III. Finance II: foreign money

1. Exchange rates, arbitrage and speculation

Exchange rate

The exchange rate (foreign exchange rate, forex rate or FX rate) between two currencies is the price of one currency relative to the other. For example, if the exchange rate between dollar and euro is $e = 2$ \$/€, then one euro can be exchanged for two dollars: the price of one euro in dollars is two dollars.

The reciprocal $e' = 1/2$ €/\$ of $e = 2$ \$/€ indicates how many euros you get for one dollar: the price of one dollar in euros is half a euro. Both e and its reciprocal e' express the same information: the price of acquiring a currency in terms of another.

The exchange rate is supposed to define a proportion between currency amounts; that is, the price between currencies does not depend on the amount bought or sold. For instance, $e = 2$ \$/€ means not just that

2 dollars exchange for 1 euro

but also that (multiplying by two)

4 dollars exchange for 2 euros

and that (dividing by two)

1 dollar exchanges for 1/2 euro.

Currencies isolate economies. The exchange rate connects economies, as an exchange rate enables members of an economy to conduct economic transactions with members of an economy in which a different currency is used. Thanks to the exchange rate, domestic consumers and producers can buy foreign goods and financial assets and sell goods and assets to foreigners. These transactions cannot be carried out directly if the two economies have different currencies (provided that sellers only accept domestic currency).

Quotation methods

An exchange rate is a peculiar price. A price is a measure of something in money terms: euros per kilogram of apples. This implies that there is a 'natural' choice of units for prices: monetary units per unit of that something (commodities, goods, services, financial assets…). What is odd about the exchange rate is that the something else is also money. As a consequence, there is no 'natural' way to quote the price: money 1 per unit of money 2 is as valid as money 2 per unit of money 1. Both choices are acceptable even if one differentiates the currencies by declaring one to be the 'domestic' currency and the other the 'foreign' currency.

The direct quotation (or price quotation) of an exchange rate expresses the exchange rate as domestic (home) currency units divided by foreign currency units. When the peseta was the Spanish currency, direct quotation was the norm: $e = 150$ Pts/\$.

The indirect quotation (or quantity quotation) of an exchange rate expresses the exchange rate as foreign currency units divided by domestic (home) currency units. For instance, taking the euro as the home currency, then $e = 2$ \$/€ quotes the exchange rate indirectly; the same exchange rate, from the US perspective, would represent a direct quotation. Just a EU citizen is more likely to be interested in knowing how many dollars one euro is worth, a US citizen is more likely to be interested in knowing how many euros one dollar is worth.

The choice of quotation method is essentially irrelevant: it only matters to get the right units when an exchange rate is part of an economic formula and, for practical purposes, to avoid confusion.

Currency appreciation

Currency X appreciates relative to (or against) currency Y when the number of units of Y that can purchase one unit of X increases. If X appreciates relative to Y, the value of X increases relative to Y.

Let the units of the exchange rate be X/Y. Then a rise in the exchange means that currency Y appreciates against currency X, since an increase in the exchange rate means that more units of X are obtained from a unit of Y. Conversely, a fall in the exchange rate so expressed means that currency X appreciates against currency Y, because a reduction in the exchange rate means that fewer units of X must be delivered to acquire one unit of Y.

For instance, when the exchange rate moves from $e = 1$ \$/€ to $e' = 2$ \$/€, the euro appreciates against the dollar. Initially, one euro was worth just one dollar; subsequently, one euro is worth two dollars. Symmetrically, in passing from e' to e the dollar appreciates against the euro: given e', two dollars are required to get one euro; given e, one dollar suffices to purchase one euro and, accordingly, the dollar has risen in value.

Currency depreciation

Currency X depreciates relative to (or against) currency Y when the number of units of Y that can purchase one unit of X decreases. When X depreciates relative to Y, the value of X diminishes relative to Y. By definition, X depreciates against Y if and only if Y appreciates against X. In passing from $e = 1$ €/¥ to $e' = 2$ €/¥, the euro depreciates with respect to the yen. Initially, one euro could buy one yen; after the fall in the exchange rate, one euro can only buy 0.5 yen and, therefore, the euro has lost value

Choice of exchange rate units

To facilitate the interpretation of exchange rate changes, it is convenient to adopt the perspective of the currency placed in the denominator; that is, if the units are X/Y, then think in terms of currency Y (that is, treat Y as the domestic currency and adopt indirect quotation). A reason for this choice is that an increase in the units X/Y means that the value of Y increases (Y appreciates against X). Focusing instead on X would lead to the potentially confusing result that X appreciates against Y when the exchange rate in X/Y units falls.

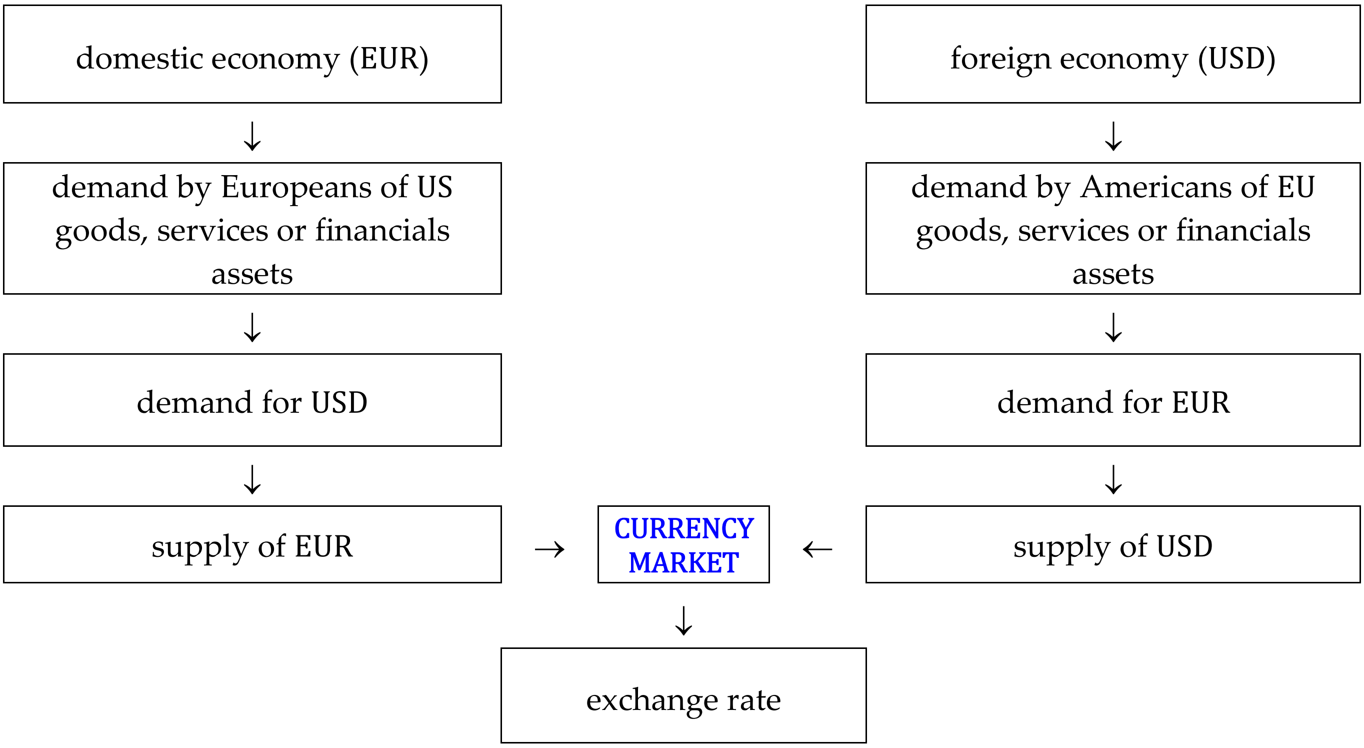
That is why, in these notes, the default choice in the exchange rate quotation is 'to put the euro down': euro exchange rates will be stated as \$/€, ¥/€, £/€… In case an exchange rate is given with these units inverted, then reverse the inversion: x €/\$ is the same rate as $1/x$ \$/€ (invert the number and invert the units: 2 €/\$ is equivalent to $1/2$ \$/€).

Simple model of exchange rate determination

Once a price is defined the basic economic question is: what pushes the price up or down? For the exchange rate there is a simple theory, which is a specific application of the general microeconomic theory based on supply and demand in a competitive setting (that is, with 'many' buyers and 'many' sellers).

The sketch below summarizes the model. Let there be just two economies: the domestic one (euro-based) and the foreign economy (dollar-based).

European residents generate a demand for foreign (US) goods, services or financial assets. To purchase such goods, services or financial assets dollars are recquired. Accordingly, the demand for US goods, services or financial assets creates a derived demand for US dollars. How do Europeans pay for those dollars? With euros. Therefore, Europeans demanding US dollars (to make purchases in the US economy) are simultaneously supplying (offering) euros. In sum, Europeans go to the currency market (where the exchange rate is determining) by selling euros and, at the same time, buying dollars.



Conversely, US residents generate a demand for foreign (European) goods, services or financial assets. To purchase them euros are necessary. Hence, the demand for European goods, services or financial assets induces a demand for euros. How do US residents pay for those euros? With dollars. As a result, US residents demanding euros (to make purchases in the EU economy) are simultaneously supplying (offering) dollars. Summing up, Americans go to the currency market by buying euros and, simultaneously, selling dollars.

The exchange rate is obtained through the interaction in the currency market between supply of euros and demand for euros (or, symmetrically, between supply of dollars and demand for dollars). The sketch above simplifies the reasons for supplying or demanding currencies: both are instrumental because the ultimate goal is to obtain foreign goods, services or financial assets.

In other words: everything inducing members of an economy to purchase more foreign goods, services or financial assets, also induces them to demand more foreign currency (and, a fortiori, supply more domestic currency). The basic supply and demand model of price determination contends that:

- a higher demand for what is bought/sold in a market tends to increase its price (and, analogously, a higher supply tends to lower it); and
- a lower demand tends to decrease the price (and a lower supply tends to increase it).

It then follows that:

- whatever makes people in an economy be willing to buy more foreign goods, services or financial assets, tends to appreciate the foreign currency (and depreciate the domestic one), since more foreign currency is demanded (and, necessarily, more domestic currency is supplied);
- whatever makes people in an economy be willing to buy fewer foreign goods, services or financial assets, tends to depreciate the foreign currency (and appreciate the domestic one), because less foreign currency is demanded (and, thus, less domestic currency is supplied).

By way of illustration, if $e_1 = 2\,\$/€$ rises to $e_2 = 3\,\$/€$, then the exchange rate between the currencies changes from

$$2\,\$ \;—\; 1€$$

to

$$3\,\$ \;—\; 1€\,.$$

This means that the euro has appreciated against the dollar: every euro can buy more dollars (previously two; now, three). Considering only the values $e_1 = 2\,\$/€$ and $e_2 = 3\,\$/€$ the euro 'is cheap' when the rate is e_1 (with e_1 fewer dollars are needed than with e_2 to buy a euro) and, symmetrically, the euro 'is expensive' when the rate is e_2.

Now, what could explain the euro appreciation (equivalently, the dollar depreciation)? The sketch above provides the only possible (general) explanations:

- European wanted fewer US goods, services or financial assets; and/or
- Americans wanted more EU goods, services or financial assets.

Empirical evidence suggests that financial reasons are much for significant than real ones; perhaps more than 90% of the purchase or sale of currencies in the currrency market is motivated by the desire to purchase or sale foreign financial assets. That is, international financial transactions (international financial investments, dominated by speculation) mostly determine exchange rates due to the fact that, in volume and value, financial transactions dwarf real transactions (those associated with international trade).

Spatial arbitrage

Arbitrage and speculation consist of buying or selling goods (or assets), when the prices of the goods (or assets) differ, with the aim of obtaining a profit. In arbitrage the profit is certain; in speculation the profit is uncertain (the intended profit may ultimately result in a loss).

