHI* can cope with industry settings involving all types of owners and rights: owners that can be internal and external to the industry; and rights that can capture financial and corporate control interests, can be direct and indirect, can be partial or not. Moreover, it nests several concentration measures outlined in the literature, of which we highlight the following.

First, in structures in which *cross- and common-ownership rights are absent*, it reduces to the standard *HHI*, since in those cases (as discussed above) we have that $\mathbf{W} = \mathbf{I}$:

$$GHHI = \mathbf{s}^{\intercal}\mathbf{s} = HHI. \tag{31}$$

Second, in structures characterized by a *cross-ownership of financial rights*, it reduces to Dietzenbacher, Smid and Volkerink (2000)'s modified *HHI*, since in those cases (as discussed above) we have that $\mathbf{W} = diag\left(\left(\mathbf{I} - \mathbf{F}^{*}\right)^{-1}\right)^{-1}\left(\mathbf{I} - \mathbf{F}^{*}\right)^{-1}$:

$$GHHI = \mathbf{s}^{\intercal} diag\left(\left(\mathbf{I} - \mathbf{F}^{*}\right)^{-1}\right)^{-1}\left(\mathbf{I} - \mathbf{F}^{*}\right)^{-1}\mathbf{s}. \tag{32}$$

Third, in structures characterized by a *common-ownership of financial and/or voting rights*, it reduces to O'Brien and Salop (2000)'s modified *HHI*, since in those cases (as discussed above), we have that $\mathbf{W} = diag\left(\mathbf{C}^{\top}\mathbf{F}\right)^{-1}\mathbf{C}^{\top}\mathbf{F}$:

$$GHHI = \mathbf{s}^{\intercal} diag\left(\mathbf{C}^{\top}\mathbf{F}\right)^{-1}\mathbf{C}^{\top}\mathbf{F}\mathbf{s}. \tag{33}$$

Finally, in structures characterized by a *cross- and common-ownership of financial and/or voting rights* in which managerial candidates choose strategy proposals so to maximize their vote share, it reduces to Azar, Raina and Schmalz (2016)'s generalized *HHI*, since in those cases (as discussed above) we have that $\mathbf{W} = diag\left(\left(\mathbf{V}^{u}\right)^{\top}\mathbf{F}^{u}\right)^{-1}\left(\mathbf{V}^{u}\right)^{\top}\mathbf{F}^{u}$:

$$GHHI = \mathbf{s}^{\intercal} diag\left(\left(\mathbf{V}^{u}\right)^{\top}\mathbf{F}^{u}\right)^{-1}\left(\mathbf{V}^{u}\right)^{\top}\mathbf{F}^{u}\mathbf{s}. \tag{34}$$

4.1.1 *GHHI* Decomposition

Adelman (1969) decomposes the standard *HHI* into two elements: the number of firms in the industry and the variance of the output market shares of the firms in the industry. In particular, he establishes that $HHI = N\sigma_s^2 + \bar{s} = N\sigma_s^2 + (1/N)$, where $\bar{s} = \sum_{j\in\Im}(s_j/N) = 1/N$ and $\sigma_s^2 = \sum_{j\in\Im}(s_j - \bar{s})^2(1/N)$ denote the mean and the variance, respectively, of the firms' output market shares. A similar decomposition can be derived for our proposed *GHHI*, as follows:

$$\begin{aligned} GHHI &= \sum_{j\in\Im}\sum_{g\in\Im} w_{jg}s_g s_j \\ &= \sum_{j\in\Im} s_j \sum_{g\in\Im} w_{jg}s_g \\ &= \sum_{j\in\Im} s_j\left(\sigma_{w_j s} + \bar{w}_j\bar{s}\right)N \\ &= N\sum_{j\in\Im} s_j\sigma_{w_j s} + \sum_{j\in\Im} s_j\bar{w}_j, \end{aligned} \tag{35}$$

where $\bar{w}_j = \sum_{g\in\Im}(w_{jg}/N)$ for all $j \in \Im$ denotes the average normalized weight that the manager of firm j places on the operating profit of the firms in the industry and $\sigma_{w_j s} = \sum_{g\in\Im} w_{jg}s_g(1/N) - \bar{w}_j\bar{s} = \sum_{g\in\Im}(w_{jg} - \bar{w}_j)(s_g - \bar{s})(1/N)$ for all $j \in \Im$ denotes the covariance between the normalized weight that the manager of firm j places on each firms' operating profit and its output market share. This implies that our proposed *GHHI* can be decomposed into three elements: the number of firms in the industry, the weighted (by output market share) average of the covariances of the normalized weights and output market shares of the firms in the industry, and the weighted average (also by output market share) of the normalized weights of the firms in the industry.

One interesting feature of the above result is that it reduces to Adelman (1969)'s decomposition in the absence of cross- and common-ownership. In order to see why, note that in those cases (as discussed above) we have that $\mathbf{W} = \mathbf{I}$, which implies $\bar{w}_j = 1/N$ and $\sigma_{w_j s} = (s_j - \bar{s})(1/N)$ for all $j \in \Im$, and hence establishes:

$$\begin{aligned} GHHI &= \sum_{j\in\Im} s_j(s_j - \bar{s}) + (1/N)\sum_{j\in\Im} s_j \\ &= \sum_{j\in\Im}(s_j - \bar{s})^2 + \bar{s} \\ &= N\sigma_s^2 + (1/N). \end{aligned} \tag{36}$$

4.1.2 Unilateral Effects Screen Indicator

Having established our proposed structural measure of concentration, we can relatively straightforward derive an indicator to screen the anti-competitive effects of an acquisition. To do so, consider now an hypothetical acquisition, which can be partial or full, prompted by internal or external owners, and involve corporate control or not. Let $\tilde{\mathbf{W}}$ denote the post-acquisition weight matrix $\mathbf{W}$, with weights given by $\tilde{w}_{jg}$ for any $j, g \in \Im$.

The idea behind our proposed screen indicator is to use information local to the pre-acquisition Cournot-Nash equilibrium $(q_1, \ldots, q_j, \ldots, q_N)$ to predict, under a setting of no efficiency gains (as in the standard *HHI*), the directional price impact of acquisitions. To do so, we assume that the industry quantity Q is the only variable that re-equilibrates after the acquisition, i.e., we ignore the re-equilibration across firm-output shares. Let $\tilde{Q}$ denote the interior Cournot-Nash equilibrium in the industry quantity *post-acquisition*, which, for an interior solution, assuming no efficiency gains and no other re-equilibration, is characterized by the following simultaneous market solution:

$$\sum_{j\in\Im}\left(\frac{p\left(\tilde{Q}\right)-mc_j}{p\left(\tilde{Q}\right)}\right)s_j=\frac{1}{\eta}\sum_{j\in\Im}\sum_{g\in\Im}\tilde{w}_{jg}s_g s_j. \tag{37}$$

The above result implies that the difference between the post- and the pre-acquisition output share-weighted margins to price ratio is given by:

$$\sum_{j\in\Im}\left(\frac{p\left(\tilde{Q}\right)-mc_j}{p\left(\tilde{Q}\right)}\right)s_j-\sum_{j\in\Im}\left(\frac{p\left(Q\right)-mc_j}{p\left(Q\right)}\right)s_j=\frac{1}{\eta}\left(\widetilde{GHHI}-GHHI\right), \tag{38}$$

where $\widetilde{GHHI}=\sum_{j\in\Im}\sum_{g\in\Im}\tilde{w}_{jg}s_g s_j$ denotes the post-acquisition *GHHI*. The higher the post-acquisition *GHHI* and the increase in the *GHHI*, the greater the unilateral effects impact of the acquisition on the output share-weighted margins to price ratio.

4.2 Bertrand Differentiated-Product Industries

In a Bertrand differentiated-product industry, we have that $x_j = p_j$ for all $j \in \Im$. Under this setting, we propose a generalized *GUPPI*, structurally constructed as follows. The manager of each firm j solves:

$$\max_{p_j}\varpi_j'=\sum_{g\in\Im}w_{jg}\pi_g\left(\mathbf{p}\right)=\sum_{g\in\Im}w_{jg}\left(\left(p_g-mc_g\right)q_g\left(\mathbf{p}\right)-c_g\right), \tag{39}$$

where $q_g\left(\mathbf{p}\right)$ is the quantity demanded for the product of firm g, which is a function of the $N\times 1$ vector $\mathbf{p}$ of prices of all the products available in the industry. This function is typically assumed downward sloping with respect to own-price and upward sloping with respect to all other rival firms' prices.

