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Document de treball n. 01 - 2021

Edita:

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Adreçar comentaris al Departament d'Economia / ECO-SOS

ISSN: 2696-5097

Market power in California's water market*

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January 31, 2021

Abstract

We estimate market power in California's thin water market. Market frictions may distort the potential welfare gains from water marketing. We use a Nash-Cournot model and derive a closed-form solution for the extent of market power in a typical water market setting. We then use this solution to estimate market power in a newly assembled dataset on California's water economy. We show that, under the assumptions of the Cournot model, market power in this thin market is limited.

Keywords: Water markets, Market power, California, Cournot-Nash.

JEL classification: C72, D43, Q25.

*We thank Marjan Hofkes, Mark Lijesen, Rick van der Ploeg, Jos van Ommeren, and seminar participants at EAERE 2020 for comments on earlier versions of this paper. We thank Clay Landry and WestWater Research, LLC for sharing the Californian water transaction data used in this paper. Françeska acknowledges financial support from the *Ministerio de Ciencia, Innovación y Universidades* and the *European Union* under grants ECO2016-75410-P BES-2017-080192 and PID2019-105982GB-I00/AEI/10.13039/501100011033, and from the *Generalitat de Catalunya/Universitat Rovira i Virgili* under grant 2019PFR-URV-53.

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1 Introduction

We estimate the extent and impact of market power in California's thin water market. In this water market, both water leases and permanent sales of water rights reallocate water from lower to higher value uses. Such reallocation is known to substantially increase the efficiency of water use (cf. Vaux Jr. and Howitt, 1984; Jenkins et al., 2004), but it may be obstructed by various market frictions. There is ample evidence, both from California and other regions, and both for ground- and surface water trading, that market power may be an important source of friction in water markets (Rosegrant and Binswanger, 1994; Easter et al., 1999; Jacoby et al., 2004; Holland, 2006; Chakravorty et al., 2009; Bruno and Sexton, 2020). Under the premise that market frictions may distort the potential welfare gains from water marketing, we seek to identify the extent and impact of such market power in California.

Inspired by the model set-up of Ansink and Houba (2012), we introduce a Nash-Cournot model of water transactions. Under two main assumptions, discussed below, this model allows us to derive a closed-form solution for the extent of market power in a typical water market setting. One novelty is that we write this solution in terms of willingness-to-pay and -accept. We subsequently apply our model to a newly assembled dataset on California's water economy by Hagerty (2019). The data that we use is 1993-2015 panel data on water transactions in California, with detailed information on quantities and prices at the water district-level, combined with detailed spatial data on locations of buying and selling districts as well as geographical factors that may affect market power. The data allows us to control a.o. for main water uses of buying and selling districts and various types of associated transaction costs. The results of our estimation allow us, ultimately, to estimate Lerner indices for California's water market.

Our main approach starts with two main assumptions, both of which will be relaxed later on. One assumption is that we fix the side of the market where market power resides. Our base model assumes buyer market power, a belief held by many stakeholders and supported by previous literature (cf. Tomkins and Weber, 2010; Hansen et al., 2014; Hagerty, 2019). To check the relevance of this assumption, we also employ a model specification where we allow for market power on both sides; we find support for buyer power only. The second main assumption is that we use linear demand, originating from a quadratic benefit function of water use. This functional form is commonplace in the water economics literature and allows for a straightforward empirical strategy to derive our results. Constant linear demand across selling districts may not be realistic, however, and therefore we relax this assumption in an alternative specification where, instead, we impose

constant price elasticity. This alternative specification, with constant price elasticity, is presented as part of a larger class of model specifications featuring homogeneous demand, for which we present a closed-form solution as well.

An important methodological advantage of our model is that we do not rely on a conjectural variations approach that employs consistent conjectures (Bresnahan, 1989). This approach is not compatible with standard notions of rational behavior since the game theory revolution (cf. Lindh, 1992).¹ In addition, the model that we propose can be adapted and applied to other endowment economies, including permit markets.

Our results show that market power in the Californian water market is limited. Our main specification implies that buyer power yields an average mark-down of 6% of the transaction price. This result is obtained for the linear model, but continues to hold for the non-linear specification and is robust to other model modifications. Our result is surprising in the sense that the thinness of water markets, including California's, is conventionally associated with higher possibilities of exploiting market power. Our result is also important in that water market reform need not take into account market power but can focus on other factors instead, most notably transaction costs (Carey et al., 2002; Regnacq et al., 2016; Hagerty, 2019; Leonard et al., 2019).

We first introduce the model and our main model specification in Section 2. Next, we present the data in Section 3 and our empirical strategy in Section 4. Subsequently, we present model results in Section 5, focusing on our estimation of Lerner indices for California's water market. This main result is compared with a conjectural variations approach in Section 6 and checked for robustness in Section 7. In Section 8, we conclude.

2 Model

2.1 A model of market power in water markets

We develop a Nash-Cournot model of water transactions in order to derive an index for the extent of market power in a typical water market setting. Consider a water market with water transactions between sellers at origins $o = 1, 2, \dots, N_o$ and buyers at destinations $d = 1, 2, \dots, N_d$. Water is a homogeneous good and purchases from different sellers are perfect substitutes. Both sellers and buyers have entitlements of water, denoted either $e_o > 0$ or $e_d \geq 0$, depending upon their role. Although variation in rainfall and snow-melt

¹In Hagerty (2019), the same dataset is analyzed, but the focus is on the impact of transaction costs in obstructing water markets. As a robustness check, the potential impact of market power as an alternative explanation for market frictions is explored, using an approach that employs consistent conjectures. Other papers, including Bruno and Sexton (2020), use this same approach.

may cause endowments to change over time, we suppress time subscripts in this section to keep notation simple. The amount of water sold by seller o to buyer d is denoted $q_{od} \geq 0$. Obviously, sellers cannot sell more water than their entitlements, i.e. $\sum_{d=1}^{N_d} q_{od} \leq e_o$.

Water use by buyers consists of their own entitlement plus purchased water: $Q_d \equiv e_d + \sum_{o=1}^{N_o} q_{od}$. Buyers' benefit from using this total sum of water equals $f_d(Q_d)$, which is increasing in the neighborhood of e_d (buyers are unsatiated at e_d), strictly concave, and twice continuously differentiable in Q_d . For later reference, we introduce the buyer's willingness-to-pay, denoted WTP, which is defined as the partial derivative of net benefits w.r.t. water use. Formally,

$$\text{WTP}_d(Q_d) = f'_d(Q_d). \quad (1)$$

In any bilateral trade, buyers do not pay more than their WTP_d through the transaction-specific price $p_{od} \leq f'_d(Q_d)$.

Water use by sellers consists of their own entitlement minus sold water: $Q_o \equiv e_o - \sum_{d=1}^{N_d} q_{od}$. Sellers' benefit from using the unsold amount of water equals $f_o(Q_o)$, which is increasing in the neighborhood of e_o (sellers are unsatiated at e_o), strictly concave, and twice continuously differentiable in Q_o . Sellers' net benefits of water use are now given by $f_o(Q_o)$ plus revenues from selling water, introduced below. For later reference, we introduce the seller's willingness-to-accept, denoted WTA, which is defined as the partial derivative of net benefits w.r.t. water use. Formally,

$$\text{WTA}_o(Q_o) = f'_o(Q_o). \quad (2)$$

In any bilateral trade, sellers must be financially compensated for these opportunity costs through the transaction-specific price $p_{od} \geq f'_o(Q_o)$.

Recall that we consider the case where buyers hold all market power. In this case, the market clearing price must equal the seller's WTA:

$$p_{od} = \text{WTA}_o(Q_o). \quad (3)$$

Buyer d 's expenditure on buying water from seller o is then given by $q_{od} \cdot \text{WTA}_o(Q_o)$. Buyers maximize over all potential sellers to purchase their water. Formally,

$$\max_{q_{1d}, \dots, q_{N_o d}} f_d(Q_d) - \sum_{o=1}^{N_o} q_{od} \cdot \text{WTA}_o(Q_o). \quad (4)$$

Using the positive relation between Q_d and q_{od} as well as the negative relation between Q_o

and q_{od} , a buyer's first-order condition w.r.t. Q_d (implicitly, q_{od}) for an interior solution is given by²

$$f'_d(Q_d) - WTA_o(Q_o) + q_{od} \cdot WTA'_o(Q_o) = 0. \quad (5)$$

Substituting (1) into (5) and rewriting yields

$$\begin{aligned} WTP_d(Q_d) &= WTA(Q_o) - q_{od} \cdot WTA'_o(Q_o) \\ &\geq WTA_o(Q_o). \end{aligned} \quad (6)$$

Substituting (3) into (6), we now have the following system that we will use in Section 4:

$$p_{od} = WTA_o(Q_o), \quad (7a)$$

$$p_{od} = WTP_d(Q_d) + q_{od} \cdot WTA'_o(Q_o). \quad (7b)$$

Recall that $WTA'_o(Q_o) < 0$ so that the last term of (7b) is negative.

