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The allocation of CO_2 emissions as a claims problem

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Abstract

This paper proposes to use claims models as a reasonable and operative alternative in order to allocate CO_2 emissions by countries (or groups), in the framework of multilateral negotiations and the fight against climate change. This framework has two characteristics which fits this type of claims models: a restrictive global endowment (the maximum world emissions permitted) and the excess of emissions (and demand) by countries. The proposed methodology consists on establishing some requirements that any admissible distribution solution should satisfy, examining a broad group of theoretical distribution solutions emerged from the specific literature and analyzing their application according to reasonable ordering criteria linked to equity and stability properties. The proposed theoretical framework is applied empirically to an analysis by groups of countries in the period 2010-2050, using various world endowments from Meinshausen et al. (2009), together with claims forecasts associated with the RCP scenarios. The results obtained point out that for intermediate claims scenarios the solutions associated with the constrained equal awards (CEA) and α-minimal (α-min) solutions are typically selected. In particular, these two solutions are clearly equity-sensitive, where the efforts to be made by Asia and OECD are very important, as a whole, and especially in the case of the CEA. Given these circumstances, and the better balance between equity and proportionality associated to the α-min allocation methodology maybe that one would be more operative and acceptable.

Keywords: Carbon emissions, claims problem, climate change policy
JEL classification: D7, H4, H8, Q58, Q54

1. Introduction

In a referential work, Rockstrom et al. (2009) established that the planet would have already exceeded its maximum threshold in terms of climate change. With a carbon dioxide concentration of 387 parts per million, it exceeded the proposed maximum limit of 350, far from the pre-industrial value, 280. Further, the documents of the working groups associated with the IPCC are especially illuminating in terms of the consequences of all this (Team et al., 2014). The existing empirical evidence indicates, in the first place, and in consistency, the great growth of global emissions of anthropogenic origin since 1950; in second place, its impact on climate change or extreme events; and, third, if we do nothing, the imbalances that will be created.

As a magic quantitative goal, the objective would be to reduce emissions by at least 50% until 2050, which is, however, very ambitious given the current patterns (Pan et al., 2014). In particular, limiting the cumulative emissions of CO_2 up to 1440Gt until 2050 would mean increasing the temperature by 2^o C in 2050 over the pre-industrial levels (Meinshausen et al., 2009) by 50%, if these were 1000 it would be 25% and if they were 745Gt 0%. In this sense, the RCP scenarios (Moss et al., 2010; IPCC, 2014) indicate that for pessimistic scenarios (RCP 8.5) the cumulative volume of emissions could exceed 2500Gt or be around 1900 even in intermediate scenarios (RCPs 6.0 and 4.5).

Therefore, it seems immediate not only the need to stop global emissions but at least to reduce them. In this sense, the making process at a global level has been described up to now by multilateral negotiations, through world conferences (COPs) and the distribution of objectives at the territorial level, since the signing of the Kyoto Protocol. The distribution, in a scenario of lower global emissions, implies, in fact, a distribution of costs and sacrifices through a potentially conflictive and complicated negotiation that entails, fundamentally, a distributive problem. These difficulties, in view of the different situations and positions of the countries and groups, have been clearly shown in the successive conferences. A typical result has been the establishment of groups of countries with common interests, with at least two realities appearing at the beginning. One of the developed countries, responsible for the bulk of the accumulated emissions, which accept reductions and require analogous sacrifices to the developing countries and those that resist for the impact that these can have on their growth pattern, having an objective of real convergence with the incomes of the first countries. In fact, over time the structuring by groups has been complicated by exacerbating, for example, the heterogeneity of the group of developing countries and emerging new associations in this group (Costantini et al., 2016).

It is this context of reduction of the global allowance of permitted emissions and heterogeneity of interests and objectives in the negotiations that has raised responses by the academic community in order to propose rational allocation mechanisms for a scarce resource context. Following the survey by Zhou and Wang (2016), different associated methods would have been proposed, for example, indicators (Miketa and Schrattenholzer, 2006), optimization methods (Cantore and Padilla, 2010), game theory (Ren et al., 2015) or hybrid methods, which can yield clearly different results depending on the solution applied (see, for instance, Akhundjanov et al., 2017; Wang and Zhou, 2017; and, He et al., 2018).

In this paper a different methodology is proposed and applied which we believe allows to reduce the degree of discrepancies prior to the negotiation processes. Thus, our approach would avoid having to agree on distribution criteria, which can be strategically used by the parties, and, instead, a neutral method is proposed. In this sense, it would be a matter of fixing some principles that must satisfy any acceptable distribution solution and analyzing different theoretical solutions developed in the literature, together with the selection of several acceptable criteria for final selection. This aseptic proceeding, which in no case speaks of indicators, we believe may be useful in order to adopt a compromise solution between parties with opposing interests. If we agree with the procedure and its solutions we should accept the results. In particular, it is proposed to extend bankrupcy models (O'Neill, 1982; Hougaard et al., 2012) for the analysis of the distribution of a maximum limit of emissions, i.e. endowment. In Giménez-Gómez et al. (2016) similar models have already been applied for this analysis. However, in this paper we propose different novelties that we believe improve their analytical capacity. In the first place, the required principles are extended to each distribution solution; secondly, the analyzable distribution solutions are also extended, going from four to seven alternatives, and therefore, increasing their capacity for representation; and thirdly, additional selection criteria are established, with the use of equity criteria having a prominent place, in line with the principle of fairness that is well-liked by the literature (Meyer, 2000; Höhne et al., 2006). The proposed method seems consistent with two of the basic principles established for a multilateral agreement to be operative and effective: legitimacy and equality (Young, 2011, Kampas, 2015). The concept of legitimacy would imply, in our case, the existence of clear and transparent principles and solutions that derive in the analyzed and proposed deals. Following Kampas (2015), the concept of basic equity used in the proposal is that of distributive justice (Koh, 1997) that implies preferences, ceteris paribus, of less unequal distributions. In addition to the previous ones, and again according to Kampas (2015), it is proposed to use the stability criterion for the selection of the most acceptable distribution solution.

On the other hand, and in addition to the presentation of the theoretical framework for the allocation of maximum emission allowances, an empirical application is made using the data of maximum endowments extracted from Meinshausen et al. (2009) together with the forecasts of various emission scenarios up to 2050 extracted from the RCP data (IPCC). In this sense, the intermediate scenarios for the evolution of RCP emissions 6.0 and RCP 4.5 are especially handled under the assumption that the countries will make some reduction effort, above all in terms of emission intensities (CO_2 / GDP).

The paper is organized as follows. Section 2 describes the main literature associated to allocation models and, in particular, from the claims problem literature. In Section 3 we provide the main aspects of the proposed allocation model, that is, principles, allocation solutions and fairness criteria. Section 4 makes the empirical implementation for regional groups over the period 2010 -2050, and, finally, Section 5 concludes the paper. Appendix gathers supplementary material.

2. The allocation approaches

The problem we are facing, then, is characterized by two basic parameters: first, an available resource budget (the allowed global CO2 emissions), and some agents' requests/needs (countries or groups of countries, that are typically going to claim a larger amount of CO_2 emissions than the envelope). Therefore, it is clear enough that this problem deals with an assignment problem with divergent and conflicting interests among the different countries or groups of countries.