The Bertrand-Nash equilibrium in prices $(p_1,\ldots,p_j,\ldots,p_N)$ for an interior solution is characterized by the following system of first-order conditions, for all $j\in\Im$:

$$q_j\left(\mathbf{p}\right)+\left(p_j-mc_j\right)\frac{\partial q_j\left(\mathbf{p}\right)}{\partial p_j}+\sum_{g\neq j\in\Im}w_{jg}\left(p_g-mc_g\right)\frac{\partial q_g\left(\mathbf{p}\right)}{\partial p_j}=0, \tag{40}$$

which makes use of the fact that $w_{jj}=1$. This result establishes that a decision by the manager of firm j to increase price by one unit has several impacts on her objective function. First, it increases the operating profits of the firm (which the manager weights by $w_{jj}=1$) by $q_j\left(\mathbf{p}\right)$, since the new price impacts the operating profit derived from the units previously produced by the firm. Second, it decreases the demand for the firm's product by $\partial q_j(\mathbf{p})/\partial p_j$, which, in turn, decreases the operating profit of the firm (which, again, the manager weights by $w_{jj}=1$) by $(p_j - mc_j)(\partial q_j(\mathbf{p})/\partial p_j)$. Third, it increases the demand for each rival firm g's product by $\partial q_g(\mathbf{p})/\partial p_j$, which, in turn, increases the operating profit of firm g (which the manager weights by $w_{jg}>0$ for $j \neq g$ if the owners of firm j hold, direct or indirectly, financial and/or corporate control rights in firm g) by $(p_g - mc_g)(\partial q_g(\mathbf{p})/\partial p_j)$.

After some rearranging, we have that:

$$p_j = mc_j - q_j(\mathbf{p})(\partial q_j(\mathbf{p})/\partial p_j)^{-1} + \sum\nolimits_{g\neq j\in\Im} w_{jg}(p_g - mc_g)\, DR_{gj}, \tag{41}$$

where $DR_{gj} = -(\partial q_g(\mathbf{p})/\partial p_j)(\partial q_j(\mathbf{p})/\partial p_j)^{-1}$ denotes the diversion ratio from the product of firm j to the product of firm g, which quantifies how much of the displaced demand for the product switches to product g if the price of product j were to rise.

4.2.1 Unilateral Effects Screen Indicator

Consider now (as discussed above) an hypothetical acquisition, which can be partial or full, prompted by internal or external owners, and involve corporate control or not. Let $\tilde{\mathbf{W}}$ denote the post-acquisition weight matrix $\mathbf{W}$, with weights given by $\tilde{w}_{jg}$ for any $j, g \in \Im$.

The idea behind our proposed screen indicator is again to use information local to the pre-acquisition Bertrand-Nash equilibrium $(p_1, \ldots, p_j, \ldots, p_N)$ to predict, under a setting of no efficiency gains (as in the standard *GUPPI*), the directional price impacts of acquisitions (in the lines of Cheung, 2011; and Jaffe and Weyl, 2013). To do so, we assume that the price of product j is the only variable that re-equilibrates after the acquisition, i.e., we ignore the re-equilibration of the remaining variables (all the other prices, all the quantities and all the price-effects). Let $(\tilde{p}_1, \ldots, \tilde{p}_j, \ldots, \tilde{p}_N)$ denote the Bertrand-Nash equilibrium in prices *post-acquisition*, which, for an interior solution and assuming no efficiency gains, is characterized by the following system of first-order conditions, for all $j \in \Im$:

$$\tilde{p}_j = mc_j - q_j(\tilde{\mathbf{p}})(\partial q_j(\tilde{\mathbf{p}})/\partial p_j)^{-1} + \sum\nolimits_{g\neq j\in\Im} \tilde{w}_{jg}(\tilde{p}_g - mc_g)\, DR_{gj}, \tag{42}$$

which, under the no re-equilibration assumption, can be rewritten as:

$$\tilde{p}_j = mc_j - q_j(\mathbf{p})(\partial q_j(\mathbf{p})/\partial p_j)^{-1} + \sum\nolimits_{g\neq j\in\Im} \tilde{w}_{jg}(p_g - mc_g)\, DR_{gj}, \tag{43}$$

since we have that $\tilde{p}_g = p_g$ for $g \neq j \in \Im$, $q_j(\mathbf{p}) = q_j(\tilde{\mathbf{p}})$ for $j \in \Im$, and $\partial q_g(\tilde{\mathbf{p}})/\partial p_j = \partial q_g(\tilde{\mathbf{p}})/\partial p_j$ for all $g \in \Im$.

The above result implies that the difference between the post- and pre-acquisition price of the product of firm $j \in \Im$ is given by:

$$(\tilde{p}_j - p_j) = \sum\nolimits_{g\neq j\in\Im} (\tilde{w}_{jg} - w_{jg})(p_g - mc_g)\, DR_{gj}, \tag{44}$$

which establishes that the product of firm j's upward pricing pressure, gross of efficiency gains, is a function of the change in the weights in matrix $\mathbf{W}$, of the pre-acquisition price-cost margins, and of the diversion ratios *for the firms involved* (i.e., for the firms whose weights exhibit changes pre- and post-acquisition). Multiplying both sides of the above result by $1/p_j$ establishes our proposal.

Definition 2 *The generalized GUPPI for the product of firm $j \in \Im$ is given by:*

$$GGUPPI_j = \sum\nolimits_{g \neq j \in \Im} (\tilde{w}_{jg} - w_{jg}) (p_g - mc_g) DR_{gj}/p_j, \tag{45}$$

where $\tilde{w}_{jg} - w_{jg}$ denotes the difference in the normalized weight that the manager of firm j places on the operating profit of firm g post- and pre-acquisition, p_j and p_g denote the pre-acquisition prices of the products of firms j and g, respectively, mc_g denotes the pre-acquisition marginal cost of the product of firm g, and DR_{gj} denotes the pre-acquisition diversion ratio from the product of firm j to the product of firm g.

The higher the level of the *GUPPI* for each product involved in the acquisition, the greater the unilateral impact of the acquisition on their corresponding prices.

The *GGUPPI* can cope with acquisition settings involving all types of owners and rights: owners that can be internal and external to the industry; and rights that can capture financial and corporate control interests, can be direct and indirect, can be partial or not. Moreover, in cases of full acquisitions within an industry structure in which cross- and common-ownership rights are absent, it reduces to the standard *GUPPI*. In order to see why note that in this setting, we have $\mathbf{W} = \mathbf{I}$ and $(\tilde{w}_{jg} - w_{jg}) = 1$ for the firms j and $g \neq j \in \Im$ involved in the acquisition, to:

$$GGUPPI_j = \sum\nolimits_{g \neq j \in \Im} (p_g - mc_g) DR_{gj}/p_j = GUPPI_j. \tag{46}$$

5 Empirical Application

This section presents an empirical application of the *GHHI* and *GGUPPI* to several acquisitions in the wet shaving industry, with the objective of providing practitioners with a step-by-step illustration of how to compute the two proposed screening indicators in antitrust cases.

On December 20, 1989, the Gillette Company, contracted to acquire the wet shaving businesses of Wilkinson Sword trademark outside of the 12-nation European Community (which included the US operations) from Eemland Management Services BV (Wilkinson Sword's parent company) for \$72 million. It also acquired a 22.9% non-voting stake in Eemland for about \$14 million. At that time, consumers in the U.S. annually purchased over \$700 million of wet shaving razor blades at the retail level. Five firms supplied all but a nominal amount of these blades: *The Gillette Company*, which had been the market leader for years, *American Safety Razor Company*, *BIC Corporation*, *Warner-Lambert Company*, and *Wilkinson Sword Inc.*.

On January 10, 1990, the DoJ instituted a civil proceeding against Gillette. The complaint alleged that the effect of the acquisition by Gillette may have been substantially to lessen competition in the sale of wet shaving razor blades in the U.S.. Shortly after the case was filed, Gillette voluntarily rescinded the acquisition of Eemland's wet shaving razor blade business in the U.S.. Gillette said it decided to settle the case to avoid the time and expense of a lengthy trial.

However, Gillette still went through with the acquisition of the 22.9% non-voting stake in Eemland, becoming a large shareholder in competitor Wilkinson Sword. The DoJ (1990) allowed the acquisition provided that:

> Gillette and Eemland shall not agree or communicate an effort to persuade the other to agree, directly or indirectly, regarding present or future prices or other terms or conditions of sale, volume of shipments, future production schedules, marketing plans, sales forecasts, or sales or proposed sales to specific customers... (page 7)

In other words, the DoJ approved the acquisition after being assured that this stake would be passive. Indeed, Gillette claimed it was merely making an investment. However, even when the acquiring firm cannot influence the conduct of the target firm, the partial acquisition may still raise antitrust concerns. The reason being that the partial acquisition may reduce the incentive of the acquiring firm to compete aggressively because it shares in the losses thereby inflicted on that rival. We examine this question by screening the unilateral effects of this acquisition. As a comparison, we also examine Gillette's initial proposed 100% acquisition of Wilkinson Sword to screen the counterfactual unilateral effects have Gillette not voluntarily rescinded the acquisition of Eemland's wet shaving razor business in the US. Finally, we screen two additional hypothetical acquisitions. We examine an hypothetical acquisition of a 22.9% voting stake in Wilkinson Sword by Gillette, in order to compare acquisitions of voting and non-voting rights. Further, we examine an hypothetical acquisition of a 22.9% voting stake in Wilkinson Sword by Berkshire Hathaway, Inc., Gillette's largest external owner, in order to compare acquisitions giving rise to a cross- and a common-ownership structure.

