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Group contest success function: The heterogeneous individuals case

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Abstract

This paper extends the axiomatic characterization of the group contest success function in Münster (2009) to groups with heterogeneous individuals (e.g., individuals with different skills or different cognitive capacities). The obtained function allows for differences in terms of effort effectiveness between the group individuals and differences in terms of returns to scale at the aggregate level.

Keywords: Group contests; Multi-issue contests; Success function; Heterogeneity; Characterization.
JEL classification: C70, D72, D74.

1. Introduction

In group contests, different groups of individuals compete for a prize (Kolmar (2013) offers a comprehensive survey of the literature).[1] For instance, in R&D races, groups of researchers work together to be the first to develop a new technology. In competitive markets, competition between firms is actually competition between different groups of individuals. In sports, players provide joint effort to increase their teams winning chances. In general, groups are composed of heterogeneous individuals with potentially different skills and backgrounds that exercise concerted physical and mental effort in order to reach a common objective. In this context, there are important differences between the individuals in terms of productivity, effort efficiency and returns to scale.

[1]The group contests literature is particularly extense and well-developed (Baik, 2008; Esteban and Ray, 2001; Hausken, 2012; Katz et al., 1990; Katz and Tokatlidu, 1996; Kolmar and Rommeswinkel, 2013; Münster, 2007; Nitzan, 1991; Nitzan and Ueda, 2009; Rinott et al., 2012; among others).

This paper presents a simple but novel extension to the axiomatic characterization of the group contest success functions in Münster (2009) to groups with heterogeneous individuals (i.e., groups composed of individuals with different skills and cognitive characteristics). This result is important because it expands the set of technical tools that can be used to study group contests and multi-issue competition.

The Tullock (1980) contest success function that is frequently employed to model competition between individuals was first characterized by Skaperdas (1996). Later, Clark and Riis (1998) drops the symmetry assumption in Skaperdas (1996) to allow for asymmetric individuals. Recently, Münster (2009) extends the axioms in Skaperdas (1996) and Clark and Riis (1998) to group contests.[2] Münster considers asymmetry between groups, but not between the members of each group. The present paper extends Münster (2009) by allowing for asymmetry between the members of each group. Moreover, it also distinguishes between the individual and the group output functions in terms of effort effectiveness and returns to scale.

Another contribution of this paper is that the proposed extension can also be applied to multi-issue contests, in which different individuals distribute effort over a set of multiple and heterogeneous issues in order to maximize their winning chances (Arbatskaya and Mialon, 2010; Epstein and Hefeker, 2003; Friedman, 1958; Osório, 2017).[3]

To summarize, I start by noting that the aggregate output of each group depends on the contributions of its members, which depends on their efforts. This is a crucial aspect because the same effort by two different individuals is likely to result in different outputs. Subsequently, I impose an axiom in which the group output remains unchanged if the output of one individual increases by the same amount as the output of another individual decreases. Consequently, the aggregate output must be an additive function of the individual outputs. Then, under the assumption that individual output functions are non-negative and continuously differentiable in effort, we can apply the Euler's homogeneous function theorem to characterize the

[2] Münster (2009) also characterizes the group logistic contest success function, while Cubel and Sanchez-Pages (2016) characterize the group difference-form contest success function. In both cases, and in this paper, the individual outputs are additive. The multiplicative output function is characterized by Arbatskaya and Mialon (2010).

[3] The main difference (with respect to group contests) is that in multi-issue contests, the distribution of efforts depends on the preferences of a single individual, while in group contests the distribution of efforts inside the group depends on the preferences of different individuals. This aspect leads to strategically different problems. However, these problems have in common the same contest success function.

individual output functions and the associated group contest success function.