In this regard, the academic literature has tried to make contributions through different approaches and methodologies, being the survey carried out by Zhou and Wang (2016) a valuable starting point in order to summarize these contributions.

On the one hand, any allocation method in this context is related those basic principles that are considered as basic requirements for any acceptable assignment. In this sense, the significant principles of efficiency (minimizing costs) and equity (equal treatment, Rose, 1990) have been emphasized, since their fulfillment promotes the legitimacy of the assigning method itself.

On the other hand, combining the idea of ensuring commonly accepted principles, and given the main features of the aforementioned context, Giménez-Gómez et al. (2016) propose a claims problem approach. A claims problem is a particular case of distribution problems in which the amount to be distributed, the *endowment* E, is not enough to cover the agents' claims on it. This model is usually used to describe the situation faced by a court that has to distribute the net worth of a bankrupt firm among its creditors. But, it also corresponds with cost-sharing, taxation, or rationing problems. In all these contexts, the main question to be solved is, how should the scarce resources be allocated among its claimants? The formal analysis of such situations, which originates in a seminal paper by O'Neill (1982), shows that a vast number of well-behaved solutions have been defined for solving claims problems, being the *proportional* and the *equal awards* (egalitarian) the two prominent concepts used in real world.[1] Furthermore, the term well-behaved reflects the idea that the considered solutions might fulfill some principles of fairness, or appealing principles. An illustrative example of claims problems is the fishing quotas reduction (Iñarra and Prellezo, 2008; Kampas, 2015), in which the agent's claim can be understood as the previous captures, and the estate is the new (lower) level of joint captures. A similar example is given by milk quotas among the EU members.[2] Another similar situation can be found when a government distributes the budget among the different needs Solís-Baltodano et al. (2018).

The current paper mainly contributes to the previous literature not only updating and improving the forecasting behaviour of regions (we consider a fifth region) in the approach introduced by Giménez-Gómez et al. (2016), but also we provide a deeply analysis of all the possible allocation in terms of the socially accepted equity principles and the kind of allocation methods that may be applied. Specifically, we extend our analysis from four different allocation methodologies up to seven, using the more vastly used solutions in real cases. Furthermore, we analyze the behavior of these potential admissible proposals of CO_2 distributions through twelve possible requirements (see

[1] The reader is referred to Moulin (2002) and Thomson (2015) as surveys of this literature.

[2] Quotas were introduced in 1984. Each member state was given a reference quantity which was then allocated to individual producers. The initial quotas were not sufficiently restrictive as to remedy the surplus situation and so the quotas were cut in the late 1980s and early 1990s. Quotas will end on April 1, 2015.

Section 3), and, given all the possible combinations of these requirements, we determine which solution should be used (see Section 4). Finally, in order to justify the choice of one of these pre-evaluated solutions, we study both, their equity behavior in terms of the Gini and Atkinson indexes, among others; and thir stability, by means of the coefficient of variation. Therefore, the current approach improves and complements the previous one with a wider and deeper analysis of the CO_2 emissions problem through a larger number or well-used principles and allocations solutions.

3. The theoretical framework

The aforementioned CO_2 emissions claims problem is formally defined as follows. Consider a set of agents $N = \{1, 2, ..., n\}$ and an amount $E \in \mathbb{R}_+$ of an infinite divisible resource, the **endowment** or budget, that has to be allocated among them. Each agent has a **claim**, $c_i \in \mathbb{R}_+$ on it. Let $c \equiv (c_i)_{i \in N}$ be the claims vector.

Then, a **claims problem** (O'Neill, 1982) is a pair (E, c) with $C = \sum_{i=1}^{n} c_i > E$.

Without loss of generality, we increasingly order the agents according to their claims, $c_1 \leqslant c_2 \leqslant \ldots \leqslant c_n$, and we denote by $\mathcal{B}$ the set of all claims problems.

3.1. The distribution method: solutions

The formal analysis of claims problems provides a vast number of well-behaved solutions (O'Neill, 1982). A solution proposes how to distribute the available resources among the different agents by satisfying the requirements of non-negativity and claim-boundedness. Formally,

A **solution** is a single-valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that $\varphi_i(E, c) \geqslant 0$, for all $i \in N$ (**non-negativity**), $\varphi_i(E, c) \leqslant c_i$, for all $i \in N$ (**claim-boundedness**), and $\sum_{i \in N} \varphi_i(E, c) = E$ (**efficiency**).

Two main ways of distribute the endowment are proposed: the proportional and the equity methods. Besides this two, the rest of the solutions that the present paper presents are by no means arbitrary, since they satisfy appealing principles and they are vastly studied in the claims problems literature, being considered a plausible alternative ways of distributing the endowment (Moulin, 2002, and Thomson, 2015). Specifically, we formally introduce the proportional, the constrained equal awards, the constrained equal losses, the Talmud, the random arrival, the adjusted proportional and the $\alpha-$minimal solutions.

The **proportional (P)** solution, one of the best-known and most used solution, simply recommends a distribution of the CO_2 emissions budget proportionally to the regions' claims. Thus, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $P_i(E, c) \equiv \lambda c_i$, where $\lambda = E / \sum_{i \in N} c_i$.

The **constrained equal awards (CEA)** solution (Maimoindes, 1135,1204) proposes an equal distribution of the CO_2 emissions, taking as an upper threshold each regions' claims. Therefore, CEA do not consider the differences between countries in terms of lost emissions rights. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E, c) \equiv \min\{c_i, \mu\}$, where μ is such that $\sum_{i \in N} \min\{c_i, \mu\} = E$.

The **constrained equal losses (CEL)** solution (Maimoindes, 1135,1204; Aumann and Maschler, 1985) applies this equal distribution criterion but taking into account the lost of emissions rights instead of the claims. In other words, CEL recommends an egalitarian distribution of the part of the aggregate cumulative CO_2 emissions that is not satisfied (i.e., $L = C - E$), given that no one can emit a negative amount. Note that this solution procures an egalitarian division of regional sacrifices. However, this may involve not allocating emissions rights to lower claimants, which cannot be done for ethical or economic reasons. Formally, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEL_i(E, c) \equiv \max\{0, c_i - \mu\}$, where μ is such that $\sum_{i \in N} \max\{0, c_i - \mu\} = E$.

The **Talmud (T)** solution, basing on the idea that "it is socially unjust for different creditors to be on opposite sides of the halfway point, $C/2$" (Aumann and Maschler, 1985), recommends a combination of CEA and CEL. Specifically, T takes the middle of the aggregate claims as a reference point. If the half of the total needs of cumulative CO_2 emissions is lower than the budget, then the CEA is applied over the half-claims; whereas, each region receives half of its expected emissions and the amount recommended by CEL, otherwise. For each $(E, c) \in \mathcal{B}$, and each $i \in N$, $T_i(E, c) \equiv CEA_i(E, c/2)$ if $E \leqslant C/2$; or $T_i(E, c) \equiv c_i/2 + CEL_i(E - C/2, c/2)$, otherwise.

Now, consider that each regions' claim is fully honored following the order of the regions' demand, until the endowment runs out. In order to remove the unfairness of the first-come first-served scheme associated with any particular order of demand, the **random arrival (RA)** (O'Neill, 1982) solution proposes to take the average of the allocations calculated in this way when all the orders are equally probable. That is, for each $(E, c) \in \mathcal{B}$, and each $i \in N$, $RA_i(E, c) \equiv \frac{1}{|N|!} \sum_{\prec \in \mathbb{R}^N} \min\{c_i, \max\{E - \sum_{j \in N, j \prec i} c_j, 0\}\}$.