The wedge between buyers' WTP and sellers' WTA reflects the possible price range for each transaction. Under our assumption of buyer power, the realized price equals the seller's WTA, the lowest possible price. We therefore use the wedge to construct our measure of market power, which can be interpreted as the Lerner index applied to our model (note the multiplication of the inverse price elasticity of sellers' WTA by the ratio of transaction volume to water use):

$$\frac{WTP_d(Q_d) - WTA_o(Q_o)}{WTA_o(Q_o)} = -\frac{q_{od}}{Q_o} \cdot \frac{Q_o WTA'_o(Q_o)}{WTA_o(Q_o)}. \quad (8)$$

This Lerner index is the main result of our theoretical model. In Section 4 we will use the system of equations (7) to estimate $WTA'_o(Q_o)$ which we then use in (8) to measure market power in California's water market.

Our model is illustrated in Figure 1. With two types of districts (buyers and sellers) and one good (water), whose supply is given, our model is an endowment economy and so we can visualize it in a chart with a secondary mirrored primary axis, while total available water is on the horizontal axis. Demand for water is displayed using the $WTA_o(Q_o)$ curve for sellers and the $WTP_d(Q_d)$ curve for buyers. Starting from water endowments e_o and e_d in Figure 1, water transactions increase buyers' water consumption and decrease sellers' water consumption, while closing the wedge between buyers' WTP and sellers' WTA. Compared

²The first-order conditions for the boundary solution $q_{od} = 0$ have the weak inequality \leq replacing the equality.

with the competitive equilibrium, buyer power implies a lower transaction volume, which leaves a positive wedge, as discussed in this section and as illustrated in the figure.

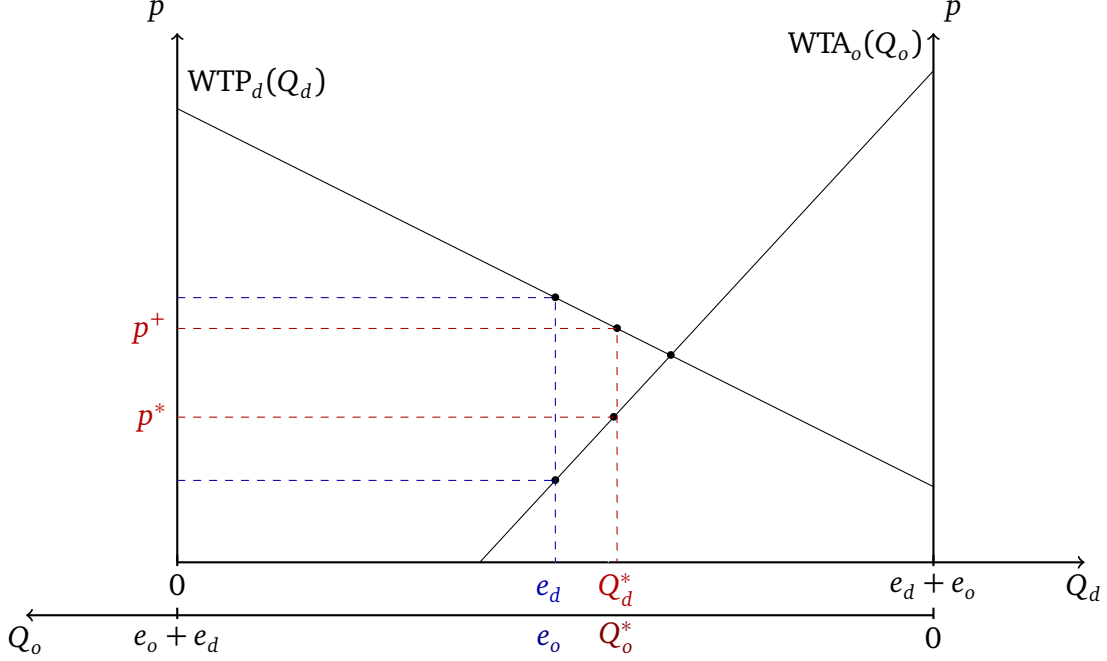


Figure 1: Stylized visualization of endowments (blue) and Nash-Cournot equilibrium (red), where p^+ equals p^* plus the wedge $WTP_d(Q_d^*) - WTA_o(Q_o^*)$.

2.2 Main specification

The preferred specification of our model uses quadratic benefit functions for both buyers and sellers. This specification allows to estimate a linear model, as explained in Section 4. Our proposed benefit functions allow for heterogeneity across buyers and sellers as well as over time, which is why we add time subscripts from here on.

For each origin we have $f_{ot}(Q_{ot}) = Q_{ot}(\alpha_{ot} - \frac{1}{2}\delta Q_{ot})$, where $\alpha_{ot} = \phi_o + \beta_t + \nu_{ot}$ captures heterogeneity between different sellers and time periods, while parameter δ is kept constant. This benefit function implies that $f'_{ot}(Q_{ot}) = \alpha_{ot} - \delta Q_{ot}$, which is the sellers' WTA in (2). Similarly, for each destination we have $f_{dt}(Q_{dt}) = Q_{dt}(a_{dt} - \frac{1}{2}\gamma Q_{dt})$, with $a_{dt} = \psi_d + \beta_t + u_{dt}$ and therefore $f'_{dt}(Q_{dt}) = a_{dt} - \gamma Q_{dt}$, which is the buyers' WTP in (1). Note that in Appendix B we generalize our main model specification to allow for asymmetry in terms of benefit parameters γ and δ . We do so after presenting the solution to the symmetric version of our main specification in Appendix A.

The sufficient and necessary condition for positive quantities in the symmetric Nash equilibrium is that $f'_{dt}(e_{dt}) > f'_{ot}(e_{ot})$, which implies

$$a_{dt} - \gamma e_{dt} > \alpha_{ot} - \delta e_{ot}. \quad (9)$$

The interpretation is that the marginal benefit of water use at the initial entitlement of each destination exceeds the marginal benefit of water use at the initial entitlement of each origin. In other words, trade is (marginally) beneficial at the initial entitlement levels.

3 Data

We apply our model using newly assembled data on California's water economy, first described by Hagerty (2019). We mainly use three datasets. The first is a proprietary dataset compiled by WestWater Research, LLC, listing prices, volumes, and other information related to Californian water transactions. The second is a dataset compiled from the archives of the California Department of Water Resources, the U.S. Bureau of Reclamation, and the State Water Resources Control Board, that combines the universe of yearly surface water entitlements and deliveries in California. The third is a geo-spatial dataset that identifies locations of buying and selling districts, and is used to estimate distances and identify other parameters related to transaction costs. Full details on each dataset, its cleaning and processing, is provided in Hagerty (2019, Section 4 and Appendix G).

The combined dataset provides panel data on a.o. water deliveries and transaction prices in California over the 23-year period 1993-2015. The panel data is unbalanced since districts can be involved in more than one transaction per year. Our unit of observation is the water district-level. This is the lowest possible level where (a) we can unambiguously match transactions to units, and (b) we have sufficient information on the units' entitlements and deliveries. It turns out that roughly 75% of all transactions in our transaction dataset can be matched to districts with complete information on entitlements and deliveries.³

The WestWater water transactions database includes a total of 6,309 transactions over the period 1990–2015. Since we will assess transactions both from the sellers' and from the buyers' perspective, we duplicate each transaction and split the dataset into two, one for buyers and one for sellers. A minority of transactions involve more than one district on each

³The alternative to districts as units of observation would be to either use planning areas or DAU-county areas (both are hydro-geographical areas defined by the California Department of Water Resources). Doing so would facilitate the matching with entitlements and deliveries. The downside, however, is that it would severely reduce the number of observations in our final dataset since transactions would be lumped into fewer units.

side of the transaction. We split up such transactions such that each observation contains one selling and one buying district. Because of our focus on market power, we choose to include in our dataset only freely-negotiated transactions of surface water in the spot market. We therefore drop transactions by excluding (1) transactions whose price is set administratively (or missing), (2) groundwater transactions, (3) transactions of permanent water rights, and (4) transactions executed before 1993 (since data on water deliveries is only available from 1993 onward). Applying these exclusion criteria, we drop 88% of our observations. We subsequently lose another 28% of our remaining observations (slightly more for buyers than for sellers) when merging our transactions dataset with our dataset on districts' entitlements and deliveries. Our final dataset contains 1131 observations, 592 for sellers and 539 for buyers.

Summary statistics (mean, standard deviation, and number of observations) on transaction for both buyers and sellers are shown in Table 1. In addition to transaction volumes and prices, this table lists statistics on six different factors that were found by Hagerty (2019) to be costly to buyers or sellers and thereby generate transaction costs. The first three are costly to sellers: (S1) transactions that cross the Sacramento-San Joaquin Delta, (S2) transactions where the buyer is primarily using water for agricultural purposes, and (S3) the total distance if water is conveyed along a river. The next three are costly to buyers: (B1) the virtual distance between buyer and seller if water is transferred against the direction of flow, and (B2) transactions that are subject to a State Water Boards review, and (B3) transactions that export water from a federal or state water project. Two factors cause differences in the data between buyers and sellers. One is that, in merging transactions with entitlements, we lose more observations for buyers than for sellers and this difference is apparently not a random draw. The second factor is that the buyer observations include a substantial share, 24%, where water is acquired for instream use, while for sellers this is only 1%. Such transactions tend to have much lower prices, roughly half of those where buyers are purchasing water for consumptive use. We will check whether inclusion of these transactions affects our results in Section 5.