Arbitrage is a conceptually interesting mechanism because, as will be shown next, it contributes to integrate markets: arbitrage operates as a force of market integration. On the other hand, it will be argued that speculation is more likely to be a market destabilizing force.

For example, suppose there are two currency markets for the same currency with different rates, $e_1 = 2$ \$/€ and $e_2 = 3$ \$/€. In this situation arbitrage can be carried out.

Geographical reasons would justify the existence of two markets for the same currencies: one market would be organized in the US (in Washington, for example, where the US Federal Reserve is headquartered) and another in the eurozone (in Frankfurt, where the European Central Bank is headquartered). Specifically, let $e_1 = 2$ \$/€ be the rate in the American market, so

$$2\,\$ \text{ —US— } 1€$$

and $e_2 = 3$ \$/€ the rate in the European market, and thus

$$3\,\$ \text{ —EU— } 1€\,.$$

The strategy for making a profit from a purchase is very simple: buy cheap and sell expensive. In the case of the euro, it is cheap in the American market: there buying a euro only requires spending two dollars, while buying it in the European market requires spending three.

Assuming transaction costs to be negligible, a US arbitrageur would make a safe profit by buying euros in the US market and selling them in the European market. For every dollar the US arbitrageur has, he can obtain half a euro in the US market, which he can exchange for 3/2 dollars in the European market, thus making a profit of half a dollar per dollar spent; see Fig. 1.

$$e_1 = 2\,\$/€ \qquad\qquad e_2 = 3\,\$/€$$

$$1\,\$ \;\xrightarrow{\;\downarrow\;}\; \tfrac{1}{2}€ \;\xrightarrow{\;\downarrow\;}\; \tfrac{3}{2}\$$$

Fig. 1. Arbitrage by a US arbitrageur (50% sure profit)

As argued in the previous section, the purchase of euros by US arbitrageurs in the US market would raise the value of the euro there: e_1 it would tend to increase (the euro would appreciate against the dollar). And their sale in the European market would cause the value of the euro to fall: e_2 would be pressured down. As long as the two values differ, the strategy of buying euros in the US and selling them in the eurozone shall be profitable. But the operations that the arbitrageurs themselves conduct in the two markets tend to bring the rates closer together, eventually producing their equality (at some value between 2 \$/€ and 3 \$/€).

Spatial arbitrage (arbitrage involving geographically separated markets) has produced a local or microeconomic outcome (profits for arbitrageurs) but also a global or macroeconomic one: the two markets need to have the same exchange rate.

It can be deduced from the above explanation that spatial arbitrage is a force that integrates geographically segmented markets: thanks to arbitrage it can be interpreted that there are not two currency markets but only one, since any discrepancy in the value of the rate will be corrected by arbitrageurs. Price differences are like banknotes on the ground, and arbitrageurs are simply those ready to pick them.

The correction will be even faster due to the participation of European arbitrageurs, who could also make a sure profit by buying cheap and selling expensive. The strategy of US arbitrageurs does not seem appropriate for Europeans, since European arbitrageurs already have euros. In any case, buying more (in the US market) would require paying for them in dollars, which is not the Europeans' own currency.

Yet, European arbitrageurs have a more appropriate strategy owing to their possession of euros. From the perspective of a European, the dollar is cheap in the European market (with an exchange rate of $e_2' = \frac{1}{3}$ €/\$) and expensive in the US market (where the rate is $e_1' = \frac{1}{2}$ €/\$).

Consequently, for every euro available for arbitrage, the European arbitrageur can obtain 3 dollars in the European market, which can then be sold in the US market in exchange for 3/2 euros; see Fig. 2.

$$e_2 = 3\ \$/€ \qquad\qquad e_1 = 2\ \$/€$$

$$1\ € \;\rightarrow\; 3\ \$ \;\rightarrow\; \tfrac{3}{2}\ €$$

Fig. 2. Arbitrage by a European arbitrageur (sure profit of 50%)

Combining the operations of the two groups of arbitrageurs:

- in the European market, Americans increase the supply of euros (they sell the euros acquired in the American market) and Europeans increase the demand for dollars (by selling the euros they initially have), growth that is equivalent to an increase in the supply of euros, so that an expanded supply of euros by Americans and Europeans depreciates the euro against the dollar and causes the rate to fall $e_2 = 3$ \$/€;
- in the American market, Americans increase the demand for euros (they sell the dollars they initially own) and Europeans increase the supply of dollars (by selling the dollars acquired in the European market), growth that is equivalent to an increase in the demand for euros, so that an expanded demand for euros by Americans and Europeans appreciates the euro against the dollar and raises the rate $e_1 = 2$ \$/€.

Triangular arbitrage

A characteristic feature of macroeconomic analysis is to consider the interaction of parts of an economy. In the conventional analysis the more relevant 'parts' of the economy are called 'markets', so the macroeconomic analysis would focus on determining how markets of all kinds are related. The more intense the relationship between mercats, the more integrated markets are to be considered. Sufficiently integrated markets would come to be seen as unique markets.

Currency markets illustrate this view. The creation of new currencies generates more markets. The more markets, the more complex the analysis of their connections and interactions becomes. On the other hand, arbitrage constitutes an 'economic force' causing market integration (and, in practice, reduces their number: two highly integrated markets are de facto a single market). An economy could be conceptualized as a set of markets that expands and contracts: expansion stems from creating new goods, services, financial assets or currencies; contraction would be the result of (sufficiently deep) integration or (sufficiently intense) market connection.

Triangular arbitrage is a mechanism that has a double result. On the one hand, it generates a private benefit for arbitrageur; on the other hand, it contributes to integrating markets and, thus, reducing their number in practice. The macroeconomic effect of triangular arbitrage is to reduce all currency markets to one: although there would still be a market for each currency, arbitragewould make it irrelevant in which specific market the currency exchange was made. The microeconomic effect of triangular arbitrage is to balance all foreign exchange markets with consistent exchange rates between them, in the sense that the price of switching, for example, euros per dollar would be the same as that of going from euros to dollars through a third currency (swapping, for example, euros for yen and next yen for dollars).

Triangular arbitrage takes advantage of price differences of three currencies to obtain sure profits. The passage of two to three currencies is a profound change: with two, there is only one way to go from one to another. The introduction of a third currency causes a radical change: now, apart from the direct path between two currencies (represented by the market of the two currencies) there is an indirect path that leads from one currency to another through the third currency.

Triangular arbitrage is possible when the two paths disagree: the rate of direct exchange between two currencies is different from the rate of indirect exchange of the two currencies obtained by the intervention of a third currency.

For example, suppose there are three currencies (euro, dollar and yen) and that their exchange rates (direct) are 2 $/€, 3 ¥/$ and 4 ¥/€. In this case the direct rate between two currencies is different from the indirect rate. Specifically, as regards euros and dollars, the direct rate is

$$2\,\$\ —\ 1\,€$$

while the indirect one would be

$$4/3\,\$\ —\ 1\,€$$

which is the result of using the other two direct rates

$$3\,¥\ —\ 1\,\$$$

$$4\,¥\ —\ 1\,€\,.$$

In fact, given the 4 ¥/€ rate, one euro can be converted into four yen; and, given the 3 ¥/$ rate, four yen can be transformed into one plus 1/3 dollars.

According to the 2 \$/€ direct rate, the price of one euro is two dollars. On the other hand, if the euro is sold for yen according to the rate 4 ¥/€, then four yen are obtained; and if these yen are sold for dollars applying the rate 3 ¥/\$, it follows that 4/3 dollars are pocketed.

One might think that the second price does not matter, as the direct rate provides more dollars (two) than the indirect rate (one and one-third). Error. Price inconsistencies always matter, because they create opportunities to make a sure gain. Just like seeing banknotes on the ground. Arbitrage is to take them, not to leave them there.

Besides, the fact that the rate discrepancy does not matter to someone who wants to sell euros does not imply that it does not matter to someone who wants to buy them. Normalizing the rates with respect to the dollar, the direct rate is

$$1\,\$ \;—\; 1/2\,€$$

while the indirect rate is

$$1\,\$ \;—\; 3/4\,€\,.$$

Obviously, someone who wants to sell dollars will be interested in using the indirect rate (selling dollars for yen and then selling yen for euros) rather than the direct rate (selling dollars for euros).

The direct rate between the euro and the dollar is

$$e_{\$€} = 2\,\$/€\,.$$

The indirect rate between euro and dollars (via the yen) is obtained by multiplying the other two exchange rates, but in such a way that, in the product, the yen cancels out. Given how the other two exchange rates have been presented,

$$e_{¥\$} = 3\,¥/\$$$

$$e_{¥€} = 4\,¥/€$$

multiplying them does not produce any exchange rate:

$$e_{¥\$} \cdot e_{¥€} = 3\,\frac{¥}{\$} \cdot 4\,\frac{¥}{€} = 12\,\frac{¥^2}{\$€}\,.$$

This little bug is not an exchange rate. There is, however, an easy solution: invert one of the two rates. Specifically, if you want the units to be the same as the direct exchange rate (\$/€), you should keep the euro in the denominator (and thus not invert the rate $e_{¥€}$) and bring the dollar to the numerator in the rate $e_{¥\$} = 3\,¥/\$$. The result of inverting this rate (both the numerical value and the units are to be inverted) is

$$e_{\$¥} = 1/3\,\$/¥.$$

The indirect rate between euro and dollar (via the yen) is

$$e_{\$¥} \cdot e_{¥€} = \frac{1}{3}\frac{\$}{¥} \cdot 4\,\frac{¥}{€} = \frac{4}{3}\frac{\$}{€}\,.$$

Clearly,

$$2 = e_{\$€} \neq e_{\$¥} \cdot e_{¥€} = 4/3$$

which is symptomatic of the difference between exchanging euros for dollars directly or doing so through the exchange of yen. In short, arbitrage opportunities exist if

$$\boxed{e_{\$€} \neq e_{\$¥} \cdot e_{¥€}\,.}$$

It does not matter whether the exchange rate is measured in \$/€ (which leads to considering the previous inequality) or in €/\$. If the previous inequality occurs, then it is true as well that

$$e_{€\$} \neq e_{€¥} \cdot e_{¥\$}$$

in view of the fact that

$$x \neq y \cdot z$$

implies

$$\frac{1}{x} \neq \frac{1}{z} \cdot \frac{1}{y}.$$

Neither the two currencies chosen to determine the direct and indirect rates do matter. It is easy to verify that if the previous inequality occurs, then

$$e_{\$¥} \neq e_{\$€} \cdot e_{€¥}$$

and, by the explanation above,

$$e_{¥\$} \neq e_{¥€} \cdot e_{€\$}$$

(the exchange between dollars and yen does not coincide with the exchange between dollars and yen going through the euro) and, moreover,

$$e_{€¥} \neq e_{€\$} \cdot e_{\$¥}$$

and, by extension,

$$e_{¥€} \neq e_{¥\$} \cdot e_{\$€}.$$

This says that the exchange rate between euros and yen is not the same as the exchange rate between euros and yen through the dollar. In other words, if any of the six exchange rate inequalities above hold, then so do the other five.

Focusing on the first inequality

$$e_{\$¥} \cdot e_{¥€} \neq e_{\$€}$$

it can be equivalently expressing by moving $e_{\$€}$ on the left (which is the same as inverting it):

$$e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} \neq 1.$$

In the numerical example, the rates are

$$e_{\$¥} = 1/3 \ \$/¥$$

$$e_{¥€} = 4 \ ¥/€$$

$$e_{€\$} = 1/2 \ €/\$$$

and the corresponding product

$$e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} = \frac{1}{3} \cdot 4 \cdot \frac{1}{2} = \frac{2}{3} \neq 1.$$

The product $e_{\$¥} \cdot e_{¥€} \cdot e_{€\$}$ represents the result of the exchange cycle where, starting from one dollar, three yen are bought ($e_{\$¥} = 1/3 \ \$/¥$), these yen are sold for 3/4 euros ($e_{¥€} = 4 \ ¥/€$) and, finally, 3/2 dollars are bought back with these euros ($e_{€\$} = 1/2 \ €/\$$).

