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The mathematics of profit maximization is incorrect

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Abstract
Profit maximization is one of the two main optimising principles of neoclassical economics, the other being utility maximization. In this paper we draw on Chapter 6 of John Maynard Keynes's *General Theory* to show that the mathematics of profit maximization is incorrect. We show, moreover, that marginal cost, a variable fundamental to neoclassical economics, cannot be calculated. We explore the implications for sticky prices, increasing returns, the shape of the supply curve, and market clearing. Finally, we argue that an important reason for the failure of neoclassical economics is that while it pays a great deal of attention to the influence of future expectations on present decisions, it completely ignores the past.

1. Introduction

Most critiques of profit maximization are empirical, arguing that firms in the real world seek to maximize, not profits, but revenue, market share or some other metric. Other critiques, e.g., by Kenneth Arrow (Arrow, 1979), have dwelt on the sociological and moral objections to the profit system. The mathematics itself is rarely challenged. The assumption is that firms can, if they wish to, abide by the mathematics of profit maximization. This paper shows that it is not possible to do so.

2. The Mathematics

We use the math of profit maximization as found in Paul Samuelson's *Foundations of Economic Analysis* (Samuelson, 1947). Current expositions do not vary from this in any significant manner.

> "I take as a datum the maximum amount of gross total revenue which can be secured for each level of output. This may be written
>
> $$R = R(x)$$
>
> Let us define profit, net revenue, as the difference between gross revenue and total expenditure,
>
> $$\pi = \pi(x, w_1, \dots, w_n) = R(x) - A - V(x, w_1, \dots, w_n)$$

[Here π is the profit, *x* is the output, w_1, w_n are variable inputs, *A* is the fixed cost, and *V* is a function of output and variable inputs.]

Output will be optimum when profit is at a maximum. Necessary conditions that this be so when all functions are differentiable are

$$\frac{\partial \pi}{\partial x} = \frac{\partial R}{\partial x} - \frac{\partial V}{\partial x} = 0$$

$$\frac{\partial^2 \pi}{\partial x^2} = \frac{\partial^2 R}{\partial x^2} - \frac{\partial^2 V}{\partial x^2} \leq 0$$

Assuming that we have a regular relative maximum, this becomes

$$\frac{\partial R}{\partial x} = \frac{\partial V}{\partial x}$$

$$\frac{\partial^2 R}{\partial x^2} \leq \frac{\partial^2 V}{\partial x^2}$$

This is the familiar theorem that at the optimal output the marginal revenue curve must intersect the marginal cost curve from above" (p. 76).

Samuelson begins by considering the fixed cost as a factor determining profit but in the final equation it vanishes, having been differentiated out of existence. So far as decisions about the final output or price are considered all that matters is the marginal cost; the fixed investment might as well not have been made.

3. Chapter 6 of the *General Theory*

Chapter 6 is probably the most difficult, most neglected and least understood part of Keynes's *General Theory* (Keynes, 1936). Rather than try and explain what Keynes was getting at in the chapter by a direct exegesis I resort to a simple example.

Imagine that you are setting up a hamburger stall. The fixed investment (cost of the structure, equipment etc) works out to $1,000. The cost of bread, meat, culinary ingredients, labour and other variable inputs is $1 per burger. You decide to price each burger at $2. If your belief that people will like your burgers is right you can recover the fixed investment after 1,000 burgers. Until then you make no profit. If your estimate about the market does not work out you will have to shut down the burger stall before you recover your fixed investment.

The first point to note is that you cannot take the price of burgers as "given". The price must be set by you and you have to do it in such a way that you recover all your costs. The second point is that your pricing has to take account of fixed costs. Believing otherwise is to imagine that a shoe manufacturer will invest millions of dollars in a new plant and then price her shoes considering only variable costs, because the cost of the plant is (in the language of economists) a sunk cost and can therefore be ignored. If she were to go by Samuelson's equations the shoe manufacturer would never survive.