Transactions mostly occur in a limited number of hydrologic regions. Sellers are mostly located in the Sacramento River and San Joaquin River regions, while buyers are mostly located in the Tulare Lake, San Joaquin River, and South Coast regions. We find only few instances of districts that both sell and buy, suggesting that we can assume fixed roles for districts as sellers or buyers. Transactions in our database cover a total of 161 districts, which implies a mean number of $592/161 = 3.7$ transactions per district over our 23-year period from the sellers' perspective and $539/161 = 3.3$ for buyers. This low number

Table 1: Summary statistics on transactions by sellers/buyers.

	Sellers			Buyers		
	Mean	SD	Obs	Mean	SD	Obs
Price (2010\$/AF)	237.49	296.79	592	185.50	173.75	539
Volume (AF)	8.74	24.70	583	8.93	26.67	530
S1: Delta crossing (1=yes)	0.33	0.47	568	.	.	0
S2: Agricultural buyer (1=yes)	0.47	0.50	592	.	.	0
S3: River distance (km)	0.09	0.10	568	.	.	0
B1: Virtual distance (km)	.	.	0	0.08	0.11	534
B2: State Water Boards review (1=yes)	.	.	0	0.42	0.49	539
B3: Export from project (1=yes)	.	.	0	0.05	0.22	539

illustrates that California's water market is thin.⁴

4 Empirical strategy

The objective of our empirical exercise is to measure market power in California's water market. We do so using the Lerner index (8). Calculation of this index requires an estimate of $WTA'_{ot}(Q_{ot})$. For the linear model specification introduced in Section 2.2, we have $WTA'_{ot}(Q_{ot}) = f''_{ot}(Q_{ot}) = -\delta$, which we will estimate using the system of equations (7). Note that this parameter δ is the only estimate that we need to measure market power using the Lerner index (8). To see this, note that our linear model specification with buyer power allows us to write this index in terms of δ as well as transaction prices and quantities p_{odt} and q_{odt} , which are present in our transaction data:

$$\begin{aligned} \frac{WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot})}{WTA_{ot}(Q_{ot})} &= -\frac{q_{odt}}{Q_{ot}} \cdot \frac{Q_{ot} WTA'_{ot}(Q_{ot})}{WTA_{ot}(Q_{ot})} \\ &= \delta \cdot \frac{q_{odt}}{p_{odt}}. \end{aligned} \quad (10)$$

Below, we present our empirical strategy to estimate parameter δ .

Given our panel data on transaction prices and quantities, we construct a fixed effects model, which exploits variation in observed transaction prices, WTA, and WTP across trading districts and across time. This approach rests on two requirements. The first is that we have sufficient variation in WTA and WTP over time. In our data, such variation over time is caused by variation in water entitlements over time, which imply movements

⁴One could argue that our data suffers from selection bias since we only observe realized transactions and these are typically from seller-buyer pairs with low transaction costs. Note, however, that we only observe equilibrium transactions and any non-observed transaction price would be 'out-of-equilibrium'.

along the benefit function of water use, thereby changing districts' marginal benefits of water use. Water entitlements are determined by the interaction of weather fluctuations with historically-determined allocation rules, which are markedly different across regions of California. The second requirement is that WTA and WTP are exogenous, conditional on unobserved district characteristics (as captured by the fixed effects). We meet this requirement by assumption, since our model dictates that WTA (and, implicitly, WTP) determines transaction prices.

There are two possible sources of endogeneity in our data, one of which is that omitted variables may cause biases. Ideally, we would control for these using both year fixed effects as well as time-invariant district-by-counterparty fixed effects. The latter would capture any variation in prices caused by unobserved heterogeneity across pairs of trading districts. Unfortunately, we do not have sufficient observations per trading district-pair to estimate such fixed effects. We resort to separate seller- and buyer fixed effects instead. The second possible source of endogeneity is reverse causality, which we discuss at the end of this section.

We substitute the linear specification of our model into the system of equations (7):

$$p_{odt} = -\delta Q_{ot} + \phi_o + \beta_t + v_{ot}, \quad (11a)$$

$$p_{odt} = -\gamma Q_{dt} - \delta q_{odt} + \psi_d + \beta_t + u_{dt}. \quad (11b)$$

An implicit assumption underlying the regression of *individual* transaction prices on (some function of) *total* water use levels is that districts face no uncertainty on their water entitlements or future prices, which may give them an incentive to hedge the risk of water shortage within each year. One example would be that districts buy 'too much' water and will try to re-sell later that same year. We find, however, that only a handful of districts in our dataset have ever been active on both sides of the market within one year. Hence, this assumption of no uncertainty seems warranted. It is also consistent with the situation in many Western US watersheds, where predictions on water availability in early spring provide 'reasonably accurate forecasts' of actual availability (Draper, 2001).

Without uncertainty, price differences across transactions for a particular district and year should not occur, except in the case of transaction costs. In model variations we therefore control for various types of transaction costs, as introduced in Section 3. Transaction costs are pair-specific and time-invariant, and they apply to either the seller or the buyer in a specific transaction as summarized in Table 1. In the regressions below, transaction costs are included as $T_{odr} = \tau_r C_{odr} + \tau_o + \tau_d + \varepsilon_{odr}$, where vector C_{odr} includes seller-, buyer-, and pair-specific transaction costs, with units (mostly dummies) as presented in

Table 1, while τ_o and τ_d represent district fixed costs. Since each transaction is assessed twice in this system, we add r to indicate whether the transaction is assessed from the seller's perspective ($r = 0$) or the buyer's perspective ($r = 1$). We expect $\tau_r \geq 0$ if $r = 0$ and $\tau_r \leq 0$ if $r = 1$. That is, transaction costs enter sellers' WTA positively, because these have to be compensated for sellers on top of the sellers' net benefits, while transaction costs enter the buyers' WTP negatively, because these decrease sellers' net benefits.

We add transaction costs to (11) and re-order and re-label terms:

$$\begin{aligned} p_{odrt} &= -\delta Q_{ot} + (\phi_o + \tau_o) + \tau_d + \beta_t + \tau_r C_{odr} + (v_{ot} + \varepsilon_{odr}) \\ &= -\delta Q_{ot} + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrt} \end{aligned} \quad (12a)$$

$$\begin{aligned} p_{odrt} &= -\gamma Q_{dt} - \delta q_{odt} + \tau_o + (\psi_d + \tau_d) + \beta_t + \tau_r C_{odr} + (u_{dt} + \varepsilon_{odr}) \\ &= -\gamma Q_{dt} - \delta q_{odt} + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrt} \end{aligned} \quad (12b)$$

Note that coefficient δ appears in both equations. We estimate both equations simultaneously by constructing two variables, R_{odtk}^o and R_{odtk}^d , that combine the coefficients on water use from (12). We also add a counter k , since there can be multiple transactions between one origin o and one destination d within one year t :

$$R_{odtk}^o = \begin{cases} Q_{ot} & \text{if } r = 0 \\ q_{odtk} & \text{if } r = 1, \end{cases} \quad \text{and} \quad R_{odtk}^d = \begin{cases} 0 & \text{if } r = 0 \\ Q_{dt} & \text{if } r = 1. \end{cases}$$

The combined regression equation, which also suppresses the intercept, is:

$$p_{odrtk} = -\delta R_{odtk}^o - \gamma R_{odtk}^d + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrtk}. \quad (13)$$

In the next section, we will estimate variations of (13) using linear regression.

Unlike standard models of supply and demand, we are estimating a system with two demand functions (with slopes given by parameters γ and δ), while the annual supply of water is determined by rainfall and snow-melt. With hydrological variation between years, the total amount of water in the system changes exogenously each year. Summed over all districts, annual supply cannot respond to changes in price. Despite this exogeneity in supply, individual districts may still respond to price changes by changing the volume of water bought or sold. We therefore also estimate (13) while instrumenting for water use with districts' entitlements, in line with Hagerty (2019).

5 Results

The estimates of regression equation (13) are shown in Table 2. Recall that the aim of this regression is to estimate the impact of market power on transaction prices via the wedge $WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot})$. Applying a model with quadratic benefit functions implies that *Seller water use* (i.e., R_{odtk}^O) is one of the independent variables, whose coefficient gives the slope of the sellers' benefit function, parameter δ . Multiplied by transaction volume, this parameter gives the wedge for each transaction.

Table 2: Estimating WTA and WTP: Linear model

Price (2010\$/AF)	(1)	(2)	(3)	(4)
	OLS	OLS	IV	IV
Seller water use (1,000 AF) (coefficient $-\delta$)	-0.0183** (0.00821)	-0.0280** (0.0130)	-0.580*** (0.208)	-0.932** (0.445)
Buyer water use (1,000 AF) (coefficient $-\gamma$)	-0.00757** (0.00336)	-0.0144** (0.00612)	-0.309** (0.132)	-0.311** (0.156)
Seller fixed effects	✓	✓	✓	✓
Buyer fixed effects	✓	✓	✓	✓
Year fixed effects	✓			
Quadratic time trend		✓	✓	✓
Transaction costs				✓
# Observations	1034	1034	879	877
# Clusters	543	337	308	307
# FE dummies	212	190	164	163
Cragg-Donald F-statistic			9.936	8.681

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using OLS and IV. Standard errors in parentheses, clustered by seller, buyer, and year (but only by seller and buyer in models (2)–(4) where year fixed effects are replaced by a quadratic time trend).