Finally, we introduce two solutions which are based on the notion of lower bound, i.e., they guarantee a minimum level of resources to each region.

The **adjusted proportional (AP)** solution (Curiel et al. (1987)), firstly, assigns to each region its minimum guaranteed amount, defined as the remaining of the CO_2 emissions budget once the rest of the regions' claims have been fully honored, if any $(m_i(E, c) = \max 0, E - \sum_{j \neq i \in N} c_j)$. Secondly, the claims are revised down by these amounts. Then, the proportional solution is applied to distribute the remaining endowment according to the revised claims. Formally, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $AP_i(E, c) = m_i(E, c) + P(E - \sum_{i \in N} m_i(E, c), c - m(E, c))$.

The **α-minimal (α-min)** solution (Giménez-Gómez and Peris, 2014) recommends to each of the regions an equal minimum amount (a survival amount). Then, when the lowest region's claim is fully satisfied, the remaining CO_2 emissions budget is distributed proportionally among the other claimants. Formally, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $c_1 > E/n$ then $\alpha - min_i(E, c) = E/n$ and if $c_1 < E/n$ then $\alpha - min_i(E, c) = c_1 + P(E - nc_1, c - c_1)$.

Obviously, among all these solutions, each region prefers the solution that provides it a larger quota of the global CO_2 emissions budget. It is noteworthy that such discussion has been fruitless, see, for instance, the Paris commitment and its real results (the USA unilateral decision of not fulfill the agreement). Hence, for the sake of facilitating the agreement, the current approach proposes to focus on general and social principles that regions may accord with for distributing the CO_2 emissions budget. That is, instead of focusing on the allocation itself, the commitment will be easier to reach if regions analyze the principles they would want to implement to distribute the endowment.

In doing so, we provide some "sensible" principles that not only naturally fits this context but also are considered as a minimal requirements of fairness. Besides this set of basic principles, we also provide a wider analysis through the proposal of different vastly used allocations requirements.

3.2. The social accepted conditions: principles

For the axiomatic analysis of the aforementioned solutions, we propose two sets of principles: Minimal requirements and additional principles. The minimal requirements set is composed by equal treatment of equals, anonymity, order preservation, resource monotonicity, continuity, no transfer paradox, and claims monotoniticy. Note that, as Table 1 depicts, all these principles are satisfied by the introduced solutions, since they are quite general accepted in the literature. Additionally, and with the sake of comparison, we propose a set of more strength appealing principles: super-modularity, composition down, composition up, linked claims resource monotonicity and order preservation under claims variation. These principles might help to determine a unique and/or a better way to distribute the CO_2 emission budget.

3.2.1. Minimal requirements

Equal treatment of equals: for each $(E, c) \in \mathcal{B}$, and each $\{i, j\} \subseteq N$, if $c_i = c_j$, then $\varphi_i(E, c) = \varphi_j(E, c)$.

This principle implies those regions with the same emitting needs should be rewarded with the same CO_2 emissions allocation.

Anonymity: for each $(E, c) \in \mathcal{B}$, each $\pi \in \Pi^N$, and each $i \in N$, $\varphi_{\pi(i)}(E, (c_{\pi(i)})_{i \in N}) = \varphi_i(E, c)$, where Π^N is the class of all permutations of N.

Anonymity states the identity of the region does not matter, so a region's emission rights should only depend on its stated emitting needs (claimed emissions).

Order preservation (Aumann and Maschler, 1985): for each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geqslant c_j$, then $\varphi_i(E, c) \geqslant \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geqslant c_j - \varphi_j(E, c)$.

Order preservation asserts that the regions with larger emissions should not receive a smaller allocation than the regions with smaller emitting needs.

Resource monotonicity (Curiel et al., 1987; Young, 1987): for each $(E, c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E', c) \geqslant \varphi_i(E, c)$, for each $i \in N$.

Resource monotonicity demands that if the CO_2 emissions budget increases, then each region receives at least what they received before the increase.

Continuity: for each sequence (E^ν, c^ν) of elements of $\mathcal{B}$ and each $(E, c) \in \mathcal{B}$, if $(E^\nu, c^\nu) \to (E, c)$, then $\varphi(E^\nu, c^\nu) \to \varphi(E, c)$.

Continuity is a technical requirement that makes the errors in specifying the data of the problem or corrections of this errors, should not affect the final distribution.

No transfer paradox (Chun, 1988): for each $(E, c) \in \mathcal{B}$, each pair $\{i, j\} \subseteq N$, each $c'_i > c_i$, and each $c'_j < c_j$, if $c_i + c_j = c'_i + c'_j$ then $\varphi_i(E, c'_i, c'_j, c_{N\setminus\{i,j\}}) \geqslant \varphi_i(E, c)$, and $\varphi_j(E, c'_i, c'_j, c_{N\setminus\{i,j\}}) \leqslant \varphi_j(E, c)$,

No transfer paradox says that if one regions transfers part of its claim to other region, the former and the latter regions should receive at most and at least as much as they did initially.

Claims monotonicity: for each $(E, c) \in \mathcal{B}$, each $i \in N$, and each $c'_i > c_i$ we have $\varphi_i(E, c'_i, c_{-i}) \geqslant \varphi_i(E, c)$[3].

Claims monotonicity states that if a region's claim increases, this region should receive at least as much as it did initially.

3.2.2. Additional principles

Super-modularity (Dagan et al., 1997): for each $(E, c) \in \mathcal{B}$, all $E' \in \mathbb{R}_+$ and each $i, j \in N$ such that $C > E' > E$ and $c_i \geqslant c_j$, then $\varphi_i(E', c) - \varphi_i(E, c) \geqslant \varphi_j(E', c) - \varphi_j(E, c)$.

Super-modularity requires that regions with larger claims experience a larger gain of the CO_2 emission budget increase.

Composition down(Moulin, 2000; Kalai, 1977): for each $(E, c) \in B$, each $i \in N$, and each $0 \leqslant E' \leqslant E$, $\varphi_i(E', c) = \varphi_i(E', \varphi(E, c))$.

Note that if once the CO_2 budget is distributed among the regions, there is reevaluation and the available CO_2 budget is reduced, there are two different ways to redistributed the new CO_2 emissions budget: (i) we can cancel the initial distribution and apply the solution in the new situation; (ii) we consider the initial allocation as the regions' claims on the revised problem and apply the solution to this new problem, since this awards are commitments that had been made but cannot be honoured. Composition

[3]The notation c_{-i} designates the vector c without the i-th coordinate, and (c'_i, c_{-i}) denotes the vector c in which the i-th coordinate has been replaced by c'_i.

down establishes that both ways of proceeding should result in the same distribution.

Composition up (Young, 1987): for each $(E, c) \in B$, each $i \in N$, and each $0 \leqslant E \leqslant E'$ such that $\sum c_i \geqslant E'$, we have $\varphi_i(E', c) = \varphi_i(E, c) + \varphi_i(E' - E, c - \varphi(E, c))$.