5.1 The Normalized Weight Matrix

In order to apply the two proposed screening indicators to the above empirical setting, we have to calculate the normalized weight matrix $\mathbf{W}$ *pre-* and *post-acquisition*.

5.1.1 Industry Ownership Structure Pre-Acquisition

We begin by calculating the normalized weight matrix $\mathbf{W}$ *pre-acquisition*, i.e., prior to December 20, 1989. To do so, we gather information on the financial and voting rights structure of the five firms in the industry from two sources. For U.S.-based firms, we analyze the proxy statements (schedule 14A) filled by firms with the Securities and Exchange Commission, while, for Europe-based firms, we analyze the official decision of the Commission of the European Communities regarding the initially proposed (full) acquisition. Two comments are in order relative to this information. First, although we are describing the industry ownership structure pre-acquisition, i.e., in 1989, we use data from 1990, which was the earliest year available.[14] This implicitly assumes that from 1989 to 1990 the financial and voting rights structure of the firms did not suffer relevant variations other than the ones described above. Second, the above public ownership data is restricted to identify large external owners, whose rights (direct or consolidated with affiliates) are greater than 5%. As a consequence, we must make an assumption relative to the number of remaining (minority) external owners and their financial and voting rights. We make the following:

Assumption 9 *The number of external owners of each firm is taken to be as small as possible consistent with the observed data.*

Assumption 10 *Minority external owners do not engage in common-ownership.*

Assumptions 9 and 10 are merely illustrative. Assumption 9 implies that the financial and voting rights of the minority external owners of a firm are, following Leech (2002), concentrated in the smallest number of external owners possible that is consistent with the observed data. In our application, this implies that the number of remaining external owners of a firm is such that their rights (financial and voting) are assumed to be all equal to 5%.[15,16] See Leech (2002) and Levy (2011) for a description of alternative approaches. Assumption 10 implies, following Brito, Ribeiro and Vasconcelos (2017) that the minority external owners of a firm agree on the strategy that the manager should pursue. Naturally, in cases involving common-ownership among minority external owners, this assumption is not innocuous, since those owners may have conflicting views on the best strategy to pursue. In those cases, the financial and voting rights of each external owner matter, which implies that a careful evaluation of the rights of *all* owners is essential.

Table 2 presents the financial and voting rights (under Assumptions 9 and 10) of each owner, both internal and external, on the different firms active in the pre-acquisition industry. Let $\Im \equiv \{1,\ldots,5\}$ denote the set of owners that are internal to the industry, each of which is indexed by j, and $\Theta\backslash\Im \equiv \{6,\ldots,67\}$ denote the set of owners that are external to the industry, each of which is indexed by k. Table 2, Panel A addresses the interests of the *five* internal owners. It suggests that, pre-acquisition, the firms in the industry did not engage in cross-ownership. Table 2, Panel B addresses the interests of the *sixty-two* external owners (including the fictitious minority owners computed as described above). It suggests that, pre-acquisition, external owners also did not engage in common-ownership at all.

Having described the financial and voting rights structure of the five firms, we must convert that information into matrices $\mathbf{F}^*$, $\mathbf{F}$, $\mathbf{V}^*$ and $\mathbf{V}$, from which we can compute the pre-acquisition

[14] The only exceptions are the data referent to American Safety Razor Company and BIC Corporation, whose earliest year available was 1994 and 1993, respectively.

[15] Naturally with the eventual exception of a single minority external owner that may hold less than 5% (if the collective rights of all the other owners in the firm do not sum up to 100%).

[16] As an example, consider the case of a firm in which cross-ownership is absent and has four (observed) large external owners that collectively hold 91% of the rights. Assumption 9 implies we consider two additional minority external owners, holding 5% and 4% of the rights, respectively.

matrices $\mathbf{F}^u$, $\mathbf{C}^u$ and $\mathbf{W}$, as described in section 3. Appendix B presents a Matlab code to compute $\mathbf{C}^u$ from $\mathbf{V}^*$ and $\mathbf{V}$. In this industry, Assumption 2 is satisfied in the sense that the fixed-point procedure yields an unique $\mathbf{V}^u$ and $\mathbf{C}^u$. Finally, Appendix C presents the step-by-step details of the computation of $\mathbf{W}$, which in this industry - wherein cross- and common-ownership rights are absent - yields: $\mathbf{W} = \mathbf{I}$. This result implies that pre-acquisition, barring any market imperfections that preclude efficient contracting between the owners and the manager, the former agree (and give the appropriate incentives) that the latter should maximize own-operating profits. This constitutes the pre-acquisition benchmark by which all the four acquisitions discussed above are going to be evaluated below.

5.1.2 Gillette Acquires a 100% Voting Stake in Wilkinson Sword

We now address the normalized weight matrix $\mathbf{W}$ after the (hypothetical) acquisition of a 100% voting stake in Wilkinson Sword by Gillette. In this acquisition, Gillette, an internal owner, acquires 100% of the financial and voting rights in a competitor, Wilkinson Sword, from an external owner, Eemland. This constitutes a full merger and gives rise to a cross-ownership structure in the industry. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{V}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{V}$, which induce (Appendix D describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix, denoted $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

This result implies that the managers of Wilkinson Sword and Gillette have, post-acquisition, the same objective function. The two firms behave, effectively, as a single entity, in the sense that their owners agree that managers should maximize their joint operating profits. Moreover, it illustrates that our framework nests full mergers as a special case.

5.1.3 Gillette Acquires a 22.9% Voting Stake in Wilkinson Sword

In order to illustrate the computational difference between full and partial acquisitions of voting rights by internal owners, we now address the normalized weight matrix $\mathbf{W}$ after the (hypothetical) acquisition of a 22.9% *voting* stake in Wilkinson Sword by Gillette. In this acquisition, Gillette, an internal owner, acquires 22.9% of the financial and voting rights in a competitor, Wilkinson Sword, from an external owner. This constitutes a partial acquisition and so gives rise to a *partial* cross-ownership structure in the industry. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{V}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{V}$, which induce (Appendix D describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix, denoted $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 1.0000 \end{bmatrix}.$$

This result implies that the manager of Wilkinson Sword should maximize solely its own-operating profits. The reason being that even after Gillette's acquisition of 22.9% of the voting rights in Wilkinson Sword, Eemland will still hold the majority of the voting rights and, as a consequence, will still fully determine the decision-making within the firm, i.e., will still hold 100% of the corporate control rights in Wilkinson Sword. And Eemland only cares about the returns of the equity it holds in Wilkinson Sword. Further, the result implies that the manager of Gillette should maximize a weighted average of the operating profits of Gillette and Wilkinson Sword. The reason being that Gillette holds *financial* rights in Wilkinson Sword.

These conclusions illustrate that acquisitions of partial voting rights by internal owners that induce no corporate control of the target firm do not align the interests of the firms involved in the acquisition in the same qualitative vein as a merger. They change the incentives of the *acquiring* firm, but not of the *acquired* firm. Naturally, the impact of acquisitions of partial voting rights by internal owners that induce some degree of corporate control of the target firm will differ. In those cases, the acquisitions would align the interests of the firms involved in the acquisition in the same qualitative vein as a merger. The only difference would be solely on the weight attributed to the involved rival firm's operations.

5.1.4 Gillette Acquires a 22.9% Non-voting Stake in Wilkinson Sword

In order to illustrate the computational difference between partial acquisitions of voting and non-voting rights by internal owners, we now address the normalized weight matrix $\mathbf{W}$ after the (actual) acquisition of a 22.9% *non-voting* stake in Wilkinson Sword by Gillette. In this acquisition, Gillette, an internal owner, acquires 22.9% of the financial rights in a competitor, Wilkinson Sword, from an external owner, Eemland, but does not acquire any corresponding voting rights. This constitutes a passive partial acquisition and gives rise to a *partial* cross-ownership structure in the industry. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{F}$, which induce (Appendix D describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix, denoted $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 1.0000 \end{bmatrix}.$$

This result implies that the manager of Wilkinson Sword should maximize solely its own-operating profits. The reason being that the operation does not involve voting rights and, as a consequence, Eemland will still fully determine the decision-making within the firm, i.e., will still hold 100% of the corporate control rights in Wilkinson Sword. And Eemland only cares about the returns of the equity it holds in Wilkinson Sword. Further, the result implies that the manager of Gillette should maximize a weighted average of the operating profits of Gillette and Wilkinson Sword. The reason being, as before, that Gillette holds *financial* rights in Wilkinson Sword.

These conclusions illustrate that acquisitions of partial non-voting rights by internal owners do not align the interests of the firms involved in the acquisition in the same qualitative vein as a merger. They change the incentives of the *acquiring* firm, but not of the *acquired* firm.

