### Aggregate supply

- Variables are measured in natural logaritms.
- Short-run aggregate supply (AS) function:

$$y_t = y^* + \alpha(p_t - E_{t-1}p_t) + u_t$$
 (1)

where  $\alpha > 0$ ,  $y^*$  is potential output,  $p_t$  is the price level,  $E_{t-1}p_t$  is the price level at t that is expected at t-1 (using efficiently all the information available at t-1), and  $u_t$  is an

independent random variable  $u_t \sim N(0, \sigma_u^2)$ .

• If price level is understimated (so  $p_t > E_{t-1}p_t$ ), then too much labour is supplied and output expands above potential.

### Aggregate demand

• Short-run aggregate demand (AD) function:

$$y_t = a + \beta(m_t - p_t) + \beta' E_{t-1}(p_{t+1} - p_t) + v_t$$
 (2)  
where  $\beta, \beta' > 0$ , the real balance term  $m_t - p_t$  captures the LM (the Keynes effect), the expected inflation rate  $E_{t-1}(p_{t+1} - p_t)$  represents a Tobin effect, and  $v_t$  is an independent random variable  $v_t \sim N(0, \sigma_v^2)$ 

 A higher rate of expected inflation implies a lower real interest rate, a higher investment rate, and a higher aggregate demand.

uncorrelated with  $u_t$ :  $E(u_t, v_t) = 0$ .

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#### Policy rule

• Monetary rule followed by the government:

$$m_t = \gamma_0 + \gamma_1 m_{t-1} + \gamma_2 y_{t-1} + z_t \qquad (3)$$

where  $z_t$  is an independent random variable  $u_t \sim N(0, \sigma_u^2)$ , uncorrelated with  $u_t$  and  $v_t$ , that captures the imperfect control of the central bank over monetary aggregates.

• Monetarists would set  $\gamma_1 = \gamma_2 = 0$  (constant money supply) or, at most,  $\gamma_1 > 0$ . A Keynesian would prefer  $\gamma_1 \ge 0$  and  $\gamma_2 < 0$  (money supply raised to stimulate output).

# **Solving the model (1), (2), (3)**

• **Step 1**: equate AS & AD and solve for  $p_t$ .

$$p_{t} = \frac{a - y^{*} + \beta m_{t} + \alpha E_{t-1} p_{t} + \beta' E_{t-1} (p_{t+1} - p_{t}) + u_{t} + v_{t}}{\alpha + \beta}$$

• **Step 2**: take the expectation of  $p_t$  at t-1.

$$E_{t-1}p_t = \frac{a - y^* + \beta E_{t-1}m_t + \alpha E_{t-1}E_{t-1}p_t}{\alpha + \beta} + \frac{\beta' E_{t-1}E_{t-1}(p_{t+1} - p_t) + E_{t-1}u_t + E_{t-1}v_t}{\alpha + \beta}$$

• Shocks are independent of themselves (not autocorrelated):  $E_{t-1}u_t = E_{t-1}v_t = 0$ . Moreover,  $E_{t-1}E_{t-1}p_t = E_{t-1}p_t \& E_{t-1}cx_t = cE_{t-1}x_t$ .

 $E_{t-1}p_{t} = \frac{a - y^{*} + \beta E_{t-1}m_{t} + \alpha E_{t-1}p_{t} + \beta' E_{t-1}(p_{t+1} - p_{t})}{\alpha + \beta}$ 

In sum.

• Step 3: compute  $p_t - E_{t-1}p_t$ .  $p_t - E_{t-1}p_t = \frac{\beta}{\alpha + \beta}(m_t - E_{t-1}m_t) + \frac{1}{\alpha + \beta}(v_t - u_t)$ 

• Price surprises 
$$(p_t \neq E_{t-1}p_t)$$
 come only from

• **Step 4**: insert the policy rule. Since  $E_{t-1}m_t = \gamma_0 + \gamma_1 E_{t-1} m_{t-1} + \gamma_2 E_{t-1} y_{t-1} + E_{t-1} z_t = \gamma_0 + \gamma_1 m_{t-1} + \gamma_2 y_{t-1}$ ,

$$y_t = y^* + \frac{\beta}{\alpha + \beta} u_t + \frac{\alpha}{\alpha + \beta} v_t + \frac{\alpha \beta}{\alpha + \beta} z_t \quad (4)$$