Consider the opposite scenario to composition down. Now, consider that after the CO_2 budget has been divided among the considered regions, the available amount of CO_2 budget increases. Then, there are two different ways to redistributed the new CO_2 emissions budget. The first one is to cancel the initial division and recalculate the awards for the revised CO_2 budget amount. The second is to let each region keeps their initial awards, revise their claims down by these awards and re-apply the solution to divide the incremental amount. Composition up requires that both ways of proceeding should give the same amount.

Linked claims resource monotonicity (Thomson and Yeh, 2001): for each $(E, c) \in \mathcal{B}$, each $i \in N$, and each $d > 0$, we have $\varphi_i(E + d, c_i + d, c_{-i}) - \varphi_i(E, c) \leqslant d$

Linked claims resource monotonicity says that if a region's claim and the totally available CO_2 budget increase by equal amounts, this region's award should increase by at most this amount.

Order preservation under claims variations: for each $(E, c) \in B$, each $i \in N$, and each $c'_i > c_i$, and each pair $\{j, k\} \subseteq N \backslash \{i\}$, if $c_j \leqslant c_k$, then $\varphi_j(E, c) - \varphi_j(E, c'_i, c_{-i}) \leqslant \varphi_k(E, c) - \varphi_k(E, c'_i, c_{-i})$. If there are at least three regions, one may be interested in the impact that a change in some region's demand of emissions has on the relative values of the allocations made to the other regions. Suppose that region i's demand increases. Our requirement is that if region j's demand is at most as large as region k's demand then, it's loss should be at most as large as region k's loss.

Table 1 depicts which of the aforementioned principles are fulfilled by the proposed solutions. So, all the solutions are satisfactory but only the P, CEA, CEL and α-min solutions satisfy all of them (also the additional principles). Additionally, it is noteworthy that, from a practical point of view, CEL is difficult to implement due to propose not very ethical and plausible allocations in fact (it may assign a zero emissions rights to some regions).

Principles / Solutions	P	CEA	CEL	T	RA	AP	$\alpha - min$
Minimal requirements:							
Equal treatment of equals	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Anonymity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Order preservation	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Resource monotonicity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Continuity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No transfer paradox	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Claims monotonicity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Additional principles:							
Super-modularity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Composition down	Yes	Yes	Yes	No	No	No	Yes
Composition up	Yes	Yes	Yes	No	No	No	Yes
Linked claims resource mon	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Order preservation under claims var	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 1: **Principles and solutions.** The table shows which principles are satisfied by the considered solutions. Each column corresponds with a solution, whereas each row corresponds with the proposed principle.

3.3. The evaluation procedure: equity and fairness

How may a solution be proposed? Besides requiring some commonly accepted principle, we should consider some equity criteria. In doing so, a way of comparing solutions is given by the equity condition of Lorenz-dominance (see Dutta and Ray, 1989). Formally, let $\mathbb{R}^n_+$ be the set of positive n-dimensional vectors $x = (x_1, x_2, \ldots, x_n)$ ordered from small to large, i.e., $0 < x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$. Let x and y be in $\mathbb{R}^n_+$. We say that x Lorenz dominates y, $x \succ_L y$, if for each $k = 1, 2, \ldots, n-1$: $x_1 + x_2 + \cdots + x_k \geqslant y_1 + y_2 + \ldots + y_k$ and $x_1 + x_2 + \ldots + x_n = y_1 + y_2 + \ldots + y_n$. If x Lorenz dominates y and $x \neq y$, then at least one of these $n-1$ inequalities is a strict inequality. Regarding to claims problems, given two solutions φ and ψ it is said that φ **Lorenz dominates** ψ, $\varphi \succ_L \psi$, if for any claims problem (E, c) the vector $\varphi(E, c)$ Lorenz dominates $\psi(E, c)$.

Note that Lorenz domination is a criterion used to check whether a solution is more favourable to smaller claimants relative to larger claimants. So, in some sense, a Lorenz dominant solution can be understood as more equitable. In a recent paper, Bosmans and Lauwers (2011) obtain a Lorenz dominance comparison among several solutions and they obtain that CEA is the more equitable solution, in the sense that it Lorenz dominates any other solution. More precisely, the dominance relation they obtain is as follows:

$$CEA \succ_L \alpha - min \succ_L P \succ_L CEL$$

However, there is no Lorenz domination between α-min, RA, T, and AP solutions. So, the proportional solution only dominates CEL, which is the most favourable solution for larger claimants relative to smaller ones (so, the less equitable one), and only CEA dominates the α-min solution, for instance.

Finally, in aiming at finding the solution that distributes in a fairer way the global CO_2 budget, we introduce some criteria of justice, that is, following Robert (1974), "the complete principle of distributive justice would say simply that a distribution is just if everyone is entitled to the holdings they possess under the distribution."

Therefore, since a Lorenz domination analysis may not induce a unique solution, we propose the next inequality indexes (the lower the index the more equality the allocation) to measure the distributive justice of each of the considered solutions: the Atkinson index (At), the Gini coefficient (Gi), and the Theil index (T). Furthermore, note that the use of these inequiality indexes is a synthetic evaluation mechanism quite implemented (see, for instance, Cowell, 2011; and, Duro, 2012). Indeed, these indexes are used when there is no Lorenz domination among the proposed distributions.

The **Atkinson index (At)** (Atkinson, 1970) is given by,

$$At = 1 - \left(\frac{1}{N} \sum_i \left(\frac{r_i}{\mu} \right)^{1-\epsilon} \right)^{1/1-\epsilon} \quad \epsilon \neq 1,$$

where, N is the total number of groups of countries (in that case $N = 5$), r_i is the i-th region's allocation of the CO_2 emissions budget induced by a particular solution, and μ is the average CO_2 emissions allocations. This index, through the choice of parameter ϵ (where ϵ ranges from 0 to ∞), can be interpreted as an index of potential gains from redistribution, which takes values from $[0, 1]$.

The **Gini index (Gi)** (Gini, 1921) is defined as,

$$Gi = \frac{1}{2N^2\mu} \sum_i \sum_{j<i} |r_i - r_{j<i}|.$$

Note that it is the commonly known inequality index, and it is considered in the literature as the best single measure of inequality. It takes values in the interval $[0, 1]$, where $Gi = 0$ means perfect equality, and $Gi = 1$ means complete inequality.

The **Theil index (T)** (Theil, 1967) is defined as,

$$T_\alpha = \frac{1}{N}\sum_i \left(\frac{xi}{\mu}\right)^\alpha ln\left(\frac{xi}{\mu}\right).$$

Note that the Theil index is quite similar to the Atkinson index. However, through the α parameter it captures the relatively sensibility to the differences in the distribution of CO_2 emissions resources.

3.4. The evaluation procedure: stability

In order to consider the historical differences among regions, depending on its weight in the historical distribution of the CO_2 budget, we propose the coefficient of variation as a measure of stability.

Formally, the **coefficient of variation (CV)**,

$$CV = \frac{\delta}{\bar{PI}},$$

where δ is the standard deviation of set and $\bar{PI}$ is the mean of the Power index (PI) value. The power index has been applied in economics literature as a measure for selecting stable solutions for cooperative problems (Dinar and Howitt, 1997; Read et al., 2014). The power index is distributed more or less equally among the agents, then the coalition is more likely to be stable. The stability measure is the coefficient of variation (CV) calculated over all players in a given allocation solution ($0 \leqslant \text{CV} \leqslant 1$). The larger the value of CV the larger the instability of the allocation solution.