2. Fundamental Axioms and the Fundamental Result

Since the present paper extends Münster (2009), I follow the same notation. There are n individuals and $G \geqslant 1$ groups. Let g denotes a particular group belonging to the set $\Gamma = \{1, ..., G\}$ and $m_g \geqslant 1$ denotes the number of members in that group. Let $x_{ig} \in \mathbb{R}_+$ denotes the effort of member $i \in I_g = \{1, ..., m_g\}$ in group $g \in \Gamma$, while $\mathbf{x}_g = \{x_{1g}, ..., x_{m_g g}\}$ denotes the vector of efforts of the group g members, and $\mathbf{x} = \{\mathbf{x}_1, ..., \mathbf{x}_G\}$ denotes the vector of all group efforts. Moreover, let M denotes a subset of groups in Γ, i.e., $M \subset \Gamma$, and $\mathbf{x}_M$ denotes the vector of all group efforts in that subset. Finally, let $p_g : \mathbb{R}^n_+ \to \mathbb{R}_+$ denotes the contest success function that expresses the group $g \in \Gamma$ winning probability (or share in the prize).

*2.1. **Fundamental Axioms***

The following axioms are extensions of the axioms proposed by Münster (2009) to group contests, which in turn are extensions of the axioms in Skaperdas (1996) (see also Clark and Riis (1998)).

Axiom 1 (probability function). $\sum_{g=1}^{G} p_g(\mathbf{x}) = 1$ *and* $1 \geqslant p_g(\mathbf{x}) \geqslant 0$ *for all* $g \in \Gamma$.

It simply states that the contest success function is a probability distribution.

Axiom 2 (monotonicity). *If* $x'_{ig} > x_{ig}$*, then* $p_g(x'_{ig}, \mathbf{x}_{-ig}) > p_g(x_{ig}, \mathbf{x}_{-ig})$ *for* $g \in \Gamma$*, and* $p_k(x'_{ig}, \mathbf{x}_{-ig}) < p_k(x_{ig}, \mathbf{x}_{-ig})$ *for* $k \neq g \in \Gamma$ *(except at* $p_g(x_{ig}, \mathbf{x}_{-ig}) = 1$*).*

Monotonicity means that the group winning probability is increasing with the effort of the group member and decreasing with the effort of other group members.

Axiom 3 (anonymity). *(between-groups) The identity of the group is irrelevant. (within-group) The identity of the group members is irrelevant.*

This property is informally stated for completeness. In our context, with heterogeneous individuals, this property must be relaxed (Clark and Riis, 1998). Nonetheless, we could consider a weaker version of this property in which the individual characteristics and effort cannot be separated. In other words, if we permute the position of two individuals, we must also permute their characteristics. In the case that individuals perform different tasks, we must also permute the characteristics of the task.

Axiom 4 (contest size *consistency*). *If $p_g^M(\mathbf{x})$ is the group g winning probability in a contest between a subset of groups $M \subset \Gamma$ with at least two non-empty groups, then $p_g^M(\mathbf{x}) = p_g(\mathbf{x})/\sum_{k \in M} p_k(\mathbf{x})$ for all $g \in M$.*

In other words, increasing or decreasing the number of groups does not change the qualitative characteristic of the contest success function.

Axiom 5 (*independence of the irrelevant efforts*). *$p_g^M(\mathbf{x})$ is independent of the efforts of the individuals belonging to groups not in M.*

Independence of the irrelevant efforts means that only the effort by the members of the competing groups matters for the final outcome.

2.2. ***Fundamental Result***

Under the previous assumptions, Münster (2009) has shown the following fundamental result, which extends Skaperdas (1996) to group contests.

Theorem 1 *(Münster, 2009) Suppose the contest success function satisfies Axioms 1, 2, 4 and 5. Let M be any proper subset consisting of at least two groups. Then, for each $g \in M$, there exists a non-negative and strongly increasing function $f_g : \mathbb{R}_+^{m_g} \to \mathbb{R}_+$ such that:*

$$p_g^M(\mathbf{x}) = \frac{f_g(\mathbf{x}_g)}{\sum_{k \in M} f_k(\mathbf{x}_k)}, \tag{1}$$

with at least some $x_{ik} > 0$ for $i \in I_k$ and $k \in M$.