In model (1), estimated using OLS, we model water use as our only explanatory variable, combined with seller-, buyer-, role-, and year fixed effects. The coefficient on *Seller water use* implies that $\delta = 0.0183$, which is more than double the size of $\gamma = 0.00757$, implied by the coefficient on *Buyer water use*. The difference indicates that selling districts have steeper demand curves than buying districts. In model (2) we attempt to improve efficiency of these estimates. Given the large number of clusters compared to observations, we replace year fixed effects by a time trend. No comparable simplification was found feasible for the other fixed effects. Particularly, there is no obvious possibility to replace seller- and buyer fixed effects with a coarser set of dummy variables. As a result of replacing the year fixed effect by the time trend, the number of clusters decreases sharply. Compared to model (1), the model (2) estimates for both δ and γ increase significantly. In model (3)

we instrument water use by districts' water entitlements. The resulting estimations of δ and γ increase sharply, in absolute terms, compared to those of models (2) and (3). Finally, in model (4), we add seller- and buyer-specific transaction costs, which do not appear to improve the model results, decreasing the F-statistic and increasing the standard error of our main coefficient of interest. The Cragg-Donald F-statistics for the first stages of the IV models (not presented here) are both higher than their critical values as reported by Stock and Yogo (2005), suggesting that models (3) and (4) do not suffer from weak instruments.

Based on these model results and interpretation, our preferred model is model (3) and we use the main coefficient of interest from this specification, $\delta = 0.580$, in the remainder of this section. The interpretation of δ is that sellers' WTA, which equals the water price in our model, increases by \$0.58/AF for each 1,000 AF sold.⁵ More important for our analysis, however, is that δ is used to calculate the wedge $WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot}) = \delta q_{od}$. Doing so we find that the average wedge, after removing one outlier, equals \$4.60/AF (SD=8.66). This wedge corresponds to about 6.4% of the transaction price, on average, with markedly higher wedges (both in absolute and relative terms) for transactions with low prices. We use transaction-specific wedges to compute the Lerner index of (10) and plot these in Figure 2. This figure shows that the Lerner index is relatively low. It is markedly higher, though, for a small set of transactions with low prices, which also tend to have the highest transaction volumes. All in all, we find that market power is relatively low in California' water market.

6 A conjectural variations approach

We proceed to compare our results to those obtained using a conjectural variations approach in order to verify whether our assumption of buyer power is warranted. In this approach, the term expressing market power is multiplied by some weight that dampens this term. A recent example that employs this approach and analyzes Californian groundwater is Bruno and Sexton (2020). Accordingly, we introduce conjectural variations using parameter $\theta \in [0, 1]$ that measures the degree of buyer power, while $\xi \in [0, 1]$ measures the degree of seller power. Allowing for both buyer- and seller power, we rewrite (7) to include these market power weights:

$$p_{odt} = WTA_{ot}(Q_{ot}) - \xi \cdot q_{odt} \cdot WTP'_{dt}(Q_{dt}), \quad (14a)$$

$$p_{odt} = WTP_{dt}(Q_{dt}) + \theta \cdot q_{odt} \cdot WTA'_{ot}(Q_{ot}). \quad (14b)$$

⁵AF: acre-foot. One acre-foot equals 1,233 m³.

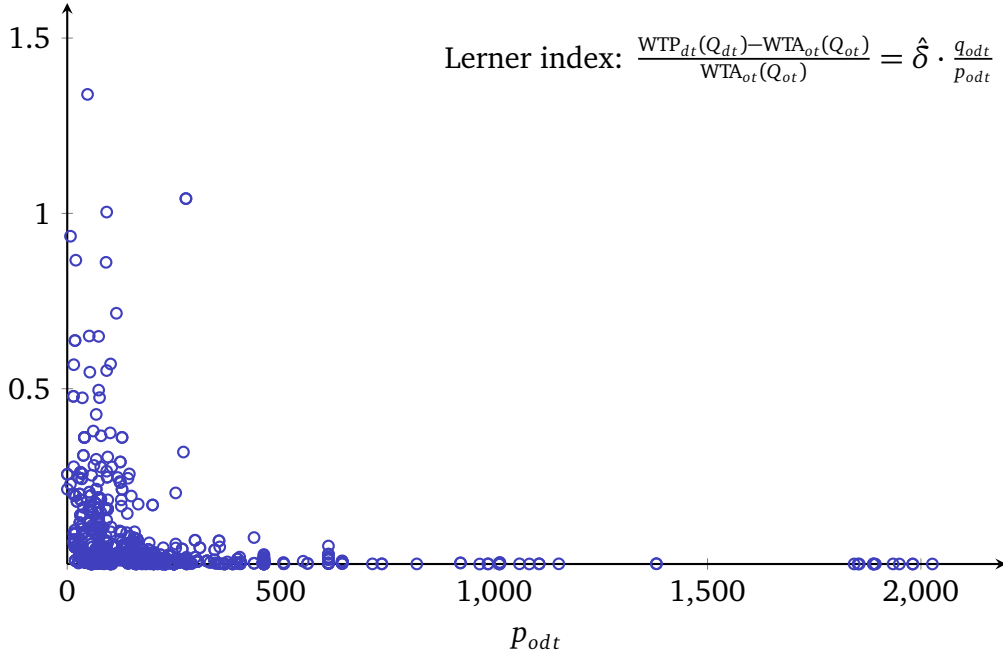


Figure 2: Scatter-plot of transaction prices and the Lerner index as given by (10) (one outlier removed).

The new terms capture districts' expectations about other districts' reactions to a change in transaction quantities. These expectations are convex combinations of expected reactions under perfect competition vs. settings with buyer or seller power. As a result, the maximum possible markups or markdowns are dampened by, respectively, θ or ξ . In our analysis so far we have assumed $(\theta, \xi) = (1, 0)$, i.e. only buyer power. Two other special cases of the model are seller power – which would imply $(\theta, \xi) = (0, 1)$ – and perfect competition, which would imply $(\theta, \xi) = (0, 0)$.

We proceed to estimate this system of equations. The resulting values of θ and ξ will verify whether our assumption of buyer power is warranted using this conjectural variations approach. Taking similar steps as before, we first substitute the linear model specification:

$$p_{odrt} = \alpha - \delta Q_{ot} + \gamma q_{odtk} - (1 - \xi)\gamma q_{odt} + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrt}, \quad (15a)$$

$$p_{odrt} = a - \delta q_{odt} - \gamma Q_{dt} + (1 - \theta)\delta q_{odtk} + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrt}. \quad (15b)$$

The combined regression equation becomes:

$$p_{odrtk} = -\delta R_{odtk}^o - \gamma \tilde{R}_{odtk}^d + (1 - \theta)\delta \hat{R}_{odtk}^o - (1 - \xi)\gamma \hat{R}_{odtk}^d + \phi_o + \psi_d + \beta_t + \tau_r C_{odr} + \epsilon_{odrtk} \quad (16)$$

with R_{odtk}^o as defined in Section 4, while \tilde{R}_{odtk}^d , \hat{R}_{odtk}^o and \hat{R}_{odtk}^d are defined as follows:

$$\tilde{R}_{odtk}^d = \begin{cases} -q_{odtk} & \text{if } r = 0 \\ Q_{dt} & \text{if } r = 1, \end{cases}, \quad \hat{R}_{odtk}^o = \begin{cases} 0 & \text{if } r = 0 \\ q_{odtk} & \text{if } r = 1, \end{cases}, \quad \hat{R}_{odtk}^d = \begin{cases} q_{odtk} & \text{if } r = 0 \\ 0 & \text{if } r = 1. \end{cases}$$

In order to get a clear view on the parameters of interest, we apply extremum estimation of the IV criterion function, transformed such that we optimize our parameters δ , γ , θ and ξ . Table 3 reports the results for the case where no restrictions were imposed on the parameters in the IV criterion function. We present three models. In model (1) we allow only buyer power, in model (2) only seller power, while model (3) allows for both. In addition to the coefficients on seller and Buyer water use, $-\delta$ and $-\gamma$, we report coefficients on both market power weights, θ and ξ , while suppressing the coefficients on the terms \hat{R}_{odtk}^o and \hat{R}_{odtk}^d , since these coefficients are combinations of the four parameters that are already reported.

Table 3: Estimating WTA and WTP: Conjectural variations

Price (2010\$/AF)	(1)	(2)	(3)
	IV Buyer power ($\xi = 0$)	IV Seller power ($\theta = 0$)	IV Both
Seller total water use (1,000 AF) (coefficient $-\delta$)	-0.577*** (0.162)	-0.610*** (0.180)	-0.573*** (0.162)
Buyer total water use (1,000 AF) (coefficient $-\gamma$)	-0.242*** (0.070)	-0.282*** (0.082)	-0.241*** (0.070)
Seller power weight (coefficient ξ)		-8.581** (3.570)	1.251 (2.422)
Buyer power weight (coefficient θ)	5.664*** (0.658)		6.155*** (1.237)
Seller fixed effects	✓	✓	✓
Buyer fixed effects	✓	✓	✓
Year fixed effects			
Quadratic time trend	✓	✓	✓
Transaction costs			
# Observations	879	879	879
# FE dummies	164	164	164

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using extremum estimation. The covariance matrix is computed as a robust sandwich covariance matrix, following the theory of extremum estimation (Cameron and Trivedi, 2005, Section 6.3.4). Standard errors in parentheses. Models (1)–(3) correspond to model (3) of Table 2, but using the conjectural variations approach.