This is the point that Keynes sought to explain in Chapter 6: a firm cannot ignore fixed costs when setting the price of its product. Not content with devoting a whole chapter to the idea he went on to elaborate it in an appendix to the chapter. Keynes could well have been anticipating Samuelson and all of neoclassical economics when he wrote: *"Indeed, the notion that the disinvestment in equipment is zero at the margin of production runs through a good deal of recent economic theory. But the whole problem is brought to an obvious head as soon as it is thought necessary to explain exactly what is meant by the supply price of an individual firm"* (p. 72).

The trouble with Samuelson's analysis is that he believes the firm takes the price of its product as set by the market and decides how much to produce. In reality the firm sets the price of its product and lives with the uncertainty of how much of its output will be bought by the market. The signals it receives from the market are not price signals but quantity signals. The output of the firm is indeed a variable but, in the final analysis, it is decided by the market, not by itself. As so often in economics, the assumptions of the problem are framed the way they are, not because they align with reality, but purely with an eye to mathematical amenability.

After that broad criticism it is time to zero down on the exact mathematical error in Samuelson's analysis. If the output is the variable being analysed then the fixed investment is not a constant but a variable. It is a decreasing function of output. With each unit of output the fixed cost is reduced by the difference between the price and the marginal variable cost. In our example, the fixed cost as seen by the second burger is not $1,000 but $999, because in selling the first burger at $2 you have recovered $1 of your fixed investment. The fixed cost after the second burger has been sold is $998, and so on. Since the fixed cost is a variable dependent on the output, differentiating it does not eliminate it. It vanishes from the equation only after it has been completely recovered.

So, the conclusion that Samuelson derives – that the optimum output occurs when the marginal revenue curve cuts the marginal cost curve from above – is not a universal one but applies only to that subset of firms which have completely recovered the cost of their investment (we assume one-product firms for convenience). Even that may not be true unless it is assumed that the firm can sell all it produces. But that assumption too is part of Samuelson's analysis, though it is not explicitly stated. Samuelson assumes that the firm is so small relative to the market that it cannot affect the market price but has to take it as given. But this also means that the firm is so small relative to the market that it has no problem selling everything it produces.

As soon as it is recognised that a firm's pricing must take account of its fixed cost, one is confronted with an insurmountable problem: it is impossible to calculate the marginal cost of output. In our example, how do we distribute the fixed cost of $1,000 among burgers? If a thousand burgers are sold at $2, we can distribute the cost among the thousand burgers. But this can be done only after the thousand burgers are sold. It may well be that the hamburger stall has to be shut down before that point because there is no market for burgers. On the other hand, the structure of the stall and the equipment may last as long as 10,000 burgers. In that case each burger would involve a fixed cost of only 10 cents. Or the equipment may last 50,000 burgers and the equipment 10,000 burgers. There is no way that the cost of a burger can be calculated at the margin.

Another point, though, can be asserted without hesitation. As the demand for burgers rises the fixed cost can be recovered faster and the burger can therefore be priced lower. If only 1,000

burgers can be sold each burger must be priced at a minimum of $2 to break even. If 2,000 burgers can be sold each burger can be sold at $1.5 and the seller could still break even. Or, to put it another way, as quantitative demand rises the price at which the stall owner is willing to sell burgers falls. This calls into question the conventional shape of the supply curve: it is not inevitably upward sloping but can be flat or even downward sloping.

From the example it is also clear that as demand rises the capitalist enjoys increasing returns. When economists talk of returns they mean technological returns. To take two commodities much loved by economists, if 2 tons of wheat and 3 tons of steel are used to produce 7 tons of steel, then producing 14 tons of steel requires 4 tons of wheat and 6 tons of steel when technological returns are assumed to be constant. When capitalists talk of returns, though, they mean monetary returns. Fixed costs do not appear in the economist's calculations. But when the capitalist produces (and is able to sell) 14 tons of steel instead of 7 tons, her fixed investment is spread over a larger output, she is able to recover it faster, and thus earn a higher profit. Her returns increase with output. Technological returns may well be constant or even diminishing while monetary returns are increasing, and for the capitalist it is the latter that matters. For the capitalist, output maximization *is* profit maximization.