Specifically,

$$PI_i = \frac{w_i(r_i^{max} - r_{ik})}{\sum_j w_j(r_j^{max} - r_{jk})},$$

where w_i is the long-run average CO_2 budget share of the region i, r_i^{max} is the ideal solution for region i across all the scenarios, and r_{ik} is the current amount received by region i.

Note that CV is a measure of the dispersion of allocations around the mean, i.e., it represents the ratio of the standard deviation to the mean. By this way, we can determine the volatility, or range of differences, is assumed in comparison to the amount of expected allocation.

4. Empirical application: The CO_2 associated claims problem

The data necessary to carry out the implementation of the previous allocation model are associated, mainly, with the claims and with the total endowments to be distributed.

For defining the claims foreseen in the 2000-2050 period, the actual observations plus the forecasts associated with the RCP (Representative Concentration Scenarios) scenarios were taken, which were in fact adopted by the IPCC in its fifth Assessment Report (IPCC, 2015). Specifically, four types of scenarios are defined: 8.5, the worst in terms of emissions (Riahi et al., 2007), two intermediate scenarios like 6.0 (Fujino et al., 2006; Hijioka et al., 2008) and 4.5 (Smith and Wigley, 2006; Clarke et al., 2007; Wise et al., 2009), where the effects of improvements in CO_2 emissions intensities are foreseen, and, finally, scenario 2.6 (Van Vuuren et al., 2007), the best in terms of emissions. Each scenario has been carried out by different research teams and also the methodologies used have not coincided. In our case, and to save space, we are going to focus on the results for intermediate RCPs, that is, 6.0 and 4.5. According to these basic data, only information for groups of countries are available. Although having more detailed data would be better, we believe that the available data would reasonably summarize the existing territorial structural differences.

Thus, the distribution analysis has been carried out for five regions: OECD (that is, Western Europe, Northern America and Pacific OECD), REF (Reforming economies, basically Eastern Europe), Asia (including China and India), MAF (Middle East and Africa) and LAM (Latin American countries). So, given these grouping we can reasonably distinguish the different interests between high historical emitters and high developed, the big emergent economies and the non-developed world. The data are available in terms of gigatons of CO_2 emissions which have been necessary to convert in terms of gigatons of CO_2 using the typical conversion factor (3.67).

In order to define the endowments, the three levels established by Meinshausen et al. (2009) have been used. Thus, they are used as referential provisions for the year 2050. Specifically, the endowment is determined by the amount of available anthropogenic cumulative CO_2 that prevents the global temperature from exceeding 2°C: 1,440 (50%), 1,000 (25%) and 745 (0%) Gt. The former two scenarios are considered sensible by the scientific community and policymakers (Rockstrom et al., 2009) being managed in both Worlds, while the latter endowment allows for a zero-risk scenario (see Table 2).

Endowment (E)	Agents (regions)	Claims (c)
1440 Gt CO_2 (50%)	LAM	RCP 8.5: bad scenario
1000 Gt CO_2 (25%)	REF	RCP 6.0 and 4.5: intermediate
745 Gt CO_2 (0%)	MAF	RCP 2.6: a more decarbonized world
	OECD	
	ASIA	

Table 2: **Specify the CO_2 emissions problem: Endowment, agents and claims**. Endowments: $1,440Gt$ CO_2, $1,000Gt$ and $745Gt$ CO_2 means the probability of exceeding 2^oC is 50%, 25% and 0, respectively.

Once we have defined the problem, we apply the aforementioned solutions to the different scenarios we analyze in the current approach. For the sake of exposition, Tables 3 and 4 depicts the results for the intermediate scenarios. The Appendix gathers the allocations proposed by all the solutions in the other possible scenarios.

By considering the set of principles aforementioned, as Table 1 shows, all the solutions are satisfactory since they satisfy the set of minimal requirements of fairness. Furthermore, as previously introduced, by considering the set of appealing principles, one region may defend the implementation of one of solutions instead of the other. However, some of the solutions could have some practical issues due to the way the CO_2 emissions are allocated (see, for instance, that CEL does not seem very ethical and plausible, in fact, since it recommends no emissions rights for R5LAM, R5MAF and R5REF in the RCP6.0 scenario for an endowment of 1,000).

Specifically, it is noteworthy that, as aforementioned, we consider five regions and the proposed CO_2 allocations as emitting right in a 50 years period. Accordingly, P divides the CO_2 emissions budget proportionally to each region's claim. The CEA proposes an egalitarian distribution of the cumulative CO_2 emissions (for instance, with $E = 745\ G_t$ CO_2), such that no group can accumulate more that its claim (with $E = 1440\ G_t$ CO_2, $R5LAM$ has honored its total claim). In fact, $R5LAM$ region, which is composed by the lowest claimers countries (despite doing their best to emit as we are in the worst feasible SRES scenario), will always prefer CEA as it is the one that, regardless the CO_2 emissions budget, allocates more emissions to them.

On the contrary, the CEL recommends an egalitarian distribution of the incurred losses (the part of the aggregate cumulative CO_2 emissions not satisfied, i.e., $\sum_{i=1}^{n} c_i - E$, given that no-one can emit a negative amount (that is, a group cannot reduce the CO_2 emissions budget). The $R5ASIA$ region, as the largest claimant, will always lobby in favor of CEL solution for the same reason as $R5REF$ countries prefer CEA. Furthermore, $R5OECD$ region, the most developed SRES countries, will vote for CEL in most of the cases.

The T is a convex combination of these latter solutions. It takes the middle of the aggregate claims as a reference point. If the half of the total needs of cumulative CO_2 emissions is lower than the CO_2 emissions budget, then the CEA is applied; whereas, each region receives half of its expected emissions and the amount recommended by CEL, otherwise. The $R5ALM$ region, among which, the most poor countries are counted, will choose this solution whenever the CO_2 emissions budget is lower than 1440 $Gt\ CO_2$.

The RA solution takes into account all the possible orderings of satisfying the regions claims, so the first regions arriving, will obtain a greater compensation of their emissions needs. Therefore, neither favours nor penalizes a region, indeed it makes the average of all the possible distributions.

The last two proposed solutions ensures a minimal amount of emissions to each region. On the one hand, AP share the endowment proportionally to the claims, once each region receives a minimal quota. On the other hand, α-min shares the budget in an egalitarian way, except in the case that the expected cumulative CO_2 emissions of the smallest groups are so small in relative terms ($E = 1440$), in which case this region is totally honored.

In order to introduce fairness criteria, first of all, we now evaluate the solutions in terms of the equity content. In this case, we will require, as a reasonable criterion, that equity will be better (larger) than the baseline case. Note that this baseline case is constructed considering the real CO_2 emissions of all the regions in 2010. That is, we compute the quota of each of the groups of countries with respect to the real global emissions in 2010. Equality (or its inverse, inequality) is cardinalized through inequality indices consistent with the Lorenz criterion. Five possible indices are attached, which are differentiated according to the weight given to the improvements of the groups with lower emissions (a kind of progressivity). In particular, we use the well-known Gini coefficient, an Atkinson index and two indices of the Theil family (Cowell, 2011; Duro, 2012). In any case, the application of these indicators in our analysis, that is, heterogeneous aggregate groups, has to be done with caution. Given the different dimension of the groups (for instance in terms of population), an equal distribution does not have to be desirable.