The function f_g is strongly increasing, if it increases when at least one member of the group $g \in M$ increases his/her effort.

This result is the starting point for the characterization of any ratio-form contest success function.

Note that if $x_{ik} = 0$ for all $i \in I_k$ and $k \in M$, then some tie-breaking rule (consistent with Axioms 1, 2, 4 and 5) must be assumed.

3. Additional Axioms and Characterization of the Group Success Function

In order to derive the exact functional form of $f_g(\mathbf{x}_g)$ further assumptions are needed.

3.1. Additional Axioms

In this context, an axiom that is usually assumed is that the contest success function is homogeneous of degree zero in effort (Clark and Riis, 1998; Münster, 2009; Skaperdas, 1996).

Axiom 6 (*homogeneous of degree zero in effort*). *$p_g(t\mathbf{x}) = p_g(\mathbf{x})$ for all $t > 0$ and $g \in \Gamma$.*

This property implies that the winning probability is independent of the units in which effort is measured. Therefore, if we scale all efforts by the same amount the winning probability remains unchanged.

Münster (2009) shows that if Axioms 1, 2, 4, 5 and 6 are simultaneously satisfied, then the function $f_g(\mathbf{x}_g)$ in (1) must be homogeneous of degree r. This result will be useful to derive our result.

Axiom 6 restricts the functional form of $f_g(\mathbf{x}_g)$, which is convenient to obtain a characterization. However, the drawback is that it limits the ways in which we can express heterogeneity. For instance, we cannot consider varying levels of complementarity and different returns to scale between groups (see, e.g., Kolmar (2013) and Kolmar and Rommeswinkel (2013), among others).

In what follows, I also distinguish between effort and the result of the effort–*the output*. This aspect is crucial in group contests because groups are composed of heterogeneous individuals with different skills and productivity. It is this aspect that motivates the present paper.

Let f_{ig} denote the output of individual $i \in I_g$ in group $g \in M$.

Axiom 7 (heterogeneous individuals). *$f_{ig} : x_{ig} \to \mathbb{R}_+$ is continuously differentiable with $f_{ig}(0) = 0$ and $f_{ig}(z) \neq f_{jg}(z)$ for $z > 0$, where $i, j \in I_g$ and $g \in M$.*

This axiom expresses heterogeneity between individuals without imposing great functional restrictions. In other words, despite providing the same effort, individuals do not perform equally well–some individuals are more productive than others. Heterogeneity is expressed in terms of the individuals' effort effectiveness, which translates into group output.[4]

[4]The function $f_g(\mathbf{x}_g)$ is frequently referred as "impact function". In the present paper, we will call it "aggregate output function".

Axiom 7 also implies that the individual i output $f_{ig}(x_{ig})$ depends only on the individual i effort x_{ig}, i.e., the individual i contribution is independent of the other individuals efforts. However, depending on the construction of the problem, the effort decision may depend on the other individuals decisions. Consequently, we can study free-riding problems, complementar and substitution effects, and also differences in terms of group behavior in competitive environments.

The continuously differentiable assumption in Axiom 7 is a regularity condition, while $f_{ig}(0) = 0$ means that no effort implies no output.

An implication of the previous axiom is that the aggregate output function $f_g(\mathbf{x}_g)$ can be expressed as a function of the individual output functions $f_{ig}(x_{ig})$ for $i \in I_g$ and $g \in M$, that is:

$$f_g(\mathbf{x}_g) = f_g(f_{1g}(x_{1g}), ..., f_{m_g g}(x_{m_g g})), \tag{2}$$

for each $g \in M$. Consequently, the distinction between the individuals is based on their outputs and not on their effort. Along this line of reasoning, I consider the following axiom, which is analogous to Axiom 8 in Münster (2009).

