An insultingly simple model of GDP determination

Model description

Conventionally, GDP is regarded as the main macroeconomic variable. If an economy had to be summarized by a number, it would be GDP. Thus a basic macroeconomic model should explain how GDP is determined. The model next starts from the savings identity, without foreign sector (or with an always balanced foreign sector):

$$NPS = PD$$

$$S - I = G - T.$$

Investment and public spending are assumed to be constant (or not substantially altered by GDP)

$$I = \bar{I}$$

$$G = \bar{G}$$

and savings and tax revenue depend linearly and positively on GDP, designated by Y,

$$S = s \cdot Y$$

$$T = t \cdot Y$$

where s is the savings rate (the fraction of GDP saved) and t is the tax rate (the fraction of GDP collected by the government). Therefore,

$$s \cdot Y - \bar{I} = \bar{G} - t \cdot Y.$$

Solving for Y,

$$\boxed{Y = \frac{\bar{I} + \bar{G}}{s + t}.}$$

This formula indicates that GDP is a multiple of autonomous spending $\bar{I} + \bar{G}$ (that is, the spending that does not depend on GDP). The term $\frac{1}{s+t}$ is called the spending multiplier: each unit of autonomous spending becomes $\frac{1}{s+t}$ units of GDP ($\frac{1}{s+t} > 1$ because s and t are supposed to be small enough).

It also indicates that GDP depends:

- positively on autonomous investment $\bar{I}$;
- positively on autonomous public expenditure $\bar{G}$;
- negatively on the savings rate s; and
- negatively on the tax rate t.

Furthermore, if the savings rate and tax rate remain constant, a change $\Delta(\bar{I} + \bar{G})$ in total autonomous spending implies a change ΔY in GDP equal to

$$\Delta Y = \frac{1}{s + t} \cdot \Delta(\bar{I} + \bar{G}). \quad (1)$$

Numerical example Let

- $A = \bar{I} + \bar{G}$ be total autonomous expenditure;
- $\alpha = s + t$ be the sum of the savings rate and the tax rate;
- the savings rate $s = \frac{1}{5}$;
- the tax rate $t = \frac{2}{5}$;
- the variation in total autonomous expenditure $\Delta A = 120$.

By (1), the 120 increase in autonomous expenditure raises GDP in the amount

$$\Delta Y = \frac{1}{s+t} \cdot \Delta(\bar{I} + \bar{G}) = \frac{1}{\frac{1}{5} + \frac{2}{5}} \cdot 120 = \frac{5}{3} \cdot 120 = 200.$$

This is the multiplier effect in action: 120 additional units of spending have generated 200 additional units of GDP. The value of the expenditure multiplier is $\frac{5}{3}$: each extra unit of autonomous expenditure generates $\frac{5}{3}$ units of GDP.

Table 1 provides an explanation of how the multiplier effect occurs, where:

- $AD = C + I + G$ is the aggregate demand (consumption C and investment I by the private sector plus public expediture G by the public sector); and
- it is assumed that GDP (the production of goods and services by firms) always adjusts to equal aggregate demand AD; that is, $\Delta Y = \Delta AD$. [That $Y = \frac{\bar{I}+\bar{G}}{s+t}$ also follows from defining $C = Y - S - T$ and assuming that GDP ('aggregate supply') equals aggregate demand: $Y = AD$.]

The change $\Delta Y - \Delta T$ in disposable income is the sum $\Delta C + \Delta S$ of the change in consumption and the change in savings. Specifically, since $\Delta T = t \cdot \Delta Y$ and $\Delta S = s \cdot \Delta Y$, the change in consumption is $\Delta C = \Delta Y - \Delta T - \Delta S = \Delta Y - t \cdot \Delta Y - s \cdot \Delta Y = (1 - t - s) \cdot \Delta Y = \frac{2}{5} \cdot \Delta Y$.