 $p_t - E_{t-1}p_t = \frac{\beta}{\alpha + \beta}z_t + \frac{1}{\alpha + \beta}(v_t - u_t)$ 

**Step 5**: substitute into AS.

- This is the stochastic steady-state solution for output, where  $u_t$  captures the random supply shocks,  $v_t$  the random demand shocks, and  $z_t$  factors affecting the money supply that the central bank cannot control.
- As there is no policy rule parameter in (4), policy is ineffective at influencing output.

#### Counterexample to policy irrelevance

- Workers sign two-period nominal wage contracts. At *t*, half of the workforce is on the wage contract signed at *t* − 2 running from *t* − 1 to *t* and the other half on those signed at *t* − 1 valid from *t* to *t* + 1.
- $w_t^s = (\text{logaritm of the}) \text{ nominal wage at } t \text{ in}$ the contract signed at  $s \in \{t - 2, t - 1\}$
- Wage setting rule  $w_t^s = E_s p_t$
- AD function  $y_t = m_t p_t$

- Firms are identical. In 50% of them, workers are on their first year contract. In the other 50%, workers are on their second (last) year.
- $y_t = \frac{1}{2}(p_t w_t^{t-1} + u_t) +$ AS function  $\frac{1}{2}(p_t - w_t^{t-2} + u_t) = \frac{1}{2}(p_t - E_{t-1}p_t) + \frac{1}{2}(p_t - E_{t-1}p_$  $E_{t-2}p_{t}) + u_{t}$
- After equating AS & AD and solving for  $p_t$  $p_t = \frac{1}{2} \left( m_t - u_t + \frac{1}{2} (E_{t-1} p_t + E_{t-2} p_t) \right).$
- Taking expectations conditional on t-2,  $E_{t-2}p_t = \frac{1}{2} \left( E_{t-2}m_t + \frac{1}{2} (E_{t-2}p_t + E_{t-2}p_t) \right)$

because  $E_{t-2}E_{t-1}p_t = E_{t-2}p_t$ .

$$E_{t-2}p_t) + u_t$$
• After equating AS & AD and solving for  $p_t$ 

$$p_t = \frac{1}{2} \left( m_t - u_t + \frac{1}{2} (E_{t-1}p_t + E_{t-2}p_t) \right). \tag{5}$$

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- Therefore,  $E_{t-2}p_t = E_{t-2}m_t$ .
- Taking expectations conditional on t-1,

$$E_{t-1}p_t = \frac{1}{2} \left( E_{t-1}m_t + \frac{1}{2} (E_{t-1}p_t + E_{t-2}p_t) \right)$$
$$= \frac{1}{2} \left( E_{t-1}m_t + \frac{1}{2} (E_{t-1}p_t + E_{t-2}m_t) \right).$$

Solving for  $E_{t-1}p_t$  yields

$$E_{t-1}p_t = \frac{2}{3}E_{t-1}m_t + \frac{1}{3}E_{t-2}m_t.$$

Autocorrelated shock:

with  $|\rho| < 1$  and  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ 

Monetary rule:  $m_t = \mu u_{t-1}$ 

 $u_t = \rho u_{t-1} + \varepsilon_t$ 

• 
$$E_{t-1}m_t = \mu E_{t-1}u_{t-1} = \mu E_{t-1}[\rho u_{t-2} + \varepsilon_{t-1}] = \mu \rho E_{t-1}u_{t-2} = \mu \rho u_{t-2} = \mu (u_{t-1} - \varepsilon_{t-1}) = m_t - \mu \varepsilon_{t-1}.$$
  