Axiom 8 (uniform output). *Fix some output value $\Delta \in [0, f_{ig}(x_{ig})]$ for all $i \in I_g$, and define:*

$$f_g(\widehat{\mathbf{x}}_g) \equiv f_g(f_{1g}(x_{1g}), ..., f_{ig}(x_{ig}) - \Delta, ..., f_{jg}(x_{jg}) + \Delta, ..., f_{m_g g}(x_{m_g g})).$$

Then, $f_g(\mathbf{x}_g) = f_g(\widehat{\mathbf{x}}_g)$ for all $g \in M$.

Individuals are heterogeneous but the units of output are homogeneous. Output can be traded between individuals without varying the aggregate output. All individual outputs have the same value, quality and importance. The difference between individuals is in the productivity of their efforts.

Münster (2009) is stated in terms of effort (i.e., in Münster (2009) $f_{ig}(x_{ig}) = x_{ig}$ for all $i \in I_g$ and $g \in M$), while Axiom 8 is stated in terms of output. By doing so, we allow for heterogeneity between the individuals. These considerations are better to capture situations of group competition. For instance, in a R&D race the objective of a research group is to be the first to achieve a particular discovery–from the research group perspective, it is indifferent which members have contributed to this objective. Similarly, in a football match, the objective of the team is to win, draw or (at least) not lose by many goals–from the team perspective, it is indifferent which player(s) scored the goal(s). However, at the individual level these considerations are relevant for each individual because in most cases the best performing individuals are more likely to receive higher individual returns and prizes.

*3.2. **Characterization of the group success function***

An implication of Axiom 6 is that in order for the aggregate output function $f_g(\mathbf{x}_g)$ to be homogeneous of degree r, as stated above, all individual output functions $f_{ig}(x_{ig})$ must be homogeneous of degree r/s. Moreover, since these functions are non-negative and homogeneous they can be characterized by the Euler's homogeneous function theorem.

Proposition 1 *If the contest success function satisfies Axioms 1, 2, 4, 5, 6, 7 and 8, then it satisfies (1) and*

$$f_g(\mathbf{x}_g) = (\textstyle\sum_{i=1}^{m_g} c_{ig} x_{ig}^{r/s})^s, \tag{3}$$

where $r > 0$, $s \neq 0$ and $c_{ig} > 0$ for $i \in I_g$ and $g \in M$.

Proof. The general structure of the proof follows from Münster (2009). If Axioms 1, 2, 4, 5 and 6 are simultaneously satisfied, the total output function $f_g(\mathbf{x}_g)$ must be homogeneous of degree r (see Theorem 2 in Münster (2009)). On the other hand, if Axioms 1, 2, 4, 5 and 8 are simultaneously satisfied, then we must be able to write the total output function in the following summation functional form:

$$f_g(\mathbf{x}_g) = f_g(f_{1g}(x_{1g}), ..., f_{m_g g}(x_{m_g g})) = f_g(\textstyle\sum_{i=1}^{m_g} f_{ig}(x_{ig})),$$

for all $g \in \Gamma$. Consequently, since Axioms 1, 2, 4, 5, 6 and 8 are simultaneously satisfied, in order for the aggregate output function $f_g(\sum_{i=1}^{m_g} f_{ig}(x_{ig}))$ to be homogeneous of degree r, and simultaneously $f_{ig}(0) = 0$, as stated in Axiom 7, each of the individual output functions $f_{ig}(x_{ig})$ must be homogeneous of degree r/s for all $i \in I_g$ and $g \in \Gamma$. Moreover, since by Axiom 7 the individual output functions $f_{ig}(x_{ig})$ are non-negative and continuously differentiable, by the the Euler's homogeneous function theorem, $f_{ig}(x_{ig})$ must satisfy the ordinary differential equation $(r/s)f_{ig}(x_{ig}) - x_{ig}\partial f_{ig}(x_{ig})/\partial x_{ig} = 0$ for all $i \in I_g$ and $g \in M$. In this context, we obtain that $f_{ig}(x_{ig}) = c_{ig} x_{ig}^{r/s}$ for all $i \in I_g$ and $g \in M$ is the unique solution that satisfies such equation, where the solution is obtained using the integrating factor approach and c_{ig} is some constant that can be specific to each individual and/or group. Therefore, $f_g(\mathbf{x}_g)$ must be uniquely given by expression (3). ■