period	ΔAD	ΔY	ΔC	$\Delta S + \Delta T$
1	120	120	48	72
2	48	48	19.2	28.8
3	19.2	19.2	7.68	11.52
4	7.68	7.68	3.072	4.608
5	3.072	3.072	1.2288	1.8432
6	1.2288	1.2288	0.49152	0.73728
7	0.49152	0.49152	0.196608	0.294912
	...	...	...	...
SUMA	200	200	80	120

Table 1. The spending multiplier process generating GDP

The increase ΔAD in period 1 corresponds to the exogenous increase $\Delta A = 120$. From period 2 onwards. $\Delta AD = \Delta C$: any increase in aggregate demand generates an increase in income (GDP) in the same period. which induces an increase in consumption in the following period.

The values in Table 1 result from applying the formulas in Table 2. with $A = 120$ i $\alpha = \frac{5}{3}$.

period	ΔAD	ΔY	ΔC	$\Delta S + \Delta T$
1	A	A	$A(1-\alpha)$	$A\alpha$
2	$A(1-\alpha)$	$A(1-\alpha)$	$A(1-\alpha)^2$	$A\alpha(1-\alpha)$
3	$A(1-\alpha)^2$	$A(1-\alpha)^2$	$A(1-\alpha)^3$	$A\alpha(1-\alpha)^2$
4	$A(1-\alpha)^3$	$A(1-\alpha)^3$	$A(1-\alpha)^4$	$A\alpha(1-\alpha)^3$
5	$A(1-\alpha)^4$	$A(1-\alpha)^4$	$A(1-\alpha)^5$	$A\alpha(1-\alpha)^4$
	...	...	...	
SUMA	**B**	**B**	**E**	**D**

Table 2. The spending multiplier process depending on parameters A and α

The sum B is

$$B = A + A(1-\alpha) + A(1-\alpha)^2 + A(1-\alpha)^3 + \cdots$$

Naming $\beta = 1 - \alpha$.

$$B = A \cdot (1 + \beta + \beta^2 + \beta^3 + \cdots).$$

Also.

$$1 + \beta + \beta^2 + \beta^3 + \cdots = 1 + \beta \cdot (1 + \beta + \beta^2 + \beta^3 + \cdots).$$

Defining $S = 1 + \beta + \beta^2 + \beta^3 + \ldots$. it follows that

$$S = 1 + \beta \cdot S.$$

Solving for S.

$$S = \frac{1}{1-\beta} = \frac{1}{\alpha}.$$

To recap. the total sum B of all increases in GDP is

$$B = A \cdot S = \frac{A}{\alpha}$$

which is the formula that establishes the increase in GDP in the model.

It is left as an exercise to verify that the sum E of all consumptions is

$$E = A \cdot \frac{1-\alpha}{\alpha}.$$

In the numerical example, $\frac{1-\alpha}{\alpha} = \frac{2}{3}$. Given that the value A representing the increase in spending is 120, the total increase in consumption is

$$\Delta C = 120 \cdot \frac{2}{3} = 80.$$

The value of the sum D is the difference $B - E$ (since $B = E + D$).

Graphical representation

In the model, the function NPS takes the form

$$NPS = S - I = s \cdot Y - \bar{I}.$$

Since this is a linear function of GDP, to represent it graphically it is enough to identify two points of the function and join them. Two easy points to calculate are those where $Y = 0$ and where $NPS = 0$. Further, as NPS depends positively on Y, the function increases with Y.

First, if $Y = 0$, then $NPS = -\bar{I}$. Therefore, the line defining the function passes through the point $(Y, NPS) = (0, -\bar{I})$. This is point a in Fig. 1. Second, if $NPS = 0$, then $Y = \bar{I}/s$. Thus, the line defining the function passes through the point $(Y, NPS) = (\bar{I}/s, 0)$. This is point b in Fig. 1.