•  $E_{t-2}m_t = \mu E_{t-2}u_{t-1} = \mu E_{t-2}[\rho u_{t-2} + \varepsilon_{t-1}] = \mu E_{t-2}[\rho u_{t-2} + \varepsilon_{t-1}] = \mu E_{t-2}[\rho u_{t-2} + \varepsilon_{t-1}]$ 

$$\mu \rho E_{t-2} u_{t-2} = \mu \rho E_{t-2} [\rho u_{t-3} + \varepsilon_{t-2}] =$$

$$\mu \rho (\rho u_{t-3}) = \mu \rho (u_{t-2} - \varepsilon_{t-2}) = \mu (u_{t-1} - \varepsilon_{t-1}) - \mu \rho \varepsilon_{t-2} = m_t - \mu \varepsilon_{t-1} - \mu \rho \varepsilon_{t-2}.$$
•  $E_{t-1} p_t + E_{t-2} p_t = \frac{2}{3} E_{t-1} m_t + \frac{4}{3} E_{t-2} m_t =$ 

$$2m_t - 2\mu\varepsilon_{t-1} - \frac{4}{3}\mu\rho\varepsilon_{t-2}.$$
• Inserting the previous result into (5),

 $p_t = \frac{1}{2} \left( m_t - u_t + \left( m_t - \mu \varepsilon_{t-1} - \frac{2}{3} \mu \rho \varepsilon_{t-2} \right) \right)$ 

or 
$$p_t = m_t - \frac{u_t}{2} - \mu \left( \frac{\varepsilon_{t-1}}{2} + \rho \frac{\varepsilon_{t-2}}{3} \right).$$

By substituting this into the AD function,

$$y_t = m_t - p_t = \frac{u_t}{2} + \mu \left( \frac{\varepsilon_{t-1}}{2} + \rho \frac{\varepsilon_{t-2}}{3} \right).$$
• This proves that output depends on the poli-

This proves that output depends on the policy rule parameter  $\mu$ . The intuition is that, while the two-period contracts are in effect, there is room for the government to react to new events that, when contracts were signed, were not foreseeable or anticipated. Hence, half of the workers have signed contracts with outdated information.

## **Designing institutions**

- Imagine that  $U_t = -\frac{1}{2}[\pi_t^2 + \alpha \cdot (y_t \bar{y})^2]$  is a utility function can be ascribed to a society, where  $\pi$  is the inflation rate and (in logs) y is real GDP, and  $\bar{y}$  the desired GDP.
- AS function:  $y_t = y^* + \beta \cdot (\pi_t \pi_t^e) + u_t$ , where  $y^*$  is potential output,  $\pi^e$  the expected inflation rate, and  $u_t$  a random variable with mean value 0 and variance  $\sigma^2$  that captures supply and demand shocks on the economy.
  - The utility function of the central bank (*CB*) is given by  $U_t^{CB} = -\frac{1}{2}[\pi_t^2 + \gamma \cdot (y_t \bar{y})^2].$

- The CB chooses  $\pi_t$  to maximize  $U_t^{CB}$ . Let the government have the power to pick  $\gamma$  (the extent to which the CB should care about the gap between output and desired output).
- Option 1:  $\gamma = 0$ . This means that the *CB* only cares about inflation. Thus,  $U_t^{CB} = -\frac{1}{2}\pi_t^2$

• Option 1: 
$$\gamma=0$$
. This means that the *CB* only cares about inflation. Thus,  $U_t^{CB}=-\frac{1}{2}\pi_t^2$  and  $EU_t^{CB}=-\frac{1}{2}E\pi_t^2=-\frac{1}{2}\pi_t^2$ . Therefore, *CB* sets  $\pi_t=0$ . This implies  $\pi_t^e=E\pi_t=0$ , so

- $EU_t^1 = -\frac{1}{2} [E\pi_t^2 + \alpha \cdot E(y_t \bar{y})^2] =$  $-\frac{1}{2} \cdot \alpha \cdot E[y^* + \beta \cdot (\pi_t - \pi_t^e) + u_t - \bar{y}]^2 =$  $-\frac{\alpha}{2} \cdot E[y^* - \bar{y} + u_t]^2 = -\frac{\alpha}{2} \cdot [(y^* - \bar{y})^2 + \sigma^2].$