The transaction cycle $ → ¥ → € → $ is Cockaigne, the land of plenty: you start with one dollar and end the round with one and a half dollars (a 50% return). Since it is a cycle, it does not matter which currency you start with. If the round starts with the yen, the sequence ¥ → € → $ → ¥ generates an output of 3/2 yen for each yen that enters at the beginning (one yen buys 1/4 euros, these buy 1/2 dollars and these buy back 3/2 yen).

And if the initial currency is the euro, the sequence € → $ → ¥ → € ultimately produces 3/2 euros for euro spent (one euro buys two dollars, these buy six yen and these buy back 3/2 euros). In all cases each initial unit (one dollar, one yen, one euro) is eventually transformed into 1.5 units.

Not every transaction cycle is profitable. As a matter of fact, with three currencies, there are only two transaction (exchange, trading) cycles: $ → ¥ → € → $ and the reverse $ → € → ¥ → $, shown in Fig. 3.

Fig. 3. The two trading cycles with three currencies

Under cycle $ → € → ¥ → $, one dollar buys half a euro, which buys two yen, which buys two thirds of a dollar. This is a ruinous business: you start with one dollar and close the exchange circuit with 2/3.

For example, starting with one euro, the cycle on the right produces losses: 1 € → 4 ¥ → 4/3 $ → 2/3 €. Conversely, the cycle on the left creates profits: 1 € → 2 $ → 6 ¥ → 1,5 €.

The inequality $e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} \neq 1$ demonstrates that there are arbitrage opportunities. If the product of the three rates were one, the transaction cycle would generate neither profit nor loss and, from the point of view of arbitrage, they would be mutually consistent rates (the path from one to another leads to the same result as the path through the third currency).

To recap: $e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} \neq 1$ means that there are arbitrage opportunities and these opportunities are exploited by implementing the sequence of transactions … → $ → € → ¥ → $ → …

What effect do these transactions have on exchange rates? They lead to the fulfilment of the condition $e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} = 1$. When this condition is fulfilled, triangular arbitrage opportunities disappear.

For the analysis in the sequel, it is more convenient to present the inequality $e_{\$¥} \cdot e_{¥€} \cdot e_{€\$} \neq 1$ in the equivalent form $e_{\$€} \neq e_{\$¥} \cdot e_{¥€}$. As previously calculated,

$$2 = e_{\$€} > e_{\$¥} \cdot e_{¥€} = 4/3$$

so that it would be sufficient to achieve equality

- for $e_{\$€}$ to decrease,
- for $e_{\$¥}$ to increase and
- for $e_{¥€}$ to increase.

Triangular arbitrage (through the exchange cycle $ → ¥ → € → $) achieves these three results.

First, the transaction € → $ means selling euros and buying dollars, which depreciates the euro against the dollar and, consequently, reduces the rate $e_{\$€}$ (since it is expressed in $/€).

Second, the transaction $ → ¥ means selling dollars and buying yen, which appreciates the yen against the dollar and, as a result, increases the rate $e_{\$¥}$ (which is expressed in $/¥).

And, third, the transaction ¥ → € means selling yen and buying euros, which appreciates the euro against the yen and, accordingly, increases the rate $e_{¥€}$ (which is expressed in ¥/€).

The ultimate consequence of arbitrage is that the execution of the trading strategy 1 € → 2 $ → 6 ¥ → 1.5 € reduces the gap between $e_{\$€}$ (direct rate) and $e_{\$¥} \cdot e_{¥€}$ (indirect rate). <u>The final outcome is a rebalancing of exchange rates that makes direct and indirect rates coincide</u>.

The analysis of triangular arbitrage shows the difference between the microeconomic and the macroeconomic approaches. The initial values of 2 $/€, 3 ¥/$ and 4 ¥/€ of the exchange rates in can be assumed as values that balance the respective markets. Therefore, these rates would define partial equilibria: considered independently each of the three markets would be initially in equilibrium ('supply equals demand').

But the macroeconomic approach leads to considering the connections between markets and, based on the interactions between the three markets, the partial equilibria can be unstable. In other words, predicting exchange rates by taking each market separately (without considering possible links between them) could be an erroneous prediction. The reason is that <u>the microeconomic analysis of partial equilibrium overlooks forces that connect the markets and can alter the equilibrium that each market would achieve if isolated from the rest of markets</u>.

The example shows that triangular arbitrage would integrate the three markets, for which reason the equilibrium in one market cannot be determined without simultaneously considering the equilibria of the other markets: the three markets are equilibrated simultaneously (macroeconomically), not separately (microeconomically).

Arbitrage amounts to imposing on exchange rate determination a <u>macroeconomic condition that keeps cross exchange rates consistent</u>: for any three currencies X, Y and Z,

$$e_{\mathrm{XY}} \cdot e_{\mathrm{YZ}} \cdot e_{\mathrm{ZX}} = 1.$$

Exchange rate regimes

There are two basic exchange rate regimes: the flexible and the fixed exchange rate regime.

- In a flexible (or floating or free floating) exchange rate regime, a public authority (government or central bank) lets the currency market set the value of the exchange rate *e*. In this case, the authority refuses to influence *e*. The preceding analysis of arbitrage presumed a flexible regime.
- In a fixed exchange rate regime (or hard peg), the corresponding public authority (government or central bank) picks an official value of the exchange rate between the domestic currency and some foreign currency (or group of them) and assumes the compromise of driving the currency market to the desired exchange rate (in normal circumstances, by buying or selling the domestic currency in that market). If the value of the domestic currency is pegged to the value of another currency, the latter is known as the anchor currency.

The rest of possible regimes are combinations of the fixed and flexible regimes (for full details on exchange rate arrangements, see <https://www.imf.org/external/np/mfd/er/2003/eng/1203.htm>). For instance:

- in a managed float exchange rate regime (or 'dirty float') the authority seeks to influence the exchange rate by buying and selling currencies at will (to limit the volatility of the exchange rate or to guide it), without any specific commitment;
- under an adjustable peg the exchange rate is fixed but can be periodically adjusted;
- in an exchange rate regime fluctuation band (or currency band) the exchange rate is allowed to to fluctuate within an interval defined around a central exchange rate (for example, ±3% around it), with the authority being free to narrow or widen the interval, or to modify the central rate.

Revaluation and devaluation

The terms appreciation and devaluation correspond to modifications of market exchange rates. The equivalent terms for modifications of official exchange rates are, respectively, revaluation and devaluation.

A devaluation is a reduction of the fixed exchange rate and occurs when the public authority accepts that the former fixed rate cannot be upheld (in essence, because the market exchange rate is undervalued with respect to the official rate: the market 'believes' that the currency is worth less than the authority claims). Depreciation is a market event; devaluation is a policy decision.

Revaluation is the opposite of devaluation: a fixed exchange rate reset at a higher level. Historically, revaluation is much less frequent than devaluation: the stylized fact is that, when market and the authority systematically disagree on the value of the home currency, the authority values the currency higher than the market (so the adjustament, when occurs, must be downwards).

Central bank intervention in the currency market

Suppose the central bank is in charge of upholding a fixed exchange rate. There are two strategies to attain that goal: (i) buying or selling the domestic currency in the currency market; (ii) modify the main interest rate the central bank sets.

To illustrate (i), assume the euro is the domestic currency, the dollar is the foreign currency, the exchange rate units are $/€ and the central bank is the European Central Bank (ECB).

Let e' be the fixed exchange rate and suppose the market exchange rate is $e < e'$: the fixed (official) rate overvalues the euro with respect to the market value.

Since the exchange rate regime is the fixed one, the ECB must induce an increase in the market rate e so that $e = e'$. Strategy (i) means that this has to be achieved by either supplying more euros or demanding more euros.

The currency market is supposed 'free' in the sense that no authority can force agents in the market to reduce their supply of euros or demand for euros; that is, 'money is free', capital mobility is perfect or there is no capital control. This implies that the ECB does not have the power to reduce the supply of euros or to reduce the demand for euros: the ECB can always add, not detract (the presumption is that the ECB was not already supplying or demanding).

What if the ECB sells more euros? This causes an increase in the supply of euros and, as a consequence, the euro depreciates and e falls. It is then plain that this intervention does not produce the desired effect: euro appreciation. Hence, when the market undervalues the currency with respect to the official value, the necessary intervention by the central bank is to buy the currency. This is in line with conventional economic reasoning: demand more of something and its price goes up.

Unfortunately, this simple policy of purchasing one's own currency is problematic. The reason is that, in the currency market, buying the home currency implies selling the foreign currency. A central bank has no problem in creating its own currency, but to the sell another central bank's currency, that currency must have been previously obtained: a central bank does not have the power to create foreign currencies. Typically, a central bank can swap currencies with another central bank, but that depends on both banks accepting the swap.

In any case, the point is that there is eventually a limit to the capacity of a central bank to uphold its own undervalued currency. That capacity depends on the stock of foreign currency at the central bank's disposal. If this stock is exhausted, and the market still undervalues the currency, it is no longer possible for the central bank to push up the value of the currency by purchasing it because it cannot pay the purchase.

• **Remark 1. Market undervaluation vs overvaluation.** It follows from the above analysis that there is asymmetry: the central bank would face no financial problem if the goal were instead to depreciate the currency (when the market value is above the official value). In that case the required intervention is to sell the domestic currency, and that the central bank can do theoretically without any limit. A by-product of this sort of intervention (that central banks value positively) is the accumulation of foreign currency (invaluable ammunition when the mandated intervention is the opposite: to buy the domestic currency). The asymmetry is that it is more difficult to uphold an undervalued currency than an overvalued one.

• **Remark 2. Central bank interest rate.** It is time for strategy (ii) when (i) fails. In the example above if $e < e'$ persists after all the dollars owned by the ECB have been used to try to appreciate the euro, then (ii) would require increasing the ECB interest rate. The reason is simple: if the ECB, directly, by itself, cannot generate enough demand for the euro to raise its value sufficiently, then the ECB could induce other market participants (private financial investors) to create the additional demand for euros. A higher ECB rate moves all the European interest rates up; higher European interest rates mean that owners of European financial assets obtain a higher rate of return; that makes European financial assets a more appealing financial investment option for foreign investors; this increases the demand for European financial assets by foreign investors; in the final analysis, the demand for euros in the currency market expands, which contributes to appreciate the euro. Yet, there is again a down side of upholding the exchange rate by means of interest rate hikes: domestic borrowers have to pay more for their loans (economic activity tends to be negatively affected by an interest rate rise: if loaning money is more costly, less money is borrowed; and a great deal of borrowing is made to spend in the real sector, that is, to purchase goods or services).

• **Remark 3. Devaluation.** What if neither (i) nor (ii) suffice to defend the fixed exchange rate? Help from the foreign central bank could be requested. If that fails, capital controls could be established, limiting the amount of buying or selling of the domestic currency in the market. Introducing capital controls sets a dangerous precedent. Remind that foreign investors purchase our assets to make a profit in our currency but with the ultimate goal of converting those profits in their currency. Capital controls put constraints on that convertion. In consequence, those investors need to trust that 'market freedom' will be respected. If capital controls are introduced now, the central bank may benefit now from that policy, but at the cost of losing financial investors in the future (which contribute decisively to give value to our currency in the market by demanding it). The lesser evil between sacrificing the central bank's reputation and sacrificing the domestic currency's value is the later. That is why <u>devaluation puts an end to all previous attempts to uphold the exchange rate</u>.