5.1.5 Berkshire Hathaway Acquires a 22.9% Non-voting Stake in Wilkinson Sword

Finally, in order to illustrate the computational difference of partial acquisitions of non-voting rights by internal and external owners, we now address the normalized weight matrix $\mathbf{W}$ after the (hypothetical) acquisition of a 22.9% *non-voting* stake in Wilkinson Sword by Berkshire Hathaway, Gillette's largest external owner. In this acquisition, Berkshire Hathaway, an external owner, acquires 22.9% of the financial rights in a competitor, Wilkinson Sword, from another external owner, Eemland, but does not acquire any corresponding voting rights. This constitutes a passive partial acquisition and gives rise to a *partial* common-ownership structure in the industry. Comparing with the pre-acquisition structure, this implies changes to matrix $\mathbf{F}$, which induces (Appendix D describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix, denoted $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.4834 & 1.0000 \end{bmatrix}.$$

This result implies that the manager of Wilkinson Sword should maximize solely its own-operating profits. The reason being that the operation does not involve voting rights and, as a consequence, Eemland will still fully determine the decision-making within the firm, i.e., will still hold 100% of the corporate control rights in Wilkinson Sword. And Eemland only cares about the returns of the equity it holds in Wilkinson Sword. Further, the result implies that the manager of Gillette should maximize a weighted average of the operating profits of Gillette and Wilkinson Sword. The reason being that Gillette's largest controlling external owner, Berkshire Hathaway, holds *financial* rights in Wilkinson Sword. The weight attributed to Wilkinson Sword's operating profits is slightly higher than in the previous case (in which it is Gillette that acquires the 22.9% stake) because, in this case, (*i*) Berkshire Hathaway is the sole Gillette's controlling external owner holding ultimate financial rights in Wilkinson Sword, (*ii*) it is the largest Gillette's controlling external owner, and (*iii*) its return depends on Wilkinson Sword's operating profits more than on Gillette's operating profits.

These conclusions illustrate that acquisitions of partial non-voting rights by common external owners change the incentives of the firms involved in the acquisition in the same qualitative vein as acquisitions of partial non-voting rights by internal owners. And that these changes in incentives may be greater than those from acquisitions of partial non-voting rights by internal owners.

5.2 The Generalized *HHI*

Having computed the normalized weight matrix, we now address the application of the *GHHI* to the empirical setting above. To do so, as in the standard *HHI*, we require additional information on the pre-acquisition output market shares of the firms in the industry. This latter information is included in the data submitted in a typical notification to a competition agency, and for that reason does not increase the information requirements of unilateral effects analyses. Table 3 presents the pre-acquisition output shares of each firm $j \in \Im \equiv \{1, \ldots, 5\}$ in our illustration. The data is adapted from the text published by the DoJ (1990) referent to the United States of America v. The Gillette Company, *et al.* case (Civil Action No. 90-005390-0053-TFH). It suggests that Gillette is the dominant firm, accounting for 50% of all razor blade units. BIC is the second biggest-selling firm with 20%, followed by Warner-Lambert with 14% of unit sales. Wilkinson and American Safety Razor have very residual output shares.

We use the normalized weight matrices $\mathbf{W}$ and $\tilde{\mathbf{W}}$ calculated above, and the DoJ (1990) output share data to compute the *GHHI* pre- and post-acquisition for each of the cases discussed. To do so, we make use of equation (29). The results are summarized in Table 4. The pre-acquisition industry has a *GHHI* of $3,106$ points $(= (1)^2 + (20)^2 + (14)^2 + (3)^2 + (50)^2)$. This result makes clear that in the absence of cross- and common-ownership rights, the *GHHI* reduces to the standard *HHI*. Further, it suggests that the wet shaving industry was, according to the DoJ's horizontal merger guidelines, highly concentrated even before December 20, 1989.

The acquisition of a 100% voting stake in Wilkinson Sword initially proposed by Gillette would have induced a post-acquisition industry with a *GHHI* of $3,406$ points $(= (1)^2+(20)^2+(14)^2+(3)^2+(3)(50) + (50)(3) + (50)^2)$. Since the acquisition constitutes a full merger, this measure coincides with the standard *HHI* post-acquisition. The results suggests that the acquisition would have induced an increase in concentration greater than the threshold of 200 points (in an already highly concentrated industry), an impact sufficiently high for the DoJ to presume that the acquisition would likely enhance market power, justifying the civil proceeding instituted against Gillette.

Gillette voluntarily rescinded the above acquisition. Had Gillette considered a partial acquisition of a 22.9% voting stake in Wilkinson Sword, the post-acquisition industry would have had a *GHHI* of approximately $3,140$ points ($\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.2290\,(50)\,(3) + (50)^2$). This result suggests that the acquisition would have involved an increase in concentration lower than the DoJ's threshold of 100 points, which implies that it was unlikely to have adverse competitive effects, a result which ordinarily requires no further analysis.

However, Gillette did not consider the acquisition of a partial voting stake, but a *non-voting* one. The acquisition of a 22.9% non-voting stake in Wilkinson Sword induced a post-acquisition industry with a *GHHI* of approximately $3,140$ points ($\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.2290\,(50)\,(3) + (50)^2$). This result seems to validate the decision of DoJ not to challenge the operation since it suggests that the impact in concentration induced by this acquisition is exactly the same as the one that would be induced by the acquisition of the 22.9% voting stake. The reason being that latter stake does not grant any corporate control rights in Wilkinson Sword.

Finally, had the 22.9% non-voting stake in Wilkinson Sword been acquired by Berkshire Hathaway, Gillette's largest external owner, the post-acquisition industry would have a *GHHI* of approximately $3,179$ points ($\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.4834\,(50)\,(3) + (50)^2$). This implies that the increase in concentration induced by the acquisition of a 22.9% non-voting stake in Wilkinson Sword by Berkshire Hathaway would have been greater than the one induced by Gillette's direct acquisition of the same non-voting stake. A result that suggests that acquisitions that give rise to common-ownership structures, in which external owners *partially* participate in more than one competitor firm, may induce a higher increase in concentration than acquisitions, involving the same rights, by internal owners that give rise to cross-ownership structures.

5.3 The Generalized *GUPPI*

We now address the application of the *GGUPPI* to the empirical setting above. To do so, as in the standard *GUPPI*, we require information on the pre-acquisition prices, margins, and diversion ratios *for the firms involved* (i.e., for the firms whose weights exhibit changes pre- and post-acquisition). We already discussed the calculation of the normalized weight matrices pre- and post-acquisition for each of the cases under examination. An analysis of the results makes clear that, in all cases, only the weights associated with Wilkinson Sword and Gillette do change. This implies that, in our application, we require information solely on the pre-acquisition prices, margins, and diversion ratios of these two firms. This information can be computed with the data submitted in a typical notification to a competition agency, and for that reason does not increase the information requirements of unilateral effects analyses. In our application, we compute this information using the demand and cost estimates in Brito, Ribeiro and Vasconcelos (2014).

Table 5 presents the pre-acquisition median prices, margins, and diversion ratios (across markets) for Wilkinson Sword and Gillette.[17] It suggests that Wilkinson Sword prices are relatively lower than Gillette's, \$1.54 versus \$4.04, although the two firms generate slightly the same margin per product. Further, it suggests that roughly one-quarter of the unit sales lost by Gillette if its price were to rise would be captured by Wilkinson Sword, while in the reverse case, the value is considerable smaller: only 0.3% of the unit sales lost by Wilkinson Sword if its price were to rise would be captured by Gillette. In other words, Gillette's customers see Wilkinson Sword products as relatively good substitutes, but the same is not true for Wilkinson Sword's customers. The reason may lay in the fact that Gillette's products are more expensive than Wilkinson Sword's.

We use the normalized weight matrices $\mathbf{W}$ and $\tilde{\mathbf{W}}$ calculated above, and Table 5's data to compute the *GGUPPI* for each of the acquisitions discussed. To do so, we make use of equation (45). The results are summarized in Table 6. According to this indicator, the acquisition of a 100% voting stake in Wilkinson Sword initially proposed by Gillette would have induced a slight upward pricing pressure in the products of the two firms. In order to see why, note that the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ is given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix},$$

which makes clear that *(i)* in fact only the elements referent simultaneously to Wilkinson Sword and Gillette, as discussed above, change, and *(ii)* in full merger cases the *GGUPPI* coincides with the standard *GUPPI*. This implies that the acquisition's *GGUPPI*s are thus given by: $GGUPPI_j = 0$ for $j = \{1, 2, 3\}$, $GGUPPI_4 = (1.0000)\,(0.447)\,(0.003)\,/\,(1.540) \approx 0.087\%$, and $GGUPPI_5 = (1.0000)\,(0.375)\,(0.225)\,/\,(4.036) \approx 2.091\%$. This result suggests that the acquisition would have induced an upward pricing pressure of approximately 0.087% and 2.091% in Wilkinson Sword and Gillette's products, respectively. Further, it confirms the idea, suggested by the *GHHI*, that the acquisition would likely enhance Gillette's market power. However the impact is relatively small, which calls into question DoJ's civil proceeding against Gillette.

Gillette voluntarily rescinded the above acquisition. Had Gillette considered a partial acquisition of a 22.9% voting stake in Wilkinson Sword, the results seem to suggest that the impact would have been even lower. In order to see why, note that, in this case, the difference between $\tilde{\mathbf{W}}$ and

[17] Since the firms in our empirical application are multi-product firms, the data in each market refers to a quantity-share weighted average across all the products of a given firm.