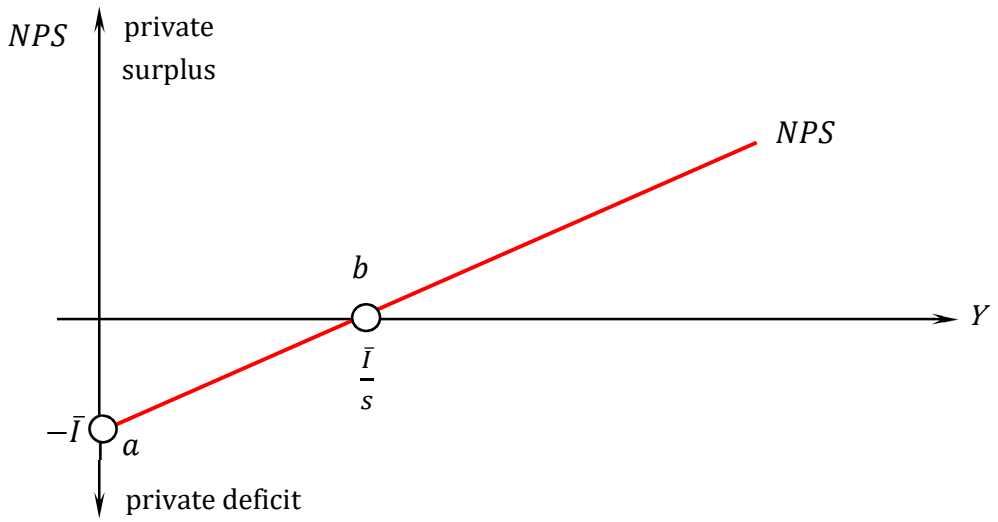

Fig. 1. Net private saving as a function of GDP

In the model, the function PD takes the form

$$PD = G - T = \bar{G} - t \cdot Y.$$

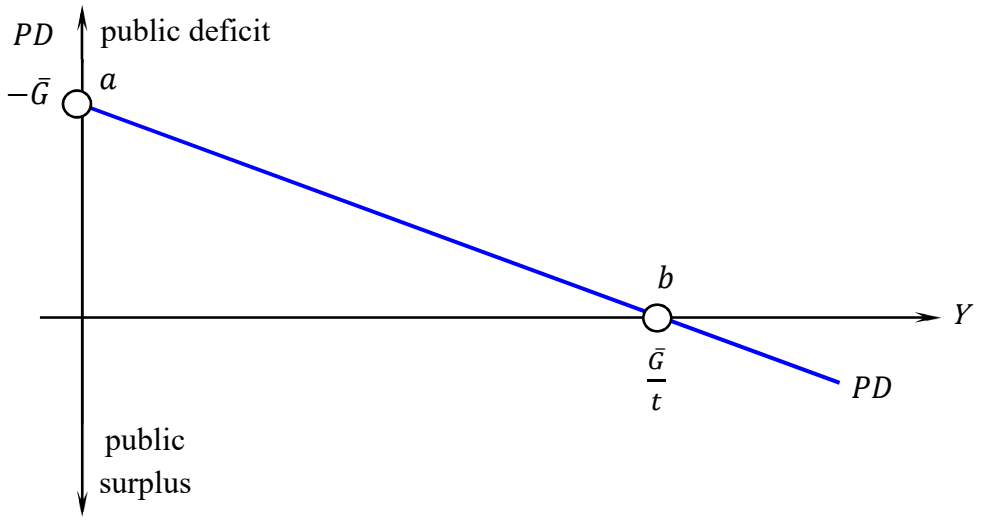

Fig. 2. Public deficit as a function of GDP

Like the NPS function, PD is a linear function of GDP. The difference is that PD decreases with Y. Fig. 2 represents the PD function.

The GDP value $Y' = \frac{\bar{I}+\bar{G}}{s+t}$ that satisfies the identity $NPS = PD$ is found at the intersection of the two lines. Fig. 3 shows this value when $\frac{\bar{I}}{s} < \frac{\bar{G}}{t}$. In view of the fact that the crossing occurs in the upper quadrant (point e), there is a public deficit (and a private surplus of the same magnitude).