• Option 2:  $\gamma = \alpha$ . That is, the preferences imposed on the *CB* are the society's. Then (assuming  $\pi_t^e$  independent of  $\pi_t$ ):

$$0 = \frac{\partial U_t^{CB}}{\partial \pi_t} = -\pi_t - \alpha \beta^2 (\pi_t - \pi_t^e) - \alpha \beta (y^* - \bar{y} + u_t)$$

• As a result,  $\pi_t = \frac{\alpha \beta^2 \pi_t^e - \alpha \beta (y^* - \bar{y} + u_t)}{1 + \alpha \beta^2}.$  (6)

• Taking expectations,
$$\pi_t^e = E\pi_t = \frac{\alpha\beta^2 E\pi_t^e - \alpha\beta E(y^* - \bar{y}) - \alpha\beta Eu_t}{1 + \alpha\beta^2}$$

$$= \frac{\alpha\beta^2 \pi_t^e - \alpha\beta (y^* - \bar{y})}{1 + \alpha\beta^2}.$$

• Solving for  $\pi_t^e$ ,  $\pi_t^e = \alpha \beta (\bar{y} - y^*)$ .

 $\pi_t = \frac{\alpha\beta^2\alpha\beta(\bar{y}-y^*) + \alpha\beta(\bar{y}-y^*) - \alpha\beta u_t}{1 + \alpha\beta^2}$ 

Accordingly, by (6),

All in all, since  $Eu_t^2 = \sigma^2$ .  $EU_t^2 = -\frac{1}{2} [E\pi_t^2 + \alpha \cdot E(y_t - \bar{y})^2] =$ 

 $y_t = y^* - \beta \cdot \frac{\alpha \beta u_t}{1 + \alpha \beta^2} + u_t = y^* + \frac{u_t}{1 + \alpha \beta^2}.$ 

 $= \alpha\beta(\bar{y} - y^*) - \frac{\alpha\beta u_t}{1 + \alpha\beta^2}.$ • Thus,  $\pi_t - \pi_t^e = \frac{\alpha \beta u_t}{1 + \alpha e^2}$ . By the AS function,

 $= -\frac{1}{2} \begin{bmatrix} E\left(\alpha\beta(\bar{y} - y^*) - \frac{\alpha\beta u_t}{1 + \alpha\beta^2}\right)^2 + \\ \alpha \cdot E\left(y^* + \frac{u_t}{1 + \alpha\beta^2} - \bar{y}\right)^2 \end{bmatrix} =$ 

higher on 
$$EU_t^2$$
 than on  $EU_t^1$ , which is due to the  $CB's$  unsuccessful attempt to stimulate GDP beyond potential.

 $= -\frac{1}{2} \left[ (\alpha^2 \beta^2 + \alpha)(\bar{y} - y^*)^2 + \frac{\alpha^2 \beta^2 + \alpha}{(1 + \alpha R^2)^2} u_t \right]$ 

 $= -\frac{\alpha}{2} \left[ (1 + \alpha \beta^2)(\bar{y} - y^*)^2 + \frac{(1 + \alpha \beta^2)}{(1 + \alpha \beta^2)^2} u_t \right]$ 

 $= -\frac{\alpha}{2} \Big[ (1 + \alpha \beta^2) (\bar{y} - y^*)^2 + \frac{1}{1 + \alpha \beta^2} u_t \Big].$ 

Since  $1 + \alpha \beta^2 > 1$ , the impact of  $(\bar{y} - y^*)^2$ [gap between desired and potential GDP] is

Since  $\frac{1}{1+\alpha R^2} < 1$ , the impact of shocks is

lower on 
$$EU_t^2$$
 than on  $EU_t^1$ , which is due to the  $CB's$  stabilization response.