$\mathbf{W}$ would have been given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 0.0000 \end{bmatrix},$$

which indicates that only the weight of Gillette's manager on Wilkinson Sword's operating profits does change. This implies that the acquisition's $GGUPPI$s are thus given by: $GGUPPI_j = 0$ for $j = \{1, 2, 3, 4\}$ and $GGUPPI_5 = (0.2290)\,(0.375)\,(0.225)\,/\,(4.036) \approx 0.479\%$. This result suggests that the acquisition would have induced an upward pricing pressure of approximately 0.479% in Gillette's products.

However, Gillette did not consider the acquisition of a partial voting stake, but a *non-voting* one. The results relative to the acquisition of a 22.9% non-voting stake in Wilkinson Sword seem to validate the decision of DoJ not to challenge the operation since the acquisition was unlikely to have had adverse competitive effects. The upward pricing pressure in Gillette's products is screened to be exactly the same as the one that would be induced by the acquisition of the 22.9% voting stake, which is in fact very small. In order to see why, note that the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ is given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 0.0000 \end{bmatrix},$$

which indicates that, in fact, only the weight of Gillette's manager on Wilkinson Sword's operating profits does change and implies that solely Gillette's products exhibited an upward pricing pressure: $GGUPPI_j = 0$ for $j = \{1, 2, 3, 4\}$ and $GGUPPI_5 = (0.2290)\,(0.375)\,(0.225)\,/\,(4.036) \approx 0.479\%$. The reason being that the acquisition of the latter 22.9% voting stake does not grant any corporate control rights in Wilkinson Sword.

Finally, had the 22.9% non-voting stake in Wilkinson Sword been acquired by Berkshire Hathaway, Gillette's largest external owner, the upward pricing pressure in Gillette's products would have been slightly higher. In order to see why, note that, in this case, the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ would have been given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.4834 & 0.0000 \end{bmatrix},$$

which indicates that, in fact, only the weight of Gillette's manager on Wilkinson Sword's operating profits changes and implies that solely Gillette's products would exhibit an upward pricing pressure. However, the acquisition's *GGUPPI*s are given by: $GGUPPI_j = 0$ for $j = \{1, 2, 3, 4\}$ and $GGUPPI_5 = (0.4834)\,(0.375)\,(0.225)\,/\,(4.036) \approx 1.011\%$. Hence the upward pricing pressure in Gillette's products induced the acquisition of a 22.9% non-voting stake in Wilkinson Sword by Berkshire Hathaway is screened to have been higher than the one induced by Gillette's direct acquisition of the same non-voting stake. A result that confirms, as suggested by the *GHHI*, that acquisitions that give rise to common-ownership structures in which external owners *partially* participate in more than one competitor firm may induce a higher upward pricing pressure than acquisitions, involving the same rights, by internal owners that give rise to cross-ownership structures.

6 Conclusions

This paper puts forward proposals to generalize the two most traditional indicators – the Herfindahl-Hirschman Index and the Gross Upward Price Pressure Index – used by competition agencies to screen potential anti-competitive unilateral effects regarding partial horizontal acquisitions. The proposed generalized indicators are endogenously derived under a probabilistic voting model in which the manager of each firm is elected in a shareholder assembly between two potential candidates who seek to obtain utility from an exogenous rent associated with corporate office. The model (i) can cope with settings involving all types of owners and rights: owners that can be internal to the industry (rival firms) and external to the industry; and rights that can capture financial and corporate control interests, can be direct and indirect, can be partial or full, (ii) yields an endogenous measure of the owners ultimate corporate control rights, and (iii) can also be used - in case the potential acquisition is inferred to likely enhance market power - to devise divestiture structural remedies.

We also provide an empirical application of the two proposed generalized indicators to several acquisitions in the wet shaving industry. An interesting result of this empirical application is that acquisitions that give rise to common-ownership structures in which external owners partially participate in more than one competitor firm may induce higher unilateral anti-competitive effects than acquisitions, involving the same rights, by internal owners that give rise to cross-ownership structures.

Appendix A: Propositions

In this appendix, we present the proofs of Propositions 1 to 4.

Proof of Proposition 1. To prove that the ultimate financial rights of external owners implied by matrix $\mathbf{F}^u$ are non-negative, note that, as discussed above, , under Assumption 1, matrix $(\mathbf{I}_N - \mathbf{F}^*)^{-1}$ exists and can be expressed as a power series of $\mathbf{F}^*$, i.e., $(\mathbf{I}_N - \mathbf{F}^*)^{-1} = \sum_{t=0}^{\infty} (\mathbf{F}^*)^t$. Given that the direct financial rights captured in $\mathbf{F}^*$ and $\mathbf{F}$ are non-negative, this implies that $\phi^u_{kj} \geq 0$ for all $k \in \Theta, k \notin \Im$ and all $j \in \Im$.

Finally, to prove that the ultimate financial rights of external owners implied by matrix $\mathbf{F}^u$ sum up to one for any given firm $j \in \Im$, note that the direct financial rights captured in $\mathbf{F}^*$ and $\mathbf{F}$ sum up to one for any given firm $j \in \Im$. This implies that:

$$\begin{aligned} (\mathbf{1}_N)\,\mathbf{F}^* + (\mathbf{1}_{K-N})\,\mathbf{F} &= \mathbf{1}_N \\ (\mathbf{1}_{K-N})\,\mathbf{F} &= (\mathbf{1}_N)\,(\mathbf{I}_N - \mathbf{F}^*)\,, \end{aligned} \tag{47}$$

where $\mathbf{1}_N$ and $\mathbf{1}_{K-N}$ denote a $(N \times 1)$ and a $((N-K) \times 1)$ vector of ones, respectively. Since $(\mathbf{I}_N - \mathbf{F}^*)^{-1}$ exists, we can right multiply both sides by it, as follows:

$$(\mathbf{1}_{K-N})\,\mathbf{F}\,(\mathbf{I}_N - \mathbf{F}^*)^{\,\mathbf{-1}} = (\mathbf{1_N})\,(\mathbf{I}_N - \mathbf{F}^*)\,(\mathbf{I}_N - \mathbf{F}^*)^{\,\mathbf{-1}}. \tag{48}$$

This yields that $(\mathbf{1}_{K-N})\,\mathbf{F}\,(\mathbf{I}_N - \mathbf{F}^*)^{\,\mathbf{-1}} = (\mathbf{1}_{K-N})\,\mathbf{F}^u = (\mathbf{1}_N)$ and hence that $\sum_{k \in \Theta, k \notin \Im} \phi^u_{kj} = 1$ for all $j \in \Im$.

Proof of Proposition 2. To prove that the ultimate voting rights of external owners implied by matrix $\mathbf{V}^u$ are non-negative, note that the direct voting rights captured in $\mathbf{V}^*$ and $\mathbf{V}$ as well as the ultimate corporate control rights captured in $\mathbf{C}^u = \mathcal{F}(\mathbf{V}^u)$ are non-negative. This implies that $\upsilon^u_{kj} \geq 0$ for all $k \in \Theta, k \notin \Im$ and all $j \in \Im$.

Finally, to prove that the ultimate voting rights of external owners implied by matrix $\mathbf{V}^u$ sum up to one for any given firm $j \in \Im$, note that the direct voting rights captured in $\mathbf{V}^*$ and $\mathbf{V}$ sum up to one for any given firm $j \in \Im$ and that the ultimate corporate control rights captured in $\mathbf{C}^u = \mathcal{F}(\mathbf{V}^u)$ also sum up to one for any given firm $j \in \Im$. This implies that for all $j \in \Im$:

$$\begin{aligned} \sum_{k \in \Theta, k \notin \Im} \upsilon^u_{kj} &= \sum_{k \in \Theta, k \notin \Im} \upsilon_{kj} + \sum_{k \in \Theta, k \notin \Im} \sum\nolimits_{g \in \Im \backslash j} \gamma^u_{kg} \upsilon_{gj} \\ &= \sum_{k \in \Theta, k \notin \Im} \upsilon_{kj} + \sum\nolimits_{g \in \Im \backslash j} \left(\sum_{k \in \Theta, k \notin \Im} \gamma^u_{kg} \right) \upsilon_{gj} \\ &= \sum_{k \in \Theta, k \notin \Im} \upsilon_{kj} + \sum\nolimits_{g \in \Im \backslash j} \upsilon_{gj} \\ &= \sum_{k \in \Theta} \upsilon_{kj} \\ &= 1. \end{aligned} \tag{49}$$

Proof of Proposition 3. Under Assumption 5, external owners are conditionally sincere, which implies that the incumbent candidate to firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. As a consequence, the first-order condition associated with her maximization problem is, independently of whether the solution is in the interior or the border of Ω_j, given by:

$$\begin{aligned} &\sum\nolimits_{\Theta^\imath_j \in \wp_j} \sum\nolimits_{k \in \Theta^\imath_j} g\left(R_k(\mathbf{x}_a) - R_k(\mathbf{x}_b)\right) \frac{\partial R_k(\mathbf{x}_a)}{\partial x_{aj}} \prod\nolimits_{h=1, h \neq k}^{\ell_j} G\left((2d_h - 1)\left(R_h(\mathbf{x}_a) - R_h(\mathbf{x}_b)\right)\right) \\ -&\sum\nolimits_{\Theta^\imath_j \in \wp_j} \sum\nolimits_{k \notin \Theta^\imath_j} g\left(R_k(\mathbf{x}_b) - R_k(\mathbf{x}_a)\right) \frac{\partial R_k(\mathbf{x}_a)}{\partial x_{aj}} \prod\nolimits_{h=1, h \neq k}^{\ell_j} G\left((2d_h - 1)\left(R_k(\mathbf{x}_a) - R_k(\mathbf{x}_b)\right)\right) \lesseqgtr 0, \end{aligned} \tag{50}$$

where $g(\cdot)$ denotes the probability density function of the random utility shocks.