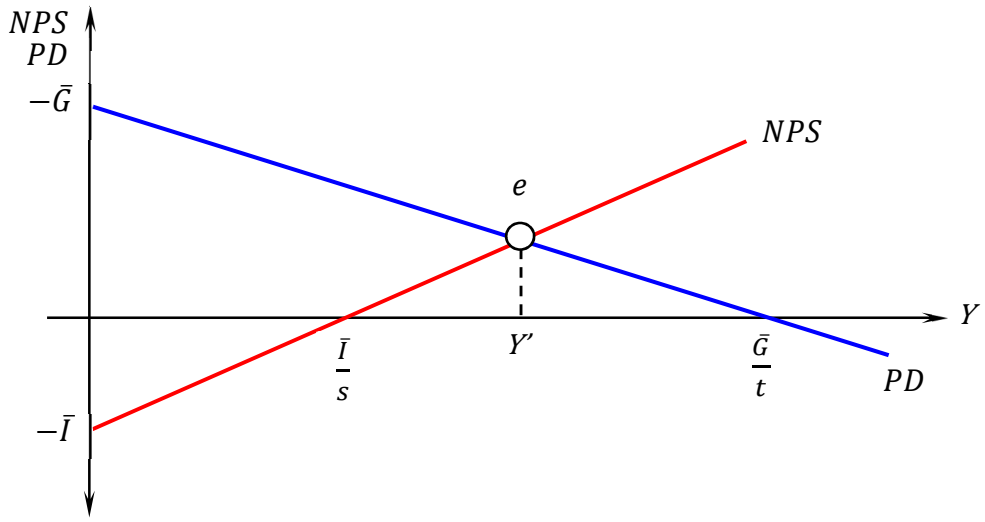

Fig. 3. GDP determination (resulting in a public deficit)

La Fig. 4 presents the complementary case $\frac{\bar{I}}{s} > \frac{\bar{G}}{t}$. Now the intersection occurs in the lower quadrant (point d) and, as a result, there is a public surplus (and private deficit of the same magnitude).

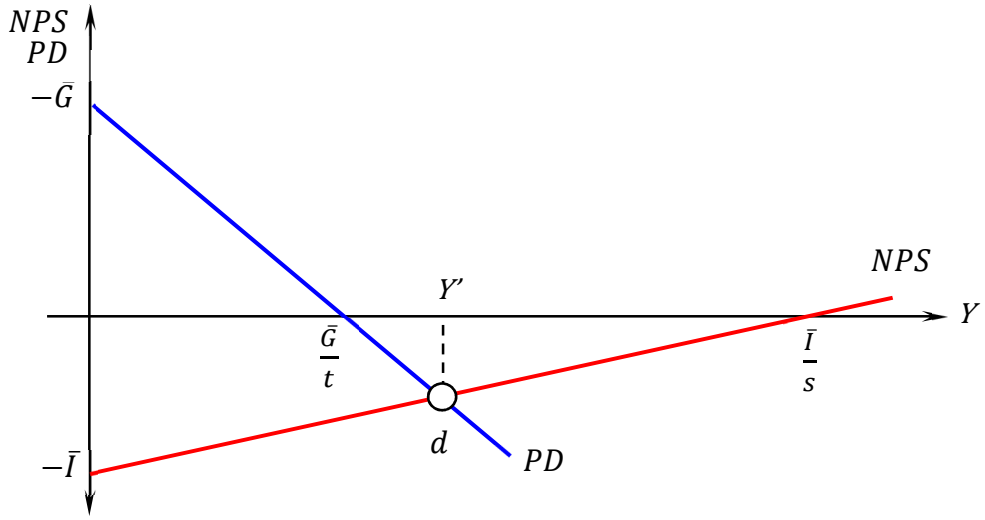

Fig. 4. GDP determination (resulting in a public surplus)

A simple lesson from the model is that a public deficit is not necessarily the result of 'irresponsible' fiscal policy decisions, but can be generated automatically by the functioning of the economy.