The parameter c_{ig} is specific to each individual and/or group. It captures individual characteristics (i.e., physical, cognitive or in terms of skills) that affect the performance of the individual and/or the group that he/she belongs.

The parameter r/s refers to the scale of the individual effort x_{ig} and measures the effectiveness of the individual effort in the individual output $f_{ig}(x_{ig})$. For instance, if $r/s > 1$ the individual output exhibits increasing returns to effort.

On the other hand, the parameter $s > 0$ is more specific to the scale of the group's total output and measures the effectiveness of the aggregate sum of individual outputs $\sum_{i=1}^{m_g} c_{ig} x_{ig}^{r/s}$ in the total output $f_g(\mathbf{x}_g)$.[5]

Note also that in the case that $s = 1/t$ and $r = 1$, the output function $f_{ig}(x_{ig})$ given in expression (3) is the Constant Elasticity of Substitution (CES) production function with constant elasticity of substitution of $1/(1-t)$ between the factors of production. In our context, that means constant elasticity of substitution between the efforts of the group members. Therefore, if $t = 1$ we have the linear or perfect substitutes function; if $t \downarrow 0$ we have the Cobb–Douglas function; and if $t \downarrow -\infty$ we have the Leontief or perfect complements function.

Finally, consider the following particular cases. Note that if we set $s = r$ and $c_{ig} = c_g$ for all $i \in I_g$, we obtain the contest success function characterized in Münster (2009). We can also separate between individual and group specific characteristics, i.e., by setting $c_{ig} = c_i c'_g$ where $c'_g = c_g^{1/s}$ for all $i \in I_g$ and $g \in \Gamma$, and consequently, writing $f_g(\mathbf{x}_g) = c_g(\sum_{i=1}^{m_g} c_i x_{ig}^{r/s})^s$. On the other hand, if we set $s = 1$, we obtain the contest success function in Osório (2017). The case $s = r$ with $r \to \infty$ corresponds to the all-pay auction contest function. In the case $m_g = 1$ for all $g \in \Gamma$, we obtain the Tullock (1980) contest success function characterized in Clark and Riis (1998) and Skaperdas (1996).[6]

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[5] In this context, the aggregate output function $f_g(\mathbf{x}_g)$ exhibits increasing or decreasing returns to effort depending on the value of the parameters r and s, and on the weights $w_{ig} \equiv (c_{ig} x_{ig}^{r/s}) / \sum_{j=1}^{m_g} c_{jg} x_{jg}^{r/s}$. In this context, the output function $f_g(\mathbf{x}_g)$ exhibits increasing returns to the effort x_{ig} if $w_{ig} r(s-1) > s - r$. For instance, (i) if $s = 1$ or $s = r$ the function $f_g(\mathbf{x}_g)$ exhibits increasing returns to effort for $r > 1$, while (ii) if $r = 1$ the function $f_g(\mathbf{x}_g)$ exhibits increasing returns to effort for $s > 1$.

[6] Axioms *7* and *8* together with Axiom 7 in Münster (2009) characterize the analogous heterogeneous individuals logistic group contest success function with the aggregate output function: $f_g(\mathbf{x}_g) = c_g \exp\{\sum_{i=1}^{m_g} f_{ig}(x_{ig})\}$ for all $g \in M$. However, we need an additional axiom in order to specify the functional form of $f_{ig}(x_{ig})$ for all $i \in I_g$ and $g \in M$.

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