Macroeconomics · 2025-26 · Exercise 1

Reminder on the 45-degree model:

$\mathbf{C}$	stands for aggregate private consumption
$\mathbf{I}$	stands for aggregate private investment
$\mathbf{G}$	stands for public expenditure
$\mathbf{EX}$	stands for aggregate exports
$\mathbf{IM}$	stands for aggregate imports
$\mathbf{NX}$	stands for net aggregate exports (the difference $\mathbf{EX} - \mathbf{IM}$, also called 'trade balance')
$\mathbf{T}$	stands for tax revenue
$\mathbf{TR}$	stands for transfers (from the public to the private sector)
$\mathbf{AD}$	stands for aggregate demand (where $\mathbf{AD} = \mathbf{C} + \mathbf{I} + \mathbf{G} + \mathbf{EX} - \mathbf{IM}$)
$\mathbf{Y}$	stands for aggregate income (or, alternatively, GDP)
$\mathbf{Y}_d$	stands d for aggregate disposable income (where $\mathbf{Y}_d = \mathbf{Y} + \mathbf{TR} - \mathbf{T}$)
$\mathbf{S}$	stands for aggregate savings (that is, $\mathbf{S} = \mathbf{Y}_d - \mathbf{C}$)
c	is the propensity to consume (so $s = 1 - c$ is the propensity to save)
β	is the sensitivity of investment to aggegate income
t	is the tax rate
m	is the propensity to import
$\bar{\mathbf{C}}$	stands for autonomous (exogenous, constant) aggregate private consumption
$\bar{\mathbf{I}}$	stands for autonomous (exogenous, constant) aggregate private investment
$\bar{\mathbf{G}}$	stands for autonomous (exogenous, constant) public expenditure
$\bar{\mathbf{T}}$	stands for autonomous (exogenous, constant, income independent) tax revenue
$\overline{\mathbf{TR}}$	stands for autonomous (exogenous, constant) transfers
$\overline{\mathbf{EX}}$	stands for autonomous (exogenous, constant) aggregate exports
$\overline{\mathbf{IM}}$	stands for autonomous (exogenous, constant) aggregate imports
$\mathbf{Y} = \mathbf{AD}$	is the model macroeconomic equilibrium condition.

1. 45-degree model

The aggregate private consumption function is

$$\mathbf{C} = \bar{\mathbf{C}} + c\,\mathbf{Y}_d.$$

With $\beta > 0$, the aggregate private investment function is

$$\mathbf{I} = \bar{\mathbf{I}} - \beta i.$$

With $t > 0$, the tax revenue function is

$$\mathbf{T} = \bar{\mathbf{T}} + t\mathbf{Y}.$$

With $m > 0$, the aggregate import function is

$$\mathbf{IM} = \overline{\mathbf{IM}} + m\mathbf{Y}.$$

In addition,

$$\mathbf{G} = \bar{\mathbf{G}}$$

$$\mathbf{TR} = \overline{\mathbf{TR}}$$

$$\mathbf{EX} = \overline{\mathbf{EX}}.$$

(1) Find the formula that gives all the GDP values for which the public deficit $\mathbf{PD} = \mathbf{G} + \mathbf{TR} - \mathbf{T}$ is zero.

(2) Find the formula that gives all the GDP values making zero the trade balance $\mathbf{NX} = \mathbf{EX} - \mathbf{IM}$.

(3) Find the IS relation and prove that, as a function relating $\mathbf{Y}$ and i, it is a decreasing function (that is, the derivative $\frac{d\mathbf{Y}}{di}$ is negative).

(4) Show that condition $\mathbf{Y} = \mathbf{AD}$ is equivalent to $\mathbf{NPS} = \mathbf{PD} + \mathbf{NX}$, where $\mathbf{NPS} = \mathbf{S} - \mathbf{I}$.

(5) Find the formula that gives all the GDP values such that $\mathbf{Y} = \mathbf{AD}$.

In questions (6)–(9) next, $\bar{\mathbf{C}} = \bar{\mathbf{I}} = \bar{\mathbf{G}} = \overline{\mathbf{T}} = \overline{\mathbf{TR}} = \overline{\mathbf{EX}} = \overline{\mathbf{IM}} = 10$, $c = 4/5$, $t = 1/5$ and $\beta = m = 0$. These questions illustrate that an exogenous negative shock to autonomous aggregate demand create a fiscal policy dilemma: to neutralize the effect on GDP of the negative shock by increasing the public deficit or to keep the pre-shock public deficit by reinforcing the negative impact on GDP the shock causes. That is, the dilemma is to fight economic contraction or respect a fiscal rule.