Nevertheless two comments can be appropriate: first, the groups are similar in terms of surface; second, ceteris paribus, a more egalitarian distribution can be conceived as superior to another. Additionally, we may demand that each solution meets the criterion of relative stability. Recall that this criterion has to do with the probable general acceptability of the solution and that, in our case, it is related to the balance in cooperative games.

As Table 3 depicts, with a CO_2 endowment of 1,440, CEA is the only appealing solution that satisfies inequality and stability criteria. Note that CEA honors all the claims of the lower emitters. Besides, it demands to exert a larger effort to larger emitters. So, it is a very equitable solution Bosmans and Lauwers (2011).

For the case of a CO_2 budget of 1,000 and 745, CEA and α-min are appealing solutions, since both of them satisfy the inequality and stability criteria. It is noteworthy that CEA demands a larger reduction to ASIA, in comparison with the α-min (the OECD efforts are not very different). So, may be, at the end we should select between a more equitable solution against a more plausible solution for high-emitters (the negotiation) like the α-min.

RCP6.0: 1,440	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	106.73	82.93	106.73	24.07	53.37	71.15	70.85	106.73	102.99
R5MAF	132.88	103.25	132.88	50.22	66.44	88.59	88.21	124.69	143.73
R5REF	163.10	126.73	163.10	80.44	81.55	108.73	108.27	145.45	144.01
R5OECD	634.39	492.92	518.64	551.73	528.43	494.87	495.45	469.14	515.25
R5ASIA	816.18	634.17	518.64	733.52	710.22	676.66	677.24	594.00	534.02
Gini Index	0.41	0.41	0.34*	0.53	0.49	0.45	0.45	0.37	0.34
Atkinson Index	0.89	0.89	0.73*	1.15	1.07	0.97	0.97	0.79	0.75
T0	0.35	0.35	0.23*	0.78	0.58	0.43	0.43	0.26	0.24
T1	0.32	0.32	0.22*	0.56	0.47	0.37	0.38	0.24	0.23
CV	0.89	0.89	0.73*	1.15	1.07	0.97	0.97	0.79	0.89
RCP6.0: 1,000	**c**	**P**	**CEA**	**CEL**	**T**	**RA**	**AP**	**α-min**	**Base**
R5LAM	106.73	57.59	106.73	0.00	53.37	59.24	57.59	106.73	71.53
R5MAF	132.88	71.70	132.88	0.00	66.44	74.49	71.70	115.97	99.82
R5REF	163.10	88.01	163.10	0.00	81.55	92.12	88.01	126.65	100.02
R5OECD	634.39	342.31	298.64	409.11	317.19	329.49	342.31	293.20	357.85
R5ASIA	816.18	440.40	298.64	590.89	481.45	444.67	440.40	357.45	370.89
Gini Index	0.41	0.41	0.22*	0.64	0.44	0.41	0.41	0.27*	0.34
Atkinson Index	0.89	0.89	0.46*	1.41	0.96	0.88	0.89	0.58*	0.75
T0	0.35	0.35	0.09*	60.24	0.40	0.34	0.35	0.13*	0.24
T1	0.32	0.32	0.08*	0.99	0.35	0.30	0.32	0.13*	0.23
CV	0.89	0.89*	0.46*	1.41	0.96*	0.88*	0.89*	0.58*	1.02
RCP6.0: 745	**c**	**P**	**CEA**	**CEL**	**T**	**RA**	**AP**	**α-min**	**Base**
R5LAM	106.73	42.90	106.73	0.00	53.37	40.91	44.62	106.73	53.28
R5MAF	132.88	53.42	132.88	0.00	66.44	49.95	55.55	110.92	74.34
R5REF	163.10	65.56	163.10	0.00	81.55	60.03	68.18	115.76	74.51
R5OECD	634.39	255.02	171.15	281.61	271.82	283.07	265.20	191.24	266.57
R5ASIA	816.18	328.10	171.15	463.39	271.82	311.04	311.44	220.35	276.31
Gini Index	0.41	0.41	0.09*	0.65	0.34*	0.42	0.40	0.17*	0.34
Atkinson Index	0.89	0.89	0.19*	1.44	0.76	0.91	0.86	0.36*	0.75
T0	0.35	0.35	0.02*	60.25	0.25	0.38	0.33	0.05*	0.24
T1	0.32	0.32	0.02*	1.00	0.23	0.34	0.30	0.05*	0.23
CV	0.89	0.89*	0.19*	1.44	0.76*	0.91*	0.86*	0.36*	1.72

Table 3: **Allocations solutions for the RCP6.0 scenario.** The rows indicates the emissions allocations that each region receives according to the different considered solutions (in columns). Furthermore, the five last rows of each scenario show the equity criteria applied for all the possible allocation solutions. Finally, the last column indicates how the "Base" benchmark, i.e., how the CO_2 emissions have been shared until 2010 in terms of real data and stars indicate the indexes below the base.

RCP4.5: 1,440	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	117.72	86.88	117.72	15.50	58.86	78.48	78.23	117.72	102.99
R5REF	166.46	122.86	166.46	64.24	83.23	110.97	110.63	148.18	143.73
R5MAF	217.38	160.44	217.38	115.16	108.69	144.92	144.47	180.00	144.01
R5OECD	598.19	441.49	469.22	495.97	468.03	426.24	426.76	417.96	515.25
R5ASIA	851.34	628.33	469.22	749.12	721.18	679.39	679.91	576.15	534.02
Gini Index	0.39	0.39	0.28*	0.53	0.47	0.42	0.42	0.33*	0.34
Atkinson Index	0.82	0.82	0.59*	1.11	1.02	0.90	0.90	0.69*	0.75
T0	0.29	0.29	0.15*	0.76	0.48	0.34	0.34	0.19*	0.24
T1	0.26	0.26	0.14*	0.52	0.41	0.31	0.31	0.19*	0.23
CV	0.82	0.82	0.59*	1.11	1.02	0.90	0.90	0.69*	0.89

RCP4,5: 1,000	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	117.72	60.34	117.72	0.00	58.86	59.76	60.34	117.72	71.53
R5REF	166.46	85.32	166.46	0.00	83.23	86.71	85.32	132.44	99.82
R5MAF	217.38	111.41	217.38	0.00	108.69	112.17	111.41	147.81	100.02
R5OECD	598.19	306.59	249.22	373.42	299.10	302.57	306.59	262.80	357.85
R5ASIA	851.34	436.34	249.22	626.57	450.12	438.80	436.34	339.23	370.89
Gini Index	0.39	0.39	0.14*	0.65	0.40	0.39	0.39	0.23*	0.34
Atkinson Index	0.82	0.82	0.29*	1.44	0.85	0.82	0.82	0.48*	0.75
T0	0.29	0.29	0.04*	60.25	0.30	0.28	0.29	0.09*	0.24
T1	0.26	0.26	0.04*	1.00	0.28	0.26	0.26	0.09*	0.23
CV	0.82	0.82*	0.29*	1.44	0.85*	0.82*	0.82*	0.48*	1.02