The unrestricted estimates for seller and buyer power weight, ξ and θ , are found to lie outside the bounds $[0, 1]$. All three models find estimates for $-\delta$ and $-\gamma$ that are very

close to those obtained in our preferred model (3) of Table 2. Looking at the estimated coefficients for ξ and θ , we find that the seller power weight ξ is negative in model (2) while it is not significantly different from zero in model (3). The buyer power weight θ has the correct sign, though the present estimates seems to be larger than 1, both in model (1) and (3). These results point to absence of seller power while they support buyer power.

Imposing the restrictions that $\xi \in [0, 1]$ and $\theta \in [0, 1]$ leads to the expected result, estimates at the boundaries of these intervals. Given that the amount of observations in the dataset is relatively low compared to the number of parameters and fixed effects, and hence both estimates and standard deviations cannot be extracted with too great precision, we take these results as an indication that buyer power is the most reasonable assumption.

7 Robustness

In this section, we report on five robustness checks. First, we check robustness when we focus on relevant sub-samples of the data. Second, we apply an alternative model specification featuring non-linear benefit functions. Third, we alter the calculation of districts' water use to account for timing of transactions within one year. Fourth, we check whether selling and buying districts can be reasonably assumed to have similar benefit functions. Finally, despite the results of our conjectural variations approach, we estimate a model with seller power.

Note that this list of robustness checks is not exhaustive. Importantly, we also checked for differential levels of market power. One such example would be differential market power occurs in wet vs. dry years. In wet years, one could imagine that buyers have better opportunities to exercise market power. Using the Sacramento Valley Water Year Hydrological Classification Index to classify years, we fail to find such differences. Another option is differential market power depending on the location of buyers and sellers. The argument would be that buyers that are more central would have more opportunities to switch to another seller and could therefore achieve higher markdowns. This argument ignores, however, that the Californian water market features an almost complete hydrological network enabling water transfers between nearly any two districts. As a result, while central buyers would probably face lower transaction costs, they do not have increased opportunities to exercise market power compared to buyers at the periphery.

7.1 Sub-sample analysis

We repeat our preferred model (3) of Table 2 for three sub-samples of interest. Table 4 shows the results of these additional regressions. For reference, we include the preferred model as model (1). In model (2) we drop all observations that involve water for environmental use, for instance buy-backs by the government. Arguably, such transactions are markedly different from transactions between districts that intend to use the water for consumptive purposes. In model (3) we include only transactions where agricultural districts are selling, which seem to represent the smaller, weaker actors in the market. Unfortunately, our sample size does not allow us to focus only on water sales from agricultural to urban districts (slightly more than 100 transactions), which seem to represent the larger, stronger actors capable of exercising market power (cf. Isaaks and Colby, 2020). By focusing on all sales from agricultural districts, we may still capture the fact that agricultural districts may have less market power than the other types of districts. Note that half of these sales are to other agricultural districts, while the other half is shared roughly equally between buying urban districts and environmental projects. In model (4) we drop outlier transactions. We exclude the 5% transactions with lowest and 5% transactions with highest transaction prices and similarly for transaction volumes.

Table 4: Estimating WTA and WTP: Sub-samples

Price (2010\$/AF)	(1)	(2)	(3)	(4)
	IV preferred	IV no env	IV ag sellers	IV no outliers
Seller water use (1,000 AF) (coefficient $-\delta$)	-0.580*** (0.208)	-0.520** (0.201)	-0.570*** (0.179)	-0.442*** (0.142)
Buyer water use (1,000 AF) (coefficient $-\gamma$)	-0.309** (0.132)	-0.278** (0.127)	-0.284*** (0.108)	-0.232** (0.0906)
Seller fixed effects	✓	✓	✓	✓
Buyer fixed effects	✓	✓	✓	✓
Quadratic time trend	✓	✓	✓	✓
# Observations	879	728	778	737
# Clusters	308	248	261	250
# FE dummies	164	149	133	132
Cragg-Donald F-statistic	9.936	7.057	14.01	10.82

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using IV. Standard errors in parentheses, clustered by seller and buyer. Model (1) corresponds to our preferred model (3) of Table 2. In model (2) we drop transactions from or to environmental use. In model (3) we keep only transactions where agricultural districts are selling. In model (4) we drop transactions that are outliers in terms of price or volume.

Coefficients of sub-sample Models (2)–(4) are not statistically different from those of the preferred model. Model (2), which discards 17% of the observations, performs similarly

in terms of precision and slightly worse in terms of the F-statistic. Unexpectedly, Model (3) does not show a higher coefficient (in absolute terms). Hence, there is no indication of more buyer power when buying from an agricultural district. Model (4) suggests that some of the market power we find is driven by outlier transactions in terms of price or volume, as one could expect. Combined, these additional regressions show that our main results are robust to including only specific sub-samples of interest.

7.2 Constant price elasticity

In the main specification of our model we have imposed a constant slope of the benefit functions and a variable price elasticity. In this section, we impose instead that these functions have a constant price elasticity and, consequently, a variable slope. In particular, we consider the class of non-linear WTA functions that are homogeneous⁶. Given our earlier assumption of differentiability we have that Euler's Homogeneous Function Theorem⁷ applies to the equilibrium conditions and Lerner index.

For arbitrary homogeneous $WTA_{ot}(Q_{ot})$ of order $-\kappa_o$, we can rewrite the wedge $WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot})$ as

$$-q_{odt} \cdot WTA'_{ot}(Q_{ot}) = -\frac{q_{odt}}{Q_{ot}} \cdot [Q_{ot} \cdot WTA'_{ot}(Q_{ot})] = \frac{q_{odt}}{Q_{ot}} \cdot [\kappa_o \cdot WTA_{ot}(Q_{ot})]. \quad (17)$$

This implies that the Lerner index of (8) can be updated to

$$\frac{WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot})}{WTA_{ot}(Q_{ot})} = \frac{q_{odt}}{Q_{ot}} \cdot \kappa_o. \quad (18)$$

The empirical strategy to estimate κ_o has many similarities to the empirical strategy proposed in Section 4, and we refer to Appendix C for details. The resulting regression equation becomes:

$$\ln p_{odrtk} = -\kappa_o \bar{R}_{odtk}^o - \kappa_d \bar{R}_{odrtk}^d + \phi_o + \psi_d + \beta_t + \ln \tau_r C_{odr} + \epsilon_{odrtk}, \quad (19)$$

where \bar{R}_{odtk}^o and \bar{R}_{odtk}^d are modifications of R_{odtk}^o , respectively, R_{odtk}^d that are defined in Appendix C.

We estimate variations of (19) using linear regression, similar to Table 2 for our main model specification. Table 5 shows the estimates of four models that are similar to models

⁶The function $f : R \rightarrow R$ is homogeneous of order $\kappa \in R$ if $f(\mu x) = \mu^\kappa f(x)$ for all x and $\mu > 0$.

⁷Let the function $f : R \rightarrow R$ be homogeneous of order $\kappa \in R$. Euler's Homogeneous Function Theorem states that $x \cdot f'(x) = \kappa f(x)$.

(1)–(4) of Table 2. Despite allowing for non-linear benefit functions, the models of Table 5

Table 5: Estimating WTA and WTP: Constant price elasticity

Log price (2010\$/AF)	(1)	(2)	(3)	(4)
	OLS	OLS	IV	IV
Log (seller water use, 1,000 AF) (coefficient $-\kappa_o$)	−0.0000147 (0.000224)	−0.000660 (0.000670)	−0.370** (0.173)	−0.623* (0.357)
Log (buyer water use, 1,000 AF) (coefficient $-\kappa_d$)	−0.000576 (0.000568)	−0.00185* (0.00106)	−0.402** (0.194)	−0.458 (0.279)
Seller fixed effects	✓	✓	✓	✓
Buyer fixed effects	✓	✓	✓	✓
Year fixed effects	✓			
Quadratic time trend		✓	✓	✓
Transaction costs				✓
# Observations	942	942	827	825
# Clusters	465	292	274	273
# FE dummies	188	166	148	147
Cragg-Donald F-statistic			7.808	5.954

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using OLS and IV. Standard errors in parentheses, clustered by seller and buyer, and year (but only by seller and buyer in models (2)–(4) where year fixed effects are replaced by a quadratic time trend). Models (1)–(4) correspond to models (1)–(4) of Table 2, but now with a non-linear model specification.

do not perform better than those of our linear model specification in Table 2 in terms of the Cragg-Donald F-statistic nor the precision of our coefficient of interest, the coefficient on *Seller water use*.

Again, we use model (3) to derive the main coefficient of interest for this model specification, $\kappa_o = 0.370$. Similar as before we use this coefficient to calculate the wedge $WTP_{dt}(Q_{dt}) - WTA_{ot}(Q_{ot}) = \frac{q_{odt}}{Q_{ot}} \cdot [\kappa_o \cdot WTA_{ot}(Q_{ot})]$ which now also depends on the ratio $\frac{q_{odt}}{Q_{ot}}$. We find that the mean value of this ratio is heavily skewed by 15 districts that sell the majority of their entitlements at least once. After removing these outlier observations we have $\frac{q_{odt}}{Q_{ot}} = 0.10$ and the corresponding average wedge equals \$ 6.97/AF (SD=14.05), which is about 50% larger than the wedge found for the linear model specification, but still small in percentage terms.