Other models

Dividing the balance identity

$$NPS = PD + NX$$

by GDP Y the identity is preserved:

$$\frac{NPS}{Y} = \frac{PD}{Y} + \frac{NX}{Y}. \tag{2}$$

Each term measures the corresponding sectoral balance as a proportion of GDP. For example, if $Y = 60$, $NPS = 20$, $PD = 30$ and $NX = -10$, then

$$\frac{NPS}{Y} = \frac{20}{60} = \frac{1}{3}$$

$$\frac{PD}{Y} = \frac{30}{60} = \frac{1}{2}$$

$$\frac{NX}{Y} = -\frac{10}{60} = \frac{1}{6}.$$

These values indicate that the net private balance is one third of GDP, the public deficit is half of GDP and the external balance is (in absolute terms) one sixth of GDP.

In percentage terms, a public deficit of 50% of GDP combined with a trade deficit of $16.\hat{6}\%$ of GDP must result in a net private balance of $33.\hat{3}\%$ of GDP.

From the new identity (2) another one is deduced: the variation $\Delta\left(\frac{NPS}{Y}\right)$ of the net private balance in relation to GDP is the variation $\Delta\left(\frac{PD}{Y}\right)$ of the public deficit in relation to GDP plus the variation $\Delta\left(\frac{NX}{Y}\right)$ of net exports in relation to GDP:

$$\Delta\left(\frac{NPS}{Y}\right) = \Delta\left(\frac{PD}{Y}\right) + \Delta\left(\frac{NX}{Y}\right). \tag{3}$$

Alternative models for determining GDP can be constructed based on either of the two new identities.

In case (2), an equation should be postulated that explains what each term $\frac{NPS}{Y}$, $\frac{PD}{Y}$ and $\frac{NX}{Y}$ depends on. In case (3), the equations should express what the change in these quantities depends on.

The savings identity could also be used as a starting point to define a model based on the rates of change of sectoral balances and where the objective is to determine the rate of change of GDP (instead of its absolute value).

Fiscal policy response to a fall in private spending

Point **1** in Fig. 5 represents the initial state of the economy. There is firstly a reduction in investment I, which shifts the net private balance NPS line to the left, to NPS'. There are many possible causes of the reduction: for example, businessmen adopt more pessimistic expectations about the dynamics of the economy and postpone investment projects until forecasts improve in the future.

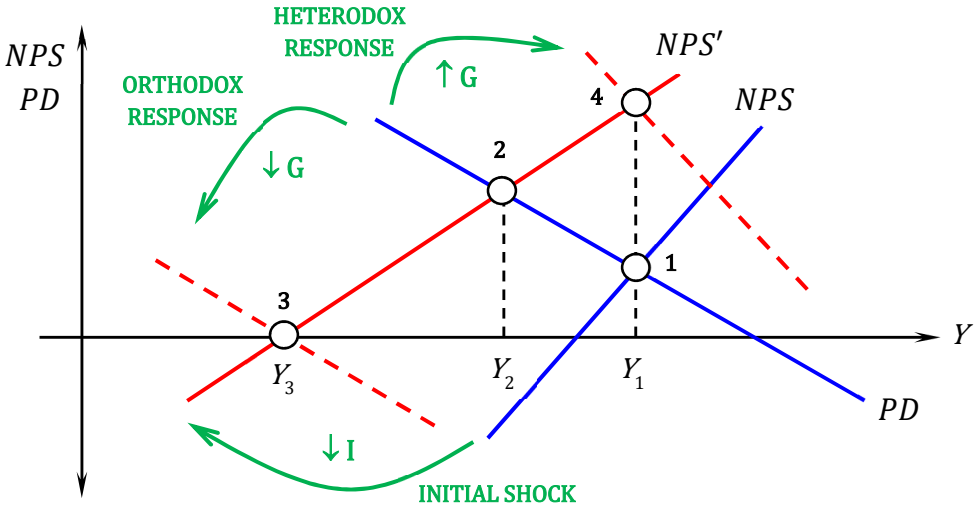

Fig. 5. GDP determination (resulting in a public surplus)

Due to the investment reduction the economy reaches point **2**, where GDP Y_2 is lower than the initial value Y_1. There are two basic fiscal policy responses to a negative private spending shock.