Speculation: friend or foe?

Speculators are sharks following the blood to get prey. In the currency market, the weaker prey is a fixed exchange rate that the market undervalues and blood is a continuous, not effective enough, central bank intervention to uphold the fixed rate. To make things more interesting, speculators may have the power to determine how 'the market' values currencies and how effective a central bank intervention can be.

A view blames speculators for chasing preys not strong enough to defend themselves; another view holds that speculators just exploit the bad decision of putting yourself in the position of a hunted prey. The first view considers speculators the cause of the problem (a fixed exchange rate devaluation unwanted by the central bank); the second that they are the symptom of a problem (and simply exploit the troubles others have created to themselves). In the first view, speculators are considered murderers; in the second, scavengers.

How to become a millionaire in a single day

• **Example.** The euro-dollar exchange rate is $e = 2$ \$/€ today. Suppose I expect the exchange to be $e' = 1.9$ \$/€ tomorrow. Let the overnight (daily) interest rate be 3‰. If my expectation is correct, tomorrow I will become a millionaire. This is the recipe.

- **Step 1.** I ask for a loan of, say, €100 million. Tomorrow I will have to return this amount plus the interest payment €300,000.
- **Step 2.** With my €100 million, and given the rate $e = 2$ \$/€, I purchase \$200 million.

- **Step 3.** I could lend those dollars for a day (or buy a US financial asset), but since the day has been hard enough I just rest and wait for tomorrow.
- **Step 4.** Tomorrow comes and, of course, I am right. I then sell the $200 million at the rate $e' = 1.9$ $/€ and get €105,263,157 (the additional cents, left as a tip). I next repay my €100 million debt plus the loan interest of €300,000.
- **Step 5.** I finally search for a fiscal paradise that would welcome my remaining €4,963,157…

What if I am wrong and, for instance, $e'' = 2.1$ $/€ instead of $e' = 1.9$ $/€? Then I have a little problem, since, at the rate e'', I can only obtain €95,238,095.23 from my $200 million. Plainly stated: I incur a big loss, given that the 95-odd million euros cannot cover the 100-odd million euros debt.

Going short and going long

Steps 1-5 in the preceding example illustrate a simple speculation strategy: I go into debt for things that I am confident enough that will lose value, for, if I am right, it will be cheaper to repay the debt and I will extract a profit from a value decline. This strategy is known as short-selling.

Short-selling (or going short) consists of

- borrowing now some good or financial asset to
- sell it now, expecting to make a profit by
- buying the good or asset later, when it is time to return it to the lender, at a smaller price.

In the example above I (as a short-seller) assumed a debt in euros because I expected a depreciation of the euro. Hence, by purchasing dollars, I expected to obtain next more euros for the same dollars, so that my debt could be repaid with cheaper euros.

Going long is the strategy opposite to short-selling: an asset or good is bought expecting its price to rise.

Speculation and currency crises

In a fixed exchange rate regime, the expression 'currency crisis' refers to a forced devaluation of a significant magnitude (in a floating regime, a currency crisis is associated with a sharp and sudden depreciation).

Severe currency crises are associated with speculative attacks on exchange rates. Those 'attacks' are implemented through short-selling. Short-selling is widely recognized as capable of triggering currency crises. It is no surprise that in September 2008 restrictions to short selling were imposed. At that time, dreadful news about the state of US banks made everyone panic, fearing what actually subsequently occurred: the deepest global financial crisis in nearly a century (the public start of the crisis is ascribed to investment bank Lehman Brothers filing for bankruptcy on 15 September 2008, then the biggest bankruptcy in US history: debt of $619 billion against $639 billion in assets). Short-selling bans were quickly introduced.

> "Washington, D.C., Sept. 19, 2008 — The Securities and Exchange Commission, acting in concert with the U.K. Financial Services Authority, took temporary emergency action to prohibit short selling in financial companies to protect the integrity and quality of the securities market and strengthen investor confidence. The U.K. FSA took similar action yesterday."

<https://www.sec.gov/news/press/2008/2008-211.htm>

In 2008, short sales were banned for stocks in Australia, Greece, Italy, Japan, South Korea, Spain …

Why banning short sales? Because they help to magnify trouble. A speculative attack against a currency is just a bet on the value of a currency. When there are serious reasons justifying the bet and pointing to the direction of the bet, a speculative attack is a sure thing.

For example, if country X has a consistently higher inflation rate than country Y, the exchange rate drifts towards the depreciation of X's currency with respect to Y's currency. The simple explanation is that prices of goods going up faster in X than in Y induces consumers in X to shift from domestic to foreign consumption, which amounts to demanding more of Y's currency and, consequently, depreciating X's currency against Y's.

Similarly, if X's grows faster than Y's, then X's imports from Y's will grow faster than Y's imports from X's and, as a result, in net terms, demand for Y's currency will rise and X's currency will depreciate.

Further, if X's interest rate falls, domestic financial investors will shift investments abroad, more demand for Y's currency and a depreciation of X's currency.

A speculative attack in a floating regime is less likely, as some speculators may interpret that there is a tendency for a depreciation whereas others may interpret the same, or related factors, as pointing to an appreciation. Their simultaneous bets could easily cancel each other and have no effect on the exchange rate.

Contrariwise, at the outset, a fixed regime makes attacks more likely because the fixed rate serves as a reference point: if the combination of factors affecting the economy move the value of its currency away from the fixed rate systematically, it tends to do it in a specific direction, either consistently above the fixed rate or consistently below. When that consistency is persistent, the expectations of speculators converge, are correlated: either all (or most) believe in an overvaluation or all (or most) believe in an undervaluation. This is dangerous because speculators join forces and make more likely what they expect to occur (and they have a strong interest in making it happen).

• **Example 1. George Soros, the financier who broke the Bank of England in 1992.** On 16 September 1992 the UK suffered from a currency crisis that forced the UK Government to leave an exchange rate agreement (the European Exchange Rate Mechanism, the precursor of the euro). George Soros became then famous for engineering a successful speculative attack against the sterling. By short selling sterling, it is claimed that Soros made a profit of over one billion pounds.

From Wikipedia (2025):

> "In the months leading up to Black Wednesday, George Soros, among many other currency traders, had been building a huge short position in sterling that would become immensely profitable if the currency fell below the lower band of the ERM. Soros believed the rate at which the United Kingdom was brought into the Exchange Rate Mechanism was too high, inflation was too high (triple the German rate), and British interest rates were hurting their asset prices."

"The UK government attempted to prop up the depreciating pound to avoid withdrawal from the monetary system the country had joined only two years earlier. John Major authorised the spending of billions of pounds worth of foreign currency reserves to buy up sterling being sold on the currency markets. These measures failed to prevent the pound falling below its minimum level in the ERM. The Treasury took the decision to defend sterling's position, believing that to devalue would promote inflation."

"Currency traders began a massive sell-off of pounds on Wednesday, 16 September 1992. The Exchange Rate Mechanism required the Bank of England to accept any offers to sell pounds (…) The Bank of England's intervention was ineffective because traders were dumping pounds far faster. The Bank of England continued to buy, and traders continued to sell."

"At 10:30 am on 16 September, the British government announced an increase in the base interest rate, from an already high 10%, to 12% to tempt speculators to buy pounds. Despite this and a promise later the same day to raise base rates again to 15%, dealers kept selling pounds, convinced that the government would not keep its promise. By 7:00 pm that evening, Lamont announced Britain would leave the ERM and rates would remain at the new level of 12%; however, on the next day the interest rate was back to 10%."

"In 1997, the UK Treasury estimated the cost of Black Wednesday at £3.14 billion, which was revised to £3.3 billion in 2005."

Wikipedia (2025): "Black Wednesday" <https://en.wikipedia.org/wiki/Black_Wednesday>

• **Example 2. The 1997 Southeast Asian Financial Crisis.** This crisis is a textbook case of fixed exchange rates becoming vulnerable to speculative attacks justified by governments pegging their currencies to the currency of a stronger economy. The stability of a hard pegs depends on the economies linked by the peg having a similar dynamics (in terms of GDP growth, inflation rates, interest rates…). A growing difference between economies translates into a growing tension on the exchange rate that, if not corrected, is an invitation to a speculative attack. The hard lesson of currency crises under a hard peg is that a viable peg demands discipline to keep the involved economies in line. Speculative attacks are reminders that discipline has to be maintained at all costs. There is a sort of trade-off: with a fixed exchange rate a problem is avoided (the volatility in the value of the currency) but, perhaps inadvertently, another one must be assumed (the currency peg makes the involved economies be also pegged, so that by the currency peg is not consistent with divergent economic paths). In the Asian crisis, substantially less developed economies pegged their currencies to the US dollar, the currency of the most advanced and powerful economy. Replicating the behaviour of the US economy proved, in the medium run, an unassuming challenge.

From Wikipedia (2025):

"The crisis began in Thailand in July 1997 before spreading to several other countries with a ripple effect, raising fears of a worldwide economic meltdown due to financial contagion (…) Originating in Thailand, on 2 July, it followed the financial collapse of the Thai baht after the Thai government was forced to float the baht due to lack of foreign currency to support its currency peg to the U.S. dollar (…) South Korea, Indonesia and Thailand were the countries most affected by the crisis. Hong Kong, Laos, Malaysia and the Philippines were also hurt."

Wikipedia (2025): "1997 Asian financial crisis"
< https://en.wikipedia.org/wiki/1997_Asian_financial_crisis >

From Google:

"The 1997 Southeast Asian (or Asian Financial) Crisis began in Thailand and rapidly spread across East and Southeast Asia, triggered by the collapse of the Thai Baht in July 1997 due to speculative attacks fueled by economic imbalances, particularly in real estate, and heavy foreign borrowing. The crisis caused currency devaluations, deep recessions, rising unemployment, and social instability in affected 'tiger economies' like Indonesia, South Korea, and Thailand. The International Monetary Fund (IMF) provided bailouts with strict conditions for economic reforms and led to a long process of financial restructuring and recovery."

Does speculation destabilize?

The success of speculative operations depends on the degree of accuracy of the guesses or expectations. The more indications that an exchange rate will move in one direction, the more incentive is given to speculation since, in general, in the short term, a rate can either increase or decrease.

Currency crises suggest that one of the dangers of adopting a fixed exchange rate is that, over time, pressures on the exchange rate tend to align in a clear direction. Inflation rates are an easy indicator of possible misalignment. For example, the currency of a country that experiences more inflation than another will tend to depreciate (relative to the currency of the other country). If a fixed exchange rate has been adopted, the natural expectation is that the rate will experience downward pressure, not upward pressure. This makes it safer to bet on the short side: speculating on the belief that the fixed exchange rate will be devaluated.

But even if the exchange rate is flexible, the same forces that push the rate in one clear direction (and not the other) provide an incentive to speculate. It is no longer a question (as with a fixed exchange rate) of making a profit by causing the government to alter the fixed exchange rate, but simply of being successful in predicting the future exchange rate; review the example of how to become a millionaire.

At least one feature makes it worrying that speculation is not subject to restrictions: the ability of speculators to provoke with their actions what they believe will happen (or rather what they want to happen). This is what lies behind a successful speculative attack.

As an illustration, resume the central bank intervention discussion: e' is the fixed exchange rate and the market exchange rate is $e < e'$. A speculative attack can be launched by borrowing euros, selling them in the currency market and getting dollars in return. By selling euros, e tends to fall; if it actually falls, speculators make a profit. Yet, having $e < e'$ forces the central bank to purchase euros in order to put e in line with e'. It is then all a matter of who is stronger: the central bank or the speculators. If the latter can mobilize more euros to sell than the central bank dollars to sell, then the speculative attack is successful (as in the 1992 Black Wednesday).