Note that we do not attach much weight to the results from this specification, both because of its sensitivity to removing outliers and also since the functional form of regression equation (19) depends on the specific implementation of a first-order Taylor expansion (see Appendix C for details), which may not be warranted. With these caveats in mind, the results of a model specification with constant price elasticity are largely consistent with those from the linear model specification.

7.3 Transaction timing

So far we have ignored information on the timing of transactions. As a result, in case of multiple transactions per district per year, each district’s water use—as captured by variables R_{odtk}^i , $i = o, d$, in (13)—is identical for each of these transactions within one year. This approach is consistent with the assumption of no uncertainty with respect to districts’ water entitlements, such that districts can foresee how much water they are going to sell or buy within a year. In this section, we take the alternative approach and update Q_{dt} and Q_{ot} after each transaction. This implies that we use counter k to calculate water use (just) after transaction $j = 1, 2, \dots$ as $Q_{otj} = e_{ot} - \sum_{k=1}^j q_{od(j)tk}$ and $Q_{dtj} = e_{dt} + \sum_{k=1}^j q_{o(j)dtk}$, where $d(j)$ is the j^{th} counterparty of o and $o(j)$ is the j^{th} counterparty of d . When multiple transactions happen to occur within the same month, we order them by transaction volume such that smaller transactions go first. In an alternative specification, we reverse this order.

Table 6 shows the results. For reference, we include the preferred model from Table 2

Table 6: Estimating WTA and WTP: Dynamic updating

Price (2010\$/AF)	(1)	(2)	(3)
	IV preferred	IV dynamic	IV dynamic reversed
Seller water use (1,000 AF) (coefficient $-\delta$)	-0.580*** (0.208)	-0.530*** (0.186)	-0.551*** (0.195)
Buyer water use (1,000 AF) (coefficient $-\gamma$)	-0.309** (0.132)	-0.306** (0.127)	-0.305** (0.128)
Seller fixed effects	✓	✓	✓
Buyer fixed effects	✓	✓	✓
Quadratic time trend	✓	✓	✓
# Observations	879	879	879
# Clusters	308	308	308
# FE dummies	164	164	164
Cragg-Donald F-statistic	9.936	11.15	10.58

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using IV. Standard errors in parentheses, clustered by seller and buyer. Model (1) corresponds to our preferred model (3) of Table 2. In models (2) and (3), water use is updated dynamically in case of multiple transactions per district per year. In model (2), multiple transactions in one month are ordered from small to large volume, in model (3) this is reversed.

as model (1). In models (2) and (3), we repeat this model using our dynamically updating measure of water use. The results shows that the effect of transaction timing on prices is negligible.

7.4 Sellers and buyers on one demand curve

So far we have estimated buyers' and sellers' demand curves separately rather than estimating a combined curve. We reject this possibility with multiple arguments. First, we test for equivalence of coefficients using our preferred model (3) of Table 2. Based on a Wald test ($F(1,356)=4.92$, $p = 0.027$), we reject equality of these coefficients. Second, we use theory and data to argue that selling and buying water districts differ in key characteristics, implying that buying districts cannot be on the same demand curve as selling districts, and hence our approach of modeling two distinct curves is correct.

Table 7 compares selling and buying districts in terms of their main type of water use (urban, agriculture, environment), levels of water entitlements and water use, as well as whether or not a district trades with more than one counterparty in any given year. Clearly,

Table 7: Key differences between sellers/buyers.

	Sellers		Buyers	
	Mean	SD	Mean	SD
District: urban (share)	0.08	0.28	0.29	0.45
District: agriculture (share)	0.91	0.29	0.47	0.50
District: environment (share)	0.01	0.09	0.24	0.43
Water entitlements (1,000 AF)	193.72	298.63	207.97	570.51
Total water use (1,000 AF)	176.22	280.61	251.65	578.29
More than one counterparty (yes=1)	0.36	0.48	0.55	0.50

selling and buying districts differ in their types of water use. Sellers are more likely to use water for agriculture, while buyers are more likely to use water for urban or environmental uses. The key variable that underlines our argument that sellers and buyers are on different demand curves for water is *Water entitlements*. Table 7 shows that buying districts have higher water entitlements than selling districts, and by purchasing water they end up with even higher levels of water use compared with selling districts. If selling and buying districts would have identical demand curves for water, then districts with higher water use would be selling water, rather than buying. In Figure 1 this implies that e_d would be located to the right of the competitive Q_d . This location implies that $WTP_d(Q_d) < WTA_o(Q_o)$, which is inconsistent with the occurrence of observed water transactions. It follows that sellers and buyers are not on one demand curve.

A final difference between selling and buying districts is related to the dummy variable that measures whether a district has *More than one counterparty*. Comparison indicates that buyers have 53% more transactions with multiple counter-parties than sellers do. This statistic points to buyer power, with sellers being on the long side of the market.

7.5 Seller power

Our main result is that buyer power is relatively low. Going against previous literature, stakeholder beliefs, and the results of our conjectural variations approach of Section 6, we now reverse our model to estimate seller power. This allows us to check if, rather counter-intuitively, a model with seller power would better explain our data than our model with buyer power. We start by adapting (7), as follows:

$$p_{od} = \text{WTA}_o(Q_o) - q_{od} \cdot \text{WTP}'(Q_d), \quad (20a)$$

$$p_{od} = \text{WTP}_d(Q_d). \quad (20b)$$

Taking similar steps as in Section 4, the resulting regression equation becomes:

$$p_{odrtk} = -\delta \tilde{R}_{odtk}^o - \gamma \tilde{R}_{odtk}^d + \tau_r C_{odr} + \phi_o + \psi_d + \beta_t + \varepsilon_{odrtk}, \quad (21)$$

where \tilde{R}_{odtk}^o and \tilde{R}_{odtk}^d are modifications of R_{odtk}^o , respectively, R_{odtk}^d that are defined as follows:

$$\tilde{R}_{odtk}^o = \begin{cases} Q_{ot} & \text{if } r = 0 \\ 0 & \text{if } r = 1, \end{cases} \quad \text{and} \quad \tilde{R}_{odtk}^d = \begin{cases} -q_{odtk} & \text{if } r = 0 \\ Q_{dt} & \text{if } r = 1. \end{cases}$$

Result of this regression are displayed in Table 8. The resulting coefficients are very similar to those of models (1)–(4) of our main specification with buyer power in Table 2. Importantly, with seller power our measure of market power is now based on the coefficient on *Buyer water use*, i.e. γ rather than δ . Restricting the comparison to our preferred model (3), we find that model (3) of Table 8 does not perform better than model (3) of Table 2 when comparing either the Cragg-Donald F-statistic or the precision of our coefficient of interest. In case one would still assume seller power, we obtain from model (3) of Table 8, that $\gamma = 0.329$, which is lower than $\delta = 0.580$ from model (3) of Table 2. This difference would imply Lerner indices to be about 50% lower under seller power than under buyer power.

8 Conclusion

Using a Nash-Cournot model, we derive a closed-form solution for the extent of market power in a typical water market setting and we construct related Lerner indices. Applying our model to surface water transactions in California over the period 1993-2015, we find

Table 8: Estimating WTA and WTP: Seller power

Price (2010\$/AF)	(1)	(2)	(3)	(4)
	OLS	OLS	IV	IV
Seller water use (1,000 AF) (coefficient $-\delta$)	-0.0162** (0.00786)	-0.0251** (0.0126)	-0.576*** (0.212)	-0.867** (0.435)
Buyer water use (1,000 AF) (coefficient $-\gamma$)	-0.00764** (0.00341)	-0.0145** (0.00620)	-0.329** (0.147)	-0.324** (0.159)
Seller fixed effects	✓	✓	✓	✓
Buyer fixed effects	✓	✓	✓	✓
Year fixed effects	✓			
Quadratic time trend		✓	✓	✓
Transaction costs				✓
# Observations	1034	1034	879	877
# Clusters	543	337	308	307
# FE dummies	212	190	164	163
Cragg-Donald F-statistic			8.507	8.231

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Coefficient estimates from fixed effects models using OLS and IV. Standard errors in parentheses, clustered by seller, buyer, and year (but only by seller and buyer in models (2)–(4) where year fixed effects are replaced by a quadratic time trend). Models (1)–(4) correspond to models (1)–(4) of Table 2, but now with seller power.

only limited market power in California’s water market, despite the thinness of this market. Our main specification implies that buyer power yields an average mark-down of 6% of the transaction price. This result is important in the context of current discussions on Californian water market reform (cf. Maples et al., 2018) which, perhaps, should focus on other distorting factors, most notably transaction costs (Carey et al., 2002; Regnacq et al., 2016; Hagerty, 2019; Leonard et al., 2019).

Our model has three main assets: (1) it features a closed-form solution, (2) it does not rely on conjectural variations, and (3) it is sufficiently flexible that it can be applied to other types of endowment economies, including permit markets. On the downside, our model requires choosing a specific functional form for WTP and WTA that may not be warranted. In addition, while our current application is quite clear in terms of the the side of the market where market power resides, this may not be the case in other applications.