RCP4,5: 745	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	117.72	44.95	117.72	0.00	58.86	45.13	47.54	117.72	53.28
R5REF	166.46	63.56	156.82	0.00	83.23	63.80	67.22	123.31	74.34
R5MAF	217.38	83.00	156.82	0.00	108.69	80.77	87.79	129.16	74.51
R5OECD	598.19	228.41	156.82	245.92	247.11	258.09	241.58	172.87	266.57
R5ASIA	851.34	325.07	156.82	499.07	247.11	297.22	300.87	201.93	276.31
Gini Index	0.39	0.39	0.04*	0.67	0.29*	0.38	0.37	0.12*	0.34
Atkinson Index	0.82	0.82	0.12*	1.50	0.61*	0.80	0.77	0.25*	0.75
T0	0.29	0.29	0.01*	60.26	0.16*	0.28	0.26	0.02*	0.24
T1	0.26	0.26	0.01*	1.03	0.15*	0.26	0.23	0.02*	0.23
CV	0.82	0.82*	0.12*	1.50*	0.61*	0.80*	0.77	0.25*	1.72

Table 4: **Allocations solutions for the RCP4.5 scenario.** The rows indicates the emissions allocations that each region receives according to the different considered solutions (in columns). Furthermore, the five last rows of each scenario show the equity criteria applied for all the possible allocation solutions. Finally, the last column indicates how the "Base" benchmark, i.e., how the CO_2 emissions have been shared until 2010 in terms of real data and stars indicate the indexes below the base.

RCP6.0: 1,440	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.78	**1**	0.23	0.50	0.67	0.66	**1**
MAF	0.78	**1**	0.38	0.50	0.67	0.66	**0.94**
REF	0.78	**1**	0.49	0.50	0.67	0.66	**0.89**
OECD	0.78	**0.82**	0.87	0.83	0.78	0.78	**0.74**
ASIA	0.78	**0.64**	0.90	0.87	0.83	0.83	**0.73**
RCP6.0: 1,000	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.54	**1**	0.00	0.50	0.56	0.54	**1**
MAF	0.54	**1**	0.00	0.50	0.56	0.54	**0.87**
REF	0.54	**1**	0.0	0.50	0.56	0.54	**0.78**
OECD	0.54	**0.47**	0.64	0.50	0.52	0.54	**0.46**
ASIA	0.54	**0.37**	0.72	0.59	0.54	0.54	**0.44**
RCP6.0: 745	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.40	**1**	0.00	0.50	0.38	0.42	**1**
MAF	0.40	**1**	0.00	0.50	0.38	0.42	**0.83**
REF	0.40	**1**	0.0	0.50	0.38	0.42	**0.71**
OECD	0.40	**0.27**	0.44	0.43	0.45	0.42	**0.30**
ASIA	0.40	**0.21**	0.57	0.33	0.38	0.38	**0.27**
RCP4.5: 1,440	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.74	**1**	0.13	0.50	0.67	0.66	**1**
MAF	0.74	**1**	0.39	0.50	0.67	0.66	**0.89**
REF	0.74	**1**	0.53	0.50	0.67	0.66	**0.83**
OECD	0.74	**0.78**	0.83	0.78	0.71	0.78	**0.70**
ASIA	0.74	**0.55**	0.88	0.85	0.80	0.80	**0.68**
RCP4.5: 1,000	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.51	**1**	0.00	0.50	0.51	0.51	**1**
MAF	0.51	**1**	0.00	0.50	0.52	0.51	**0.80**
REF	0.51	**1**	0.0	0.50	0.52	0.51	**0.68**
OECD	0.51	**0.78**	0.62	0.50	0.51	0.51	**0.44**
ASIA	0.51	**0.55**	0.74	0.53	0.52	0.51	**0.40**
RCP4.5: 745	P	**CEA**	CEL	TAL	RA	AP	**α-min**
LAM	0.38	**1**	0.00	0.50	0.38	0.40	**1**
MAF	0.38	**0.94**	0.00	0.50	0.38	0.40	**0.74**
REF	0.38	**0.72**	0.0	0.50	0.37	0.40	**0.59**
OECD	0.38	**0.26**	0.41	0.41	0.43	0.40	**0.29**
ASIA	0.38	**0.18**	0.59	0.29	0.35	0.35	**0.24**

Table 5: **Claims percent honored**. The values in columns show the percentage of each region's claims that are satisfied by all the aforementioned solutions.

Similar comments are applied to the RCP4.5 and RCP2.6 scenarios (see Appendix). Therefore, in summary,

	RCP6.0	RCP4,5	RCP2.6
1,440	CEA	CEA & $\alpha-$min	CEA & $\alpha-$min
1,000	CEA & $\alpha-$min	CEA & $\alpha-$min	CEA & $\alpha-$min
745	CEA & $\alpha-$min	CEA & $\alpha-$min	CEA & $\alpha-$min

Table 6: **Summary results of allocation in terms of equity criteria.** For each of the three different CO_2 budget, the solutions providing a more equitable distribution of emissions than the actual one are presented.

Synthesizing technically, as Table 7 shows, two solutions appear as the most attractive because of their principles and the results they would produce. The CEA and the α-min solutions. The first one is very equitable, practically providing all the claims to the groups with the lowest emissions, that is, R5LAM, R5REF and R5MAF and big reduction efforts to OECD and Asia. In fact, no other solution provides more equally distributed results Hougaard et al. (2012). This happens because it does not take into account the losses that occur between groups. However, note the larger grade of adjust demand to the larger emitters, which provokes, indeed, can complicate its real application in a negotiation process where precisely the groups most affected by the reduction typically have more weight, not only in geopolitical terms but in terms of global GDP generation and population.

Alternatively, the α-min solution emerges mostly as technically interesting. In fact, the latter is an interesting solution that combines the strength of the adoption of a principle of equity, materialized by minimal rights, and proportionality. In addition, this solution could be more easily admissible in practice given that the reductions in the allocations, based on the claims, of a group such as Asia are minor and therefore also the possible impacts of this effort on economic parameters.

Finally, in the claims problems literature there has been always presented the idea of establishing a lower bound on awards that ensures a minimal guarantee of the endowment to each claimant (see Table 5). Indeed, the definition of a solution already establishes a lower bound on awards by demanding non-negativity. In this regard, all the aforementioned solutions, except the CEL one, guarantee a minimal quota of the CO_2 emission budget to all the regions. Specifically, the CEA and the α-min solutions ensure a greater amount of emissions rights to each region, being the CEA solution the most egalitarian allocation.

CEA	$\alpha-$min
· Satisfies basic principles.	· Satisfies basic principles.
· Satisfies additional principles.	· Satisfies additional principles.
· The most equitable measure.	· A mix of minimal guarantees (equity) and proportionality.
· Satisfies stability.	· Satisfies stability.
· Makes important reductions to larger emitters.	· Makes feasible reductions to larger emitters.

Table 7: **Summary insights of the CEA and the α-min solutions.**

5. Conclusions

It is noteworthy that through our claims approach we model the behaviour in a very natural way, since we are not considering the strategies behavior of each of the regions. In contrast, we propose a set of minimal requirements of fairness that should be satisfied by any distribution solution. Furthermore, we facilitate the commitment through a more detailed choice of additional principles and the analysis of the behaviour of different solutions according to previous principles and other criteria, like equity and stability. As a consequence, from this point of view, the CEA solution and α-min are typically the solutions selected because they satisfy all the principles, they are equitable criteria and also full fill the stability principle. Nevertheless, and if we want to make a step forward, the CEA solution is more equitable (lorenz-domination) and can be selected through voting by more groups (if all the groups have the same weight). However, if we consider that developed countries and Asia are more important in the agreements under some circumstances possibly the α-min solution can win relevance (because guarantees a minimal right (equality) and it is also proportional to the CO_2 emissions needs.