Under Assumption 8 we have that her maximization problem has an unique maximum. This implies, given the symmetry of the maximization problem of the challenger candidate to firm j, that the two candidates will choose the same best-response function, i.e., the same strategy proposal for the firm, conditional on the strategies of the candidates to the remaining firms. We now show that this best-response function is the same as the best-response function that arises while maximizing a weighted average of the returns of the firm's ultimate external owners conditional on the strategies of the candidates to the remaining firms, with normalized Banzhaf (1965) power indices as weights. To do so, note that since the two candidates will choose the same best-response function, in equilibrium, we have $R_k(\mathbf{x}_a) = R_k(\mathbf{x}_b) = R_k(\mathbf{x})$ for all $k \in \Theta \backslash \Im$. This implies that the first-order condition reduces to:

$$\frac{1}{2^{\ell_j - 1}} \sum\nolimits_{\Theta_j^{\imath} \in \wp_j} \sum\nolimits_{k \in \Theta_j^{\imath}} \frac{\partial R_k(\mathbf{x})}{\partial x_j} - \frac{1}{2^{\ell_j - 1}} \sum\nolimits_{\Theta_j^{\imath} \in \wp_j} \sum\nolimits_{k \notin \Theta_j^{\imath}} \frac{\partial R_k(\mathbf{x})}{\partial x_j} \lesseqgtr 0, \tag{51}$$

which makes use of the fact that the probability distribution of the random utility shocks is symmetric with mean zero: $G(0) = 1/2$, and that $g(0) > 0$. This first-order condition can, in turn, be rewritten as:

$$\frac{1}{2^{\ell_j - 1}} \sum\nolimits_{k \in \Theta \backslash \Im} \left(\lambda_{jk} \frac{\partial R_k(\mathbf{x})}{\partial x_j} - \left(2^{\ell_j - 1} - \lambda_{jk}\right) \frac{\partial R_k(\mathbf{x})}{\partial x_j} \right) \lesseqgtr 0, \tag{52}$$

where λ_{jk} denotes the number of subsets in $\wp_j$ in which owner k enters and $\left(2^{\ell_j - 1} - \lambda_{jk}\right)$ denotes the number of subsets in $\wp_j$ in which owner k does not enter. Finally, consider that λ_{jk} can be divided in two terms: the number of subsets in $\wp_j$ in which owner k enters and is pivotal, λ_{jk}^{p}, and the number of subsets in $\wp_j$ in which owner k enters and is not pivotal, $\lambda_{jk}^{\tilde{p}}$.[18] The latter is, by construction, equal to the number of subsets in $\wp_j$ in which owner k does not enter. This implies that $\lambda_{jk}^{\tilde{p}} = \left(2^{\ell_j - 1} - \lambda_{jk}\right)$ and that the first-order condition can be rewritten as:

$$\sum\nolimits_{k \in \Theta \backslash \Im} \left(\frac{\lambda_{jk}^{p}}{2^{\ell_j - 1}} \right) \frac{\partial R_k(\mathbf{x})}{\partial x_j} \lesseqgtr 0, \tag{53}$$

where $\lambda_{jk}^{p}/2^{\ell_j - 1}$ denotes the Banzhaf (1965) power index associated to external owner k in firm j. This establishes that, in equilibrium, the candidates to each firm converge to the same strategy, which maximizes the following weighted average of the returns of the external owners with ultimate voting rights in the firm, conditional on the strategies of the candidates to the remaining firms:

$$\max_{x_j} \sum\nolimits_{k \in \Theta \backslash \Im} \alpha_{kj} R_k(\mathbf{x}), \tag{54}$$

where $\alpha_{kj} = \left(\lambda_{jk}^{p}/2^{\ell_j - 1}\right) / \left(\sum_{k \in \Theta \backslash \Im} \left(\lambda_{jk}^{p}/2^{\ell_j - 1}\right)\right)$ denotes the weight assigned by firm j's manager to the return of external owner k, measured by the normalized Banzhaf (1965) power index of external owner k in firm j.

Finally, given that the strategy proposal of each candidate to the different firms is, under Assumption 4, defined in a convex set and $R_k(\mathbf{x})$ is, under Assumption 6, continuous, the best-response functions of the candidates to the different firms are guaranteed to be upper-hemicontinuous, which implies that we can apply Kakutani's fixed point theorem to ensure that the Nash equilibrium exists.

Proof of Proposition 4. To prove that the objective function of the manager associates a zero weight to the ultimate financial rights of an external owner with no ultimate corporate control rights in the firm, note that if

[18] External owner k is pivotal if for some subset $\Theta_j^{\imath}$ which does not include owner k, we have $\sum_{h \in \Theta_j^{\imath}} \upsilon_{hj}^{u} \leq 0.5$, but if we include owner k, $\sum_{h \in \Theta_j^{\imath}} \upsilon_{hj}^{u} > 0.5$.

external owner k has $\gamma^u_{kj} = 0$, then her ultimate financial rights in any firm g, ϕ^u_{kg}, are not weighted at all by the manager of firm j.

Finally, to prove that, as the ultimate corporate control of an external owner in the firm increases, the objective function of the manager associates a higher weight to the ultimate financial rights of that owner, note that the marginal effect of the ultimate financial rights of external owner k in any firm g in the objective function of the manager of firm j is increasing in the ultimate corporate control rights of that owner in firm j: $\partial^2 \varpi_j / \partial \phi^u_{kg} \partial \gamma^u_{kj} = \pi_g > 0$.

Appendix B: Ultimate Voting and Corporate Control Rights

In this appendix, we present a Matlab code to compute the ultimate voting and corporate control rights of external owners. The main code is the following:

```
Cu0 = zeros(K-N,N);
Vu0 = V + Cu0*Vstar;
diff = 1;
while diff>0.000001;
    tmp0 = [Cu0; Vu0];
    Cu1 = banzafh(Vu0,N,K);
    Vu1= V + Cu1*Vstar;
    tmp1 = [Cu1; Vu1];
    diff = max(max(tmp1-tmp0));
    Cu0 = Cu1;
    Vu0 = Vu1;
end;
```

The auxiliary function that computes the Banzafh power index is the following:

```
function [CN] = banzafh(a,b,d)
C = zeros(size(a,1),size(a,2));
for i = 1:1:b
    tmp = find(a(:,i));
    tmp2 = size(tmp,1);
    den = 2^(tmp2-1);
    if tmp2==1; niter = 1; else niter = 1+ tmp2*(tmp2-1); end;
    for k1 = 1:1:tmp2
        num = 0;
        if a(tmp(k1),i)>0.5; num = num + 1; end;
        for k2 = 1:1:(tmp2-1)
            tmp3 = tmp;
            tmp3(k1) = [];
            tmp4 = a(tmp3,i);
```

```
            tmp5 = combntns(tmp4,k2);
            tmp6 = sum(tmp5,2); clear tmp5;
            tmp7 = tmp6 + a(tmp(k1),i);
            tmp8 = (tmp7>0.5)-(tmp6>0.5);
            num = num + sum(tmp8);
        end;
        C(tmp(k1),i) = num/den;
    end;
end;
tmp = sum(C);
CN = zeros(size(C,1),size(C,2));
for i = 1:1:b;
    if tmp(i)==0;
        CN(:,i) = C(:,i);
    else
        CN(:,i) = C(:,i)./(ones(size(C,1),1)*tmp(i));
    end;
end;
```

Appendix C: Normalized Weight Matrix Pre-Acquisition

In this appendix, we present the step-by-step computation details of the pre-acquisition normalized weight matrix. We begin by describing how to convert the financial and voting rights of the five firms in the industry into the four matrices that are instrumental in computing the weight matrix $\mathbf{W}$: matrices $\mathbf{F}^*$ and $\mathbf{V}^*$, which capture eventual cross-ownership by internal owners, and matrices $\mathbf{F}$ and $\mathbf{V}$, which capture eventual common-ownership by external owners.