One explanation for the limited extent of market power in California is that transaction quantities are, generally, small. These quantities enter our Lerner index linearly such that small quantities imply low mark-downs. By the same line of reasoning, high prices also imply low mark-downs. This effect was illustrated clearly in Figure 2. Another explanation for the limited extent of market power is that, although California’s water market is ‘thin’ in trades, it is ‘thick’ in possibilities to trade. Recall from Section 7 that California features an almost complete hydrological network such that nearly any two districts can trade water.

The fact that many do not does not imply that such trades are not feasible. Rather, it implies that such districts have high pair-specific transaction costs, which causes a relatively low WTP or a relatively high WTA. The threat of a counterparty switching to a competing district limits the possibility to exercise market power (Funaki et al., 2020). The extent to which such threats affect equilibrium outcomes is an avenue for future research.

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Appendices

A Solution for the symmetric model

In this appendix we provide a solution to this main specification of our model in terms of quantities and prices, assuming $\alpha_{ot} = \alpha$ for all o and $a_{dt} = a$ for all d to keep the analysis simple. We maintain condition (9) which, suppressing time subscripts, can now be written as $a - \gamma e_d > \alpha - \delta e_o$.

For each individual buyer, we can write the maximand of equation (4), i.e. the buyer's profit function, as

$$\begin{aligned} \pi_d &= f_d(Q_d) - \sum_{o=1}^{N_o} q_{od} \cdot [\text{WTA}(Q_o)] \\ &= \left(e_d + \sum_{o=1}^{N_o} q_{od} \right) \left(a - \frac{1}{2} \gamma \left(e_d + \sum_{o=1}^{N_o} q_{od} \right) \right) - \sum_{o=1}^{N_o} q_{od} \cdot \left[\alpha - \delta \left(e_o - \sum_{d=1}^{N_d} q_{od} \right) \right]. \end{aligned} \quad (\text{A.1})$$

Applying (5), we take the derivative of the buyer's profit function (A.1) with respect to q_{od} and, by symmetry, simplify the resulting condition by writing $q_{od} = q$:

$$a - \gamma(e_d + N_o q) - \alpha + \delta(e_o - N_d q) - \delta q = 0, \quad (\text{A.2})$$

This condition implies $a - \gamma e_d - \alpha + \delta e_o = [N_o \gamma + (N_d + 1) \delta] q > 0$. Thus, the equilibrium quantity from seller to buyer q_{od} equals

$$q^* = q_{od}^* = \frac{a - \gamma e_d - \alpha + \delta e_o}{N_o \gamma + (N_d + 1) \delta}, \quad (\text{A.3})$$

which is positive for all o and d by (9).

In case the number of available buyers N_d and/or the number of available sellers N_o increases, then each buyer would buy less water from each individual seller. The quantity in equilibrium can be expressed differently by substituting $S = \alpha - \delta e_o$ and $B = a - \gamma e_d$. Thus, $q^* = (B - S) / (N_o \gamma + (N_d + 1) \delta)$. The numerator of this expression consists of the marginal benefits of water use at the initial entitlements. If the buyers' marginal benefit B increases, trade will increase. In contrast, if the sellers' marginal benefit S increases, trade will decrease. The effects on trade of parameters a , α , γ , δ and initial entitlements e_d and e_o follow immediately through their effects on either B or S . For example, an increase in the initial entitlement e_d of individual buyers implies that individual buyers buy less. Similarly,

an increase of the initial entitlement e_o of individual sellers implies that individual sellers sell more.

We use equilibrium quantities as derived in (A.3) to derive the sellers' and buyers' equilibrium (marginal) benefits as well as prices. Using (2), we have that $WTP(Q_d) = a - \gamma Q_d$ and $WTA(Q_o) = \alpha - \delta Q_o$. By symmetry, we can therefore write the marginal benefit for, respectively, each buyer and each seller in equilibrium:

$$WTP(Q_d^*) = a - \gamma(e_d + N_o q_{od}^*) = \frac{(N_d + 1)\delta B + N_o \gamma S}{N_o \gamma + (N_d + 1)\delta}, \quad (\text{A.4a})$$

$$WTA(Q_o^*) = \alpha - \delta(e_o - N_d q_{od}^*) = \frac{(N_o \gamma + \delta)S + N_d \delta B}{N_o \gamma + (N_d + 1)\delta}. \quad (\text{A.4b})$$

From the WTP function we directly obtain $Q_d = (a - WTP(Q_d))/\gamma$. The other component of benefit function f_d is $(a - \frac{1}{2}\gamma Q_d)$ and it can also be expressed in terms of this WTP: $(a - \frac{1}{2}\gamma Q_d) = \frac{1}{2}(a + a - \gamma Q_d) = \frac{1}{2}(a + WTP(Q_d))$. Combining these expressions yields the buyers' benefit function

$$\begin{aligned} f_d(Q_d^*) &= Q_d^* \left(a - \frac{1}{2}\gamma Q_d^* \right) = \frac{1}{2\gamma} \left[a^2 - (WTP(Q_d^*))^2 \right] \\ &= \frac{1}{2\gamma} \left[a^2 - \left(\frac{(N_d + 1)\delta B + N_o \gamma S}{N_o \gamma + (N_d + 1)\delta} \right)^2 \right]. \end{aligned} \quad (\text{A.5a})$$

Similar steps are applied to obtain the sellers' benefit function

$$\begin{aligned} f_o(Q_o^*) &= Q_o^* \left(\alpha - \frac{1}{2}\delta Q_o^* \right) = \frac{1}{2\delta} \left[\alpha^2 - (WTA(Q_o^*))^2 \right] \\ &= \frac{1}{2\delta} \left[\alpha^2 - \left(\frac{(N_o \gamma + \delta)S + N_d \delta B}{N_o \gamma + (N_d + 1)\delta} \right)^2 \right]. \end{aligned} \quad (\text{A.5b})$$

Given buyer power, equilibrium price equals the marginal willingness to accept. Using (A.4b), we have

$$p^* = WTA(Q_o^*) = \frac{(N_o \gamma + \delta)S + N_d \delta B}{N_o \gamma + (N_d + 1)\delta}. \quad (\text{A.6})$$

This completes the derivation of the symmetric version of the main specification of our model.

For completeness, we also verify that equation (6) holds. This equation states that the difference between WTP and WTA equals $-q_{od} WTA'_o(Q_o) = \delta q_{od}^* > 0$. Substitution of our

equilibrium expressions (A.4b) and (A.4a) gives

$$\begin{aligned}
\text{WTP}(Q_d^*) - \text{WTA}(Q_o^*) &= \frac{(N_d + 1)\delta B + N_o\gamma S}{N_o\gamma + (N_d + 1)\delta} - \frac{(N_o\gamma + \delta)S + N_d\delta B}{N_o\gamma + (N_d + 1)\delta} \\
&= \delta \frac{B - S}{N_o\gamma + (N_d + 1)\delta} \\
&= \delta q_{od}^*.
\end{aligned} \tag{A.7}$$

Therefore, equation (6) holds, as it should.

B Main specification with asymmetry

In this appendix, we generalize the linear model specification introduced in Section 2.2 to allow for asymmetry in terms of benefit parameters γ and δ .

Consider a setting where all buyers are asymmetric, all sellers are asymmetric and buyers are on the short side of the market. We update our benefit functions to allow for asymmetry in terms of benefit parameters γ and δ , while suppressing time subscripts to keep notation simple. For each destination we now have $f_d(Q_d) = Q_d(a_d - \frac{1}{2}\gamma_d Q_d)$ and for each origin we now have $f_o(Q_o) = Q_o(\alpha_o - \frac{1}{2}\delta_o Q_o)$. Therefore $f'_d(Q_d) = a_d - \gamma_d Q_d$, which is the WTP in (1), while $f'_o(Q_o) = \alpha_o - \delta_o Q_o$, which is the WTA in (2). We number sellers as $o = 1, 2, 3, \dots$ and buyers as $d = -1, -2, -3, \dots$. Subscript $od = 2 - 1$ implies that seller 2 delivers to buyer 1.

Three simplified settings are representative for almost all transactions that occur in the data, as introduced in Section 3: (a) 1 seller–1 buyer, (b) 1 seller– y buyers with $y \geq 2$, and (c) x sellers–1 buyer with $x \geq 2$. Transaction networks with x sellers and y buyers pertain to only 3% of transactions in our database, illustrating again that California’s water market is thin.