(6) Find equilibrium GDP and the associated public deficit and trade balance.

(7) Find equilibrium GDP and the associated public deficit if autonomous private investment is reduced by half.

(8) Continuing with (7), what change in autonomous public expenditure $\bar{\mathbf{G}}$ restores the public deficit level in (6)? Find the corresponding equilibrium GDP.

(9) Continuing with (7), what change in autonomous public expenditure $\bar{\mathbf{G}}$ restores the GDP value in (6)? Find the corresponding public deficit.

(10) Answer (6)–(9) above, if the aggregate functions are redefined as follows, where $\alpha = 1/6$ and $\gamma = 1/4$:

$$\mathbf{C} = \bar{\mathbf{C}}$$

$$\mathbf{I} = \bar{\mathbf{I}} + \alpha\mathbf{Y}$$

$$\mathbf{T} = \overline{\mathbf{T}} \text{ and } \mathbf{TR} = \mathbf{0}$$

$$\mathbf{G} = \bar{\mathbf{G}} + \gamma\mathbf{Y}$$

$$\mathbf{NX} = \mathbf{0}.$$

Two economies constitute a monetary union. The monetary union constitutes a closed economy (so exports in one economy equal imports of the other economy). Economy 1 is described by equations

$$\mathbf{C} = \bar{\mathbf{C}} + c\,(\mathbf{Y} - \mathbf{T})$$

$$\mathbf{I} = \bar{\mathbf{I}} - \beta i$$

$$\mathbf{G} = \bar{\mathbf{G}}$$

$$\mathbf{TR} = \mathbf{0}$$

$$\mathbf{T} = t\mathbf{Y}$$

$$\mathbf{EX} = \overline{\mathbf{EX}}$$

$$\mathbf{IM} = \overline{\mathbf{IM}} + m\mathbf{Y}.$$

Economy 2 is described by equations

$$\mathbf{C}' = \bar{\mathbf{C}}' + c'\,(\mathbf{Y}' - \mathbf{T}')$$

$$\mathbf{I}' = \bar{\mathbf{I}}' - \beta' i$$

$$\mathbf{G}' = \bar{\mathbf{G}}'$$

$$\mathbf{TR}' = \mathbf{0}$$

$$\mathbf{T}' = t'\mathbf{Y}'$$

$$\mathbf{EX}' = \overline{\mathbf{EX}}'$$

$$\mathbf{IM}' = \overline{\mathbf{IM}}' + m'\mathbf{Y}'.$$

(11) Find the formula la relates the GDPs $\mathbf{Y}$ and $\mathbf{Y}'$ of the two economies using the condition that (being the monetary union a closed economy) $\mathbf{NX} = -\mathbf{NX}'$. Does this formula represent an increasing or a decreasing function? Represent this function graphically.

(12) Find the formula la relates the GDPs $\mathbf{Y}$ and $\mathbf{Y}'$ of the two economies using the condition that in a monetary union the interest rate i is the same. Does this formula represent an increasing or a decreasing function? Represent this function graphically.

(13) Use the formulas in (11) and (12) to calculate the GDPs $\mathbf{Y}$ and $\mathbf{Y}'$ of the two economies.

(14) Given the result in (13), ascertain the effect on $\mathbf{Y}$ and $\mathbf{Y}'$ of an increase in the tax rate t of economy 1.

(15) Answer (11)–(14) above if $\bar{\mathbf{C}} = \bar{\mathbf{C}}' = \bar{\mathbf{I}} = \bar{\mathbf{I}}' = \bar{\mathbf{G}} = \bar{\mathbf{G}}' = \overline{\mathbf{EX}} = \overline{\mathbf{EX}}' = \overline{\mathbf{IM}} = \overline{\mathbf{IM}}' = 10$, $\beta = t = m' = \frac{1}{10}$, $\beta' = t' = m = \frac{1}{5}$, $i = 5$ and the tax rate (of economy 1) duplicates.

(16) Using the data in (15) answer (14) if the tax rate of economy 2 duplicates.

(17) Answer again (11)–(16) if exports are not autonomous and the equations defining exports in both economies are

$$\mathbf{EX} = \mathbf{IM}' = \overline{\mathbf{IM}}' + m'\mathbf{Y}'$$

$$\mathbf{EX}' = \mathbf{IM} = \overline{\mathbf{IM}} + m\mathbf{Y}.$$