We address first the former. Matrices $\mathbf{F}^*$ and $\mathbf{V}^*$ denote, in our application, (5×5) matrices. The diagonal elements are, by definition, zero. The off-diagonal elements, ϕ_{jg} and υ_{jg}, represent the financial and voting rights of firm j in firm g, respectively, for all $j, g \in \Im$ and $j \neq g$. In both cases, the rows and columns are ordered from $j = 1$ to $j = 5$. Given that, pre-acquisition, firms in the industry do not engage in cross-ownership, we have that $\phi_{jg} = 0$ and $\upsilon_{jg} = 0$ for all $j, g \in \Im$ and $j \neq g$. This implies that $\mathbf{F}^*$ and $\mathbf{V}^*$, pre-acquisition, constitute null matrices:

$$\mathbf{F}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \mathbf{V}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

We now address the latter. Matrices $\mathbf{F}$ and $\mathbf{V}$ denote, in our application, (62×5) matrices. The typical element is given by ϕ_{kj} and υ_{kj}, respectively, for all $j \in \Im$ and all $k \in \Theta \backslash \Im$. The rows are ordered from $k = 6$ to $k = 67$, while the columns are ordered from $j = 1$ to $j = 5$. For instance, external owner Berkshire Hathaway, indexed as

$k = 49$, holds 10.80% of financial and voting (both) rights in Gillette, indexed as $j = 5$. As a consequence, we have that $\phi_{49,5} = 0.1080$ and $\upsilon_{49,5} = 0.1080$. Formally, pre-acquisition matrices $\mathbf{F}$ and $\mathbf{V}$ are given by:

$$\mathbf{F} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix}.$$

Having constructed matrices $\mathbf{F}^*$, $\mathbf{V}^*$, $\mathbf{F}$, and $\mathbf{V}$, we have all the necessary information to compute matrices $\mathbf{F}^u$ and $\mathbf{C}^u$, which capture the ultimate ownership rights, as described in section 3. Appendix B presents a Matlab code to compute $\mathbf{C}^u$ from $\mathbf{V}^*$ and $\mathbf{V}$. In this industry, Assumption 2 is satisfied in the sense that the fixed-point procedure yields an unique $\mathbf{V}^u$ and $\mathbf{C}^u$. Finally, we can use $\mathbf{F}^u$ and $\mathbf{C}^u$ to compute the pre-acquisition matrices $\mathbf{L}$ and $\mathbf{W}$. This computation yields:

$$\mathbf{L} = \begin{bmatrix} 0.0775 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0575 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Appendix D: Normalized Weight Matrix Post-Acquisition

In this appendix, we present the step-by-step computation details of the post-acquisition normalized weight matrix for each of the cases considered in our application.

Gillette Acquires a 100% Voting Stake in Wilkinson Sword

The (hypothetical) acquisition of a 100% voting stake in Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry. In this acquisition, Gillette, an internal owner, acquires 100% of the financial and voting rights in a competitor, Wilkinson Sword, from an external owner, Eemland. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{V}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{V}$.

We address first the former. Let $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{V}}^*$ denote matrices $\mathbf{F}^*$ and $\mathbf{V}^*$ *post-acquisition.* The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial and voting rights of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ and $\tilde{\upsilon}_{jg} = \upsilon_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial and voting rights of Wilkinson Sword in Gillette also remain unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$ and $\tilde{\upsilon}_{4,5} = \upsilon_{4,5}$. However, the financial and voting rights of Gillette in Wilkinson Sword increase to $\tilde{\phi}_{5,4} = 1$ and to $\tilde{\upsilon}_{5,4} = 1$, respectively. This implies the following post-acquisition matrices $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{V}}^*$:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix} \tilde{\mathbf{V}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$ denote matrices $\mathbf{F}$ and $\mathbf{V}$ *post-acquisition.* All elements relative to the financial and voting rights in American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ and $\tilde{\upsilon}_{kj} = \upsilon_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial and voting rights in Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ and $\tilde{\upsilon}_{k4} = \upsilon_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{48\}$. However, the financial and voting rights of Eemland in Wilkinson Sword are reduced to $\tilde{\phi}_{48,4} = 0$ and to

$\tilde{v}_{16,4} = 0$, respectively. This implies the following post-acquisition matrices $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix} \quad \tilde{\mathbf{V}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{V}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$, we have all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$, which capture the post-acquisition ultimate ownership rights. Appendix B presents a Matlab code to compute $\tilde{\mathbf{C}}^u$ from $\tilde{\mathbf{V}}^*$ and $\tilde{\mathbf{V}}$. In this industry, Assumption 2 is satisfied in the sense that the fixed-point procedure yields an unique $\tilde{\mathbf{V}}^u$ and $\tilde{\mathbf{C}}^u$. Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the post-acquisition matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.0775 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0570 & 0.0570 \\ 0.0000 & 0.0000 & 0.0000 & 0.0575 & 0.0575 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

Gillette Acquires a 22.9% Voting Stake in Wilkinson Sword

The acquisition of a 22.9% voting stake in Wilkinson Sword by Gillette gives rise to a *partial* cross-ownership structure in the industry. In this acquisition, Gillette, an internal owner, acquires 22.9% of the financial and voting rights in a competitor, Wilkinson Sword, from an external owner, Eemland. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{V}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{V}$.

We address first the former. Let $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{V}}^*$ denote the cross-ownership matrices *post-acquisition*. The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial and voting rights of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ and $\tilde{\upsilon}_{jg} = \upsilon_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial and voting rights of Wilkinson Sword in Gillette also remain unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$ and $\tilde{\upsilon}_{4,5} = \upsilon_{4,5}$. However, the financial and voting rights of Gillette in Wilkinson Sword increase to $\tilde{\phi}_{5,4} = 0.229$ and to $\tilde{\upsilon}_{5,4} = 0.229$, respectively. This implies the following post-acquisition matrices $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{V}}^*$:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix} \tilde{\mathbf{V}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$ denote the common-ownership matrices *post-acquisition*. All elements relative to the financial and voting rights in American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ and $\tilde{\upsilon}_{kj} = \upsilon_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial and voting rights in Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ and $\tilde{\upsilon}_{k4} = \upsilon_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{16\}$. However, the financial and voting rights of Eemland in Wilkinson Sword are reduced to

$\tilde{\phi}_{16,4} = 0.771$ and to $\tilde{v}_{16,4} = 0.771$, respectively. This implies the following post-acquisition matrices $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix} \quad \tilde{\mathbf{V}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{V}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{V}}$, we have all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$, which capture the post-acquisition ultimate ownership rights. Appendix B presents a Matlab code to compute $\tilde{\mathbf{C}}^u$ from $\tilde{\mathbf{V}}^*$ and $\tilde{\mathbf{V}}$. In this industry, Assumption 2 is satisfied in the sense that the fixed-point procedure yields an unique $\tilde{\mathbf{V}}^u$ and $\tilde{\mathbf{C}}^u$. In particular, we have that $\tilde{\mathbf{C}}^u = \mathbf{C}^u$ since even after Gillette's acquisition of 22.9% of the voting rights in Wilkinson Sword, Eemland will still hold the majority of the voting rights and, as a consequence, will still fully determine the decision-making within the firm, i.e., will still hold 100% of the corporate control rights in Wilkinson Sword. Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the post-acquisition matrices

$\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.0775 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0132 & 0.0575 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 1.0000 \end{bmatrix}.$$

Gillette Acquires a 22.9% Non-voting Stake in Wilkinson Sword

The acquisition of a 22.9% non-voting stake in Wilkinson Sword by Gillette gives rise to a *partial* cross-ownership structure in the industry. In this acquisition, Gillette, an internal owner, acquires 22.9% of the financial rights in a competitor, Wilkinson Sword, from an external owner, Eemland, but does not acquire any corresponding voting rights. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{F}$.

We address first the former. Let $\tilde{\mathbf{F}}^*$ denote matrix $\mathbf{F}^*$ *post-acquisition.* The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial rights of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial rights of Wilkinson Sword in Gillette also remains unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$. However, the financial rights of Gillette in Wilkinson Sword increase to $\tilde{\phi}_{5,4} = 0.229$. This implies the following post-acquisition matrix $\tilde{\mathbf{F}}^*$:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 0.0000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ denote matrix $\mathbf{F}$ *post-acquisition.* All elements relative to the financial rights in American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial rights in Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{48\}$. However, the financial rights of Eemland in Wilkinson Sword are reduced to $\tilde{\phi}_{16,4} = 0.771$. This implies the following post-acquisition matrix $\tilde{\mathbf{F}}$:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{F}}$, we have all the necessary information to compute the corresponding matrix $\tilde{\mathbf{F}}^u$, which captures the post-acquisition ultimate financial rights. Finally, we can use the post-acquisition $\tilde{\mathbf{F}}^u$ and the pre-acquisition $\mathbf{C}^u$ to compute the post-acquisition matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.0775 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0132 & 0.0575 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 1.0000 \end{bmatrix}.$$

Berkshire Hathaway Acquires a 22.9% Non-voting Stake in Wilkinson Sword

The (hypothetical) acquisition of a 22.9% *non-voting* stake in Wilkinson Sword by Berkshire Hathaway gives rise to a *partial* common-ownership structure in the industry. In this acquisition, Berkshire Hathaway, an external owner, acquires 22.9% of the financial rights in a competitor, Wilkinson Sword, from another external owner, Eemland, but does not acquire any corresponding voting rights. Comparing with the pre-acquisition structure, this implies changes to matrix $\mathbf{F}$.