# **Dynamic inconsistency**

- <u>Lucas supply curve</u>:  $y_t = y^* + \alpha(\pi_t \pi_t^e) + u_t$ , with  $u_t \sim N(0, \sigma^2)$ ,  $\alpha > 0$ , &  $\pi_t^e = E_{t-1}\pi_t$ .
- Policy maker's (*PM*) cost function:  $C_t = \frac{1}{2}(y_t \bar{y})^2 + \frac{\beta}{2}\pi_t^2$ , where  $\beta > 0$  is a measure of the inflation aversion by the *PM*.
- Information asymmetry: the PM knows  $u_t$  but people do not.
- The *PM* chooses  $y_t$  and  $\pi_t$  to minimize  $C_t$  subject to the Lucas curve. In view of this, the temporal subindex t will be omitted.

• Lagrangian of the problem: 
$$\mathcal{L} = \left[\frac{1}{2}(y_t - \bar{y})^2 + \frac{\beta}{2}\pi^2\right] + \lambda[y - y^* - \alpha(\pi - \pi^e) - u].$$

- First-order conditions (*FOC*):  $0 = \frac{\partial \mathcal{L}}{\partial y} = y \bar{y} + \lambda$  and  $0 = \frac{\partial \mathcal{L}}{\partial \pi} = \beta \pi \alpha \lambda$  (where the *PM* takes  $\pi^e$  as given).
- The *FOC* gives the pairs  $(\pi, y)$  that minimize the PM's cost:  $\pi = -\frac{\alpha}{\beta}(y \bar{y})$ .
- Combining this with the Lucas curve,  $\pi_u = \frac{\alpha^2 \pi^e + \alpha(\bar{y} y^* u)}{\alpha^2 + \beta}$

which is the  $\underline{PM's}$  choice of  $\pi$  knowing  $\underline{u}$ .

•  $\frac{\partial \pi_u}{\partial \pi^e} = \frac{\alpha^2}{\alpha^2 + \beta} > 0$ : higher inflation expectations makes inflation higher.

• 
$$\frac{\partial \pi_u}{\partial (\bar{y} - y^*)} = \frac{\alpha}{\alpha^2 + \beta} > 0$$
: the more ambitious the *PM* (the higher the difference  $\bar{y} - y^*$  between desired output  $\bar{y}$  and the long-run sustainable output  $y^*$ ), the higher the inflation rate.

- $\frac{\partial \pi_u}{\partial u} = -\frac{\alpha}{\alpha^2 + \beta} < 0$ : adverse aggregate supply shocks cause a surge in the inflation rate.
- If people knows that the *PM* chooses  $\pi_u$ , rational inflation expectations are  $\pi^e = E\pi_u = \frac{\alpha^2 E \pi^e + \alpha(\bar{y} y^* Eu)}{\alpha^2 + \beta} = \frac{\alpha^2 \pi^e + \alpha(\bar{y} y^*)}{\alpha^2 + \beta}$ .

$$\pi^e = \frac{lpha}{eta} (ar{y} - y^*) \,.$$

• Inserting this into  $\pi_u$ ,  $\left(\frac{\alpha^2 + \beta}{\alpha}\right) \pi_u = \frac{\alpha^2}{\beta} (\bar{y} - y^*) + (\bar{y} - y^* - u)$ 

and, therefore, 
$$\pi_u = \frac{\alpha}{\beta} (\bar{y} - y^*) - \left(\frac{\alpha}{\alpha^2 + \beta}\right) u.$$

• This and either Lucas curve or the optimality condition  $\pi = -\frac{\alpha}{\beta}(y - \bar{y})$  yield

condition 
$$\pi = -\frac{u}{\beta}(y - \bar{y})$$
 yield 
$$y_u = y^* - \left(\frac{\beta}{\alpha^2 + \beta}\right)u.$$

- The equation for y<sub>u</sub> implies that the *PM* partially accommodates supply shocks: without any intervention, by the Lucas curve, y = y\* u; with intervention, the impact of -u on y is not 1 but β/(α²+β) < 1.</li>
   A flat Lucas curve (α large) or a "leftist" *PM*
- generate a large degree of accomodation.
  Problem: (π<sub>u</sub>, y<sub>u</sub>) is <u>suboptimal</u>. To see this, suppose *PM* follows the zero inflation rule π<sub>r</sub> = 0. If people trust the *PM*, π<sup>e</sup> = 0. By the

( $\beta$  small, indicating slow aversion to  $\pi$ )

Lucas curve,  $y_r = y^* - u$ .