What makes the attack more likely to be successful? Having had previous rounds of attacks in which the ability of the central bank to appreciate its currency has been tested. Each such attack is a step further to deplete the central bank foreign currency reserves. If speculators anticipate that the central bank lacks enough foreign currency, the success of a further attack will almost be certain and will be launched.

A second factor contributing to make an attack successful is the dynamics of the economy itself. If speculators identify structurals reasons in the economy that make the currency automatically depreciate (the short to medium run tendency is for e to go down and enlarge the gap with e'), it is plain the cost for the central bank to bridge the gap is increasing. It is then just a matter of estimating when the cost will be unacceptable high for the central bank and, thus, the defense of the fixed rate will become insurmountable. A firm belief that the central bank will have to give up is all that is needed to engineer a speculative attack and bring to the present what shall eventually happen.

Observe that the more speculators believe that their bet will succeed, the more they will invest in their bet and the more likely their success. This observation may justify the view that speculation is destabilizing. Again: are the speculators' actions making the peg unsustainable or rather a symptom that the peg was already unsustainable (and, as such, constitutes an invitation to take advantage of it)?

2. Interest rate parities and other parities

Uncovered interest rate parity

In orthodox macroeconomics, parities are theoretical relationships that justify macroeconomic outcomes: they present the final stage of some underlying process. Specifically, parities summarize the economic result of some type of arbitrage.

Interest rate parities express relationships between interest rates and exchange rates. As such, they can be viewed as simple theories of exchange rate determination.

The uncovered interest rate parity is a theoretical relationship between four variables:

- the exchange rate e (measured in \$/€);
- the domestic interest rate i;
- the foreign interest rate i*;
- the expectation e^e about the future exchange rate.

The relationship assumes a series of relatively restrictive conditions:

- for each economy, an interest rate representative of the set of rates in the economy can be defined (for example, the interest rate of a savings or term deposit, or the main one set by the corresponding central bank);
- interest rates are not significantly affected by currency purchases or sales and are the only factor in which investors are interested;
- there is no exchange rate risk;
- there is no risk in collecting interest payments;
- there are no significant transaction costs;
- the two representative rates refer to the same time interval, between times t and $t + 1$;
- there is a generally accepted expectation about the future exchange rate (in $t + 1$), for which reason there is no uncertainty about the future value of the exchange rate;
- financial investors have only two investment options (buy a financial asset with a return equal to the domestic interest rate i or buy a financial asset with a return equal to the foreign interest rate i*).

Uncovered parity is the condition under which two investment options (domestic and foreign) provide the same expected return, measured in the same currency. Expressing the returns in euros, each option would have the following return per euro.

- Option 1: Invest in domestic assets. Each euro invested in t in domestic financial assets would provide $(1 + i\,)$ euros in $t + 1$.

- Option 2: Invest in the foreign asset. Given the exchange rate of e \$/€ , in t each euro could be converted into e dollars. If this amount is invested, also in t, in the foreign financial asset, then $e(1 + i^*)$ dollars are obtained in $t + 1$. To express this yield in euros, it must divided by the exchange rate e^e that is expected for $t + 1$. Thus, option 2 generates $e(1 + i^*)/e^e$ euros for each euro spent.

The uncovered parity condition is satisfied when the sure return $1 + i$ of option 1 matches the expected return $e(1 + i^*)/e^e$ of option 2. Therefore:

$$1 + i = \frac{e(1 + i^*)}{e^e}. \qquad (1)$$

Fig. 4 summarizes the construction of uncovered parity. The parity is 'uncovered' because the term of the righ in (1) is an expectation, not an actual value: the option of investing abroad is not covered against the risk of predicting the exchange rate wrongly.

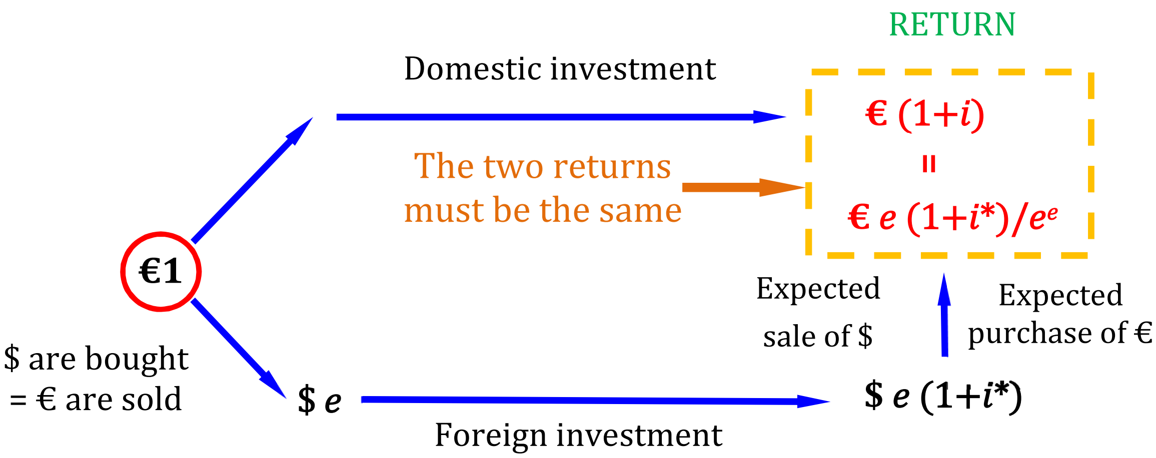


Fig. 4. Sketch of the uncovered interest rate parity

Uncovered parity implicitly defines a very short-term model of the exchange rate. Condition (1) expresses the equality between two return functions:

- the domestic return function $R_d = 1 + i$; and
- the foreign return function $R_f = e\frac{1+i^*}{e^e}$.

Function R_d does not depend on the exchange rate e. Function R_f depends linearly on e, taking the term $\frac{1+i^*}{e^e}$ as a parameter. Fig. 5 represents the two functions, in the space where the exchange rate e is measured on the vertical axis and the expected return in euros is measured on the horizontal axis.

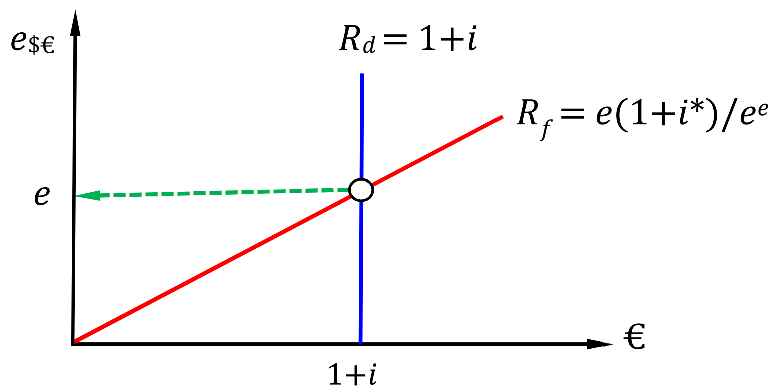


Fig. 5. Exchange rate determination model based on the uncovered parity

The intersection of the two lines in Fig. 5 identifies the value of the exchange rate that equalizes returns and, therefore, satisfies the uncovered parity. The model in Fig. 5 is useful to predict the effect on the exchange rate of variations in any of the other three variables (the two interest rates and the expected exchange rate).

Fig. 6 on the right illustrates the effect on the exchange rate e of a reduction in the domestic interest rate i. Point 1 (where functions R_d and R_f ar) determines the initial exchange rate. Since i affects function R_d but not function R_f, a change in i only modifies R_d. Specifically, a decrease in i shifts R_d to the left. In Fig. 6 the equality between the new R_d and the old function R_f occurs at point 2. The passage from point 1 to point 2 means that the exchange rate decreases, meaning a depreciation of the domestic currency.

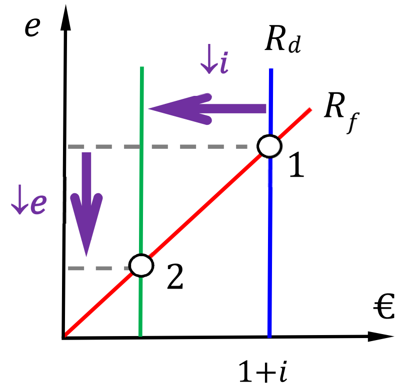


Fig. 6. Effect on e of a fall in i

The conclusion is that

- if the domestic interest rate falls, then compliance with the uncovered parity demands a reduction in the exchange rate: a falling domestic interest rate depreciates the domestic currency.

The move from point 2 to 1 would represent the effect of an increase in the domestic interest rate. Therefore,

- if the domestic interest rate increases, then compliance with the uncovered parity requires an increase in the exchange rate: a rising domestic interest rate appreciates the domestic currency.

Without knowing the uncovered parity, it is possible to explain why a reduction in the domestic interest rate depreciates the domestic currency on the basis of the demand for domestic financial assets made by foreigners.

If the domestic interest rate goes down, domestic financial assets are less profitable and foreign investors will reduce the demand for domestic financial assets and, by extension, reduce the demand for domestic currency, which then depreciates (domestic financial assets will also be less profitable than before for domestic investors, who will buy more foreign financial assets, for which reason they will demand more foreign currency, offer more of the domestic currency and contribute to its depreciation).

The uncovered parity argument is based on the idea that (under the conditions of the model) the return on the purchase of domestic assets must be equal to the return on the purchase of foreign assets (once the returns are expressed in the same currency). Specifically, it is necessary that

$$1 + i = e\frac{1 + i^*}{e^e}.$$

Since the model takes the exchange rate as the expained variable, it is a question of determining how the exchange rate e need to react to a domestic interest rate i decline if the goal is to maintain the equality of yields. Reducing i makes the left side of the equality smaller. Consequently, if only e is to be modified and equality is to be preserved, the right side will also have to be made smaller, which requires a decrease in the exchange rate e.

The economic explanation is as follows. If the return on domestic investment decreases (so that domestic investment becomes less attractive) it is also necessary to make investment abroad less attractive (causing an equivalent reduction in the return on investment abroad) by making it more expensive to acquire foreign currency (through the depreciation of the domestic currency).

For example, if $i = 10\%$, $i* = 5\%$ and $e^e = 2$ \$/€, then

$$e = e^e \frac{1+i}{1+i^*} = 2\frac{1{,}1}{1{,}05} = \frac{220}{105} = \frac{44}{21} \approx 2{,}1 \text{ \$/€}\,.$$

Suppose that i falls to 5%. Now the new exchange rate e' satisfies

$$e' = e^e \frac{1+i'}{1+i^*} = 2\frac{1{,}05}{1{,}05} = 2 \text{ \$/€}\,.$$

Interpretation: to avoid the flight of investments abroad ('capital flight') due to the decrease in the domestic interest rate i, the domestic currency must depreciate (so that the profitability in euros of the investment abroad also decreases appropriately: the depreciation of the euro makes the purchase of American financial assets more expensive).

Fig. 7 on the right shows the effect on the exchange rate of an increase in the foreign interest rate i*. Point 1 indicates the initial exchange rate. Since i* affects R_f but not R_d, a change in i* only alters R_f. Specifically, an increase in i* shifts R_f to the right (or downwards): if i* goes up, the value $e\frac{1+i^*}{e^e}$ (given e and e^e) is larger. In Fig. 7 the equality between the new R_f and old function R_d occurs at point 2. The passage from point 1 to point 2 means that the exchange rate e decreases: the domestic currency depreciates.

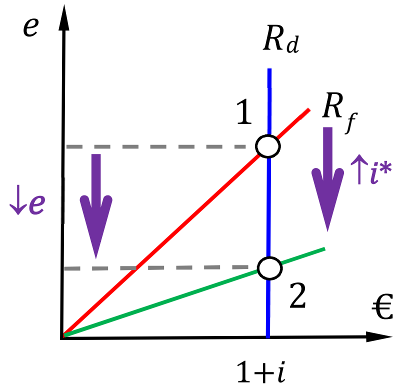


Fig. 7. Effect of i* on e

The explanation of this result is similar to that following a decrease in i. Both an rise in i* and a fall in i make financial investment abroad more attractive. The parity condition requires disincentivizing investment abroad. To achieve this by only modifying the exchange rate, it is necessary an exchange rate reduction: the decline in the value of the domestic currency makes it more expensive to acquire foreign currency and, therefore, makes it more expensive to purchase foreign financial assets and thus makes investment abroad less profitable.