Consider setting (a) of a single seller and a single buyer who only trade with each other and non-traders in the background as potential alternative trading partners. The simplest situation consists of one non-trader on each side of the market. If we number seller 1 and buyer -1 as the trading parties with $q_{1-1} > 0$, then seller 2 and buyer -2 do not trade, i.e. $q_{1-2} = q_{2-1} = q_{2-2} = 0$. The equilibrium conditions are derived from buyer -1 who maximizes over quantities q_{1-1} and q_{2-1} and from buyer -2 who maximizes over quantities q_{1-2} and q_{2-2} . After adding subscripts d and o , we take the derivative of the buyer’s profit function (A.1) with respect to q_{od} and obtain:

$$a_d - \gamma_d(e_d + q_{1d} + q_{2d}) - \alpha_o + \delta_o(e_o - q_{o-1} - q_{o-2}) + \delta_o q_{od} \leq 0. \tag{B.1}$$

For $o = 1, 2$ and $d = -1, -2$, we have $q_{1-1} > 0$ and $q_{1-2} = q_{2-1} = q_{2-2} = 0$, so we obtain four equilibrium conditions:

$$a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}) - \alpha_1 + \delta_1(e_1 - q_{1-1}) + \delta_1 \cdot q_{1-1} = 0, \quad (\text{B.2a})$$

$$a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}) - \alpha_2 + \delta_2 e_2 \leq 0, \quad (\text{B.2b})$$

$$a_{-2} - \gamma_{-2} e_{-2} - \alpha_1 + \delta_1(e_1 - q_{1-1}) \leq 0, \quad (\text{B.2c})$$

$$a_{-2} - \gamma_{-2} e_{-2} - \alpha_2 + \delta_2 e_2 \leq 0. \quad (\text{B.2d})$$

Before solving, we combine and rewrite these four equilibrium conditions in terms of equilibrium WTP or WTA. In doing so, note that because $q_{1-1} \geq 0$, we can rewrite the first condition as a weak inequality: $a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}) - \alpha_1 + \delta_1(e_1 - q_{1-1}) = -\delta_1 q_{1-1} \leq 0$. We obtain

$$\begin{aligned} \alpha_1 - \delta_1(e_1 - q_{1-1}) &\geq \max\{a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}), a_{-2} - \gamma_{-2} e_{-2}\}, \\ \alpha_2 - \delta_2 e_2 &\geq \max\{a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}), a_{-2} - \gamma_{-2} e_{-2}\}, \\ a_{-1} - \gamma_{-1}(e_{-1} + q_{1-1}) &\leq \min\{\alpha_1 - \delta_1(e_1 - q_{1-1}), \alpha_2 - \delta_2 e_2\}, \\ a_{-2} - \gamma_{-2} e_{-2} &\leq \min\{\alpha_1 - \delta_1(e_1 - q_{1-1}), \alpha_2 - \delta_2 e_2\}. \end{aligned}$$

The first two conditions indicate that, *in equilibrium*, the seller's WTA must be equal to or larger than the highest WTP for all buyers in the market, independent whether these sellers trade or not. The last two lines indicate that, *in equilibrium*, the buyers' WTP must be equal to or lower than the highest WTA from sellers in the market, independent whether these buyers trade or not. These insights generalize to any market with N_o sellers and N_d buyers independent whether these trade or not. These conditions imply that none of the buyer-seller pairs has incentives to expand equilibrium trade.

We now check each of the equilibrium conditions in (B.2). Solving condition (B.2a) gives equilibrium trade between seller $o = 1$ and buyer $d = -1$. We obtain

$$q_{1-1}^* = \frac{a_{-1} - \gamma_{-1} e_{-1} - \alpha_1 + \delta_1 e_1}{\gamma_{-1}}. \quad (\text{B.3})$$

Under $a_{-1} - \gamma_{-1} e_{-1} > \alpha_1 - \delta_1 e_1$, which is a straightforward modification of (9), this quantity is positive. Substitution of q_{1-1}^* into condition (B.2b) yields $\alpha_1 - \delta_1 e_1 \leq \alpha_2 - \delta_2 e_2$. Evaluated at the initial entitlements, seller 1's WTA is lower than that of seller 2, making seller 1 more efficient in supplying water. Rewriting after substitution of q_{1-1}^* into condition (B.2c)

yields

$$a_{-2} - \gamma_{-2}e_{-2} \leq \left(1 - \frac{\delta_1}{\gamma_{-1}}\right)(\alpha_1 - \delta_1 e_1) + \frac{\delta_1}{\gamma_{-1}}(a_{-1} - \gamma_{-1}e_{-1}), \quad (\text{B.4})$$

For $\frac{\delta_1}{\gamma_{-1}} \in [0, 1]$, the right-hand side is the convex combination of seller 1's WTA and buyer 1's WTP, both evaluated at the initial entitlements. For the boundary case $\delta_1 = \gamma_{-1}$, the right-hand side simplifies to $a_{-1} - \gamma_{-1}e_{-1}$. Evaluated at the initial entitlements, buyer -1 's WTP is higher than that of buyer -2 , making buyer -1 more efficient in purchasing water. If the gap in WTP between the two buyers is positive, then condition (B.4) also holds for δ_1 almost equal to γ_{-1} . Finally, condition (B.2d) specifies the condition that non-trading seller $o = 2$ and non-trading buyer $d = -2$ do not want to trade with each other. If rewritten as $a_{-2} - \gamma_{-2}e_{-2} \leq \alpha_2 - \delta_2 e_2$, it is the complement of modified condition (9).

To summarize, the configuration in which seller 1 exclusively trades with buyer -1 arises naturally in case seller 1 has a substantially lower WTA than competing seller 2, while buyer -1 has a substantially larger WTP than competing buyer -2 . By the preceding discussion of the equilibrium conditions in (B.2), a sufficient condition for water trade between seller 1 and buyer -1 is the following:

$$\alpha_2 - \delta_2 e_2 > a_{-1} - \gamma_{-1}e_{-1} > \alpha_1 - \delta_1 e_1 > a_{-2} - \gamma_{-2}e_{-2}. \quad (\text{B.5})$$

Equilibrium (marginal) benefits and prices for the asymmetric case can be determined similarly to the symmetric case as was done in Appendix-A.

Cases (b) and (c) can be analyzed in a similar way when trading buyers are symmetric and trading sellers are symmetric. This involves a lot of repetition of case (a) without generating new insights. Asymmetry within the groups of trading buyers and sellers can also be included. This requires solving a linear system of equations in order to obtain the unique equilibrium quantities, which is cumbersome for the general case of x asymmetric sellers and y asymmetric buyers.

C Non-linear demand

In this appendix, we present our empirical strategy for the model of Section 7.2 featuring a non-linear WTA function that is homogenous. Our aim is to estimate κ_o so that we can measure the Lerner index for this model specification.

The strategy is largely similar to that of Section 4 for the linear model specification. We start with the following system of regression equations, based on (7), and substitute (17)

to obtain

$$p_{od} = \text{WTA}(Q_o), \quad (\text{C.1a})$$

$$p_{od} = \text{WTP}(Q_d) - \kappa_o \cdot \frac{q_{od}}{Q_o} \cdot \text{WTA}(Q_o). \quad (\text{C.1b})$$

Substituting p_{od} for $\text{WTA}_o(Q_o)$, we solve the last equation for p_{od} , which yields the non-linear system

$$p_{od} = \text{WTA}(Q_o), \quad (\text{C.2a})$$

$$p_{od} = \left(1 + \frac{q_{od}}{Q_o} \cdot \kappa_o\right)^{-1} \text{WTP}_d(Q_d). \quad (\text{C.2b})$$

This system can be written in logarithmic form as

$$\ln p_{od} = \ln \text{WTA}(Q_o), \quad (\text{C.3a})$$

$$\ln p_{od} = \ln \text{WTP}(Q_d) - \ln \left(1 + \frac{q_{od}}{Q_o} \cdot \kappa_o\right). \quad (\text{C.3b})$$

To extract parameter κ out of the last term, we approximate it by the first-order Taylor expansion of the logarithmic function around 1.⁸ This yields the following non-linear system:

$$\ln p_{od} = \ln \text{WTA}(Q_o), \quad (\text{C.4a})$$

$$\ln p_{od} = \ln \text{WTP}(Q_d) - \kappa_o \cdot \frac{q_{od}}{Q_o}. \quad (\text{C.4b})$$

We proceed to estimate (C.4) for the specification $A_i(Q_i)^{-\kappa_i}$, $i = o, d$ and $\kappa_i > 0$, that features constant price elasticity equal to $-1/\kappa_i$. Substitution, rewriting and including multiplicative transaction costs in the factor A_i , $i = o, d$, as well as seller-, buyer-, and year fixed effects, yields

$$\ln p_{odrtk} = -\kappa_o \ln Q_{ot} + \ln \tau_r C_{odr} + \phi_o + \psi_d + \beta_t + \varepsilon_{odrtk}, \quad (\text{C.5a})$$

$$\ln p_{odrtk} = -\kappa_o \frac{q_{odtk}}{Q_{ot}} - \kappa_d \ln Q_{dt} + \ln \tau_r C_{odr} + \phi_o + \psi_d + \beta_t + \varepsilon_{odrtk}. \quad (\text{C.5b})$$

Similar to the procedure used in deriving the regression equation for our linear model specification, we combine both equations. This combination requires the construction of

⁸The first-order Taylor expansion of $\ln(1+x)$ around $x_0 = 0$ is given by $\ln(1+x_0) + \frac{1}{1+x_0}(x-x_0) = x$. In our case $x = \frac{q_{od}}{Q_o} \cdot \kappa$.

two new variables that are defined by

$$\bar{R}_{odtk}^o = \begin{cases} \ln Q_{ot} & \text{if } r = 0 \\ q_{odtk}/Q_{ot} & \text{if } r = 1, \end{cases} \quad \text{and} \quad \bar{R}_{odtk}^d = \begin{cases} 0 & \text{if } r = 0 \\ \ln Q_{dt} & \text{if } r = 1. \end{cases}$$

The combined regression equation is:

$$\ln p_{odrkt} = -\kappa_o \bar{R}_{odtk}^o - \kappa_d \bar{R}_{odrkt}^d + \ln \tau_r C_{odr} + \phi_o + \psi_d + \beta_t + \varepsilon_{odtk}. \quad (\text{C.6})$$

Results of the estimation of this regression equation are presented in Table 5 and discussed in Section 7.2.