• Consider the case 
$$u=0$$
. Then  $(\pi_u, y_u)=\left(\frac{\alpha}{\beta}[\bar{y}-y^*], y^*\right)$  and  $(\pi_r, y_r)=(0, y^*)$ .

• The corresponding costs are 
$$C_u = \frac{1}{2}(y^* - \bar{y})^2 + \frac{\alpha^2}{2\beta}(\bar{y} - y^*)^2 = \frac{1}{2}\left(\frac{\alpha^2 + \beta}{\beta}\right)(y^* - \bar{y})^2$$

$$C_r = \frac{1}{2}(y^* - \bar{y})^2 + \frac{\beta}{2}0^2 = \frac{1}{2}(y^* - \bar{y})^2$$
• Since  $\frac{\alpha^2 + \beta}{\beta} > 1$ , it follows that  $C_u > C_r$ . As a result,  $(\pi_u, y_u)$  is not maximizing  $C$ .

• But the problem with the rule  $\pi_r = 0$  is that the *PM* has an incetive to break it.

In fact, if people believe that the rule  $\pi_r = 0$  is followed and adopt  $\pi^e = 0$  accordingly, then, recalling that  $\pi_u = \frac{\alpha^2 \pi^e + \alpha(\bar{y} - y^* - u)}{\alpha^2 + \beta}$  determines the optimal response to  $\pi^e$ , the *PM* has an incetive to choose  $\tilde{\pi}_u = \frac{\alpha(\bar{y} - y^* - u)}{\alpha^2 + \beta}$ .

- Output is  $\tilde{y}_u = \frac{\beta}{\alpha^2 + \beta} y^* + \frac{\alpha^2}{\alpha^2 + \beta} \bar{y} + \frac{\beta}{\alpha^2 + \beta} u$ .
- Considering again the case u = 0, the resulting cost is

$$\tilde{C}_u = \frac{1}{2} \left( \frac{\beta}{\alpha^2 + \beta} \right) (y^* - \bar{y})^2.$$

• It is then plain that  $C_u > C_r > \widetilde{C}_u > 0$ .

- In the cheating solution, the *PM* announces the rule  $\pi_r = 0$  and, if people believe the announcement, the *PM* creates an inflation surprise  $\tilde{\pi}_u > \pi_r = 0$  so that output is expanded:  $\tilde{y}_u > y_r = y^*$ . Summarizing:
  - the solution  $(\pi_u, y_u)$  based on <u>discretion</u> is credible, consistent with rational expectations, but <u>not optimal</u>;
  - the solution  $(\pi_r, y_r)$  based on the <u>zero</u> inflation rule is not credible (there is an incentive to break it), consistent with rational expectations, and optimal;
  - the <u>cheating</u> solution  $(\tilde{\pi}_u, \tilde{y}_u)$  is credible, <u>inconsistent</u> with rational expectations, but closest to the bliss point of C.

#### Reputation

- Reputation may solve dynamic inconsistency.
- To illustrate the importance of reputation effects, let the government get elected, for a two-period term (t, t + 1), between the <u>leftist party</u> l (adopts a left wing ideology) and the <u>rightist party</u> r (has a right wing ideology). l and r do not care about t + 2, t + 3, ...
- Party l's utility function is  $U_t^l = -\frac{1}{2}\pi_t^2 + \delta(y_t \bar{y}) + \beta \left[ -\frac{1}{2}\pi_{t+1}^2 + \delta(y_{t+1} \bar{y}) \right]$ , so l cares about inflation and unemployment.