Fig. 8 below represents the impact on the exchange rate of a decrease in the expected exchange rate (a depreciation of the euro is expected). A decrease in e^e increases the value $e\frac{1+i^*}{e^e}$, so the function R_f shifts to the right (for each value of e, a decrease in e^e enlarges R_f). The passage from point 1 to point 2 means a depreciation of the euro.

This result is significant: the fulfilment of the uncovered parity transforms an expected, future, depreciation into a real, present, depreciation. Expecting the euro to depreciate in the future makes the euro depreciate now. If the euro is expected to lose value in the future, obtaining returns in dollars will lead (after being exchanged for euros at the expected rate) to have more euros than before the expected value loss. This stimulates investment abroad (the fall in e^e shifts R_f to the right). As in the other two cases, parity requires that the shock be counterbalanced by a euro depreciation now (which thus makes it more expensive to obtain dollars to invest abroad).

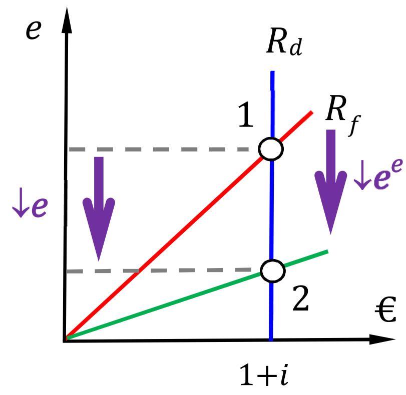


Fig. 8. Effect of e^e on e

This case makes evident the importance of expectations regarding the evolution of economic variables: since expectations can change quickly and significantly, the variables that depend on expectations (the exchange rate, in this case) become volatile and potentially unstable.

In fact, the fall of e^e analyzed in Fig. 8 could be due to errors in the interpretation of reality, bad faith, fraud intent, aggressiveness in the search for benefits or hypersensitivity to economic information. The modification in e^e could even not be justified by any objective reason of substance. Unfortunately, the cause of the change in e^e is irrelevant because the crux of the matter is that the modification in e^e is transferable to e. It is then not surprising that some market participants could be interest in manipulating information to induce less informed participants to believe that certain market events will occur. Speculation feeds on that possibility of expectation manipulation.

It is more common to present the uncovered parity condition in an approximate version of equation (1). Define

$$E^e = \frac{e^e - e}{e}$$

as the expected rate of appreciation of the exchange rate (expressed as a multiple of one; to obtain a percentage, multiply the formula by 100). For example, if $e^e = 3$ \$/€ and $e = 2$ \$/€, then the expectation is that the euro will appreciate by

$$E^e = \frac{e^e - e}{e} = \frac{3-2}{2} = \frac{1}{2} = 0{,}5 = 50\%$$

with respect to the dollar. If $e^e < e$, then E^e would be an expected rate of depreciation. For example, if $e^e = 3$ \$/€ and $e = 4$ \$/€,

$$E^e = \frac{e^e - e}{e} = \frac{3-4}{4} = \frac{-1}{4} = -0{,}25 = -25\%$$

means that it is believed that the euro will depreciate by 25% against the dollar.

Starting from (1) ,

$$\frac{e^e}{e}(1+i) = 1+i^*$$

$$\left(\frac{e^e}{e} - 1 + 1\right)(1+i) = 1+i^*$$

$$(E^e+1)(1+i) = 1+i^*$$

$$1 + i + E^e + E^e i = 1 + i^*$$

$$E^e + E^e i = i^* - i.$$

When the values of i and E^e are small enough, their product is almost negligible. All this justifies the approximate version of uncovered parity, according to which

$$\boxed{E^e \approx i^* - i.} \qquad (2)$$

By the approximate version of the uncovered parity, the expected rate of appreciation of the euro against the dollar is approximately equal to the foreign interest rate (the dollar's) minus the domestic interest rate (the euro's). In (2) the three variables can be expressed in percentages.

As an illustration, if $i^* = 5\%$ and $i = 3\%$, then the uncovered parity requires an appreciation of the euro relative to the dollar of approximately 2%. Since the exact formula is

$$E^e + E^e i = i^* - i$$

an error equal to $E^e i$ is made. Specifically, if $e = 2$ \$/€, by (1), it is necessary that $e^e = \frac{e(1+i^*)}{1+i} = \frac{2(1+0{,}05)}{1+0{,}03} = \frac{210}{103} = 2{,}03883$ \$/€. The step from $e = 2$ \$/€ to $e^e = 2{,}03883$ \$/€ represents a change of 1.9417%, close enough to the approximate value of 2%.

Sometimes, it is reasonable to assume that the two investments, domestic and foreign, are not perfect substitutes. One reason is that one of the two is considered riskier; for example, because the characteristics of one country are less well known than the other or, simply, because investing in one of them is subject to greater uncertainty.

The inclusion of this asymmetry leads to a generalization of the parity condition (1) by applying a risk premium p to one of the two investments. In particular, (1) would take the form

$$1 + i = \frac{e(1+i^*)}{e^e} + p. \qquad (3)$$

The premium p (which can be positive or negative) would serve to compensate for the possible costs, disadvantages, risks or uncertainties of investing in one of the countries in comparison with the other. For example, investment in less developed countries is considered riskier than investment in developed countries; that is why, without a sufficient compensatory premium, there would be no investment by foreigners in less developed countries.

Covered interest rate parity

The exchange rate considered so far is the spot exchange rate, meaning that the exchange of currencies is made relatively immediately (within a few hours or perhaps, at most, a couple of days). This means that in each time period there is a spot currency market, with its own spot exchange rate. Fig. 9 represents the set of spot currency markets, one for each time, with the corresponding spot exchange rate: e_1, e_2, e_3, … The expectations formed in the two initial periods about future spot exchange rates are also indicated: ${}^1e_2^e$ is the expectation formed in $t = 1$ about the spot exchange rate e_2 in $t = 2$; ${}^1e_3^e$ is the expectation formed in $t = 1$ about the spot exchange rate e_3 in $t = 3$; … ; ${}^2e_3^e$ is the expectation formed in $t = 2$ about the spot exchange rate e_3 in $t = 3$; … It may be that ${}^1e_3^e \neq {}^2e_3^e$ because relevant information for $t = 3$ in may emerge in $t = 2$ that was ignored in $t = 1$.

time	1	2	3	4	5	6	7	…
spot exchange rate	e_1	e_2	e_3	e_4	e_5	e_6	e_7	…
expectation in $t = 1$	–	${}^1e_2^e$	${}^1e_3^e$	${}^1e_4^e$	${}^1e_5^e$	${}^1e_6^e$	${}^1e_7^e$	…
expectation in $t = 2$	–	–	${}^2e_3^e$	${}^2e_4^e$	${}^2e_5^e$	${}^2e_6^e$	${}^2e_7^e$	…

Fig. 9. Proliferation of exchange rates based on time and expectations

The uncovered parity links spot rates with expected rates. For instance, at time $t = 1$, the parity relates the values in the yellow boxes and, consequently, one degree of freedom is lost: the values of the two variables do not arise independently.

Contractual freedom makes it possible to sign a currency exchange contract now to be executed in the future (a week, a month, a year, a decade, a century, a millennium...). This type of contract is called a 'futures contract': it creates an obligation now (to deliver a good, an asset or whatever at a future time at a price agreed upon now) to be fulfiled in a preset future.

Fig. 10 shows the forward exchange rates (${}^te_r^F$) created at time t and which establish the terms of exchange between currencies in futures contracts signed at t but which must be fulfilled at time r.

time	1	2	3	4	5	6	7	…
spot exchange rate	e_1	e_2	e_3	e_4	e_5	e_6	e_7	…
forward rate in $t = 1$	–	${}^1e_2^F$	${}^1e_3^F$	${}^1e_4^F$	${}^1e_5^F$	${}^1e_6^F$	${}^1e_7^F$	…
forward rate in $t = 2$	–	–	${}^2e_3^F$	${}^2e_4^F$	${}^2e_5^F$	${}^2e_6^F$	${}^2e_7^F$	…
forward rate in $t = 3$	–	–	–	${}^3e_4^F$	${}^3e_5^F$	${}^3e_6^F$	${}^3e_7^F$	…
forward rate in $t = 4$	–	–	–	–	${}^4e_5^F$	${}^4e_6^F$	${}^4e_7^F$	…

Fig. 10. Forward foreign exchange rate markets (between only two currencies)

Given that the passage of time modifies the information available on everything that could influence an exchange rate and can also modify both the expectations of the future or the participants in the markets, it is reasonable that each time defines new forward rates. Specifically, ${}^1e_4^F$ is the exchange rate determined at time 1 for exchanges at time 4 and ${}^2e_4^F$ is the exchange rate determined at time 2 for exchanges at time 4. The two values could be equal, but this would be unlikely, given that more information is available at time 2 than at time 1 (or the underlying economic reality may have changed significantly). Therefore, it is justified to interpret each box containing an exchange rate value as a different market.

The most significant macroeconomic consequence of arbitrage is the reduction (in practice) of the number of markets (number that spontaneously increases over time). The connection that arbitrage establishes between prices in different markets is commonly known as a parity, since the action of arbitrage ends up being specific in linking the values of the prices involved in the arbitrage.

Specifically, the uncovered interest parity related (restricting attention to exchange rates) to the shaded values in Fig. 9: the spot exchange rate e_1 and the expectation of the spot exchange rate for the next period ${}^1e_2^e$. In fact, there is a parity that relates e_1 to the entire set of expectations formed at time 1 (e_1 and ${}^1e_2^e$; e_1 and ${}^1e_3^e$; e_1 and ${}^1e_4^e$; …), if the period to which the interest rates refer is appropriately adjusted (for example, the uncovered parity that relates e_1 and ${}^1e_2^e$ would require considering interest rates for two periods).

Similarly, the covered interest parity relates the shaded values with the same color in Fig. 10. Specifically, the covered parity that connects the spot exchange rate e_1 to forward exchange rate${}^1e_2^F$ of the next period, but the explanation will be valid for connecting e_2 and ${}^2e_3^F$, e_3 and ${}^3e_4^F$… (and adjusting the period of the interest rates e_1 and ${}^1e_3^F$, e_1 and ${}^1e_4^F$, …).

The covered interest rate parity is a relationship between four variables:

- the exchange rate (spot exchange rate) e (expressed in \$/€);
- the domestic interest rate i;
- the foreign interest rate i*;
- the forward exchange rate e^F.

The forward rate e^F is associated with a futures contract, where the parties agree to exchange a currency for the other at a future time according to the price established by e^F.

Covered parity is analogous to uncovered parity. The only difference is that the future exchange rate is not an expectation but a certain, known value. As with uncovered parity, covered interest parity is the condition under which two financial investment options (domestic and foreign) provide the same certain return, measured in the same currency. Measuring returns in euros, each option would have the following return per euro.

- Option 1: Invest in a domestic asset. Each euro invested in t in domestic financial assets would provide $(1 + i)$ euros in $t + 1$.
- Option 2: Invest in a foreign asset. Given the exchange rate of e \$/€, in t, each euro can be converted into e dollars. If these dollars are invested, also in t, in the foreign financial asset, $e(1 + i^*)$ dollars will be received in $t + 1$. The forward rate e^F allows these euros to be sold in $t + 1$ and get $e(1 + i^*)/e^F$ euros for every euro initially spent.