If firms love to merge and increase their scale of production it is primarily for this reason. A larger share of the market means that uncertainty about demand is reduced and fixed costs can be spread over a larger output. Unit costs therefore come down, though that is evident only in the aggregate, not at the margin. Prices can be reduced to boost demand so that even if the rate of profit falls, total profit increases. A small firm may have a very high rate of profit but if its business cannot be scaled, its profits are limited. What matters is total profit, not the rate of profit.

A similar misunderstanding explains why economists are surprised that wages are sticky during a recession. Just as the capitalist is not interested in the rate of profit but only in total profits, so she is not interested in the wage rate but only in the total wage bill. In a recession, the latter can be reduced by leaving the wage rate unchanged but cutting the number of workers on the rolls.

There are other implications as well. When fixed costs are taken into account it is possible to explain why prices are sticky downwards during a recession. In the world of neoclassical economics, when a capitalist finds some of her output unsold, all she has to do is reduce prices until her unsold inventory disappears. In the real world it is not so easy. When demand (and output) falls the capitalist needs to *raise* prices; she cannot afford to cut them. The reason for downward price stickiness is not menu costs or aggregate-demand externality or anything else but the presence of fixed costs that have to be recovered.

Fixed costs also explain why reducing interest rates during a recession has very little effect on investment but merely raises the prices of financial assets. The fall in demand means that even existing investments take longer to be recouped so there is no point in making further investments; the advantage of lower interest rates is dwarfed by the disadvantage of lower demand.

4. What do real capitalists think of marginal cost?

Between April 1990 and March 1992 Alan Blinder, a prominent US economist (he was to be vice chairman of the Federal Reserve Board from 1994 to 1996), interviewed 200 large firms representing 85% of US private, nonfarm, for-profit, unregulated gross domestic product (GDP). The objective was to understand why prices are sticky, not by constructing yet another theoretical model, but by asking the players in the market themselves.

Our interest here is not in the entire study (Blinder, 1994) but in that part of it which concerns marginal cost and the shape of the marginal cost curve. The answers turned out to be so surprising that Blinder (and the interviewers) were not sure that the interviewees had understood what marginal cost really meant. Apparently, the term marginal cost was "not in the lexicons of most businesspeople" (p. 141). The question was therefore translated to "How would you characterize the behaviour of your own variable costs of producing additional units as production rises?" (ibid).

Blinder wrote:

> "This proved a difficult question. It often had to be repeated, rephrased, or explained. Even so, 10 of our 200 respondents were unable to answer it. The other 190 executives answered in their own words, sometimes at great length, and interviewers classified the responses into one of five categories offered on the questionnaire" (ibid).

And:

> "When juxtaposed against the standard neoclassical assumption that panel e [rising marginal cost] is the rule, the answers are stunning. Only 11 percent of firms reported that their MC [marginal cost] curves are rising (panel e). By contrast, 40.5 percent claimed that their MC curves are falling, presumably *globally* (panels a and b). The good news for the constant marginal-cost theory is that approximate constancy of MC (panels c and d) is the modal case — encompassing 48.4 percent of GDP. The bad news is that this group accounts for less than half of GDP and that almost as many firms say they have *falling* MC" (ibid).

Blinder found the answers difficult to stomach, going as they did against conventional wisdom.

> "My own experience as an interviewer leads me to discount these results somewhat because many executives had difficulty understanding the question. Some may have confused marginal cost with average cost (AC), and it is surely not surprising that many firms have falling AC curves. Nonetheless, the discount would have to be pretty severe before we read figure 4.1 as saying that rising MC is the norm" (ibid).

In the light of what we said previously there is nothing surprising in the responses that Blinder got. Firms do not understand what marginal cost is because the thing does not exist. On the

other hand, when firms calculate cost per additional unit of output, they take into consideration fixed costs as well. And when that is done it is obvious that costs fall or remain constant for most firms as production increases.