- Party r's utility function is  $U_t^r = -\frac{1}{2}\pi_t^2 \beta \frac{1}{2}\pi_{t+1}^2$ , which is a reflection of the fact that r only cares about inflation.
- Since t + 1 is closest to the next election,  $\beta > 1$  in both  $U_t^l$  and  $U_t^r$ .
- The economy is represented by the Phillips curve  $y_t = y^* + \alpha(\pi_t \pi_t^e)$ , with expectations formed rationally:  $\pi_t^e = E_{t-1}\pi_t$ .
- People ignore the government's preferences. They initially attribute probability  $p_r = \frac{1}{2}$  to the event that the government is rightist.

ment would set 
$$\frac{\partial U_t^r}{\partial \pi_t} = 0 = \frac{\partial U_t^r}{\partial \pi_{t+1}}$$
, which would imply  $\pi_t^r = \pi_{t+1}^r = 0$ .

• Given this, party  $l$  knows that (i) by choosing

• To maximize its utility, a rightist govern-

$$\pi_t > 0$$
, people will know at  $t+1$  that the government is leftist and (ii) by choosing  $\pi_t = 0$ , people will still hold  $p_r = \frac{1}{2}$ .

Once inserted the Phillips curve into  $U_t^l$ , l's utility function is given by

 $+\beta \left[ -\frac{1}{2}\pi_{t+1}^2 + \delta([y^* + \alpha(\pi_{t+1} - \pi_{t+1}^e)] - \bar{y}) \right].$ 

• The condition 
$$\frac{\partial U_t^l}{\partial \pi_{t+1}} = 0$$
 yields  $-\beta \pi_{t+1} + \beta \delta \alpha = 0$ . Hence,  $l$  chooses  $\pi_{t+1}^l = \delta \alpha$  at  $t+1$ .

• To maximize  $U_t^l$  with respect to  $\pi_t$ , it cannot be that  $\pi_t < 0$  (setting  $\pi_t = 0$  is better ).

• If 
$$l$$
 chooses  $\pi_t^l=0$ , then people cannot distinguish  $l$  from  $r$ . Therefore,  $\pi_t^e=p_r\cdot \pi_t^r+(1-p_r)\cdot \pi_t^l=0$  and  $\pi_{t+1}^e=p_r\cdot \pi_{t+1}^r+(1-p_r)\cdot \pi_{t+1}^l=\delta\alpha/2$ . Consequently,

$$U_t^l(\pi_t^l = 0) = \delta(1+\beta)(y^* - \bar{y}).$$
• For  $l$  to choose  $\pi_t^l > 0$ ,  $\frac{\partial U_t^l}{\partial \pi_t} = 0$ . That is,

$$-\pi_t + \delta \alpha = 0$$
. As a result,  $\pi_t^l = \delta \alpha$ .

- In this case, people know at t+1 that the government is leftist, so  $\pi_{t+1}^e = \pi_{t+1}^l = \delta \alpha$  and  $\pi_t^e = p_r \cdot \pi_t^r + (1-p_r) \cdot \pi_t^l = \frac{1}{2}0 + \frac{1}{2}\delta \alpha = \frac{\delta \alpha}{2}$ .
- The corresponding utility for party is  $U_t^l(\pi_t^l = \delta\alpha) = \delta(1+\beta)(y^* \bar{y}) \frac{\beta}{2}(\delta\alpha)^2.$
- As  $U_t^l(\pi_t^l = 0) > U_t^l(\pi_t^l = \delta \alpha)$ , the conclusion is that it pays a leftist government to pretend at period t (the initial one) that it is rigthist.
- The leftist government builds up at t a rightist reputation exploited at t + 1 with a preelection reflation that boosts the economy.

- This situation constitutes a <u>pooling</u> equilibrium at *t*, since both parties choose the same zero-inflation policy. This makes parties indistinguishable to people at *t*.
- In the above formulation, party r did not care about being distinguishable from l (this follows from the fact that  $U_t^r$  is not directly affected by  $\pi_{t+1}^e$ ).
- If it cared (for instance, if  $U_t^r = -\frac{1}{2}\pi_t^2 + \beta \left[ -\frac{1}{2}\pi_{t+1}^2 + \tilde{\delta}(y_{t+1} \bar{y}) \right]$ , with  $\tilde{\delta} < \delta$ ), then a separating equilibrium (where l does not pretend to be r at t) would arise.