Let $\tilde{\mathbf{F}}$ denote matrix $\mathbf{F}$ *post-acquisition*. All elements relative to the financial rights in American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial rights in Wilkinson Sword remain unchanged *for all external owners except Eemland and Berkshire Hathaway*: $\tilde{\phi}_{k4} = \phi_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{48, 49\}$. However, the financial rights of Eemland in Wilkinson Sword are reduced to $\tilde{\phi}_{48,4} = 0.771$, while the financial rights of Berkshire Hathaway in Wilkinson Sword increase to $\tilde{\phi}_{49,4} = 0.229$. As a consequence the post-acquisition matrix $\tilde{\mathbf{F}}$ is given by:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.1440 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0780 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0700 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0610 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0510 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0500 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0220 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0230 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2290 & 0.1080 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0500 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0320 \end{bmatrix}.$$

Having constructed matrix $\tilde{\mathbf{F}}$, we have, jointly with the pre-acquisition $\mathbf{F}^*$, all the necessary information to compute the corresponding matrix $\tilde{\mathbf{F}}^u$, which captures the post-acquisition ultimate financial rights. Finally, we can use the post-acquisition $\tilde{\mathbf{F}}^u$ and the pre-acquisition $\mathbf{C}^u$ to compute the post-acquisition matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.0775 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7770 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.7710 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0278 & 0.0575 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.4834 & 1.0000 \end{bmatrix}.$$

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Table 1
*Literature on Screening Indicators**

	Cross Ownership	Common Ownership	Cross- and Common Ownership
Panel A: Extensions to HHI			
Direct, F	Reynolds and Snapp (1986)	–	–
Direct, F and CC	Bresnahan and Salop (1986)	O'Brien and Salop (2000)[a]	–
Direct and Indirect, F	DSV (2000)	–	–
Direct and Indirect, F and CC	–	–	Current Paper, ARS (2016)[b]
Panel B: Extensions to GUPPI			
Direct, F and CC	–	O'Brien and Salop (2000)[a],[c]	–
Direct and Indirect, F	–	–	–
Direct and Indirect, F and CC	–	–	Current Paper

* F and CC denote financial rights and corporate control rights, respectively. DSV (2000) denotes Dietzenbacher, Smid and Volkerink (2000). ARS (2016) denotes Azar, Raina and Schmalz (2016), (a) O'Brien and Salop (2000) model common-ownership of direct F and CC rights, but do not address the question of how to measure the owners' CC rights. (b) ARS (2016) model cross- and common-ownership of direct and indirect F and CC rights, but do not address the question of how to measure the owners' CC rights nor the assumptions required to apply their proposed indicator to full acquisitions by internal owners. (c) O'Brien and Salop (2000) propose a PPI, a measure which lead afterwards to the GUPPI, but was never generalized to partial horizontal acquisition settings.

TABLE 2

*Financial and Voting Rights**

j/k		ASR		BIC		WL		WS		G	
		F	V	F	V	F	V	F	V	F	V
Panel A: Internal Owners											
01	American Safety Razor Company	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
02	BIC Corporation	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00
03	Warner-Lambert Company	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00
04	Wilkinson Sword, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	–	–	0.00	0.00
05	The Gillette Company	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–
Panel B: External Owners											
06	Equitable (a)	14.40	14.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
07	Allsop Venture Partners III, LP	12.40	12.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
08	Goldman Sachs Group, LP	7.80	7.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
09	Scudder Stevens and Clarck	7.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	Leucadia-Mezzanine (b)	6.10	6.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	Grantham Mayo Van Otter	5.10	5.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	ASR Minority Owner 1	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	ASR Minority Owner 2	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	ASR Minority Owner 3	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	ASR Minority Owner 4	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	ASR Minority Owner 5	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	ASR Minority Owner 6	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	ASR Minority Owner 7	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	ASR Minority Owner 8	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	ASR Minority Owner 9	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	ASR Minority Owner 10	2.20	2.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	Bruno Bich	0.00	0.00	77.70	77.70	0.00	0.00	0.00	0.00	0.00	0.00
23	BIC Minority Owner 1	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
24	BIC Minority Owner 2	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
25	BIC Minority Owner 3	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
26	BIC Minority Owner 4	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
27	BIC Minority Owner 5	0.00	0.00	2.30	2.30	0.00	0.00	0.00	0.00	0.00	0.00

Table 2

Extended

j/k		ASR		BIC		WL		WS		G	
		F	V	F	V	F	V	F	V	F	V
Panel B: External Owners											
28	WL Minority Owner 1	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
29	WL Minority Owner 2	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
30	WL Minority Owner 3	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
31	WL Minority Owner 4	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
32	WL Minority Owner 5	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
33	WL Minority Owner 6	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
34	WL Minority Owner 7	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
35	WL Minority Owner 8	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
36	WL Minority Owner 9	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
37	WL Minority Owner 10	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
38	WL Minority Owner 11	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
39	WL Minority Owner 12	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
40	WL Minority Owner 13	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
41	WL Minority Owner 14	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
42	WL Minority Owner 15	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
43	WL Minority Owner 16	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
44	WL Minority Owner 17	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
45	WL Minority Owner 18	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
46	WL Minority Owner 19	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
47	WL Minority Owner 20	0.00	0.00	0.00	0.00	5.00	5.00	0.00	0.00	0.00	0.00
48	Eemland Management Services BV	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00

Table 2

Extended

j/k		ASR		BIC		WL		WS		G	
		F	V	F	V	F	V	F	V	F	V
Panel B: External Owners											
49	Berkshire Hathaway, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.80	10.80
50	State Street Bank and Trust Co.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.00	6.00
51	G Minority Owner 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
52	G Minority Owner 2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
53	G Minority Owner 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
54	G Minority Owner 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
55	G Minority Owner 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
56	G Minority Owner 6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
57	G Minority Owner 7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
58	G Minority Owner 8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
59	G Minority Owner 9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
60	G Minority Owner 10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
61	G Minority Owner 11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
62	G Minority Owner 12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
63	G Minority Owner 13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
64	G Minority Owner 14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
65	G Minority Owner 15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
66	G Minority Owner 16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	5.00
67	G Minority Owner 17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.20	3.20

* Figures are in percentage points. Adapted from Schedule 14A (proxy statement) information and 93/525/EEC decision. F and V denote, under Assumptions 9 and 10, each owner's financial and voting rights, respectively. ASR, B, WL, WS and G denote American Safety Razor Company, BIC Corporation, Warner-Lambert Company, Wilkinson Sword, Inc., and The Gillette Company, respectively. (a) Equitable denotes the cumulative ownership of Equitable Capital Partners, LP, Equitable Deal Flow Fund, LP, Equitable Capital Partners (Retirement Fund), LP, and The Equitable Life Assurance Society of the United States. (b) Leucadia-Mezzanine denotes the cumulative ownership of Leucadia Investors, Inc. and Mezzanine Capital and Income Trust 2001 PLC.

Table 3

*Firm Pre-Acquisition Output Market Shares**

American Safety Razor Company	1%
BIC Corporation	20%
Warner-Lambert Company	14%
Wilkinson Sword, Inc.	3%
The Gillette Company	50%

* Figures adapted from DoJ (1990).

Table 4

*GHHI**

	WS	WS acquired by			
	independent	G	G	G	BH
	shareholder	100%	22.9%	22.9%	22.9%
	structure	voting	voting	non-voting	non-voting
GHHI	3,106	3,406	3,140	3,140	3,179
ΔGHHI	–	300	34	34	73

* WS, G, and BH denote Wilkinson Sword, Gillette, and Berkshire Hathaway, respectively. ΔGHHI denotes the change in GHHI pre- and post-acquisition.

Table 5

*Median Prices, Margins and Diversion Ratios**

	WS	G
Panel A: Prices and Margins ($)		
Price	1.540	4.036
Margin	0.375	0.447
Panel B: Diversion Ratios		
Wilkinson Sword, Inc.	-1.000	0.225
The Gillette Company	0.003	-1.000

* Information computed using the demand and cost estimates in Brito, Ribeiro and Vasconcelos (2014). Price and margin figures are in USD. WS and G denote Wilkinson Sword and Gillette, respectively.

Table 6

*GGUPPI**

	WS acquired by			
	G	G	G	BH
	100%	22.9%	22.9%	22.9%
	voting	voting	non-voting	non-voting
American Safety Razor Company	0.000%	0.000%	0.000%	0.000%
BIC Corporation	0.000%	0.000%	0.000%	0.000%
Warner-Lambert Company	0.000%	0.000%	0.000%	0.000%
Wilkinson Sword, Inc.	0.087%	0.000%	0.000%	0.000%
The Gillette Company	2.091%	0.479%	0.479%	1.011%

* WS, G, and BH denote Wilkinson Sword, Gillette, and Berkshire Hathaway, respectively.