Therefore, by departing from the basis of a claims problem so that we can benefit from techniques that have already been proven useful in other similar bankruptcy situations, and by discussing on the principles that an allocation should satisfy instead on the allocation itself, the commitment among the different regions cannot be that easily biased with respect to their particular interests, as easily happens in purely ethical discussions. In this regard, Finus and Pintassilgo (2013) shows that a 'veil of ignorance' in international climate negotiations, which means that distributional output is unknown for participants, might be conducive to the success of international cooperation. Finally, we would like to mention some extensions and limitations of the current analysis:

- By examining the welfare effects of the different solutions we could evaluate solutions in a more complete way.
- Evaluating this exercise with more realistic and disaggregated groups.
- Evaluate the implications of the results in terms of emissions intensities, growth, etc.

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Appendix

Next, we provide the implemention of he analysed solutions to other schemes.

RCP8.5: 1,440	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	181.74	101.53	181.74	0.00	90.87	102.81	101.53	181.74	102.99
R5REF	294.29	164.40	294.29	55.29	147.15	168.46	164.40	217.57	143.73
R5MAF	311.50	174.01	311.50	72.50	155.75	178.50	174.01	223.05	144.01
R5OECD	826.18	461.53	326.24	587.18	454.20	449.55	461.53	386.88	515.25
R5ASIA	964.02	538.53	326.24	725.02	592.04	540.67	538.53	430.76	543.02
Gini Index	0.33	0.33	0.09	0.55	0.36	0.32	0.33	0.19	0.34
Atkinson Index	0.69	0.69	0.21	1.18	0.77	0.67	0.69	0.39	0.75
T0	0.20	0.20	0.02	20.48	0.25	0.19	0.20	0.06	0.24
T1	0.19	0.19	0.02	0.65	0.23	0.18	0.19	0.06	0.23
CV	0.69	0.69	0.21	1.18	0.77	0.67	0.69	0.39	0.89
RCP8.5: 1,000	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	181.74	70.50	181.74	0.00	90.87	71.07	70.50	181.74	71.53
R5REF	294.29	114.17	204.56	0.00	147.15	108.59	114.17	187.90	99.82
R5MAF	311.50	120.84	204.56	0.00	155.75	114.32	120.84	188.84	100.02
R5OECD	826.18	320.51	204.56	431.08	303.12	335.78	320.51	216.99	357.85
R5ASIA	964.02	373.98	204.56	568.92	303.12	370.24	373.98	224.53	370.89
Gini Index	0.33	0.33	0.02	0.63	0.23	0.33	0.33	0.05	0.34
Atkinson Index	0.69	0.69	0.05	1.39	0.49	0.71	0.69	0.10	0.75
T0	0.20	0.20	0.00	60.24	0.10	0.21	0.20	0.00	0.24
T1	0.19	0.19	0.00	0.98	0.10	0.20	0.19	0.00	0.23
CV	0.69	0.69	0.05	1.39	0.49	0.71	0.69	0.10	1.02
RCP8.5: 745	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	181.74	52.53	149.00	0.00	90.87	59.16	59.45	149.00	53.28
R5REF	294.29	85.05	149.00	0.00	147.15	96.68	96.26	149.00	74.34
R5MAF	311.50	90.03	149.00	0.00	155.75	102.42	101.89	149.00	74.51
R5OECD	826.18	238.78	149.00	303.58	175.62	243.37	243.70	149.00	266.57
R5ASIA	964.02	278.62	149.00	441.42	175.62	243.37	243.70	149.00	276.31
Gini Index	0.33	0.33	0.00	0.64	0.11	0.28	0.28	0.00	0.34
Atkinson Index	0.69	0.69	0.00	1.41	0.23	0.59	0.59	0.00	0.75
T0	0.20	0.20	-0.00	60.24	0.03	0.15	0.15	0.00	0.24
T1	0.19	0.19	-0.00	0.99	0.02	0.14	0.14	0.00	0.23
CV	0.69	0.69	0.00	1.41	0.23	0.59	0.59	0.00	1.72

RCP2.6: 1,440	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	114.74	114.26	114.74	113.52	113.52	113.52	113.52	114.74	102.99
R5REF	122.60	122.08	122.60	121.38	121.38	121.38	121.38	122.54	143.73
R5MAF	170.34	169.62	170.34	169.12	169.12	169.12	169.12	169.95	144.01
R5OECD	457.49	455.56	457.49	456.27	456.27	456.27	456.27	455.09	515.25
R5ASIA	580.94	578.49	574.83	579.72	579.72	579.72	579.72	577.67	543.02
Gini Index	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34
Atkinson Index	0.75	0.75	0.74	0.75	0.75	0.75	0.75	0.74	0.75
T0	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.24
T1	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.23
CV	0.75	0.75	0.74	0.75	0.75	0.75	0.75	0.74	0.89

RCP2.6: 1,000	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	114.74	79.34	114.74	25.52	57.37	76.49	75.36	114.74	71.53
R5REF	122.60	84.78	122.60	33.38	61.30	81.73	80.53	118.58	99.82
R5MAF	170.34	117.79	170.34	81.12	85.17	113.56	111.88	141.91	100.02
R5OECD	457.49	316.36	296.16	368.27	336.35	302.38	304.39	282.22	357.85
R5ASIA	580.94	401.73	296.16	491.72	459.81	425.83	427.84	342.55	370.89
Gini Index	0.35	0.35	0.21	0.51	0.43	0.37	0.37	0.25	0.34
Atkinson Index	0.75	0.75	0.45	1.08	0.93	0.78	0.79	0.53	0.75
T0	0.23	0.23	0.08	0.65	0.39	0.25	0.26	0.11	0.24
T1	0.22	0.22	0.08	0.49	0.34	0.24	0.24	0.11	0.23
CV	0.75	0.75	0.45	1.08	0.93	0.78	0.79	0.53	1.02

RCP2.6: 745	c	P	CEA	CEL	T	RA	AP	α-min	Base
R5LAM	114.74	59.11	114.74	0.00	57.37	58.75	59.11	114.74	53.28
R5REF	122.60	63.16	122.60	0.00	61.30	62.88	63.16	116.28	74.34
R5MAF	170.34	87.75	169.22	15.75	85.17	90.21	87.75	125.66	74.51
R5OECD	457.49	235.69	169.22	302.90	228.75	233.78	235.69	182.04	266.57
R5ASIA	580.94	299.29	169.22	426.35	312.41	299.37	299.29	206.28	276.57
Gini Index	0.35	0.35	0.08	0.62	0.36	0.35	0.35	0.13	0.34
Atkinson Index	0.75	0.75	0.19	1.35	0.77	0.74	0.75	0.28	0.75
T0	0.23	0.23	0.01	40.50	0.25	0.23	0.23	0.03	0.24
T1	0.22	0.22	0.01	0.88	0.23	0.22	0.22	0.03	0.23
CV	0.75	0.75	0.19	1.35	0.77	0.74	0.75	0.28	1.72