The covered parity condition is satisfied when the safe yield $1 + i$ of option 1 matches the yield, also safe, $e(1 + i^*)/e^F$ of option 2. Therefore:

$$1 + i = \frac{e(1 + i^*)}{e^F}. \tag{4}$$

Fig. 11 schematically summarizes the articulation of uncovered parity.

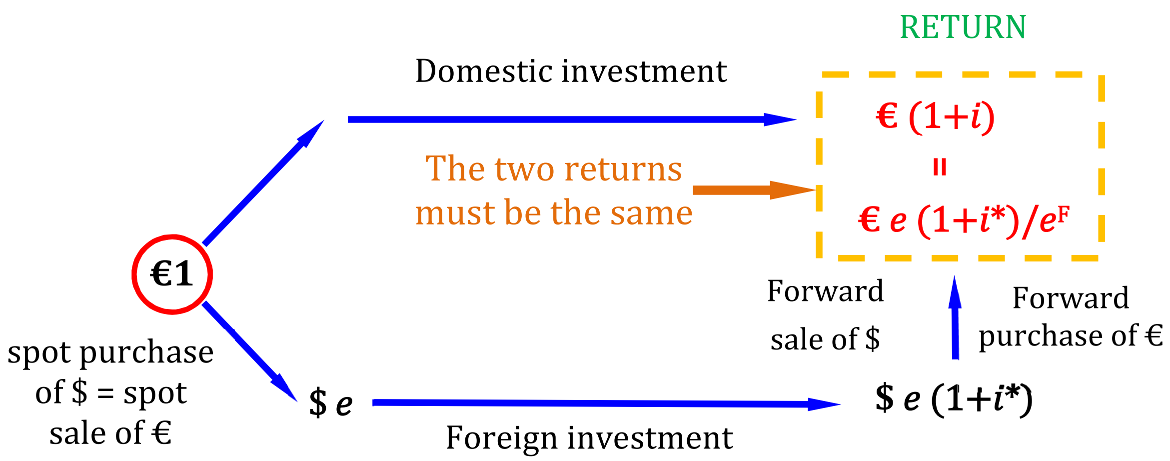


Fig. 11. Scheme of interest-covered parity

A forward exchange rate model can be derived from the covered parity. Condition (4) expresses the equality between two return functions:

- the domestic return function $R_d = 1 + i$; and
- the foreign return function $R_f = e\frac{1+i^*}{e^F}$.

Function R_d does not depend on the forward rate of exchange e^F. Function R_f depends hyperbolically on e^F (the term $e(1 + i^*)$ is now considered a parameter).

Fig. 12 on the right represents functions R_d and R_f graphically, in the space where the forward exchange rate e^F is measured on the vertical axis and the expected return in euros is measured on the horizontal axis.

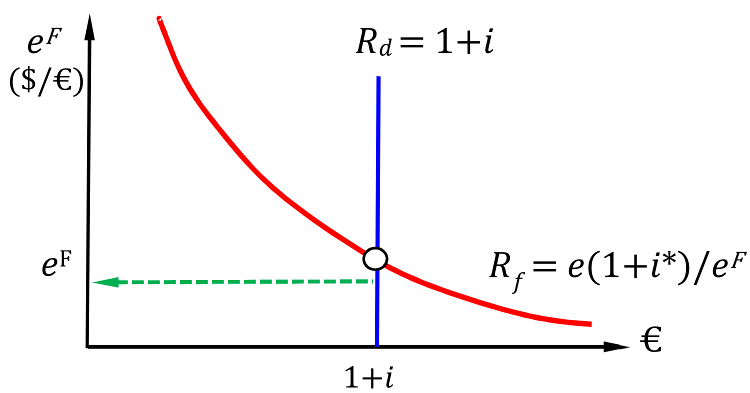


Fig. 12. Exchange rate determination model based on the covered parity

In the uncovered parity model, the exchange rate e was supposed determined by three other variables: i , i^* and e^e. Now, the forward exchange rate e^F is assumed determined by the spot rate e and the interest rates i and i^*.

Expression (4) can be manipulated to obtain a condition analogous to (2), that is, an approximate version of the covered parity:

$$E^F \approx i^* - i \tag{5}$$

where

$$E^F = \frac{e^F - e}{e}$$

represents the forward premium: the excess of the forward rate over the spot rate, relative to the spot rate (the percentage that separates forward and spot rates). If $e^F < e$, then the premium is negative: the euro depreciates in the forward rate relative to its spot value (present value).

The equality $1 + i = \frac{e(1+i^*)}{e^F}$ that characterizes the covered parity can be justified thus. Suppose the equality is not fulfilled: let $1 + i > \frac{e(1+i^*)}{e^F}$. As an exercise, try to adapt the following argument to the case $1 + i < \frac{e(1+i^*)}{e^F}$. The aim is to demonstrate that, when the equality of returns does not occur:

- there are arbitrage opportunities (safe profits can be made); and
- taking advantage of these opportunities reduces the inequality of returns.

If the above occurs, then the inequality of returns is not a stable situation and, moreover, arbitrage will tend to eliminate the inequality. The conclusion would be that only with equality of returns the resulting situation is stable (as there would be no opportunities for arbitrage).

The exploitation of any arbitrage opportunity involves buying cheap and selling expensive. Having $1 + i > \frac{e(1+i^*)}{e^F}$ is equivalent to $e^F > \frac{e(1+i^*)}{1+i}$. This suggests that the euro is worth more in the future than in the spot market, which suggests buying euros in the spot market and selling them in the futures market. The following set of operations aims to check whether this strategy works, even assuming no funds are initially avaiable to make the arbitrage. For ease of exposition, assume:

$$e^F = 3 \text{ \$/€}$$
$$e = 2 \text{ \$/€}$$
$$i = 10\%$$
$$i^* = 20\%.$$

- Step 0: <u>Sell euros on a forward basis</u>. Initially, €55 is sold on a forward basis , which must be sold in exchange for dollars in the future at the price set by the forward rate, $e^F = 3$ \$/€. The following steps will make clear the reason for this operation.
- Step 1: <u>Request a loan of dollars</u>. Specifically, you borrow \$100. This means you will have to pay \$100 plus 20% in the future, that is, \$120.
- Step 2: <u>Sell the dollars for euros in the spot market</u>. You get €50 from the sale of the \$100.
- Step 3: <u>Invest the euros</u>. Given that the domestic rate is 10%, the €50 loan would generate a gross return of €50 plus 10%: €55.

- Step 4: Execute the initially agreed forward sale of euros. Step 3 has generated the €55 that, by contract in step 0, had to be delivered now. At price $e^F = 3$ \$/€, you receive \$165 in exchange for €55.

- Step 5: Pay off the debt in dollars. Step 1 created the obligation to pay \$120 now. Step 4 has produced \$165.

- Step 6: pocket the profits. After deducting the \$120 to be paid as repayment of the loan in step 1, there are \$45 left of the \$165 obtained in step 5. In short, \$45 has been obtained out of nothing. By multiplying the scale of the operations, \$45 would be obtained for every \$100 borrowed in step 1 (it would be necessary to determine the corresponding volume of euros to be sold on a forward basis in step 0). If having dollars instead of euros were more convenient, the \$45 could be exchanged for dollars at the then spot rate.

The above sequence of operations shows how to exploit arbitrage opportunities when $e^F > \frac{e(1+i^*)}{1+i}$ (or, equivalently, $e^F(1+i) > e(1+i^*)$). What effects would the operations have on these variables?

- First, the forward sale of euros at step 0 would tend to lower the forward value of the euro. Consequently, e^F would tend to fall and, in this way, the higher value $e^F(1+i)$ would approach the lower value $e(1+i^*)$.

- Step 1 contributes to increasing the demand for dollar loans, which would put upward pressure on i^*. This would contribute to the smaller value $e(1+i^*)$ of the inequality moving closer to the higher value $e^F(1+i)$.

- The purchase of euros in the spot market in step 2 represents an increase in the demand for euros and, hence, the euro would tend to appreciate against the dollar. As a result, e would increase and again there would be pressure for the smaller term $e(1+i^*)$ of the inequality to approach the higher value $e^F(1+i)$.

- Finally, the euro loan in step 3 is equivalent (in a competitive setting) to an increase in the supply of liquidity in the domestic liquidity market, so the domestic rate i would tend to decrease. Thus, the upper value $e^F(1+i)$would approach the lower value $e(1+i^*)$.

All in all, the operations carried out in steps 0, 1, 2 and 3 would tend to reduce the difference between the upper value $e^F(1+i)$ and the lower value $e(1+i^*)$. As long as the difference is maintained, the previous steps will continue to generate sure profits. This manna ends when equality between $e^F(1+i)$ and $e(1+i^*)$ is achieved.

The simultaneous existence of spot and forward exchange rates offers the possibility of forward speculation. Forward speculation has an advantage over spot speculation: in principle, it can be conducted without having cash or credit lines to pay for the transactions. By definition, forward purchases are paid in the future, not when agreed (unlike spot purchases).

As an illustration, let $e^F = 2$ \$/€ and suppose that the expected future spot exchange rate (the exchange rate at the time when purchases at the rate are to be made e^F) is $e^e = 3$ \$/€. These values suggest that (it is believed that) in the future, the euro will be cheaper at the forward rate and more expensive at the spot rate. As always, the profitable strategy is to buy the euro where it is cheap and sell it where it is expensive.

Specifically, suppose €100 are bought on a forward basis. For now, this means signing a contract where one commits to buying €100 in the future at the rate $e^F = 2$ \$/€ (consequently, \$200 will have to be paid for the €100).

When the time comes to fulfil the contract, the €100 are purchased and, if one gets it right with the spot exchange rate at that time, the €100 can be sold at the rate $e^e = 3$ \$/€. This means getting \$300, enough to pay the \$200 of the futures contract. The net profit is \$100 (for every €100 purchased in advance).

[How is it that one gets the €100 and makes use of that money before paying for it with dollars? One explanation is that both operations are done with a single agent, such as a bank. You sign the futures contract with the bank, when the time comes you make the spot sale and the bank is responsible for settling the two operations net. This practice (if the bank does not require some kind of guarantee deposit) reinforces the idea that futures speculation does not require having initial means to invest in the speculation. Naturally, if your prediction is seriuosly wrong, the balance of the operation is negative and you incur a debt with the bank.]

This example illustrates how (in this case) speculation would connect the shaded values in the table in Fig. 13.

time	1	2	3	4	5	6	7	...
spot rate	e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
expectations in $t = 1$	–	${}^1e_2^e$	${}^1e_3^e$	${}^1e_4^e$	${}^1e_5^e$	${}^1e_6^e$	${}^1e_7^e$	...
forward rate in $t = 1$	–	${}^1e_2^F$	${}^1e_3^F$	${}^1e_4^F$	${}^1e_5^F$	${}^1e_6^F$	${}^1e_7^F$	...

Fig. 13. Values connected by speculation (expected spot rate and forward rate)

Parities and triangular (spot and forward) arbitrage

Parity analysis done with two currencies can be extended to more currencies. With more currencies there are more markets and, hence, more markets where arbitrage opportunities can arise. Specifically, what about triangular forward arbitrage? Is it necessary (as in the case of spot rates) to ensure consistency between direct and indirect forward rates?

The following remarkable result is demonstrated below (because it facilitates the achievement of consistent values in forward markets): if covered parity is satisfied for each currency pair and if there are no triangular arbitrage opportunities in the spot market, then there are no triangular arbitrage opportunities in the forward market. Schematically,

Covered parity ⇒ (No triangular spot arbitrage ⇒ No triangular forward arbitrage). (6)

This result illustrates once more the idea that arbitrage connects markets (in this case, the spot and forward markets). In particular, the result states that (satisfied the covered parity between every two currencies) if triangular arbitrage opportunities are exhausted in spot exchange markets, then it is not necessary to ask whether there will be wasted triangular arbitrage opportunities in the forward exchange markets: there are none.