(i) The orthodox response to the shock is to apply austerity measures aimed at balancing the public sector balance. Since GDP has decreased as a result of the decline in investment spending (it falls from Y_1 to Y_2) and since tax collection is proportional to GDP, there is a decrease in public sector revenue. This increases the public deficit. A quick way to try to reduce it is to cut public spending G (the alternative way would be to raise the tax rate, which generally involves legislative procedures that delays the implementation of the policy measure).

If the fiscal policy objective is to reduce the deficit to zero, the model represents this austerity measure by shifting the PD line until it intersects both the new NPS' line and the horizontal axis (the axis indicating zero public deficit). The result is that the economy would reach point **3**, where GDP contracts even more (by moving from Y_2 to Y_3)

(ii) The heterodox response is to neutralize the negative effect on GDP of the fall in I with an increase in G. This measure would shift the PD line to the right, until it intersects the new NPS' line and the vertical line drawn through the initial value Y_1 of GDP. The economy would now be at point **4**, where the negative effect on GDP caused by the contraction in private spending has been offset. In this case, public spending has replaced the lost private spending in order to keep the initial value Y_1 of GDP.

The initial reduction in private investment creates a dilemma for the government, as it faces two policy goals or objectives with a single instrument.

The instrument is fiscal policy, with which it can shift PD line to the left (contractionary fiscal policy) or to the right (expansionary fiscal policy).

One of the policy goals is to increase GDP, given that the investment contraction has had the collateral effect of decreasing GDP, from Y_1 to Y_2 in Fig. 5. The other goal is to reduce the public deficit, because the reduction in investment has had another collateral effect: increasing the public deficit (the public deficit at point **2** in Fig. 5 is higher than the deficit at point **1**).

The orthodox view gives priority to the policy objective of reducing the public deficit.

The heterodox view, considering the public deficit as instrumental, advocates increasing GDP. GDP itself is an instrument for achieving socially desirable final objectives: maintaining employment and the business fabric.

The priority given to GDP over the public deficit in the heterodox view finds justification in the asymmetry of GDP dynamics: contracting GDP is much easier than expanding it. GDP expansion is even more difficult after a relatively deep contraction. One reason is that the contraction of GDP not only represents unfulfilled productive activity (production not done) but also the destruction of jobs and the productive structure and, quite often, of people's life plans.

Recovering a company that has closed is not easy: even if there are favorable macroeconomic conditions and the government can grant aid, rebuilding what has been destroyed requires a willpower that the period of contraction may have diminished or eliminated. Similarly, a worker who has lost his job and for a while has not found alternative employment incurs costs (economic costs, psychological costs, health costs, training costs …) that make it difficult to return to work. Therefore, it is not only, nor primarily, a question of avoiding the contraction of GDP but of avoiding all the associated losses and costs, from which, the longer they last, the more difficult it is to recover losses and costs.

One of the proposals associated with Modern Monetary Theory is to make the government an employer of last resort, analogous to how a central bank acts as a lender of last resort (the government would ultimately support the non-banking private sector, just like the central bank currently ultimately supports the banking private sector). According to this proposal, everyone (willing and able) could apply for a job in the public sector. The range of government jobs would be very wide and varied: small public works, infrastructure maintenance, care and elderly care services, support for public services, attention to needs, provision of training, development of projects or companies of public interest... The work assigned by the government can be used to train and prepare to obtain a more preferred job in the private sector, gain experience, ensure a minimum income... In parallel, the implementation of this proposal would make it unnecessary to legally set minimum wages: the minimum wage that the government wanted to set would already be the guaranteed remuneration in any of the assigned public jobs. If the private sector did not pay this minimum wage, workers would migrate from the private to the public sector, where they would receive it.