5. Market clearing, or its absence

Blinder's survey also tells us that market clearing is not the inevitable occurrence that neoclassical economics assumes but a relatively rare phenomenon.

The quantity of goods sold always equals the quantity of goods bought. But that is not market clearing. It is a tautology by virtue of the definition of buying and selling. The concept of market clearing is better understood if we ask ourselves what we mean by the absence of market clearing. And an appropriate definition is: *If firms in an industry are willing to sell more than they do at present if the price of their product remains constant or even falls, then we can say that the market for that industry is in a state of non-clearing.* And conversely, if firms in an industry are not willing to sell more than they do at present unless the price of their product rises then we can say that the market for that industry is in a state of clearing.

In Blinder's survey only 11 percent of firms reported that their costs were increasing with output. Or what amounts to the same thing, 89 percent of firms would have been willing to sell a greater quantity at the current price or a lower price if the demand was forthcoming. But that amounts to saying that only 11 percent of firms experienced market clearing; the rest did not.

6. The absence of a past in neoclassical economics

Many of the errors in neoclassical economics, such as the mathematics of profit maximization, can be attributed to a single cause: the absence of a past.

In formulating the theory of profit maximization, neoclassical economics assumes that the firm makes its daily decisions unencumbered by the past and purely considering the future. But, of course, this is not the case. As soon as the firm makes a fixed investment its primary concern is to recover the cost of that investment. Its past decisions weigh heavily on the pricing decisions it makes today. The fixed investment it has made affects the shape of its supply curve (flat or downward sloping) and the leeway it has in cutting prices in the event of a fall in demand (very little). In neoclassical economics a firm suffers no change in its profit if it cuts prices, but in the real world that is not the case.

A parallel assumption runs through consumer choice theory, which assumes that prices are the only variables the consumer takes into account while making decisions about what and how much to buy. Thus, a common assumption is that if all prices in the economy, including the price the consumer gets for her own labour, are doubled or halved the amount of goods bought will remain unchanged. In practice, we know that this is not true. During the Great Recession, to take a recent example, prices fell at the same time that GDP fell.

The reason was the fall in asset prices, mainly of housing and stocks. An indicator of the loss that consumers suffered in the crash is that the net worth of the median US family in 2010 fell to the level it had in 1992 (Federal Reserve Bulletin, 2012). To make up for that 18-year loss in accumulated saving, families cut their consumption for years with a resultant fall in the growth

rate of GDP. The primary concern of households was to recover their lost saving. To that end they cut the quantity of their purchases. When firms, confronted with a higher inventory, then cut the prices of their goods, households did not respond by increasing their consumption in quantitative terms; instead, they used the opportunity to further increase savings. If they were maximizing anything at all at that point, it was saving. The destruction of their accumulated saving in a market crash that occurred in the past played a controlling part in their behaviour for several years into the future.

It is this behaviour by consumers and firms that violates a key assumption of neoclassical economics. To quote from Arrow's Nobel Prize lecture (Arrow, 1972): "Clearly, if all prices are multiplied by the same positive constant, the budget constraint for households is really unchanged, and hence so are the consumer demands. Similarly, the profits are multiplied by a positive constant, so that the profit-maximizing choice of a firm is unchanged. Hence, the functions $x_{hi}(p_1, \dots, p_n)$ and $y_{fi}(p_1, \dots, p_n)$ are homogeneous of degree zero in their arguments" (p. 114).

This assumption, that household demand functions and firm production functions are homogeneous of degree zero, runs through all of neoclassical economics, from proofs of general equilibrium to expositions of consumer choice theory. Since it is untrue it follows that most of neoclassical economics is incorrect.

If I am permitted to end this section on a somewhat facetious note: if neoclassical economics has no future, it is because it has no past.

6. Conclusion

In making their pricing decisions firms have necessarily to keep their fixed costs in mind. As soon as this fundamental truth is recognised, we can understand why the math of profit maximization is incorrect, why prices are sticky during economic downturns, why the conventional shape of the supply curve must be questioned, why firms normally experience increasing returns, and why market clearing is not a given.

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