Indeed, the converse also holds: covered parity plus absence of triangular forward arbitrage opportunities imply absence of triangular spot arbitrage opportunities. That is,

Covered parity ⇒ (No triangular forward arbitrage ⇒ No triangular spot arbitrage). (7)

By combining both results, the corollary is that the covered parity makes the absence of triangular arbitrage in the spot and the forward markets equivalent. Assuming covered parity, if all possibilities of triangular arbitrage in the forward have been exhausted, then there is no need to worry about triangular arbitrage in spot markets. And vice versa: if all possibilities of triangular arbitrage in the spot markets have been exhausted, then there is no need to worry about triangular arbitrage in the forward markets. To prove (6), take three currencies ($, € and ¥), assume the covered parity (between each pair of currencies) and, furthermore, that there are no triangular arbitrage opportunities in the spot markets. The last assumption means that the direct exchange rate between any two currencies coincides with the indirect exchange rate between them. Formally,

$$e_{\$€} \cdot e_{€¥} = e_{\$¥} .$$

This condition can be expressed equivalently as

$$\boxed{e_{\$€} \cdot e_{€¥} \cdot e_{¥\$} = 1.}$$

If, in addition, the covered parities between each currency pair are met, the covered parity between dollar and euro will be satisfied,

$$\frac{e_{\$€}}{e^F_{\$€}} = \frac{1 + i_€}{1 + i_\$},$$

between euro and yen,

$$\frac{e_{€¥}}{e^F_{€¥}} = \frac{1 + i_¥}{1 + i_€},$$

and between yen and dollar,

$$\frac{e_{¥\$}}{e^F_{¥\$}} = \frac{1 + i_\$}{1 + i_¥}.$$

By isolating the spot rates in the last three equations and introducing the result into the condition of no spot arbitrage, $e_{\$€} \cdot e_{€¥} \cdot e_{¥\$} = 1$, the following equation results:

$$\left(e^F_{\$€} \cdot \frac{1 + i_€}{1 + i_\$}\right) \cdot \left(e^F_{€¥} \cdot \frac{1 + i_¥}{1 + i_€}\right) \cdot \left(e^F_{¥\$} \cdot \frac{1 + i_\$}{1 + i_¥}\right) = 1.$$

Since the all gross interest rates $(1+i)$ cancel out, the condition of absence of triangular forward arbitrage emerges:

$$e^F_{\$€} \cdot e^F_{€¥} \cdot e^F_{¥\$} = 1.$$

What makes (6) and (7) remarkable is that the outcome of triangular arbitrage in forward markets is equivalent (or can be replicated) by the combined efforts of covered parity and triangular arbitrage in spot markets.

Other parities

Arbitrage sustains many other parities. One has already been presented: the formula relating the interest rate and the price of T-bills, itself justifiable by financial arbitrage. Commercial arbitrage justifies an old theory on exchange rates: purchasing power parity (PPP) theory. This theory holds that exchange rates moves to equate the purchasing power of money: that a given amount of domestic currency can buy the same amount of goods at home and abroad. This theory requires a lot of presumptions: negligible transport and transaction costs, similarity between the representative baskets of consumption goods in the economies involved, eventual irrelevance of financially-motivated transactions in the currency market…

The Economist computes and publishes the Big Mac Index, which is a "lighthearted guide to whether currencies are at their 'correct' level" (https://www.economist.com/interactive/big-mac-index; https://worldpopulationreview.com/country-rankings/big-mac-index-by-country). The index is inspired by PPP theory and is used to predict the dynamics of exchange rates under the presumption that they should converge to the values that guarantee that the same amount of money buys the same amount of Big Macs everywhere. There is supplementary material on the course's page illustrating the logic of the Big Mac index.

Empirical evidence does not support PPP theory. There is nonetheless a generalization known as relative purchasing power parity theory, which claims that exchange rates move to neutralize inflation differentials. To be more precise, define

$$E = \frac{e' - e}{e}$$

to be the variation in the exchange rate between two currencies: e is the initial value of the exchange rate and e' is the final value. Then relative PPP theory can be reduced to the formula (itself an approximation of an exact formula):

$$\boxed{E \approx \pi^* - \pi}$$

where is the foreign inflation rate and is the domestic inflation rate. Under relative parity, the domestic currency appreciates when the foreign consumer price index grows faster than the domestic consumer price index, and the magnitude of the appreciation corresponds to the difference in the rate of index growth. The parity logic is this: if the foreign inflation rate is higher than the domestic inflation rate ($\pi^* > \pi$), it is more costly for domestic consumers to buy abroad the same amount of goods as before; to regain purchasing power through the exchange rate, the domestic currency must appreciate (which means $e' > e$ and, hence, $E > 0$).

3. The open economy trilemma

Open economy trilemma

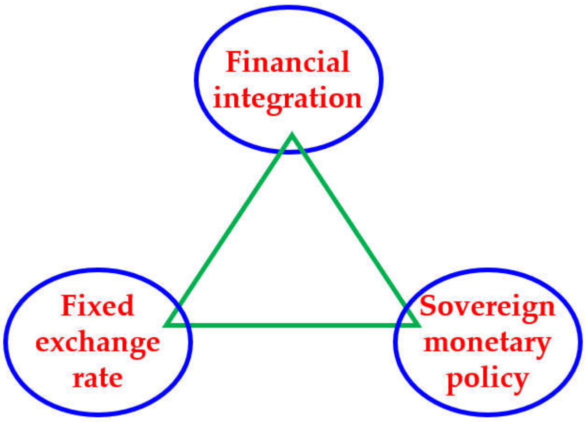
The open economy trilemma is arguably the cornerstone macroeconomic result. It was independently discovered in the 1960s by Marcus Fleming and Robert Mundell. Mundell was awarded in 1999 the Nobel Prize in Economics (The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel).

> "Prize motivation: 'for his analysis of monetary and fiscal policy under different exchange rate regimes and his analysis of optimum currency areas' (…) Exchange rates among different currencies are an important factor in business and the economy. In the beginning of the 1960s, Robert Mundell analyzed the effects of political actions on this area, including the consequences if exchange rates remain fixed or are allowed to be governed by the market. Mundell maintained that it could be advantageous for several countries to introduce a common currency, provided that the labor force has a great deal of mobility. Mundell's theories had relevance for the introduction of the euro as the European Union's currency."

Robert A. Mundell – Facts. NobelPrize.org. Nobel Prize Outreach 2025. Mon. 29 Sep 2025. <https://www.nobelprize.org/prizes/economic-sciences/1999/mundell/facts/>

The open economy trilemma provides a theoretical explanation of currency crises (like the 1997 Asian crisis): governments try to achieve policy goals that are impossible to achieve simultaneously. The trilemma is also know as 'the impossible trinity'.

The 'strict' version of the open economy trilemma contends that it is not possible (or at least no sustainable) to, at the same time,



- have a fixed exchange rate,
- choose the domestic interest rate (or to have an independent monetary policy) and
- allow free international capital mobility (that is, no capital control is imposed on the currency market: anyone can sell or buy any amount of currency anytime).

The free flow condition is often presented as 'be financially integrated with the foreign economy having the currency with respect to which the exchange rate is defined'.

A milder version says that the three goals can be simultaneously achieved but only up to some degree: if the aim is to get closer to some of the three goals, then one has to move away from at least another goal.

There is an apt hydraulic simile. Suppose each goal is represent by a water tank. Three pipes connect each two tanks. Tanks and pipes are filled with water, but not completely. The hydraulic circuit is subject to no external force. Then the more one tries to fill two tanks completely by rotating the whole circuit, the emptier will end the third tank. The attempt to replenish the tank causes some other tank to be less filled.

The trilemma dictates that it is between hard and impossible for a government to

- set the foreign value of its currency against some foreign currency (the exchange rate),
- set the domestic value of its currency (the interest rate: informally, the price of money) and
- let the domestic currency be free (money can cross borders without any constrain).

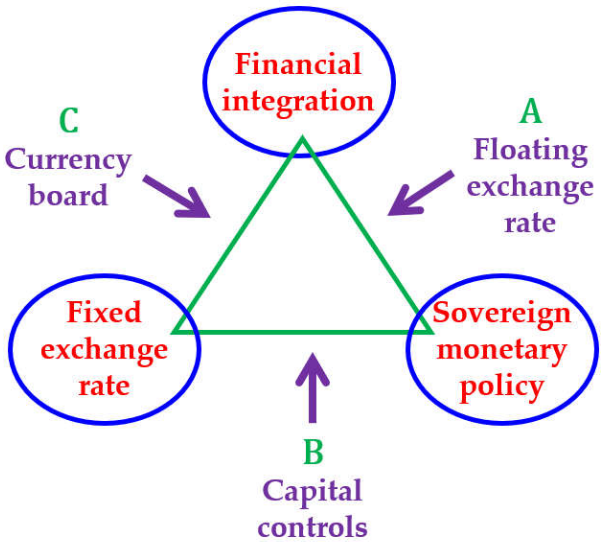
Currrency crises can be seen as failed attempts to have everything: financial integration, exchange rate stability and sovereign monetary policy. By choosing financial integration, monetary policy and exchange rate policy are no longer independent.

Jutifying the trilemma

Suppose e is fixed and that, exercizing the independence of the monetary policy, the central bank lowers the domestic interest rate i. The fall in i makes financial investment in the domestic economy less attractive and, hence, encourages financial investment in the foreign economy. Thus investors reduce financial investment in the domestic economy by selling domestic financial assets in the domestic financial markets, use the proceeds of the sales to obtain foreign currency in the currency market and spend the foreign currency so obtained in the purchase of foreign financial assets. The sale of domestic currency in the currency market lowers the exchange rate e. Since e is fixed, to defend the desired value of e, the central bank has to buy domestic currency (reserves) in the currency market. With fewer reserves around, there is an upward pressure on interest rates, which jeopardizes the original central bank's intent to lower interest rates in the economy. The final result is that the central banks is actually not free to implement the monetary policy they wanted.

Policy options

There are three policy options consistent with the open economy trilemma, displayed on the sketch on the right.



• **Option 1. Floating exchange rate.** If having an independent monetary policy and adopting no capital control are the policies chosen, then the exchange rate must float (UK, Canada).

• **Option 2. Currency board.** Fixing the exchange rate and allowing the free mobility of capital imply that monetary policy cannot be independent. It can be interpreted that the eurozone countries chose this option: their monetary policy was handed to a supranational authority, the European Central Bank. When a single country takes this option, the resulting monetary authority is called a currency board and its goal is merely to adopt the monetary policy of the country (or countries) to which the exchange rate is pegged and be willing to convert into the pegged currency any request of conversion of any amount of domestic currency. Argentina had a currency board in the 1990s when the exchange rate was fixed at one Argentinian peso per US dollar.

• **Option 3. Capital controls.** Capital controls must be established (like China) when the policy options chosen are to control both the foreign value of the domestic currency (by fixing the exchange rate) and, by conducting a sovereign monetary policy, its domestic value (the interest rate).

The euro

Among themselves, the countries that have adopted the euro are financially integrated (money flows free) and have accepted a fixed exchange rate (the new currency, the euro, is the expression of the exchange rate being held permanently fixed; for instance, forever, 166,386 Spanish pesetas = one euro = 1.95583 German marks). Consequently, no country has a sovereign monetary policy: it is a supranational organization, the European Central Bank, that conducts monetary policy for all eurozone members).

Yet, against the rest of the world, the eurozone's choices are different: again financial integration is accepted (the euro can be traded freely against any other currency), the ECB is sovereign to decide on monetary policy issues and, as a result, the euro must float against the currencies of those economies with which the eurozone is financially integrated (US, UK, Canada, Japan…). In view of this, the eurozone has not adopted a fixed exchange rate regime.